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Plasma sheath modeling with a high order Discontinuous Galerkin Method

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Since the beginning of the 20th century, research in low-temperature plasmas has grown into a major field of plasma science. In particular, confined plasmas are of great interest for a large range of technological applications: from domestic with lightning and plasma displays panels (PDPs), to electric space propulsion and, more recently, medicine. This vast domain of application gives rise to many numerical models whose specificities are adapted to current needs.

The main difficulties in plasma modelling is the very large mass disparity between electrons and the other species that compose the ionised gas. This intrinsic multiscale property makes the numerical problem very stiff. Paving the way to the need of highly accurate methods.

This work proposes the numerical simulation of a two-fluid low-temperature plasma with the use of high order Discontinuous Galerkin finite element Methods (DG-FEM). For this purpose, the coupled resolution of the electrons and ions transports equations with a Poisson's equation for the electrical potential is carried out through a fully-implicit strategy. This choice permits to overcome the strict stability constraints coming from the electrons/ions mass disparity that affect explicit schemes.

Starting from the general Galerkin variationnal formulation, this coupled resolution of hyperbolic-elliptic equations is addressed with implementation of specific strategies: An entropy-consistent Roe numerical flux is developed alongside incomplete internal penalty methods being applied only to the electrostatic system. An extension to the classical formulation of the implicit ESDIRK scheme is provided. Indeed, the inertia terms associate to the potential is deactivated by the introduction of a Boolean parameter β which is equal to 1 for all the parameters excepts for the potential. As a result, this modification of the implicit method permits to treat the Poisson's equation alongside the fluid equations during the Newton iterator, enhancing then its convergence. An automatic evaluation of the Jacobian matrix using first order central difference is also described with a specific treatment made onto the linearisation of the ionisation contributions. All of these strategies were implemented in the ForDGe solver, an immersed boundary Cartesian DG-FEM solver developed at the University of Liege.

Tested on two practical cases, namely: the two-stream periodic perturbation and the sheath problem, the current fully-coupled implicit DG-FEM solver is able to tackle accurately the physics of low-temperature collionsionless plasma. In the case of the two stream instability, the current implicit solver demonstrated its increased stability with resolution using time step that is more than two order of magnitude bigger than the electron plasma frequency, one of the most stringent stability constraints for plasma flows. Non-linear behaviour of the solution is encountered when dealing with longer simulation. This can be addressed to a lack of consistency of the Roe flux at low-mach regime.

For the plasma sheath problem, the current solver allows us to reach a steady state solution that is in very good agreement with state of the art solution, but also gives a good representation of the physics of the sheath. The steady state is achieved with the simple use of time integration, at the cost of considerable computational effort.

Keywords: plasma, Discontinuous Galerkin, multifluid formulation, two-fluid model, potential equation, low-temperature plasma, implicit time integration
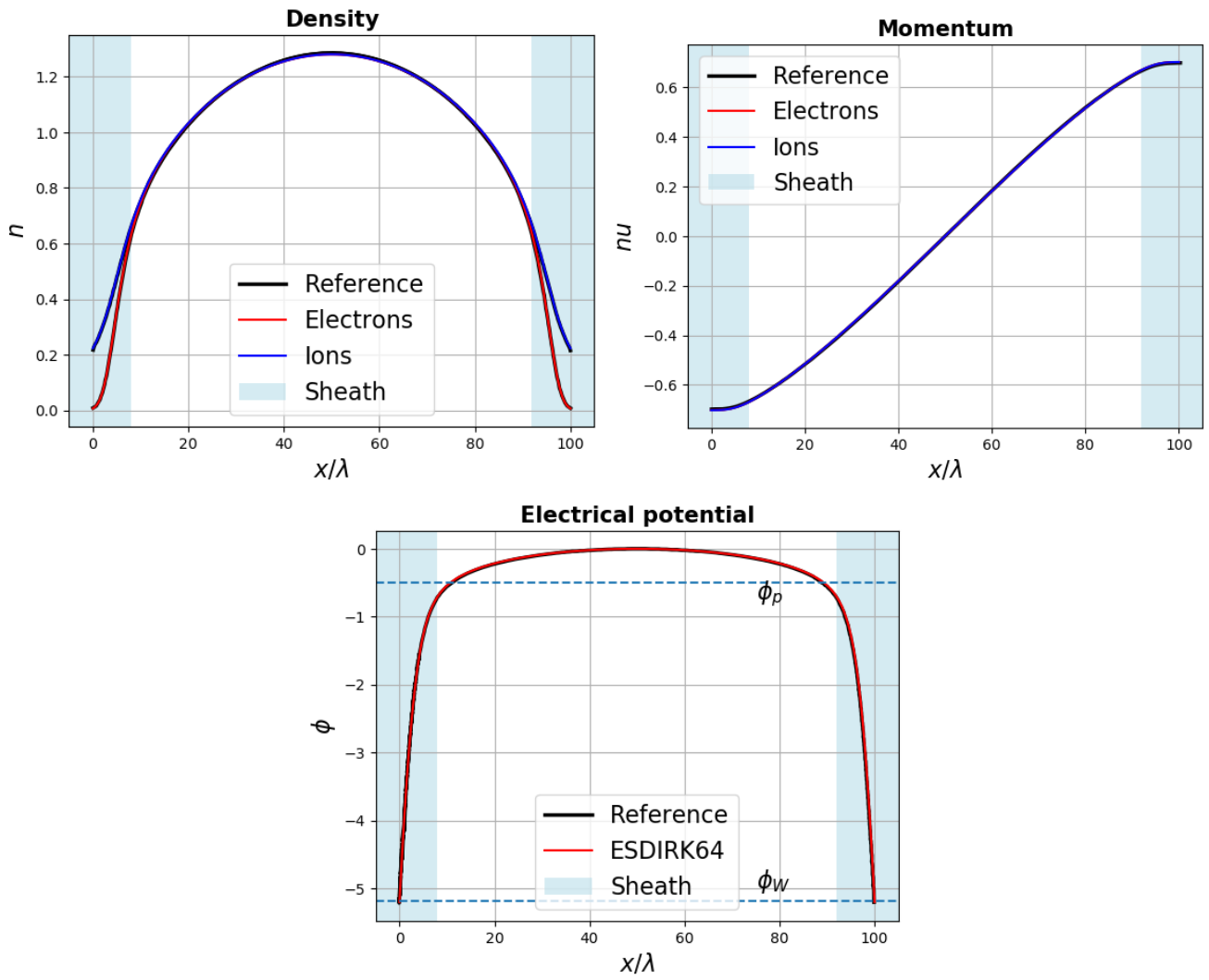
Figure 1: Comparison of the reference and the numerical solution obtained with 100 equally spaced element of order 4 and $\Delta t = 1.5 \times 10^{-5}$ sec. The final time is $t = 2$ sec. A steady state is achieved and gives results that are in very good agreement with the reference ones. The potential at the wall ϕ_W derived by Lieberman *et al.*(2005) is also very well estimated by the current simulation.