Master thesis and internship[BR]- Master's thesis : Development of a methodology for VLEO intake performance testing in a low density facility[BR]- Integration internship

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Appendix A

Expression Analytic used for the slow flux transmissivities

Thanks to various tests, it is possible to determine the parameters no measured by a probe. It is the case of the slow flux transmission probabilities. They are computed thanks to different measurements: the pressures, the PFG flow, and the temperatures, and thanks to the values estimated during previous tests: leak rates and pumping speeds. Unlike all other tests, the model equations are not sufficient, and further simplification is required. If the temperatures of the two chambers are considered equal, then it is possible to use the model from the book [15].

If two vacuum chambers are linked by a succession of N components The model considers that the transmission probabilities of this connection respects

$$\frac{1}{A_{in}}\left(\frac{1}{\tau_{1N}}-1\right)=\sum_{k=1}^{N}\frac{1}{A_k}\left(\frac{1}{\tau_k}-1\right)+\sum_{k=1}^{N-1}\left(\frac{1}{\bar{A}_{k+1}}-\frac{1}{A_k}\right)\delta_{k,k+1} \tag{A.1}$$

where k denotes the component number, A_{in} is the cross section of intake to the component series, A_k is the cross-section of component k, τ_{1N} is the total transmission probability of the connection, τ_k is the transmission probability for component k, and $\delta_{k,k+1}$ is a factor depending on the cross section dimensions. If the next component has a smaller cross-section ($A_{k+1} < A_k$), then $\delta_{k,k+1} = 1$. But if the next component has an equal or larger section ($A_{k+1} \geq A_k$), then $\delta_{k,k+1} = 0$. To apply this model, the system connecting the two chambers has three components. Starting from the main chamber, it is composed of a valve with the known section A_1, the intake, and a duct of section A_2. This latter is also known and lower than A_1. Although the model also allows considering more complex geometry as the intake, the reasoning presented here only uses the average cross-section of each piece of equipment. As shown by the drawing A.1, there are three possible scenarios. Either the section of the intake $\bar{A}_{intake}$ is greater than or equal to A_1, or $A_1 > \bar{A}_{intake} \geq A_2$, or is strictly less than A_2.

By considering Equation A.1, the two transmission probabilities of the slow flows

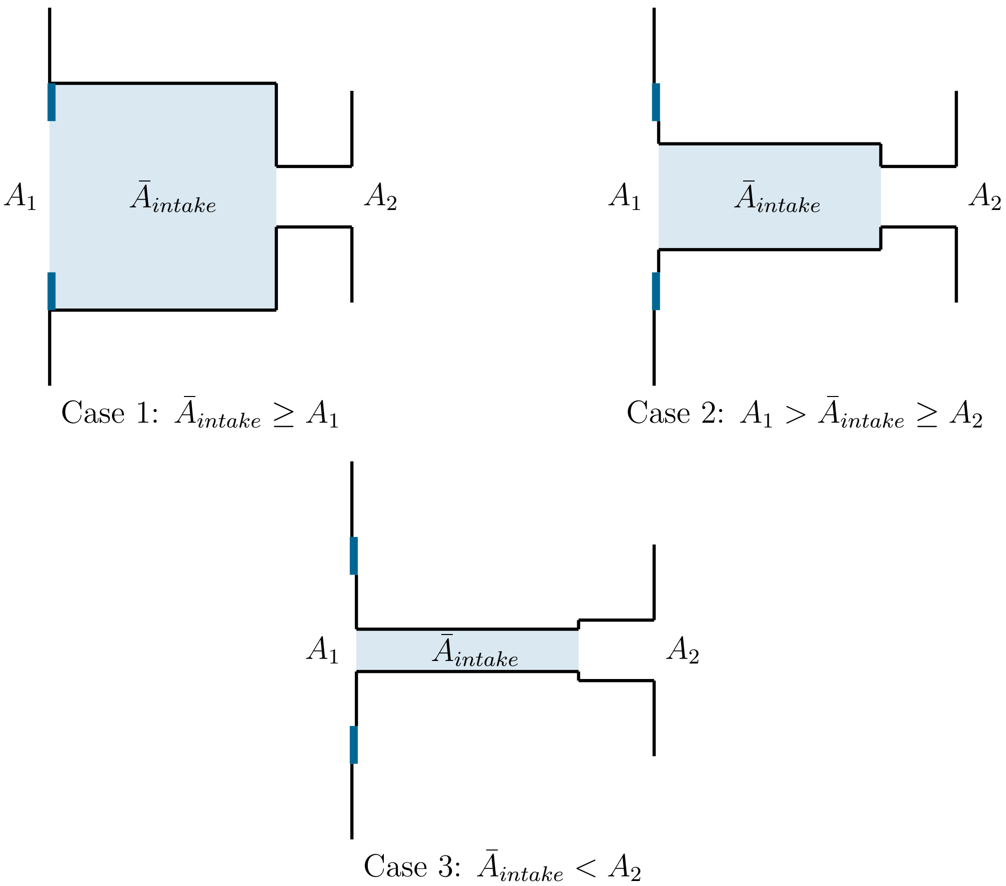

Figure A.1: Analytical model used to model the connection between both chambers belong to the relations

$$\begin{aligned}
\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}}-1\right) =& \frac{1}{A_1}\left(\frac{1}{\tau_1}-1\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-1\right)+\frac{1}{A_2}\left(\frac{1}{\tau_2}-1\right) \\
&+\left(\frac{1}{\bar{A}_{intake}}-\frac{1}{A_1}\right)\delta_{1,intake}+\left(\frac{1}{A_2}-\frac{1}{\bar{A}_{intake}}\right)\delta_{intake,2} \\
=& \frac{1}{A_1}\left(\frac{1}{\tau_1}-1-\delta_{1,intake}\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-1+\delta_{1,intake}-\delta_{intake,2}\right) \\
&+\frac{1}{A_2}\left(\frac{1}{\tau_2}-1+\delta_{intake,2}\right),
\end{aligned} \tag{A.2}$$

$$\begin{aligned}
\frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}}-1\right) =& \frac{1}{A_2}\left(\frac{1}{\tau_2}-1\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-1\right)+\frac{1}{A_1}\left(\frac{1}{\tau_1}-1\right) \\
&+\left(\frac{1}{\bar{A}_{intake}}-\frac{1}{A_2}\right)\delta_{2,intake}+\left(\frac{1}{A_1}-\frac{1}{\bar{A}_{intake}}\right)\delta_{intake,1} \\
=& \frac{1}{A_2}\left(\frac{1}{\tau_2}-1-\delta_{2,intake}\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-1+\delta_{2,intake}-\delta_{intake,1}\right) \\
&+\frac{1}{A_1}\left(\frac{1}{\tau_1}-1+\delta_{intake,1}\right),
\end{aligned} \tag{A.3}$$

where A_1 and A_2 are the cross section of the valve (in dark blue) and the duct linked to the second chamber, $\bar{A}_{intake}$ is the intake mean section (in light blue), τ_1, τ_{intake}, and τ_2 are the transmission probabilities of the valve, the intake and the duct respectively.

A.1 Case 1: $\bar{A}_{intake} \geq A_1$

In the first case, the section of the intake is larger than A_1 and A_2. The terms depending on the geometry are

$$\delta_{1,intake}=0 \qquad \delta_{2,intake}=0 \qquad \delta_{intake,1}=1 \qquad \delta_{intake,2}=1.$$

And the relation A.2 and A.3 become:

$$\begin{aligned}
\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}}-1\right) =& \frac{1}{A_1}\left(\frac{1}{\tau_1}-1-0\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-1+0-1\right) \\
&+\frac{1}{A_2}\left(\frac{1}{\tau_2}-1+1\right) \\
=& \frac{1}{A_1}\left(\frac{1}{\tau_1}-1\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-2\right)+\frac{1}{A_2}\frac{1}{\tau_2},
\end{aligned}$$

$$\begin{aligned}
\frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}}-1\right) =& \frac{1}{A_2}\left(\frac{1}{\tau_2}-1-0\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-1+0-1\right) \\
&+\frac{1}{A_1}\left(\frac{1}{\tau_1}-1+1\right) \\
=& \frac{1}{A_2}\left(\frac{1}{\tau_2}-1\right)+\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}}-2\right)+\frac{1}{A_1}\frac{1}{\tau_1}.
\end{aligned}$$

By isolating the terms $\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 2\right)$, these two expression provide

$$\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1\right) - \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1\right) - \frac{1}{A_2}\frac{1}{\tau_2} = \frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1\right) - \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1\right) - \frac{1}{A_1}\frac{1}{\tau_1}$$
$$\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1 - \frac{1}{\tau_1} + 1 + \frac{1}{\tau_1}\right) = \frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1 - \frac{1}{\tau_2} + 1 + \frac{1}{\tau_2}\right)$$
$$\frac{1}{A_1}\frac{1}{\tau_{1,slow}} = \frac{1}{A_2}\frac{1}{\tau_{2,slow}}.$$

It yields to the relation

$$A_1\tau_{1,slow} = A_2\tau_{2,slow}. \tag{A.4}$$

In reality this expression is general for any case. As the source does not provide a demonstration for the most general situation, the same reasoning for the other two cases is done in this annex.

A.2 Case 2: $A_1 > \bar{A}_{intake} \geq A_2$

For the second case, the factors take values

$$\delta_{1,intake} = 1 \qquad \delta_{2,intake} = 0 \qquad \delta_{intake,1} = 0 \qquad \delta_{intake,2} = 1.$$

With these values, the equations A.2 and A.3 are

$$\begin{aligned}\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1\right) =& \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1 - 1\right) + \frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1 + 1 - 1\right)\\ &+ \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1 + 1\right)\\ =& \frac{1}{A_1}\frac{1}{\tau_1} + \frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1\right) + \frac{1}{A_2}\frac{1}{\tau_2},\end{aligned}$$

$$\begin{aligned}\frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1\right) =& \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1 - 0\right) + \frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1 + 0 - 0\right)\\ &+ \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1 + 0\right)\\ =& \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1\right) + \frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1\right) + \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1\right).\end{aligned}$$

Again, the terms $\frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1\right)$ can be isolated from the rest in both formulas. They can be combined to obtain

$$\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1\right) - \frac{1}{A_1}\frac{1}{\tau_1} - \frac{1}{A_2}\frac{1}{\tau_2} = \frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1\right) - \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1\right) - \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1\right)$$
$$\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1 - \frac{1}{\tau_1} + \frac{1}{\tau_1} - 1\right) = \frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1 - \frac{1}{\tau_2} + 1 + \frac{1}{\tau_2}\right)$$
$$\frac{1}{A_1}\frac{1}{\tau_{1,slow}} = \frac{1}{A_2}\frac{1}{\tau_{2,slow}}.$$

This case, like the previous one, leads to the same final relation

$$A_1\tau_{1,slow} = A_2\tau_{2,slow}.$$

A.3 Case 3: $\bar{A}_{intake} < A_2$

Finally when the intake cross-section $\bar{A}_{intake}$ is lower than A_2 and A_1. The δ coefficients become

$$\delta_{1,intake} = 1 \qquad \delta_{2,intake} = 1 \qquad \delta_{intake,1} = 0 \qquad \delta_{intake,2} = 0.$$

And the transmission probabilities respect the relation

$$\begin{aligned}\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1\right) =& \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1 - 1\right) + \frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1 + 1 - 0\right) \\ &+ \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1 + 0\right) \\ =& \frac{1}{A_1}\left(\frac{1}{\tau_1} - 2\right) + \frac{1}{\bar{A}_{intake}}\frac{1}{\tau_{intake}} + \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1\right),\end{aligned}$$

$$\begin{aligned}\frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1\right) =& \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1 - 1\right) + \frac{1}{\bar{A}_{intake}}\left(\frac{1}{\tau_{intake}} - 1 + 1 - 0\right) + \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1 + 0\right) \\ =& \frac{1}{A_2}\left(\frac{1}{\tau_2} - 2\right) + \frac{1}{\bar{A}_{intake}}\frac{1}{\tau_{intake}} + \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1\right).\end{aligned}$$

As previously the terms $\frac{1}{\bar{A}_{intake}}\frac{1}{\tau_{intake}}$ are separated from the rest these equations can be assembled to give

$$\begin{aligned}\frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} + 1 - \frac{1}{\tau_1}\right) - \frac{1}{A_2}\left(\frac{1}{\tau_2} - 1\right) &= \frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} + 1 - \frac{1}{\tau_2}\right) - \frac{1}{A_1}\left(\frac{1}{\tau_1} - 1\right) \\ \frac{1}{A_1}\left(\frac{1}{\tau_{1,slow}} - 1 - \frac{1}{\tau_1} + 2 + \frac{1}{\tau_1} - 1\right) &= \frac{1}{A_2}\left(\frac{1}{\tau_{2,slow}} - 1 - \frac{1}{\tau_2} + 2 + \frac{1}{\tau_2} - 1\right) \\ \frac{1}{A_1}\frac{1}{\tau_{1,slow}} &= \frac{1}{A_2}\frac{1}{\tau_{2,slow}} \\ A_1\tau_{1,slow} &= A_2\tau_{2,slow}.\end{aligned}$$

In this case also the same relation is obtained. This confirms that it remains valid for the installation, independent of the intake dimension.

Appendix B

Partial derivatives of the Error Propagation Analysis

B.1 Slow flux transmission probabilities

This appendix lists all the partial derivatives used for the two transmission probability uncertainties of $\tau_{1,slow}$.

B.1.1 First hypothesis

In the case of the first hypothesis, $\tau_{1,slow}$ is given by

$$\tau_{1,slow} = \sqrt{\frac{2\pi m}{kT_{c1}}} \frac{S_2 P_{c2} - Q_{2,leak}}{P_{c1} 1000 A_1}.$$

And the corresponding partial derivatives are

$$\frac{\partial \tau_{1,slow}}{\partial T_{c1}} = -\sqrt{\frac{2\pi m}{kT_{c1}^3}} \frac{S_2 P_{c2} - Q_{2,leak}}{P_{c1} 2000 A_1}$$

$$\frac{\partial \tau_{1,slow}}{\partial S_2} = \sqrt{\frac{2\pi m}{kT_{c1}}} \frac{P_{c2}}{P_{c1} 1000 A_1}$$

$$\frac{\partial \tau_{1,slow}}{\partial P_{c2}} = \sqrt{\frac{2\pi m}{kT_{c1}}} \frac{S_2}{P_{c1} 1000 A_1}$$

$$\frac{\partial \tau_{1,slow}}{\partial Q_{2,leak}} = \sqrt{\frac{2\pi m}{kT_{c1}}} \frac{-1}{P_{c1} 1000 A_1}$$

$$\frac{\partial \tau_{1,slow}}{\partial P_{c1}} = -\sqrt{\frac{2\pi m}{kT_{c1}}} \frac{S_2 P_{c2} - Q_{2,leak}}{P_{c1}^2 1000 A_1}$$

B.1.2 Second hypothesis hypothesis

The second hypothesis yields to the expression

$$\tau_{1,slow} = \frac{\sqrt{2\pi m}}{1000 A_1 \sqrt{k}} \left(\frac{S_2 P_{c2} - Q_{2,leak}}{P_{c1}\sqrt{T_{c1}} - P_{c2}\sqrt{T_{c2}}} \right),$$

with the partial derivatives

$$\begin{aligned}
\frac{\partial \tau_{1,slow}}{\partial T_{c1}} &= \frac{\sqrt{2\pi m}}{1000A_1\sqrt{k}}\left(\frac{-P_{c1}(S_2P_{c2}-Q_{2,leak})}{2\sqrt{T_{c1}}(P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}})^2}\right)\\
\frac{\partial \tau_{1,slow}}{\partial S_2} &= \frac{\sqrt{2\pi m}}{1000A_1\sqrt{k}}\left(\frac{P_{c2}}{P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}}}\right)\\
\frac{\partial \tau_{1,slow}}{\partial P_{c2}} &= \frac{\sqrt{2\pi m}}{1000A_1\sqrt{k}}\left(\frac{S_2(P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}})+(S_2P_{c2}-Q_{2,leak})\sqrt{T_{c2}}}{(P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}})^2}\right)\\
\frac{\partial \tau_{1,slow}}{\partial Q_{2,leak}} &= \frac{\sqrt{2\pi m}}{1000A_1\sqrt{k}}\left(\frac{-1}{P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}}}\right)\\
\frac{\partial \tau_{1,slow}}{\partial P_{c1}} &= \frac{\sqrt{2\pi m}}{1000A_1\sqrt{k}}\left(\frac{-\sqrt{T_{c1}}(S_2P_{c2}-Q_{2,leak})}{(P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}})^2}\right)\\
\frac{\partial \tau_{1,slow}}{\partial T_{c2}} &= \frac{\sqrt{2\pi m}}{1000A_1\sqrt{k}}\left(\frac{P_{c2}(S_2P_{c2}-Q_{2,leak})}{2\sqrt{T_{c2}}(P_{c1}\sqrt{T_{c1}}-P_{c2}\sqrt{T_{c2}})^2}\right)
\end{aligned}$$

B.2 Fast flux transmission probability

The expression of the probability $\tau_{1,fast}$ is

$$\tau_{1,fast} = \frac{1}{Q_{1,fast}}\left(P_{c2}1000A_2\tau_{2,slow}\sqrt{\frac{kT_{c2}}{2\pi m}}+S_2P_{c2}-P_{c1}1000A_1\tau_{1,slow}\sqrt{\frac{kT_{c1}}{2\pi m}}-Q_{2,leak}\right).$$

The partial derivative of this expression are

$$\frac{\partial \tau_{1,fast}}{\partial Q_{1,fast}} = \frac{-1}{Q_{1,fast}^2}\left(P_{c2}1000A_2\tau_{2,slow}\sqrt{\frac{kT_{c2}}{2\pi m}} + S_2P_{c2} - P_{c1}1000A_1\tau_{1,slow}\sqrt{\frac{kT_{c1}}{2\pi m}} - Q_{2,leak}\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial P_{c2}} = \frac{1}{Q_{1,fast}}\left(1000A_2\tau_{2,slow}\sqrt{\frac{kT_{c2}}{2\pi m}} + S_2\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial \tau_{2,slow}} = \frac{1}{Q_{1,fast}}\left(P_{c2}1000A_2\sqrt{\frac{kT_{c2}}{2\pi m}}\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial T_{c2}} = \frac{1}{2Q_{1,fast}}\left(P_{c2}1000A_2\tau_{2,slow}\sqrt{\frac{k}{2\pi mT_{c2}}}\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial S_2} = \frac{P_{c2}}{Q_{1,fast}}$$

$$\frac{\partial \tau_{1,fast}}{\partial P_{c1}} = \frac{-1}{Q_{1,fast}}\left(1000A_1\tau_{1,slow}\sqrt{\frac{kT_{c1}}{2\pi m}}\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial \tau_{1,slow}} = \frac{-1}{Q_{1,fast}}\left(P_{c1}1000A_1\sqrt{\frac{kT_{c1}}{2\pi m}}\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial T_{c1}} = \frac{-1}{2Q_{1,fast}}\left(P_{c1}1000A_1\tau_{1,slow}\sqrt{\frac{k}{2\pi mT_{c1}}}\right)$$

$$\frac{\partial \tau_{1,fast}}{\partial Q_{2,leak}} = \frac{-1}{Q_{1,fast}}$$