Bitcoin hedge capacity for high risk portfolio.

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Bitcoin hedge capacity for high risk portfolio

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Abstract

Bitcoin hedge capacity for high risk portfolio

by Nicolas LECLÈRE

This master's thesis proposes an analysis of the hedge capacities of Bitcoin in the particular case of high risk portfolio. The impact of Bitcoin on the portfolio optimisation and efficient frontier is also studied.

The statistical models GARCH, eGARCH, GJR-GARCH and DCC-GARCH are build in order to compute the correlation between Bitcoin and the portfolio using the software *Matlab*. Univariate models are compared and provide volatility of assets, while a multivariate model is built to assert the hedge capacity of Bitcoin.

Montecarlo simulations are performed for the volatility of Bitcoin and correlation are established between the portfolio and Bitcoin. Portfolio are built, efficient frontiers drawn and optimal portfolio studied using the Sharpe ratio.

Final results suggests that Bitcoin can have hedge capacities over a portfolio composed of risky assets depending on the time period considered. At least it is of great interest in the construction of portfolio. The efficient frontier is displaced upward, meaning higher returns, when Bitcoin are considered in the portfolio.

Acknowledgements

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Chapter 1

Introduction

For the last few years, Bitcoin and cryptocurrency in general have been mentioned on a daily base in the media. The reason being the significant profit that it can provide but also the stories about huge losses that exist around. It quickly became an asset that everyone knew about and could play with full of hope. For experienced traders and finance enthusiasts Bitcoin arrived like a UFO on the market and in the portfolio. No one knew the potential effect it may or may not have on the optimisation of portfolios or if it could hedge risky assets. It is in this context that this work is proposed, where the goal is to identify if Bitcoin present hedge capacities when a portfolio is considered risky and how it impacts the expected returns of a portfolio.

The master's thesis is separated in three different chapters. The first chapter concerns an introduction to the world of cryptocurrencies, when and how they appeared, how do they work and what effect they have on the world. The second chapter lays down the mathematical basis that are used to obtain the results. Statistical tools, theory of portfolio optimisation and the data that are used are presented. The third chapter concerns the results. The data are fed to the models and the obtained figures and solutions are discussed. A conclusion is then drawn about the work that is proposed.

In the scope of the master's thesis, the software *Matlab* and *Excel* have been used to compute and model.

Chapter 2

Cryptocurrencies

In this chapter we present Bitcoin, the origins and the technologies that made it a reality are presented. The drawbacks of Bitcoin are also addressed.

2.1 A short history of cryptocurrencies

The first cryptocurrency ever created and holding a defined monetary value is also the most famous one, Bitcoin (Figure 2.1). In 2009, a person or group of people, named Satoshi Nakamoto, their real identity remains unknown to this day, created Bitcoin. The emergence of the bitcoin and other cryptocurrencies is directly connected to the 2008 financial crisis that plunged the world into dark times of bankruptcy and debts (Kostakis and Giotitsas, 2014).

FIGURE 2.1: Physical representation of bitcoin *credits : Pixabay*.

At first, bitcoin was not recognised as a valid form of payment. It was disregarded for the first few year of its existence. This clearly appears on

Figure 2.2 where the trading volume in USD is represented. It corresponds to the amount of Bitcoins that are bought and sold on specific exchanges. It also helps with deriving the general interest in the crypto market. More and more companies started to accept it as payment, this lead to the famous *pizza day*, where a man named Laszlo Hanyecz happily paid for two pizzas with 10 000 bitcoins. At the time of the transaction, in may 2010, it represented 41 USD. Nowadays, those two pizzas would have had a value of hundreds of millions USD. This small story shows by itself how big the market of cryptocurrencies have grown in merely one decade.

However the bitcoin market is still considered to be high stakes gambling.There are countless stories of people that became rich thanks to investing in bitcoin. Sadly, there are even more stories about people that lost a lot of money. The high volatility of the price of bitcoin may lead to significant profits but significant losses as well. Exterior actors may have a real impact on the price variations of bitcoin, one such case can be seen in notorious CEO of Tesla and SpaceX, Elon Musk (Ante, 2021). The simple act of modifying his bio on Twitter by adding *#bitcoins* highly impacted the market. Figure 2.3 displays the variation of bitcoin price over the day on which the bio Twitter was changed, the impact of the event was sudden and noticeable.

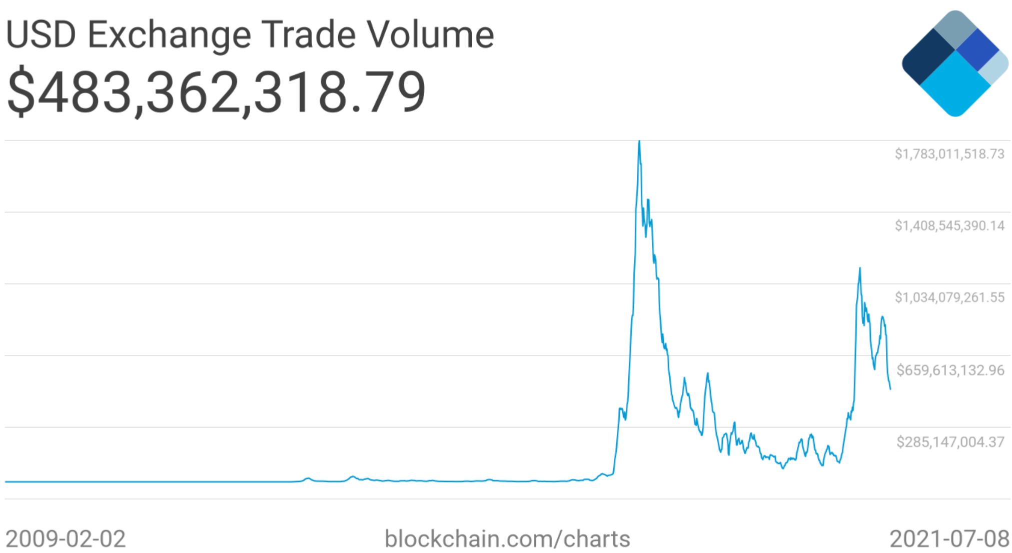


FIGURE 2.2: Exchange Trade Volume of Bitcoins in USD from *blockchain.com/charts*.

FIGURE 2.3: Impact of Elon Musk's change of twitter bio on the price of bitcoin from *https://coinmarketcap.com/fr/currencies/bitcoin/*.

There are three main criterion that make bitcoin and cryptocurrencies interesting assets to have, hold or trade with compared to the fiat currencies, which is another word for paper money (Marella et al., 2020). Firstly, bitcoin is a non-centralized money (Scott, 2016). Secondly, bitcoins are generated and managed via a block-chain technology (Zheng et al., 2017). Thirdly, the digital nature of bitcoin as all of the cryptocurrencis makes them easy to transfer across the world. the first two concepts are developed in the following sections.

2.2 Centralised or decentralised ?

As mentioned in the previous section, bitcoins and more generally cryptocurrencies are non-centralised. A non-centralised system does not depend on a unique system that would gather all the information at the same place. In the case of Bitcoins, the information is accessible to all of the users. In the case of hacking, the centralised system is much more at risk than the decentralised system since all the data is available to all users. A hacker would need to access all the users together, which is nearly impossible. The decentralisation is great for anonymous operation, in order to access the system you just need the private and public key. Figure 2.4 gives a graphical representation of the two systems.

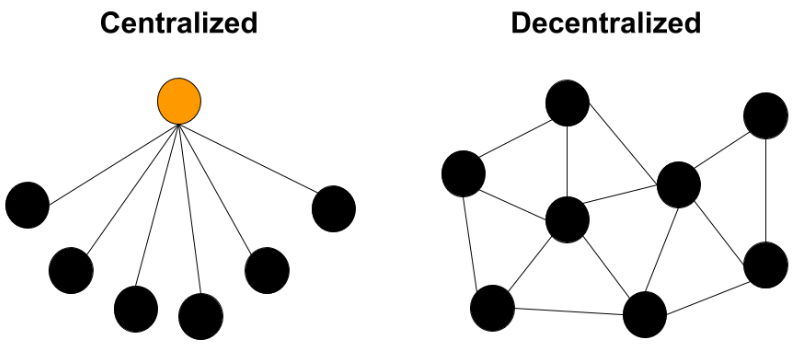


FIGURE 2.4: Graphical representation of the centralised and decentralised system

The decentralisation is made possible thanks to block-chains (Y. Chen, 2021)

2.3 Block-chain technology

The block-chain is the most important aspect that made bitcoins and all cryptocurrencies that followed possible. A block-chain is made of data sets that are composed of chains of data packages called blocks. Those blocks include multiple transactions. The addition of blocks extends the block-chain and therefore provides a complete history about all past transactions. Not only are the transactions stored in a block but there is a value corresponding to the previous block, the block parent as well as a timestamp and a random number that insures the integrity of the block-chain (Nofer et al., 2017). A graphical representation of what a block-chain looks like is given on figure 2.5.

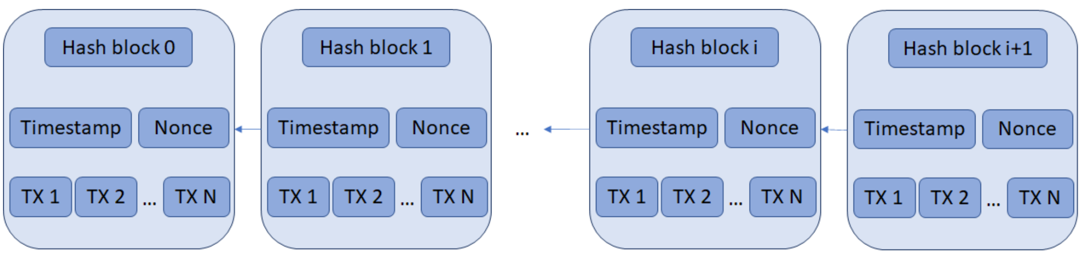


FIGURE 2.5: Graphical representation of the block-chain

So how does it work in practice? Let us assume Alice makes a transaction with Bob. All the transactions that happen during a given period of time are gathered together to create a block, including the one that Alice just made. Then the block is validated by the nodes of the network, the so called miners that must solve mathematical problems of cryptography. The new block is then added to the chain, only then Bob will receive the transaction that Alice previously made. A graphical summary of the process can be found on figure 2.6

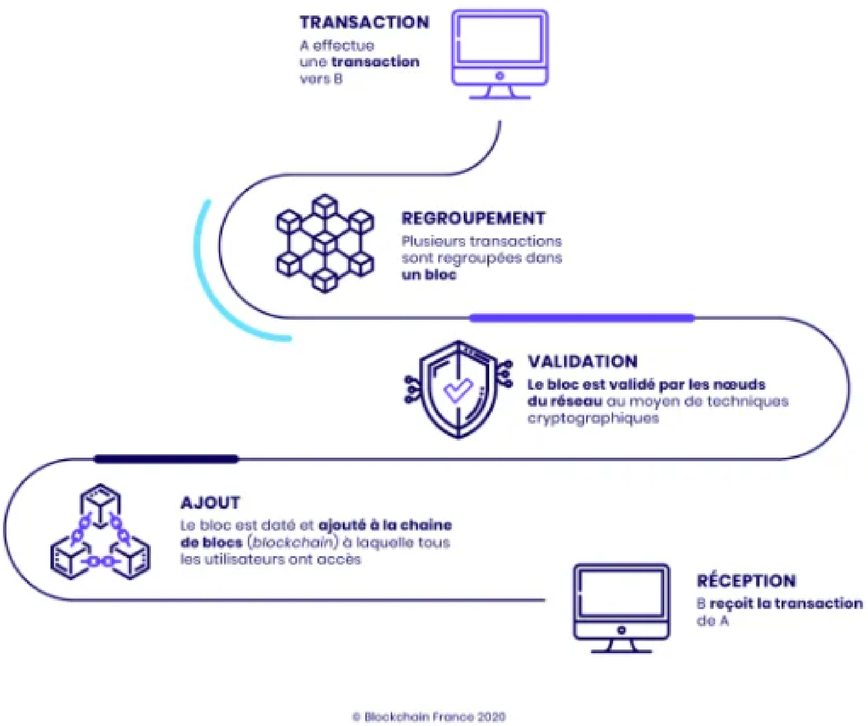


FIGURE 2.6: Blockchain process extracted from *https://blockchainfrance.net/decouvrir-la-blockchain/c-est-quoi-la-blockchain/*.

Now that we understand what the basis of Bitcoin is and how it works, it will be interesting to look at how it is perceived nowadays.

2.4 Bitcoin nowadays: its usage and impact

As stated at the beginning of this chapter the perception of the bitcoin has changed a lot over the years. From a fraction of a cent to more than thirty thousand dollars, the bitcoin has seen its value increase massively. But what drove this significant increase? There are internal and external factors that can act on the price of bitcoin. As part of the internal factors one can observe the supply and the demand that directly impacts the price of bitcoin, the more people want bitcoin the more pricey it becomes. The external factors are related to political decisions, the global market and the speculation about bitcoin evolution. A short list of a few of those factors is given on figure 2.7 (Poyser, 2017).

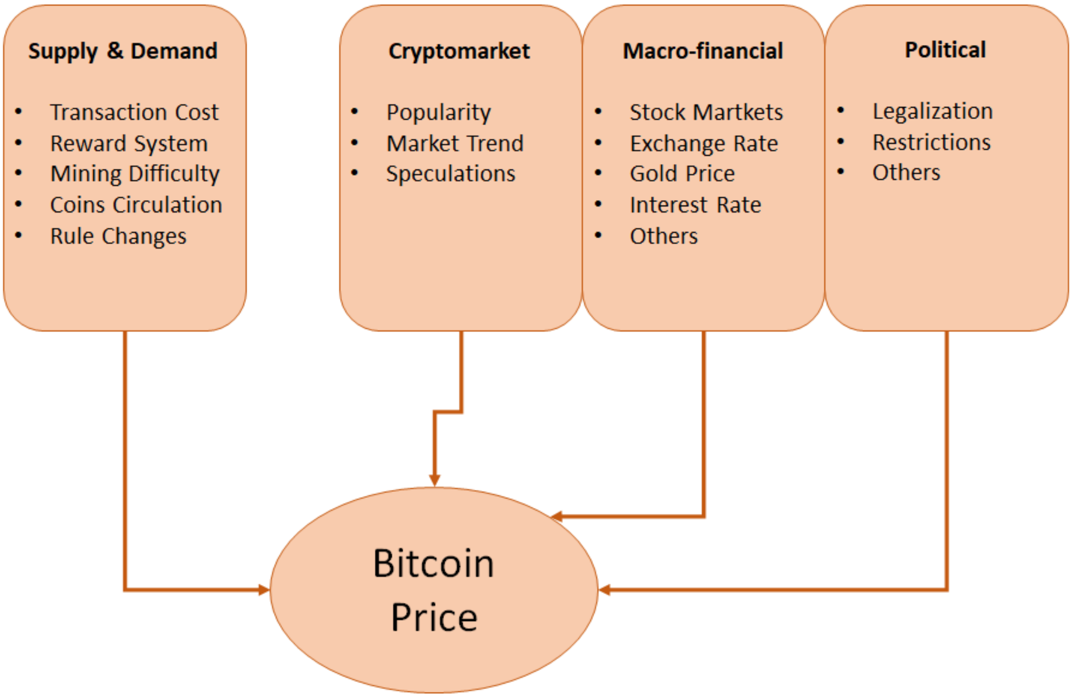


FIGURE 2.7: Factors influencing bitcoin prices (Sovbetov, 2018)

The growing popularity of bitcoins is probably the main factor that drove up its price but it is without a doubt the combination of all those factors that made the bitcoin what it is nowadays.

When the block-chain was presented, the term of miners and mining appeared but what does it refer to? As mentioned, it consists of solving complex mathematical problems, this creates bitcoins (thus the term mining, apparenting to ore extraction from the ground). A simple analogy of how it works is *"guess what number I am thinking of"* with no limit of guesses. The trick is that the pool of numbers is extremely large, up to 64-digit hexadecimal numbers. The one that guesses correctly gets the bitcoins. In 2009, there was few miners and every block created gave 50 bitcoins. Today, it only gives 6.25 bitcoins, figure 2.8. With the value of each block created decreasing, the difficulty of creating a block increases. It corresponds to how many hashes must be generated to to find the solution to the problem. The evolution of the difficulty over the years can be found on figure 2.9

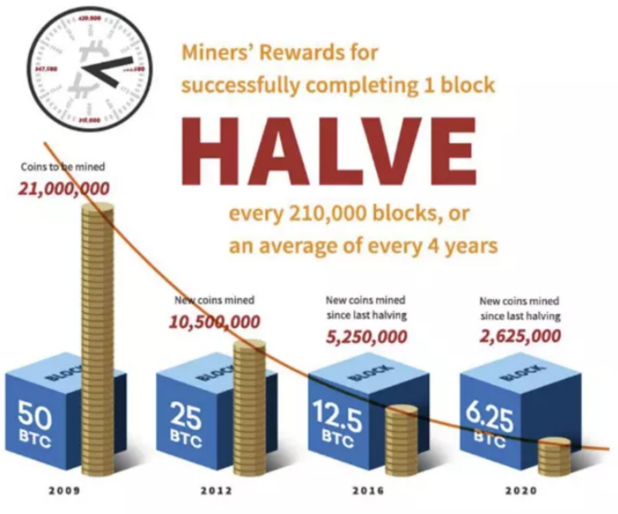


FIGURE 2.8: Miner's reward for completing a block *https://www.investopedia.com/terms/b/bitcoin-mining.asp*.

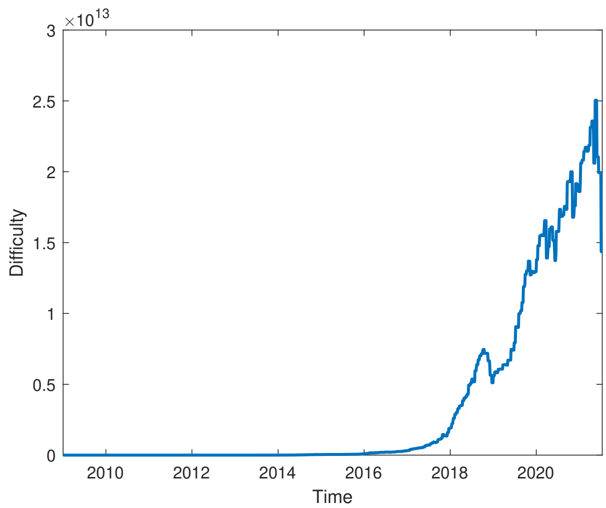


FIGURE 2.9: Difficulty of mining bitcoin over the years

With the difficulty increasing over the years, the need for highly optimized computer power became more and more important. While in 2009 mining could have been achieved with a family computer, in 2020 it requieres full GPU servers that can cost up to a few ten of thousand dollars. Obviously the energy consumption and the environmental impact are considerable (Bondarev, 2020). The energy consumption of bitcoin usage and production has increased over the years, figure 2.10 represents the cambridge bitcoin electricity consumption index.

FIGURE 2.10: Bitcoin electricity consumption (*Cambridge Bitcoin Electricity Consumption Index*)

Another environmental aspect that can be measured and compared is the carbon footprint associated to the bitcoin, figure 2.11 represents the comparison to the annual consumption of some countries. It is important to emphasise the fact that the bitcoin carbon footprint is worldwide, but it still helps to grasp to size and the importance of the carbon footprint.

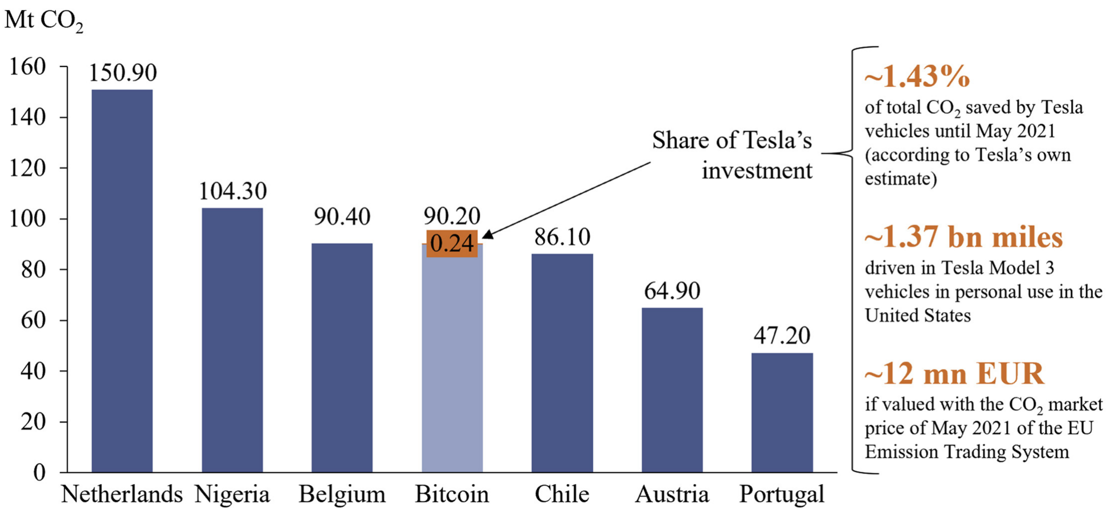


FIGURE 2.11: Comparison of Bitcoin's carbon footprint to national annual CO_2 emissions (deVries et al., 2021)

To conclude this chapter a SWOT analysis can be seen in table 2.1. It includes what has already been presented but also helps to present a clearer picture of Bitcoin strengths, weaknesses, opportunities and threats.

One can observe that overall, Bitcoins presents a wide range of advantages, but some of them involve drawbacks. For instance, the anonymity may lead to illegal activities. The list of opportunities and threats is also full of different aspects. This proves that there is still place for the Bitcoin to grow or perish.

In this chapter, the Bitcoin has extendedly been introduced, from its creation and introduction into the market to the present day with the impact it produces economically and ecologically. It went over the technological foundations and briefly detailed the underlying processes. Lastly, a SWOT analysis of the Bitcoin in its globality is proposed. The next chapter introduces the mathematical concepts that have been applied in this thesis.

Strengths	**Weaknesses**
• Bitcoin saves the time and physical space of those involved in transactions, using the virtual environment	• Increased volatility in all markets
• It created the impression of freedom, not implying the existence of a central authority	• Prohibition of the use of Bitcoin in certain countries
• It is not controlled by any authority, being able to circulate freely, directly between people, without intermediaries imposing transaction costs similar to commissions charged by banks. However, there are charged some miners fees.	• Representatives of several banks around the world believe that investig in Bitcoin is risky
• It does not involve the payment of commissions that banks usually charge	• High environmental costs, generated by electricity consumption and CO_2 emissions
• The price of Bitcoin results from the confrontation of demand with supply	• Increased vulnerability generated by the use of online environment, in which security breaches can also occur
• It does not involve bureaucracy in any of the stage of obtaining or using	• Lack of an institution/central bank to protect users in case of speculative attacks
• Beacause it exists in limited quatities, it does not generate inflation	• Accessibility conditioned by level of training compatible with new communication technologies
• It maintains the anonymity of economic operators that carry out transactions and are interested in this issue	• Limited trust, due to use of illegal activities - cryptocurrency in general can encourage gambling, tax evasion, terrorism, transactions with goods prohibited by law (drugs, weapons), money laundering, etc.
• Bitcoin illustrates the free market model that spontaneously self-orders	• Lack of intrinsic value for correlation with the price of traded goods and services
• It meets the requirements of the "IT generation"	
• It is compatible with the globalization of financial markets	
• The number of traders accepting Bitcoin is growing	

Opportunities	Threats
• The use of Bitcoin-based technology can lead to unsuspected performance in the virtual environment, associated with different areas of activity	• Authorities publicly expressed concern about the possibility of using cryptocurrencies for money laundering and other ilegal activities
• It simulates older generations to adapt to new technologies	• Losses suffered by states due to non-taxation of transactions/use in illegal activities with Bitcoin may lead in time to a ban or its use
• It is not related to issues of a patriotic nature, anthem or state, without thus arousing disputes of a nationalist nature	• High costs for purchasing the technology needed to obtain Bitcoin
• The number of those who accept BTC is increasing all over the world - restaurants, cafes, shops, universities, etc.	• The attraction to using Bitcoin is a sufficient cause for concern for traditional, conservative and rigid markets
• Unlike traditional currency, Bitcoin has no material basis, therefore it does not require a very elaborate process for issuing money	• The pressure exerted by the foollowers of the classical monetary canons, especially in the direction of Bitcoin recognition through a political act of the state
• Being still unregulated in many countries, it leaves the impression of a real freedom	• Lack of intrinsic value for correlation with the price of traded goods and services
	• Human errors, like losing the password, losing the memory, etc
	• Cyber risks

TABLE 2.1: Bitcoin SWOT L.Badea and Mungiu-Pupazan, 2021

Chapter 3

Method

In this chapter, the mathematical basis of this thesis is presented. A detailed look at statistics in the scope of finances alongside the usage of numerical methods are proposed. The main models considered in the scope of the work, the GARCH model family, are laid out. The theory of portfolio optimisation is also presented.

3.1 Fundamentals

Numerical methods are usually associated to engineering and scientific fields rather than finance but the amount of papers, research and books on the subject keeps on growing (Brandimarte, 2006). In finance, one word can be used to describe the general feeling, *uncertainty*, which is a common term within the field of statistics (Stockhammer and Grafl, 2008). To illustrate mathematically what uncertainty is, we assume that a variable, for instance the price of a stock, is modelled as a random variable. To this random variable, a probability distribution is associated, which gives a picture of the uncertainty associated to the price of the stock. However, this simple approach assumes that in terms of the probability distribution the history of the stock will repeat itself as future probabilities are equivalent to past probabilities.

For a more mathematical perspective, one can consider the current price of a stock, denoted S_0. This price can either go up, S_1^u, or down, S_1^d, with a an associated probability p for either possibilities. The basic rules of probability can be gathered from equation 3.1

$$\sum_{k=1}^{n} p^{(k)} = 1; \quad \text{with} \quad 0 \leq p^{(k)} \leq 1; \qquad k = 1, \ldots, n \tag{3.1}$$

This approach can be considered as "*discrete*" in which only a limited amount of time instances are taken into account. It is opposed to a "*continuous*" approach, in which the time step trends towards zero.

All of all the combined values of the variable as well as the associated probabilities are considered and plotted. This is known as the *probability density function*. It can take various shapes depending on the problem studied. One of the most famous probability density functions is the normal distribution, or the gaussian distribution (Ahsanullah, Kibria, and Shakil, 2014). It is a distribution centred around the mean, μ. The "*sharpness*" of the curve depends on the standard deviation, σ also called the volatility in finance, or more precisely of the square of σ known as the variance of the distribution. It describes the dispersion of the results. One can look at the associated mathematical expressions equation 3.2 for the mean and 3.3 for the variance.

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \ldots + x_n}{n} \tag{3.2}$$

$$\sigma_X^2 = Var(X) = E[(X-\mu)^2] = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{3.3}$$

The variance is the expected value of the squared deviation from the mean, where the expected value takes the mathematical form of equation 3.4, which is also known as the standard deviation. Graphically the probability density function of a normal distribution results in figure 3.1. Another famous distribution law is the Student's t-distribution. It is symmetric and bell-shaped like the normal distribution but the tails are heavier. It means that the extremes are more reduced.

$$E[X] = \sum_{i=1}^{n} x_i p_i = x_1 p_1 + x_2 p_2 + \ldots + x_n p_n \tag{3.4}$$

When two variables are considerate, the variance between the two can be expressed as the covariance. If the two variables have great values together, the covariance is positive, if the first variable is high while the second is low the covariance is negative. Equation 3.5 is the mathematical expression of the covariance.

$$\Gamma_x y = Covar(X) = E[(X - E[X])(Y - E[Y])] \tag{3.5}$$

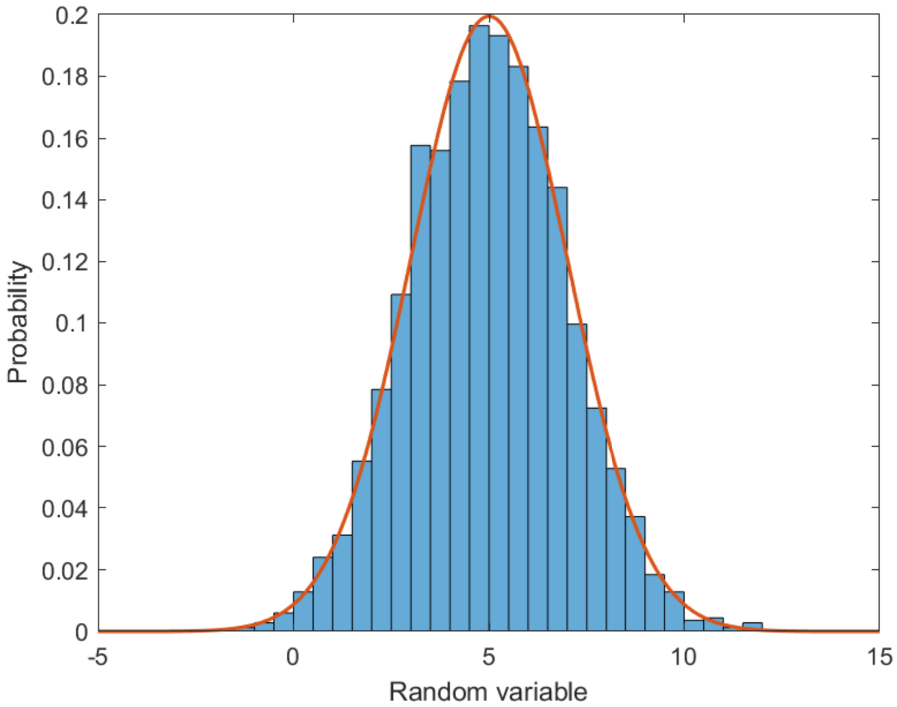


FIGURE 3.1: Histogram and interpolated probability density function of a random number generated 5000 times with a mean of 5 and a standard deviation of 2

The definition of the covariance allows the definition of an important statistical element, the correlation between two variables. It corresponds to the degree of linear relation two variables have together. It ranges between -1 and 1, in which a correlation of -1 indicates an *anti-correlation*, whereas a correlation of 1 indicates a *perfect correlation*. A correlation of 0 means that the two variables are *uncorrelated*. Naturally the correlation between a variable and itself is strictly equal to 1. Mathematically, the correlation coefficient is a relation that depends on the covariance and the standard deviations of two variables X and Y.

$$\rho_{xy} = Corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X - E[X])(Y - E[Y])]}{\sigma_X \sigma_Y} \tag{3.6}$$

There is another formulation for the correlation coefficient that depends on the number of observations N, the mean (μ) and the standard deviation (σ) of two variables X and Y. This formulation is known as the Pearson's correlation coefficient.

$$\rho(X,Y) = \frac{1}{N-1} \sum_{i=1}^{N} \left(\frac{X_i - \mu_X}{\sigma_X} \right) \left(\frac{Y_i - \mu_Y}{\sigma_Y} \right) \tag{3.7}$$

Two additional parameters are interesting to look at for probability density functions, the skewness and the kurtosis. The skewness measures the asymmetry of a pdf around its mean. It can either be positive or negative depending on the side on which the *tail* of the graph is. The kurtosis also describes the shape of the pdf, it provides information on the tail distribution. A phenomenon known as the *kurtosis risk* exists for investors which describes an event in which investors experience more extreme returns than usual, good or bad. Their respective mathematical expressions are corresponding to 3.8 and 3.9.

$$S = \tilde{\mu}_3 = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] \tag{3.8}$$

$$Kurt[X] = K = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] \tag{3.9}$$

3.1.1 Jarque-Bera Test

The Jarque-Bera test is a process that verifies if the sample data match a normal distribution as the one represented in figure 3.1. It is based on the skewness and the kurtosis. The equation used for the test is:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{3.10}$$

In which n is the number of observations. The test returns a value that can be compared to a critical value, if the returned value is larger than the critical one, the data does not fit a normal distribution.

The basic concepts of statistics have been laid out in this section; Thanks to this short introduction the more complex method that is used in this thesis can now be presented. This method is known as the GARCH method.

3.2 ARCH/GARCH Model

The ARCH (Autoregressive Conditional Heteroskedasticity) model (Engle, 1982), is a statistical model that is used to compute and study the volatility in time series. The goal of the method is to somehow predict the future behaviour of the volatility. Obviously, it is of great interest in the financial world, where the method is used to estimate the risks of the markets (Lamoureux and Lastrapes, 1990, Dyhrberg, 2016). The general idea behind the method, is to consider the volatility at a previous time in the determination of the time series. Mathematically it corresponds to equation 3.11.

$$a_t = \epsilon_t \sigma_t \tag{3.11}$$

Whereas a_t is the solution at time t of the time series, ϵ_t is a stochastic part associated to noise (close to 0) and σ_t is the volatility at time t, or yet again the standard deviation. It can also be written with the value of the time series at the previous time, $t-1$, and two undetermined constants as in equation 3.12.

$$a_t = \epsilon_t \sqrt{\omega + \alpha_1 a_{t-1}^2} \tag{3.12}$$

The issue with this model is that it can be characterised as "*bursty*", meaning that it can produce burst in the time series before going back to a nominal value. In order to avoid such behaviour, one can consider the GARCH model (Francq, Horvath, and Zakoïan, 2011).

The GARCH model is a particular case of the ARCH model. The model is often use to show the hedging capabilities of gold (Dyhrberg, 2016,Basher and Sadorsky, 2016).

$$\sigma_t = \omega + \alpha_1 a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3.13}$$

This last equation is the particular case of the GARCH model, referred as GARCH(1,1) in which $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \beta \leq 1$. The general case is written GARCH(p,q), in which one considers the "p" previous results of the time series and the "q" previous volatility forecasts which gives equation 3.14.

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3.14}$$

Another method directly derived from the GARCH method is the eGARCH method, standing for exponential GARCH. The corresponding equation is Equation 3.15, in which one can observe that the result is the logarithm of the variance. Contrarily to the classical GARCH method, it can be negative or positive which removes condition on the parameters α and β. $g\left(Z_{t-p}\right) = \theta Z_t + \lambda(|Z_t| - E(|Z_t|)$, with Z_t being a generalised error distribution and θ a coefficient.

$$\log \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i g\left(Z_{t-p}\right) + \sum_{j=1}^{q} \beta_j \log \sigma_{t-j}^2 \tag{3.15}$$

The last GARCH model studied in this thesis is the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model. Its expression is close to the classical GARCH, it incorporates another parameter γ and I_{t-1} that depends on the expected return μ such that

$$\sigma_t^2 = \omega + (\alpha_1 + \gamma I_{t-1}) a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3.16}$$

$$I_{t-1} = \begin{cases} 0 & \text{if} \mu + \epsilon_{t-1} \geq \mu \\ 1 & \text{if} \mu + \epsilon_{t-1} < \mu \end{cases}$$

To select the most suited model statistical tests exist. Two criterions are considered in this thesis, the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). Both are model selectors that compare models to each other. These methods are based on the maximum of the likelihood function $\hat{L}$, which is already a measurement for the good fit of data and models, k the number of estimated parameters in the model and n the number of observations for the BIC. Equations 3.17 and 3.18 are the respective equations for the AIC and BIC. The lower the value of the AIC and BIC the more suited the model.

$$AIC = 2k - 2\ln \hat{L} \tag{3.17}$$

$$BIC = k \ln n - 2\ln \hat{L} \tag{3.18}$$

The usage of a univariate model helps to describe the volatility of one type of asset. The main interest lays in the interaction of one item with another, in the case of the thesis, Bitcoin with stocks (Silvennoinen and Teräsvirta, 2008). This new tool is useful for the optimisation of portfolios since it helps to determine correlations and volatility between the different assets of a portfolio.

The classical method to do so is known as the Dynamic Conditional Correlation, or DCC (Engle, 2002,Celik, 2012)). Engle's idea is to consider a covariance matrix filled with returns, H_t:

$$H_t = D_t R_t D_t \tag{3.19}$$

in which

$$D_t = \begin{bmatrix} a_1, t & \dots & 0 \\ \vdots & \ddots & \dots \\ 0 & \dots & a_n, t \end{bmatrix} \tag{3.20}$$

and R_t is a $n \times n$ correlation matrix containing the conditional correlations that varies in time:

$$R_t = \begin{bmatrix} 1 & q_{12,t} & q_{13,t} & \dots & q_{1n,t} \\ q_{21,t} & 1 & q_{23,t} & \dots & q_{2n,t} \\ q_{31,t} & q_{32,t} & 1 & \dots & q_{3n,t} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_{n1,t} & q_{n2,t} & q_{n3,t} & \dots & 1 \end{bmatrix} \tag{3.21}$$

Let us note that the matrix H_t must be positive definite, which means that R_t has to be as well. D_t is already positive due to its diagonal form filled with variances. Another requirement is that the elements of the matrix R_t must be less or equal to one, it takes the form:

$$R_t = (diag(Q_t))^{*-1} \, Q_t \, (diag(Q_t))^{*-1} \tag{3.22}$$

In which the matrix Q_t is the $n \times n$ time varying covariance matrix of standardised residuals in the case of a DCC-GARCH(1,1):

$$Q_t = (1 - \Psi - \zeta) \, \bar{Q}_t + \Psi \delta_{j,t-1} \delta_{i,t-1} + \zeta Q_{t-1} \tag{3.23}$$

In equation (3.25), $\bar{Q}_t$ is the unconditional correlations of $\delta_{i,t}\delta_{j,t}$, and Ψ and ζ are scalars greater or equal to zero that must satisfy $\Psi + \zeta < 1$. The matrix Q_t^* is a diagonal matrix with the square root of the i^{th} diagonal element of Q_t.

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & \dots & 0 \\ 0 & \sqrt{q_{22,t}} & \dots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \dots & \sqrt{q_{nn,t}} \end{bmatrix} \tag{3.24}$$

The most general form of the DCC-GARCH(p,q) is:

$$Q_t = \left(1 - \sum_{i=1}^{p} \Psi_i - \sum_{j=1}^{q} \zeta_j \right) \bar{Q}_t + \sum_{i=1}^{p} \Psi_i \delta_{j,t-1} \delta_{i,t-1} + \sum_{j=1}^{q} \zeta_j Q_{t-1} \tag{3.25}$$

Typically the DCC-GARCH method is used for modelling hedging capabilities of gold or cryptocurrencies (Chu et al., 2017). It is worth mentioning that other models derived from the GARCH model exist (Siu, Nawar, and Ewald, 2014). Models such as IGARCH, QGARCH, for an exponential approach, integrated or even quadratic. The comparison of the precision between GARCH and eGARCH for normal and student's t distributions is proposed in the following chapter.

3.3 Modern Portfolio Theory

In 1952, Harry Markowitz (Markowitz, 1982) presented a theory about how risk-averse investors can build a portfolio to maximise the expected return based on the risk, this theory is known as the Modern Portfolio Theory (MPT). His work has been used as a reference for decades now (Fabozzi, Gupta, and Markowitz, 2002), Markowitz even won a Nobel Prize for the MPT.

The basic hypothesis behind the MPT is that any investors will prefer and prioritise a less risky portfolio if given the opportunity. This means that in order to consider a portfolio with a higher risk, it must be compensated by higher expected results. This approach can also be constructed the other way around; To increase the expected result, one must be willing to accept more risk. Using equation 3.4, the expected return takes the form

$$E(R_p) = \sum_i w_i E(R_i) \tag{3.26}$$

In which R_p is the return of the portfolio, w_i is the weight parameter of the asset i or yet again the percentage of asset i one has in his portfolio, and R_i is the return of the asset i.

Under the assumption that assets are not perfectly correlated, investors can reduce the risk of holding only one type of assets by diversifying the portfolio. The relation between the risk and the return is known as the efficient frontier.

3.3.1 Efficient Frontier

The key to the MPT is to determine how to diversify the portfolio, what percentage of each asset should be considered and how many assets should compose the portfolio. The efficient frontier presents all the possible combinations of portfolio with the optimal levels of return compared to the associated risk (Broadie, 1993). Any portfolio in a region outside of the efficient frontier is considered sub-optimal as they present too great of risk in comparison to the expected return. Figure 3.2 is a graphical representation of the efficient frontier.

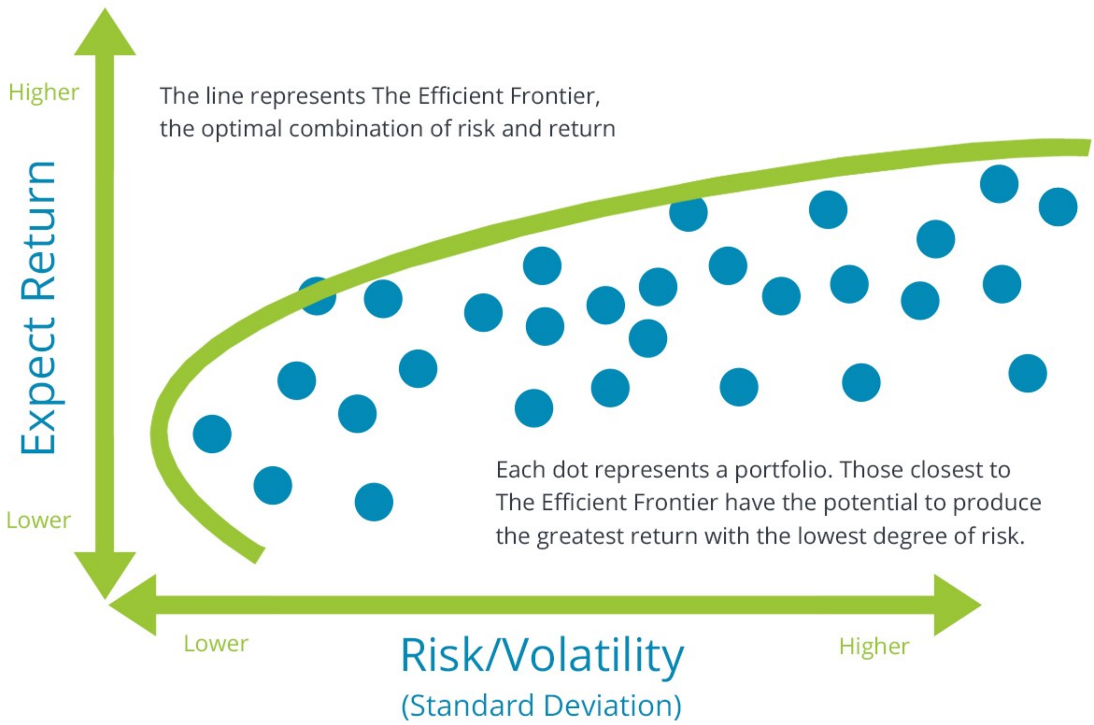


FIGURE 3.2: Efficient frontier *https://www.guidedchoice.com/video/dr-harry-markowitz-father-of-modern-portfolio-theory/*

The efficient frontier is built by minimising the matricial problem given by equation 3.27 given a certain risk tolerance parameter q that must be greater or equal to 0.

$$w^T \Gamma w - qR^T w \tag{3.27}$$

In which w still represents the portfolio weights, Γ is the covariance matrix of the returns of the various assets in the portfolio, R is the vector of the expected results. In other words, this equation can be read as the minimisation of the difference between the variance of the portfolio return and the expected return modulated by a risk tolerance parameter.

A parameter of interest when working with efficient frontier is known as the Sharpe ratio. It helps measuring the performance of a portfolio. When maximised the associated portfolio is the portfolio tangent to the efficient frontier (Sharpe, 1994). It mathematically corresponds to the ratio between the expected value of the difference between the asset return and the *risk-free return*, and the standard deviation of the asset (Eq. 3.28).

$$S_a = \frac{E[R_a - R_b]}{\sigma_a} \tag{3.28}$$

The key element of the modern portfolio theory lays in the diversification of assets. Risk will always exist in the stock market; it can be divided in two distinct types. While the first type of risk does not depend on the level of diversification and the amount of assets in the portfolio, the second type depends on the asset itself and the company it is related to, the more diversified the portfolio the lower the second type of risk becomes. An illustration of the effect of the diversification can be seen in figure 3.3.

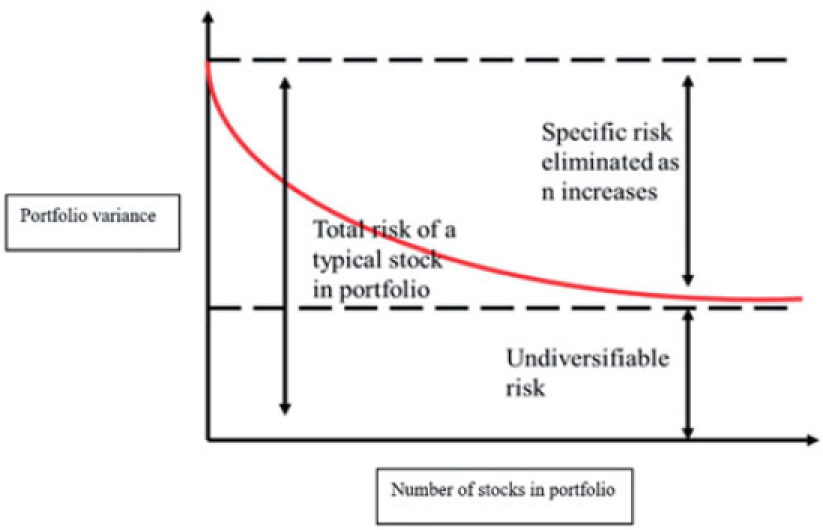


FIGURE 3.3: Effect of diversification over the risk (Hundal, Eskola, and Tuan, 2019).

3.4 Hedging or Safe Haven

The term of *hedge* has appeared multiple times the introduction and throughout the previous chapters, but it has not been clearly defined to this point. A hedge is a type of investment made in order to reduce the risk associated to the price movements of an asset. It can be viewed as an insurance against significant losses. In (Baur and McDermott, 2009), Baur proposes a definition for what exactly a hedge is: *"A strong (weak) hedge is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio on average"*. This definition provides a mathematical and quantifiable way of what a hedge is. An historical example of hedge can be found in Gold and US Dollar (Kunkler and MacDonald, 2016, Cappie, Mills, and Wood, 2004). Figure 3.4 depicts the relation between gold and USD during the first decade of the 21st century. During this period, the price of gold and the value of the USD seem to behave almost as opposites to each other which indicates a strong negative correlation.

FIGURE 3.4: Gold vs Hedge *https://seekingalpha.com/article/259249-why-gold-is-no-longer-an-effective-usd-hedge*

A *safe haven* is similar to a hedge, the only difference lays in the timing of the phase of negative correlation. The definition according to Baur (Baur and McDermott, 2009) of a safe haven is: *"A strong (weak) safe haven is defined as an asset that is negatively correlated (uncorrelated) with another asset or portfolio in certain periods only, e.g. in times of falling stock markets"*. The crisis caused by Covid-19 had a significant impact on financial markets and lead to a massive movement from risky assets to safe havens (Cheema, Faff, and Szulczyk, 2004).

3.5 Data

For most of this thesis the prices and the data are transformed into the logarithmic return, which is the ratio of the price at the time t and the price at the time $t-1$, the logarithmic return takes the form:

$$R = ln\frac{P_t}{P_{t-1}} \tag{3.29}$$

For instance, the logarithmic return of Bitcoin takes the form represented in figure 3.5.

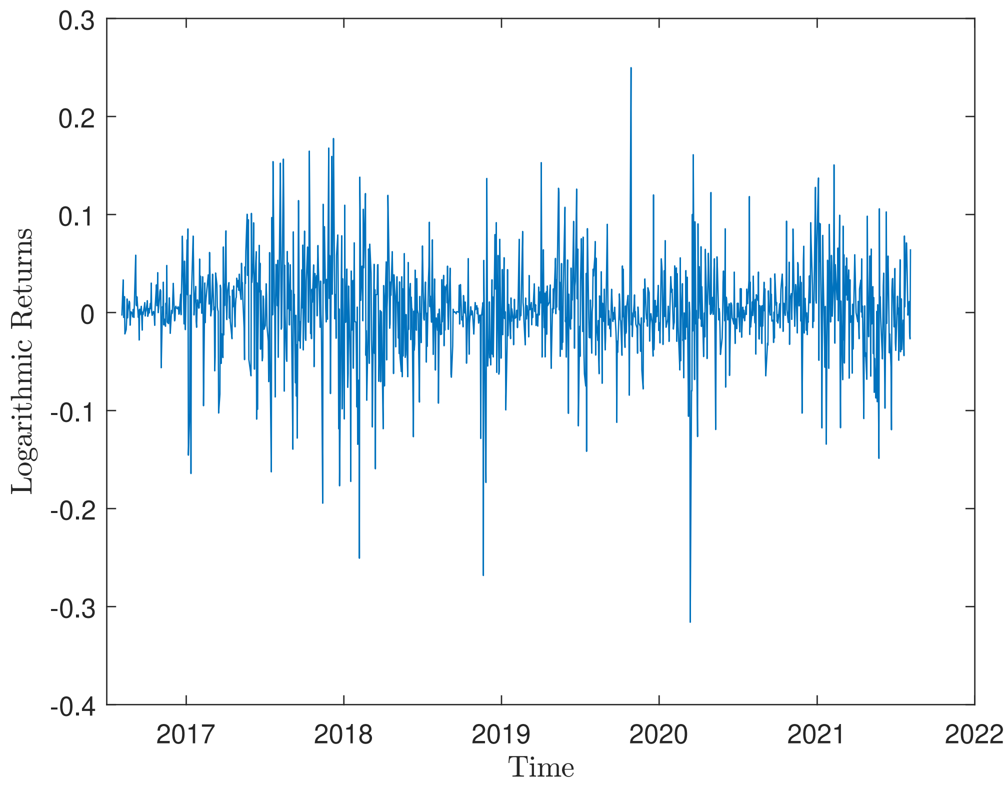


FIGURE 3.5: Logarithmic return of Bitcoin

The time scale considered for this thesis, spans from the early 2017 up to month of June 2021. Concerning the portfolio construction, 20 different stocks belonging to the $S\&P500$ are considered. This allows the construction of a portfolio solely composed of stocks that are risky assets. For the second part of the study on portfolio optimization and Bitcoin impact, more general indices are taken into accound ($S\&P500$, Gold, Oil, USD, ...). The statistics associated to the different assets are gathered in table 3.1 and the evolution of the prices of the stocks over almost five years are displayed in figure 3.6.

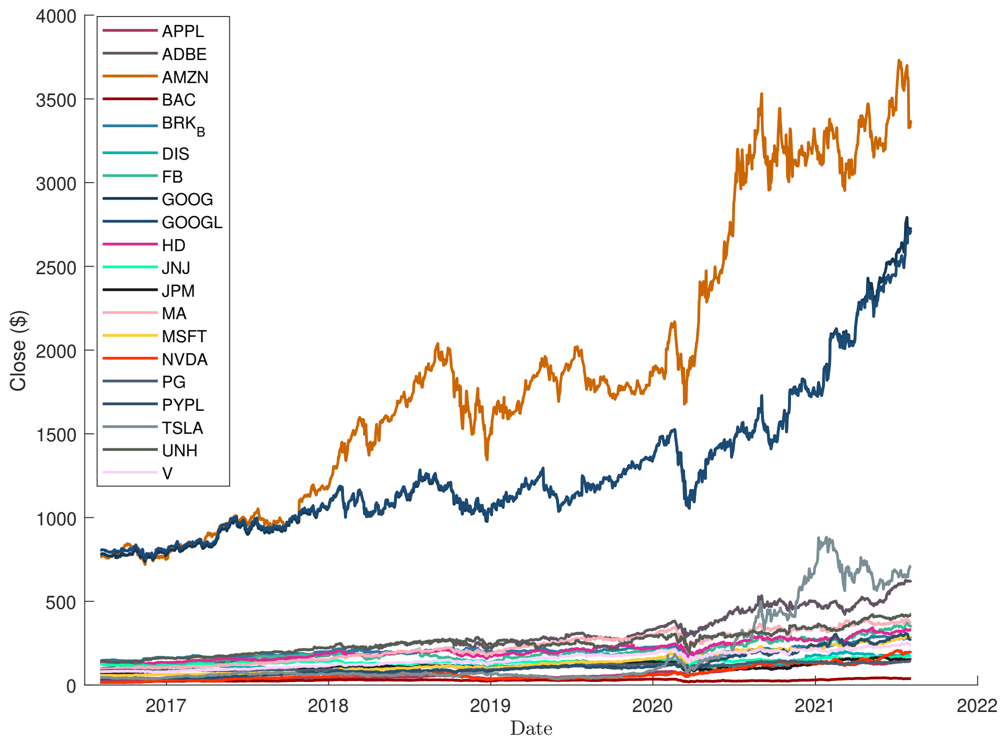


FIGURE 3.6: Evolution of the prices of the stocks of the assets

The returns of the assets can be shaped as histograms as has been done in figure 3.8. At first glance, the returns seem to follow a normal distribution. Using the Jarque-Bera Test (Eq. 3.10) and representing the results on Quantile-Quantile plots like what is proposed in figure B.1 in annexes, show that the returns can not be approximated as normal laws.

To reject the normal distribution assumption, the Jarque-Bera result must be lower than a critical value. The critical value is equal to 5.94 in this case. Figure 3.7 displays the results of the test for each asset. One can clearly see that none of the asset respect this criterion, hence the rejection of the hypothesis.

Asset	**Mean** (%)	**Median** (%)	**Variance** ($\times 10^{-4}$)	**Kurtosis**	**Skewness**
Apple (**APPL**)	0.14	0.11	3.623	10.21	-0.338
Adobe (**ADBE**)	0.15	0.21	4.052	13	-0.032
Amazon (**AMZN**)	0.12	0.14	3.412	7.41	0.049
Bank of America (**BAC**)	0.11	0.073	4.517	15.81	-0.108
Berkshire Hathaway (**BRK**$_B$)	0.074	0.067	1.835	16.57	-0.386
The Walt Disney Company (**DIS**)	0.046	0.009	3.152	17.12	0.548
Facebook (**FB**)	0.084	0.1	4.297	16.43	-1.063
Alphabet (**GOOG**)	0.099	0.13	2.828	9.47	-0.278
Alphabet (**GOOGL**)	0.096	0.13	2.843	9.49	-0.340
Home Depot (**HD**)	0.070	0.1	2.670	37.6886	-2.089
Johnson & Johnson (**JNJ**)	0.026	0.048	1.601	14.40	-0.663
JPMorgan Chanse (**JPM**)	0.0656	0.047	3.545	19.75	-0.1
Mastercard (**MA**)	0.18	0.18	3.460	13.99	-0.002
Microsoft (**MSFT**)	0.11	0.11	2.947	15.06	-0.346
Nvidia (**NVDA**)	0.27	0.27	9.08	12.90	-0.144
Procter & Gamble (**PG**)	0.057	0.057	1.635	17.19	0.212
PayPal (**PYPL**)	0.16	0.19	4.814	10.52	-0.029
Tesla (**TSLA**)	0.22	0.13	14	8.81	-0.11
UnitedHealth Group (**UNH**)	0.086	0.09	3.222	20.58	-0.554
Visa (**V**)	0.086	0.17	2.662	15.83	-0.25
Bitcoin (**BIT**)	0.34	0.28	0.24	7.67	-0.4837

TABLE 3.1: Mean, Median, Variance, Kurtosis and Skewness of the different assets

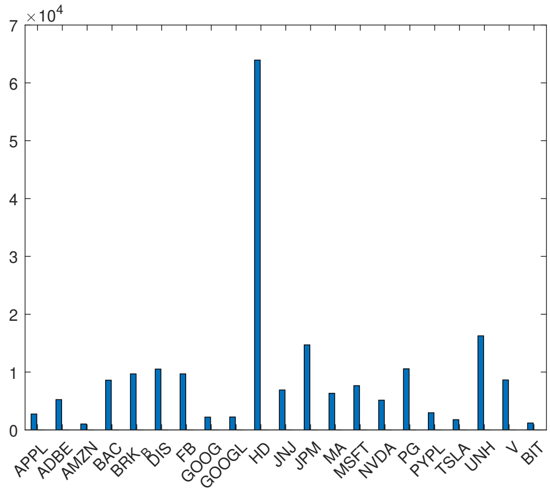
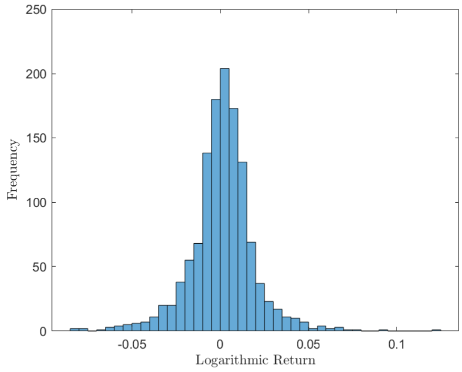
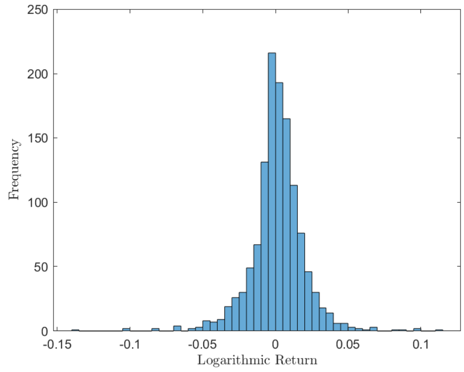


FIGURE 3.7: Results of the Jarque-Bera test for every asset

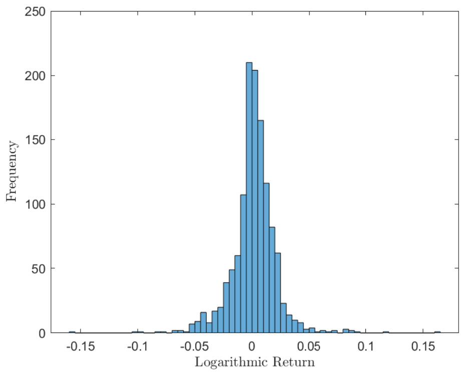
In this chapter, the mathematical basis of statistics has been presented, the fundamentals about the mean, variance, correlation, volatility and the probability density functions have been introduced. A summary of the GARCH method family, which are the main methods used in this thesis, has been given. At the end of the chapter the modern portfolio theory is explained alongside the efficient frontier. The concepts of hedge and safe haven have also been introduced.



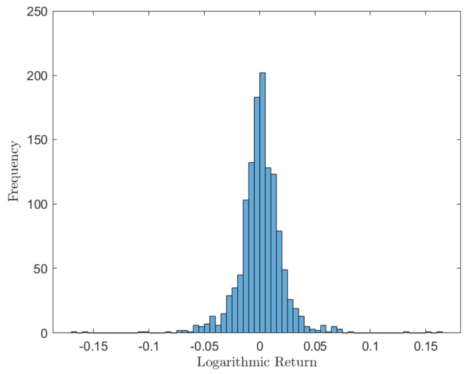
(A) APPL Returns



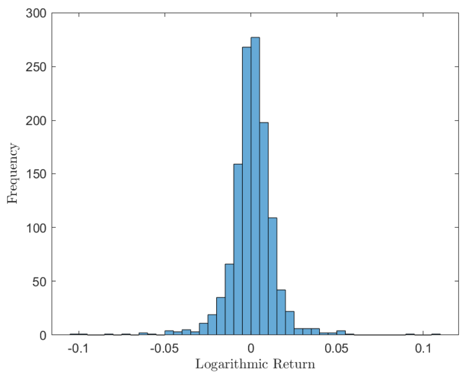
(B) ADBE Returns



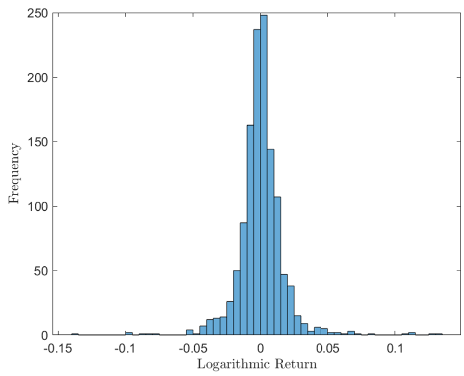
(C) AMZN Returns



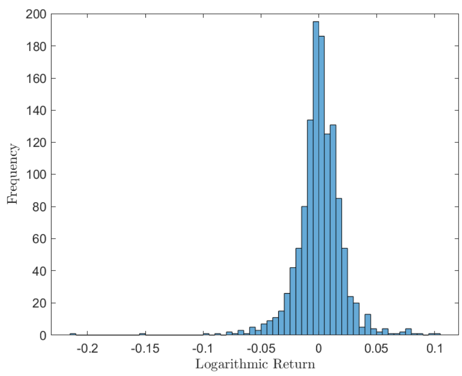
(D) BAC Returns



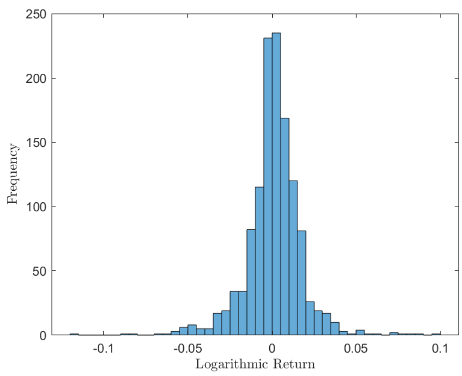
(E) BRK_B Returns



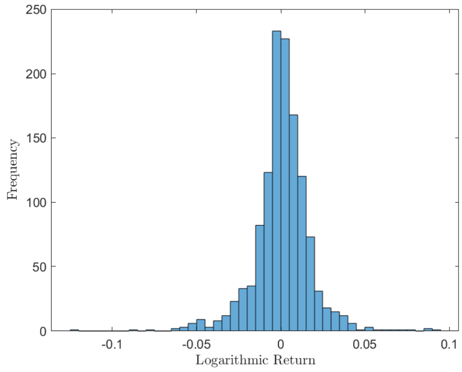
(F) DIS Returns



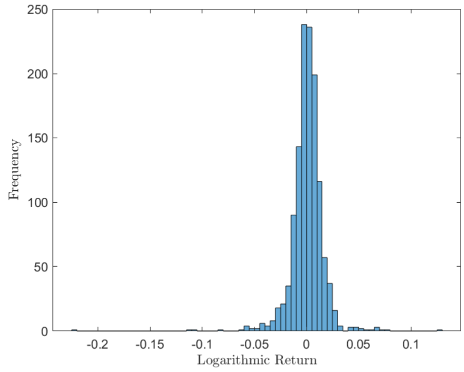
(G) FB Returns



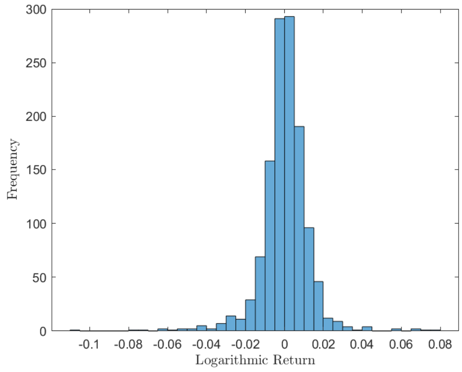
(H) GOOG Returns



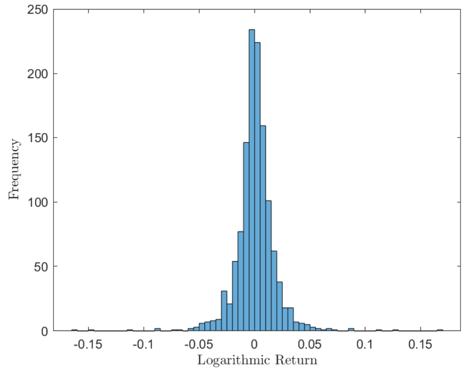
(I) GOOGL Returns



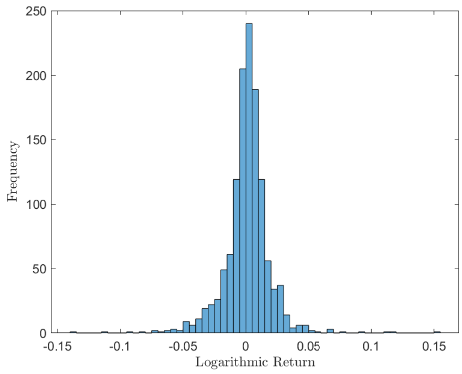
(J) HD Returns



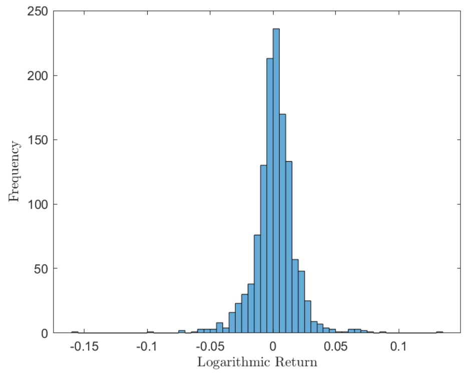
(K) JNJ Returns



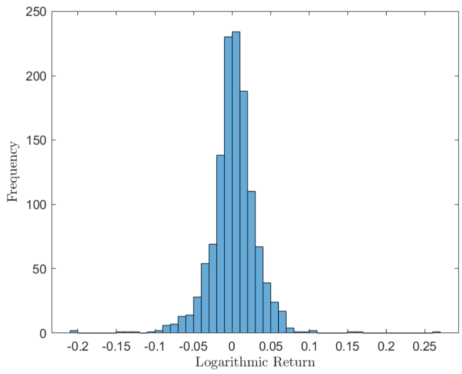
(L) JPM Returns



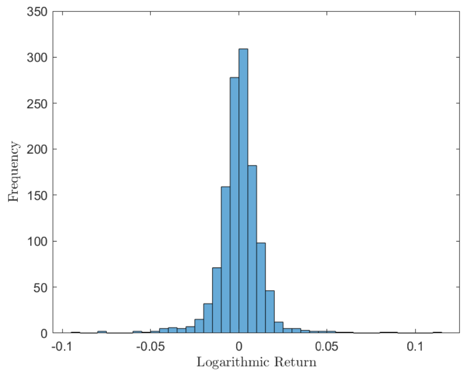
(M) MA Returns



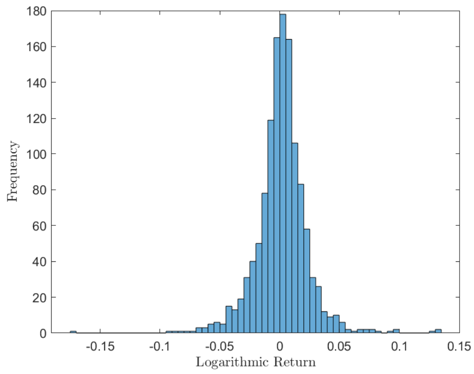
(N) MSFT Returns



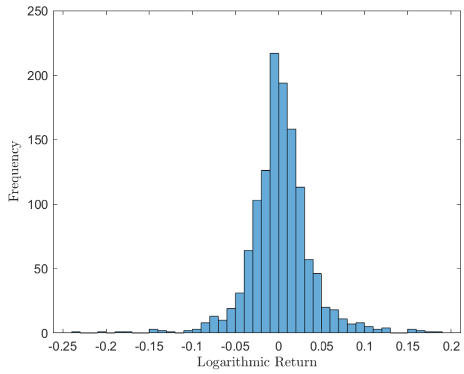
(O) NVDA Returns



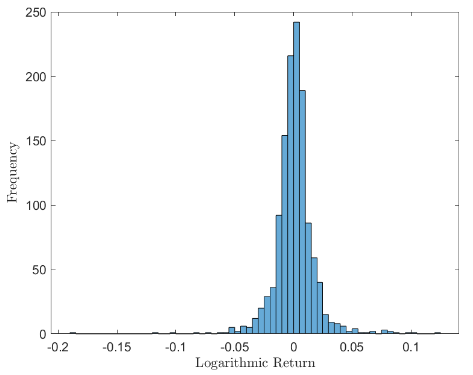
(P) PG Returns



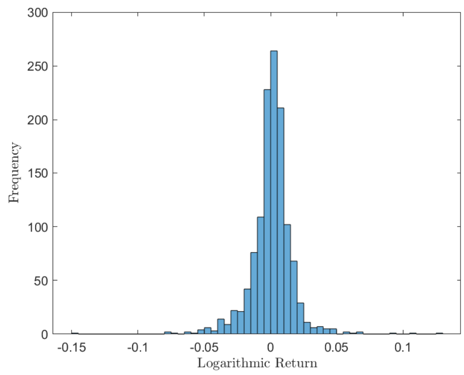
(Q) PYPL Returns



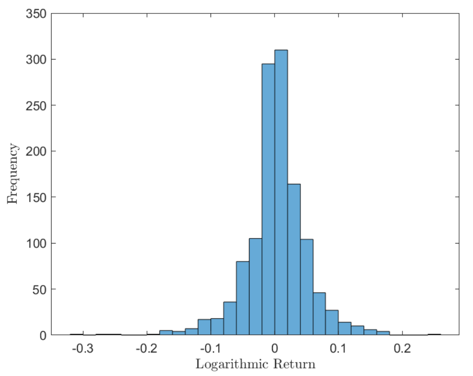
(R) TSLA Returns



(S) UNH Returns



(T) V Returns



(U) BIT Returns

FIGURE 3.8: Histograms of the logarithmic returns of the assets.

Chapter 4

Results

In order to evaluate the hedge capacities of Bitcoin and the impact of its introduction into a portfolio of stocks, a GARCH model of the various assets must be created. The results given by the GARCH model will help to build the DCC-GARCH that provides a clear insight into the hedge capacities Bitcoin in the case of the studied portfolio.

A simpler method that provides a first insight into the potential hedging capacities of Bitcoin consists of a direct computation of the covariances between the returns of the portfolio and the returns of Bitcoin. The results of this simple approach must be compared with the ones given by the more accurate DCC-GARCH model.

The direct introduction of Bitcoin into the portfolio helps to grasp the full extent of the effect that Bitcoin has on the efficient frontier.

4.1 Direct Approach

As mentioned in the previous section, the definition of a hedge is related to the correlation between two assets, if the correlation is a negative integer one asset can be considered as a hedge for the other. The direct approach consists in simply estimating the correlation between the return of Bitcoin and the return of the portfolio.

The most rudimentary and simplest approach is to first consider an equal distribution of the assets in the portfolio, each asset corresponds to 5% of the portfolio. The prices are represented in figure 3.6, averaging those prices gives us figure 4.2 representing the average price at close of the portfolio. The significant drop associated with the COVID-19 crisis can be clearly identified on this plot.

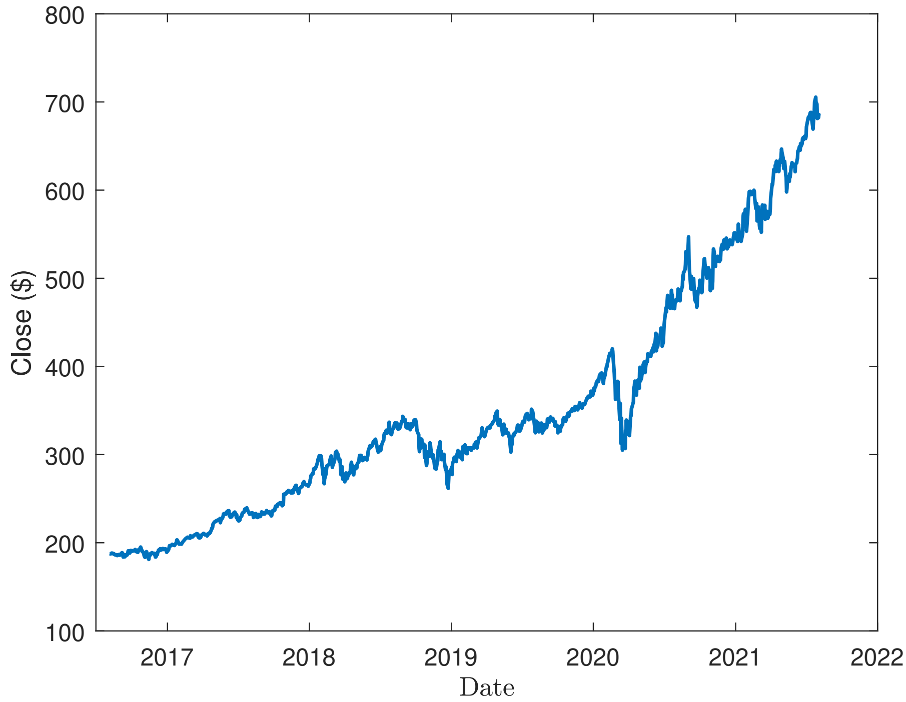


FIGURE 4.1: Average of the price of the assets at close that compose the portfolio

Using the relation of the return (Eq. 3.29) the logarithmic returns of the averaged portfolio and Bitcoin are evaluated across the studied period of time. The correlations are then obtained via the Pearson correlation coefficient equation (Eq 3.7), which returns the following correlation matrix A:

$$A = \begin{pmatrix} 1 & 0.1053 \\ 0.1053 & 1 \end{pmatrix}$$

From this result, a first interesting conclusion can be drawn. The correlation between the averaged portfolio and Bitcoin is slightly above zero, which indicates a tendency to not react like a hedge. However, this conclusion was reached under assumption of the composition of the portfolio and using a simplistic model.

The model can be extended by sorting the returns from its minimum value to its maximum value, which correspond to significant loss to significant gain. The correlation coefficient between the portfolio and Bitcoin is then computed over those specific ranges of losses and gains. The coefficients are computed for every slice of 5%. The results are available in table 4.1.

Bounds	Correlation coefficient
[0%,1%[	0.5775
[1%,5%[	0.1761
[5%,10%[	-0.1153
[10%,15%[	0.1685
[15%,20%[	-0.0484
[20%,25%[	0.2128
[25%,30%[	-0.0495
[30%,35%[	-0.0490
[35%,40%[	-0.0890
[40%,45%[	0.2556
[45%,50%[	0.0376
[50%,55%[	-0.0607
[55%,60%[	0.1313
[60%,65%[	-0.0283
[65%,70%[	-0.1819
[70%,75%[	0.1348
[75%,80%[	-0.2600
[80%,85%[	0.0554
[85%,90%[	-0.0506
[90%,95%[	0.0888
[95%,99%[	-0.0430
[99%,100%[	-0.0770

TABLE 4.1: Correlation coefficients between the averaged portfolio and Bitcoin with respect to market bonds

From the table one can observe that for about half of the cases Bitcoin seems to hedge the averaged portfolio, especially when the returns of the portfolio are higher which is not a good thing to have. Indeed, it means that Bitcoin goes down when the portfolio goes up. The fact that correlation coefficients are maximum for the worst part of the portfolio indicated that Bitcoin is the opposite of a safe haven. If one looks at the mean of the correlation coefficient gathered in the table, the resulting correlation is 0.0357 which does not say much about Bitcoin hedge capacities. The data of table 4.1 can be plotted to better visualise the hedging capacities. Figure 4.2 represents on the same graph the results from table 4.1 in orange and the returns of the averaged portfolio in blue.

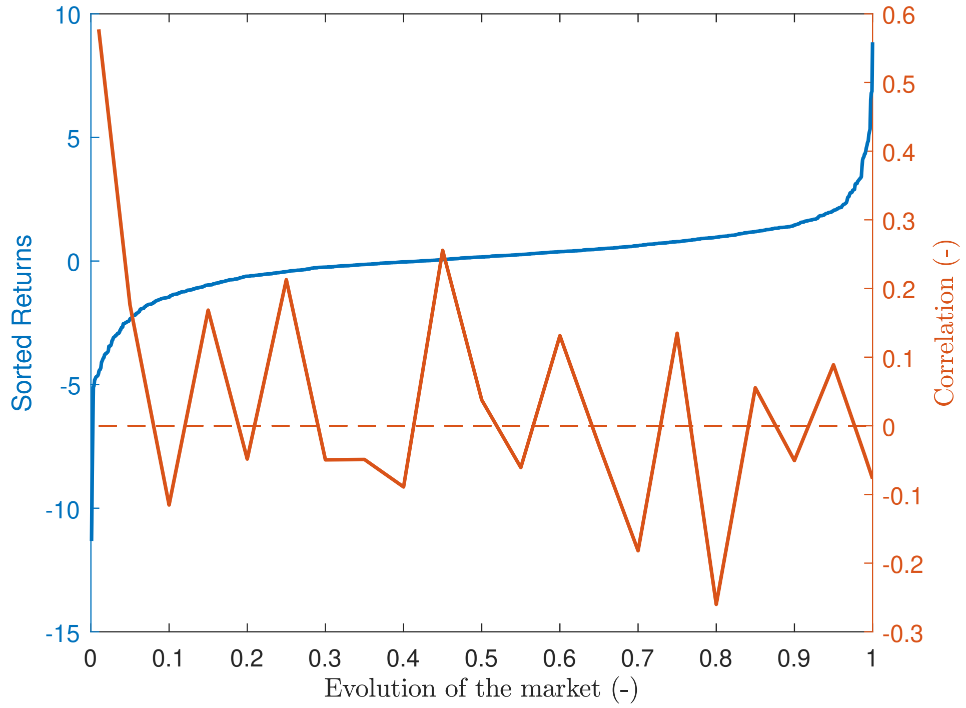


FIGURE 4.2: Evolution of the correlation coefficient (orange) and sorted returns of the portfolio (blue) with respect to the evolution of the market

In figure 4.2, the tendencies of Bitcoin to have hedging potential are more apparent. Overall, the orange curve decreases with the evolution of the market. There is an alternating pattern between spikes above zero and below zero which indicates that Bitcoin does not hedge all situations. The main tendency that can be observed is that Bitcoin hedges or not more significantly in edge cases corresponding to extremely poor and extremely good results.

Even though this method does not provide very accurate results and strongly depends on the observation period studied, it provides a first idea of the expected results. The GARCH and DCC-GARCH methods employed in the following sections help to clarify the true hedging potential of Bitcoin for our portfolio.

4.2 GARCH

Different models are applied to both the averaged portfolio and Bitcoin in order to capture the volatility of both assets. Tables 4.2 and 4.3 present the results of AIC and BIC for GARCH, eGARCH and GJR-GARCH models with a normal distribution and Student's t distribution.

Averaged Portfolio						
Distribution	Normal			Student's t		
Volatility Model	GARCH	eGARCH	GJR-GARCH	GARCH	eGARCH	GJR-GARCH
AIC	-7.5528	-7.5454	-7.5528	-7.6529	-7.6527	-7.6544
BIC	-7.5374	-7.5300	-7.5374	-7.6324	-7.6321	-7.6339

TABLE 4.2: Comparison of AIC and BIC for the averaged portfolio with different models

Bitcoin						
Distribution	Normal			Student's t		
Volatility Model	GARCH	eGARCH	GJR-GARCH	GARCH	eGARCH	GJR-GARCH
AIC	-4.1782	-4.1733	-4.1782	-4.4549	-4.4819	-4.4549
BIC	-4.1628	-4.1579	-4.1628	-4.4343	-4.4613	-4.4343

TABLE 4.3: Comparison of AIC and BIC for the Bitcoin with different models

From these tables, it clearly appears that Student's t distribution is more suited than normal distribution for modelling the averaged portfolio and Bitcoin. Furthermore, one can observe that for the volatility of the averaged portfolio the GJR-GARCH(1,1) model is more appropriate while for Bitcoin eGARCH(1,1) is more suited.

The models associated to the averaged portfolio and Bitcoin can be written using equations (3.16) and (3.15) which gives equations (4.1) and (4.2) and are graphically represented on figure 4.3.

$$\sigma_t^2 = -4.43^{-6} + (0.1481 + \gamma I_{t-1}) + 0.8458\sigma_{t-1}^2 \tag{4.1}$$

$$\log \sigma_t^2 = -0.0504 + 0.3059\,(Z_{t-1}) + 0.9902 \log \sigma_{t-1}^2 \tag{4.2}$$

The graphical representation shows that the GJR-GARCH and GARCH model capture the volatility of both Bitcoin and the averaged portfolio well. The high peaks of returns are clearly marked on the volatility curve. An interesting aspect worth mentioning is the difference of scale between the volatility of the portfolio and Bitcoin. Indeed, Bitcoin volatility maximum is almost 4 times larger than the one of the portfolio, which can largely be accredited to the COVID-19 crisis and the peak of interest for Bitcoin in 2018. This result was expected due to the volatile nature of Bitcoin.

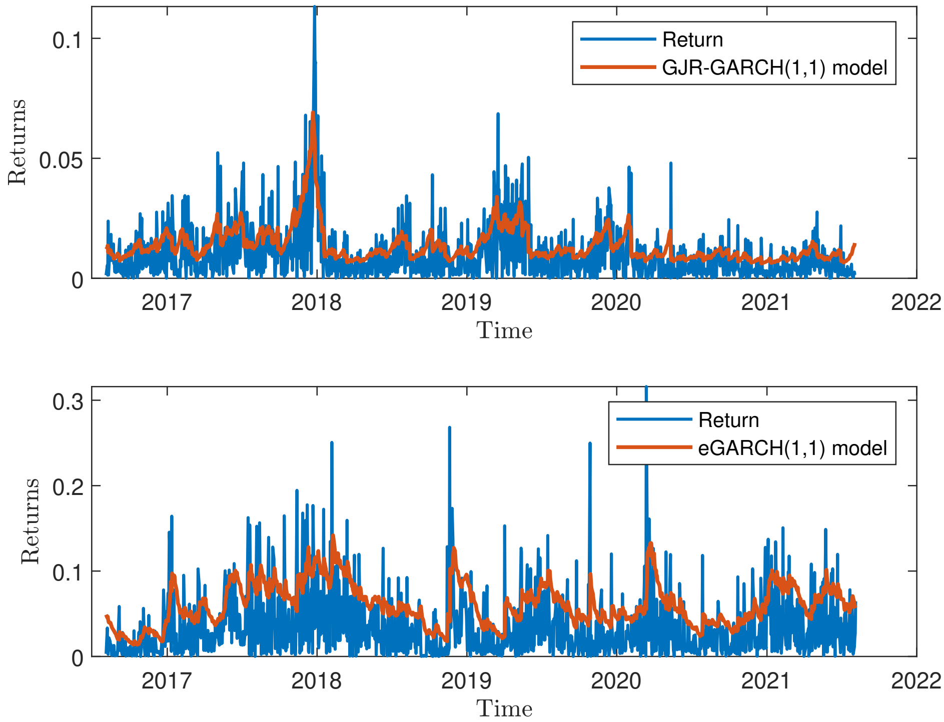


FIGURE 4.3: Volatility with GJR-GARCH model (up) and GARCH model (down) in orange and averaged porfolio (up) and Bitcoin (down) returns in blue

Obviously, this result is one among many. In order to obtain a better picture of the possibilities, a Monte Carlo simulation is performed on the GARCH model obtained for Bitcoin. One hundred paths for the volatility are simulated through the GARCH model. The results are corresponding to the grey curves on figure 4.4. The red curves are the confidence bounds at 2.5% percentile for the lower one and 97.5% percentile for the upper one. The black dashed line is the mean result. The conditional variances simulated are the nominal returns simulated squared. In direct agreement with what has been presented earlier, the volatility, corresponding to the upper plot, presents results that strongly vary outside of the bounds.

The GARCH models have helped to identify Bitcoin as a volatile asset and helped to demonstrate the necessity to build a model as close as possible to reality to forecast and predict the evolution of Bitcoin in the future. The

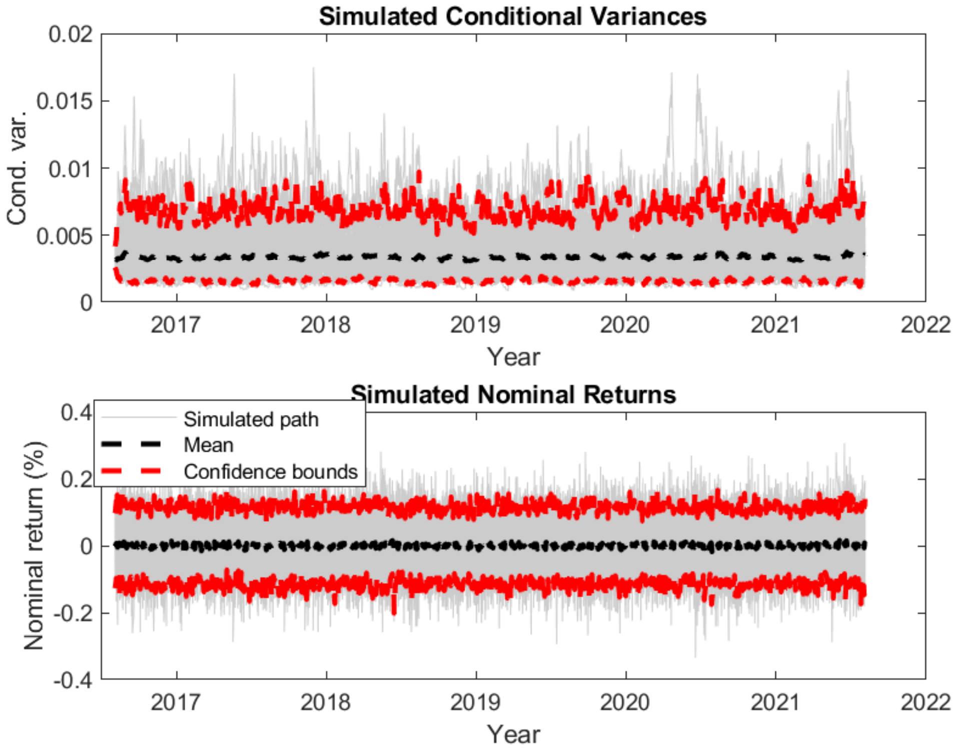


FIGURE 4.4: 100 Monte Carlo simulation for GARCH model of Bitcoin

next step is to build a DCC-GARCH model that will produce the covariance matrix between the portfolio and Bitcoin in order to confirm the associated hedge capacities.

4.3 DCC-GARCH

The construction of the GARCH model is used to validate the DCC-GARCH model. The conditional variance (volatility) computed via the DCC-GARCH is compared with the one computed with the GJR-GARCH for Bitcoin. Figure 4.5 corresponds to the comparison of the two models.

The two models are consistent, validating the results obtained with the DCC-GARCH method. The DCC-GARCH(1,1) model seems to better model the ups and downs without bursting when there is a peak. It is interesting to look at the relative error between the two curves. They are drawn in Figure 4.6. With a maximum relative error of 1% the model can be considered validated.

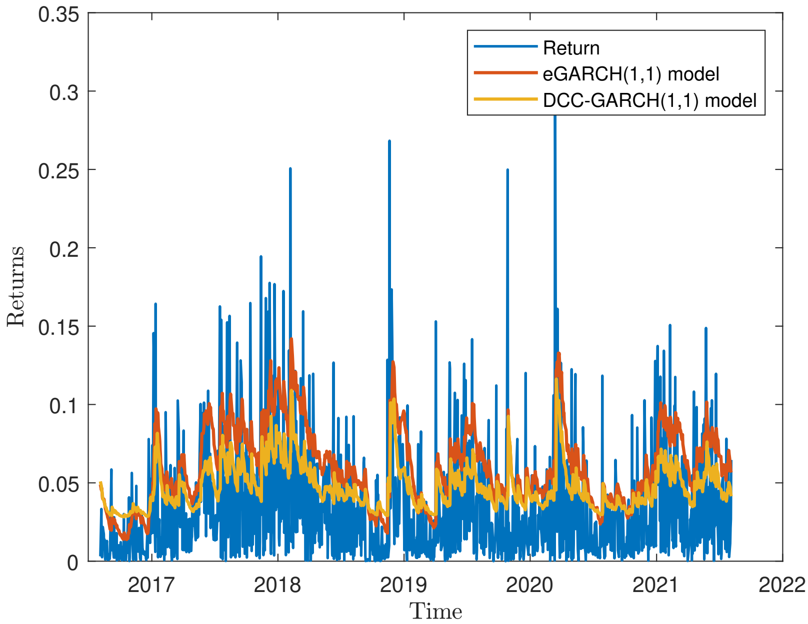


FIGURE 4.5: Comparison of the GJR-GARCH and DCC-GARCH on the averaged portfolio

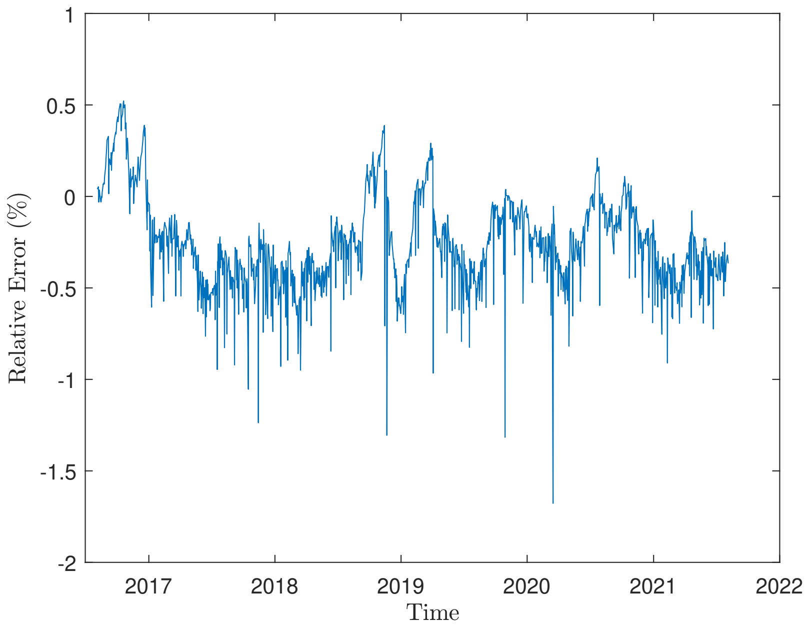


FIGURE 4.6: Relative error between the GJR-GARCH and DCC-GARCH on Bitcoin

Remembering the equations presented in the previous chapter about the method, the matrix R_t (Eq 3.21) containing the conditional correlations is of interest for the determination of hedges. This matrix and the equations globally are solved using a gradient descent algorithm with the goal of maximising the likelihood ratio over the studied period. The correlations over time are plotted on figure 4.7.

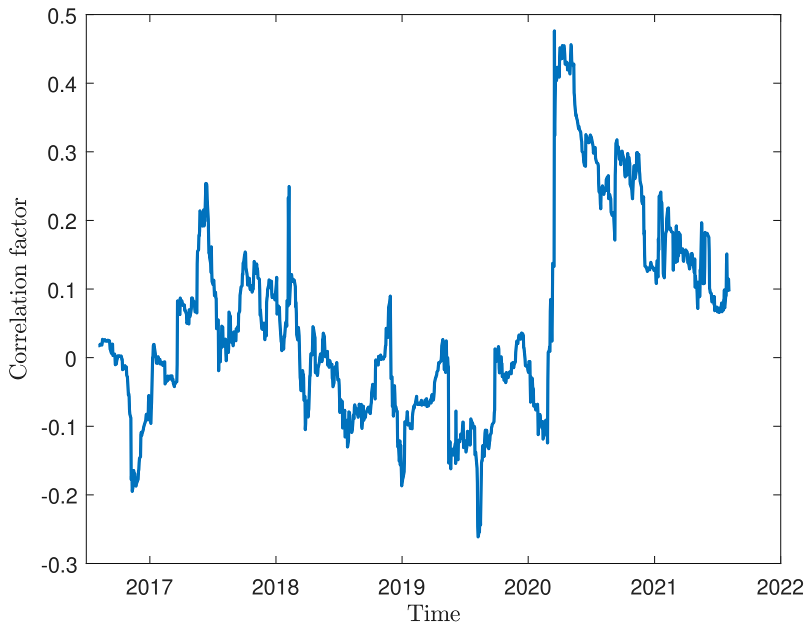


FIGURE 4.7: Evolution of the correlation between the averaged portfolio and Bitcoin.

The first thing to stand out is that the correlation is overall negative before the COVID-19 crisis early 2020 and beside the year 2018, indicating the tendency of Bitcoin to act like a hedge. The second thing that can be exposed is the trend of the correlation factor after the COVID-19 crisis, despite remaining positive it gets closer and closer to 0. The correlation will eventually become negative early 2022. The last thing to note is that the averaged correlation is about 0.0526. This is consistent with what has been observed in the beginning of this chapter in which the averaged correlation was found equal to 0.0357.

4.4 Portfolio optimisation

In this section, the assets are first considered together in order to study the general behaviour of the portfolio without the impact of Bitcoin. Figure 4.8

places the 20 assets that have been considered in the thesis on a graph displaying the standard deviation of the returns versus the mean of the returns, or in other words the risk versus the gain.

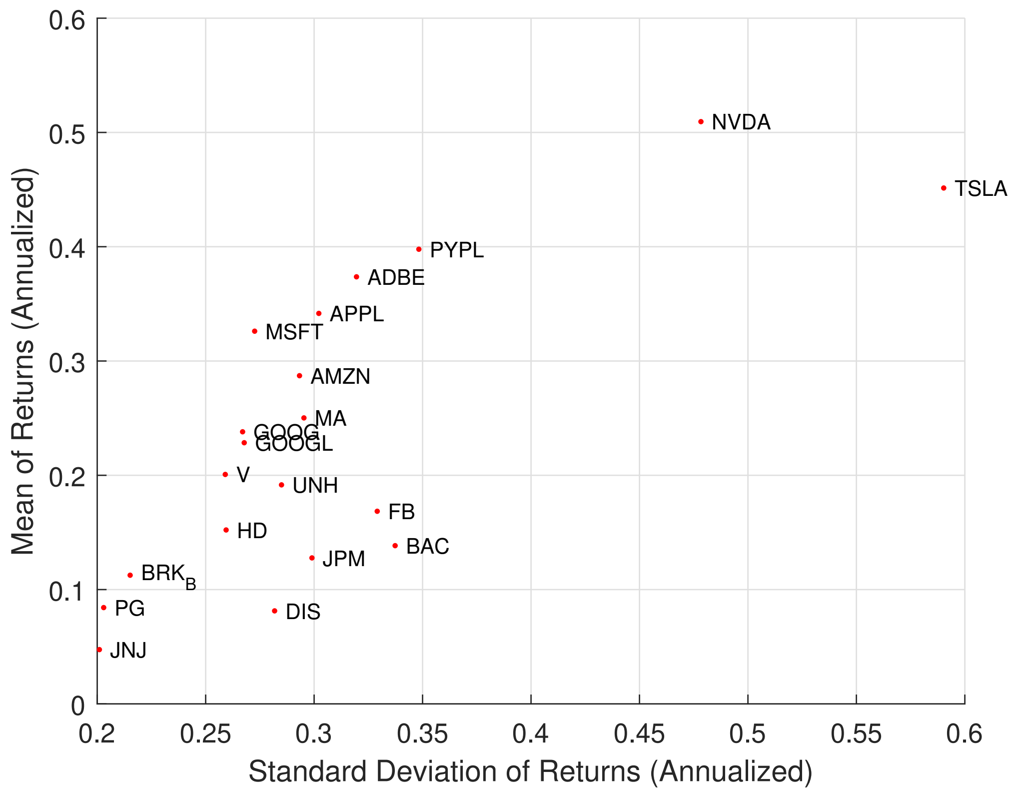


FIGURE 4.8: Portfolio risks and returns for the different assets

Quite obviously, the lower the risk the lower the gain. Most of the assets are grouped together around the same risk level. Two assets are separated from the pack, the stock associated to Nvidia which exhibited significant rise of the price of the share in the last 5 years. The company is mainly selling graphic cards so this level of potential gain can in part be explained by the rise of cryptocurrency mining as explained in the first chapter of the thesis. The other stray stock is Tesla, a famous electric cars company that saw the price of the share significantly increase in recent years.

The association of these assets build a portfolio, depending on the different combinations of the weight associated to each asset, an infinite number of portfolios can be created. A 1000 simulations of possible portfolios have been considered, in which the weights are decided randomly with the only restriction being that the sum of the weights for one portfolio must be equal to 100%. The results are plotted on figure 4.9.

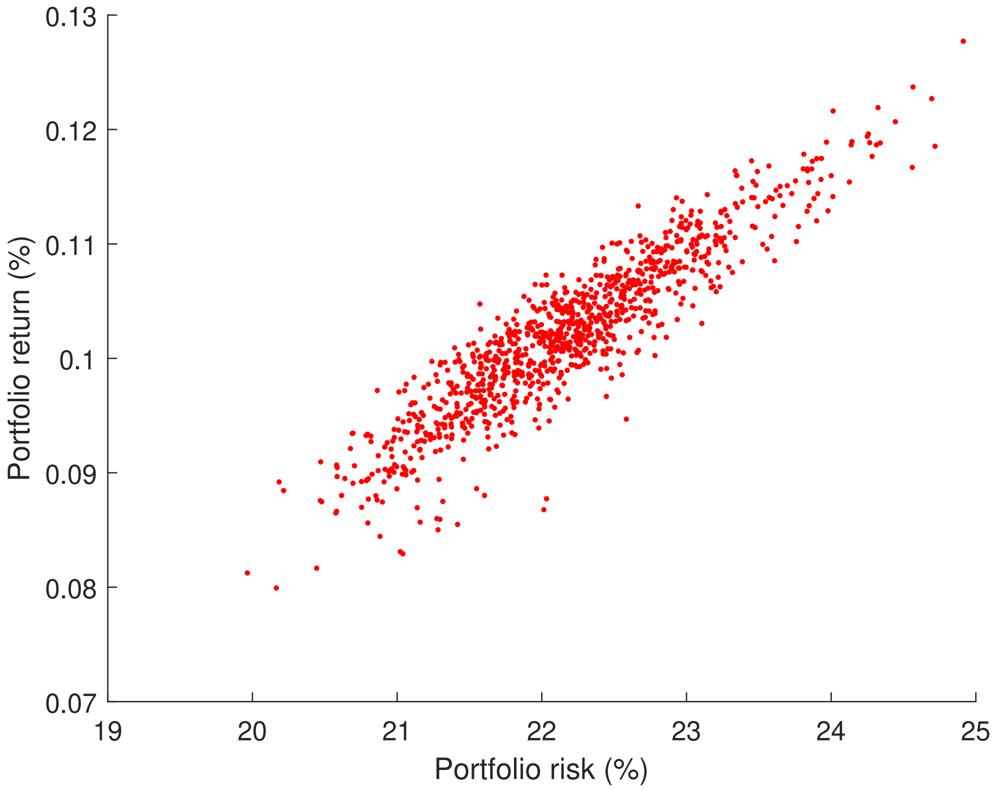


FIGURE 4.9: 1000 simulations of the portfolio for different weights

Globally, the portfolios are in the region of the majority of the assets which was expected due to the method. Three different scenarios are studied further, the first one corresponds to the case of minimised risk portfolio, the second one is an intermediate portfolio with balanced risk and return and the last portfolio is one in which the return is maximised. The composition of the weights can be found in table 4.4.

Concerning the first case, in the portfolio the least risky of the main assets is also the least risky stock, JNJ. In fact, more than a quarter of the portfolio consists in a combination of the three assets offering the least risk. It is also worth mentioning that the second largest asset in the portfolio, with almost 10% is UNH which is about 10% riskier than JNJ. This clear evidence of the power of the diversification of assets in the modern portfolio optimisation. Lastly, the two riskier assets, NVDA and TSLA are the least represented in

Asset	Minimised Risk	Intermediate	Maximised return
APPL	0.69%	1.54%	3.92%
ADBE	4.76%	6.96%	7.20%
AMZN	5.29%	0.96%	10.53%
BAC	3.82%	6.46%	2.62%
BRK_B	8.81%	0.58%	2.01%
DIS	6.86%	4.84%	0.53%
FB	3.29%	8.26%	0.09%
GOOG	2.67%	5.19%	7.57%
GOOGL	7.24%	8.32%	0.32%
HD	5.06%	8.28%	3.18%
JNJ	11.46%	6.61%	6.88%
JPM	3.95%	5.70%	0.39%
MA	8.82%	0.60%	2.73%
MSFT	3.75%	7.17%	9.99%
NVDA	0.16%	0.22%	11.47%
PG	6.91%	4.66%	1.14%
PYPL	4.42%	8.33%	5.46%
TSLA	0.30%	7.93%	11.18%
UNH	9.15%	3.48%	2.47%
V	2.58%	3.91%	10.33%

TABLE 4.4: Weight of the different assets for a portfolio with minimised risk, intermediate and maximised risk

this portfolio with less than half of a percent combined.

The second case does not reveal much, the results weights are roughly equally distributed among the assets. Some are less represented than others, but it depends on the portfolio. A general representation of the number of occurrences of the different value of the weight is proposed in Appendix B. One can observe that for less than 10% the level of occurrence is approximately the same.

The final scenario is the one in which the portfolio offers the maximum gain. As was the case for the first scenario, the two main assets are NVDA and TSLA which have a high return for a high risk. It is notable as well that more than 50% of the portfolio are made up of only 5 different assets. This portfolio provides almost the double of the gain compared to the less risky portfolio for only 5% more of risk.

This approach is effective for grasping the general idea of diversification. However, the selection of random numbers to determine weight with only

20 different assets limits the possibilities of construction. It is statistically impossible to randomly end up with 10 assets with a weight of 0. In order to compensate for this problem, the efficient frontier can be drawn, and risk/gain target can be fixed. It corresponds to figure 4.10.

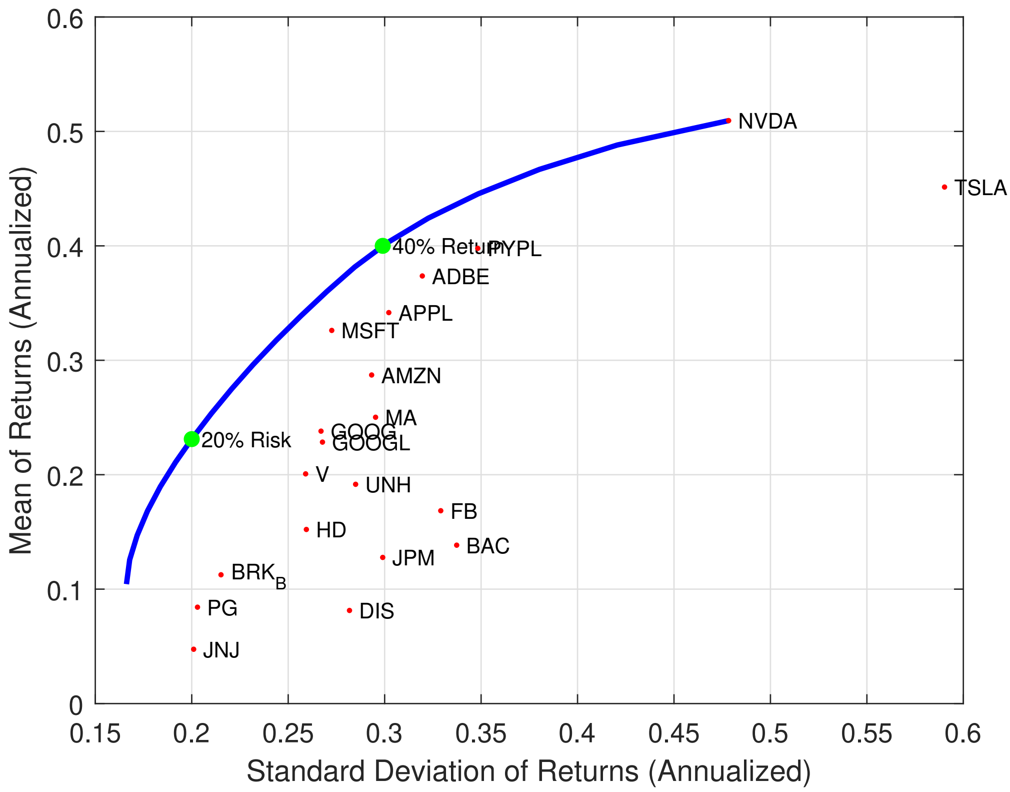


FIGURE 4.10: Efficient frontier with targeted portfolio

As presented in the previous chapter, the efficient frontier represents the portfolio with the optimal level of return in relation to the associated risk. The asset NVDA belongs to the frontier, the associated level of return corresponds to a portfolio with 100% of this asset. Two particular cases are proposed, a portfolio with a targeted risk of 20% which is about the same level of risk of a portfolio full of asset JNJ but with a return about 5 times higher and a portfolio with 40% of return which is higher than almost every assets considered in the portfolio with a comparable risk level. Respectively table 4.5 and 4.6.

It requires only 6 out of the 20 assets considered in this thesis to obtain a return of 40% with a risk of approximately 30%. All 6 of these assets are the ones with the highest individual return in the pool of assets and the three

Asset	Weight
APPL	21.47%
ADBE	21.55%
MSFT	6.86%
NVDA	18.46%
PYPL	24.66%
TSLA	7.00%

TABLE 4.5: Weights associated to the portfolio with 40% of return

Asset	Weight
APPL	10.62%
ADBE	8.65%
AMZN	14.94%
BRK_B	9.60%
JNJ	6.07%
MSFT	3.37%
NVDA	1.49%
PG	25.65%
PYPL	9.75%
TSLA	3.11%
UNH	6.75%

TABLE 4.6: Weights associated to the portfolio with 20% of risk

closest to the 40% represent $\approx$ 75% of the portfolio. On the other hand, to achieve 20% of risk, 11 assets are mandatory. One quarter of the portfolio is made of stock of PG, which is a low-risk low-return asset that helps to balance and to mitigate risk associated with the other assets.

The last analysis that can be conducted concerning the portfolio optimisation is to build the tangency portfolio, which corresponds to the maximised Sharpe ratio. Figure 4.11 shows the Sharpe ratio on the efficient frontier.

The weight associated to the Sharpe ratio can be found in table 4.7.

Despite being closer to the 40% return portfolio, the maximised Sharpe ratio presents a weight distribution closer to the one of the 20% risk one, with the same assets. Which shows how significant the weight distribution among assets can be.

The introduction of Bitcoin in the portfolio impacts the efficient frontier

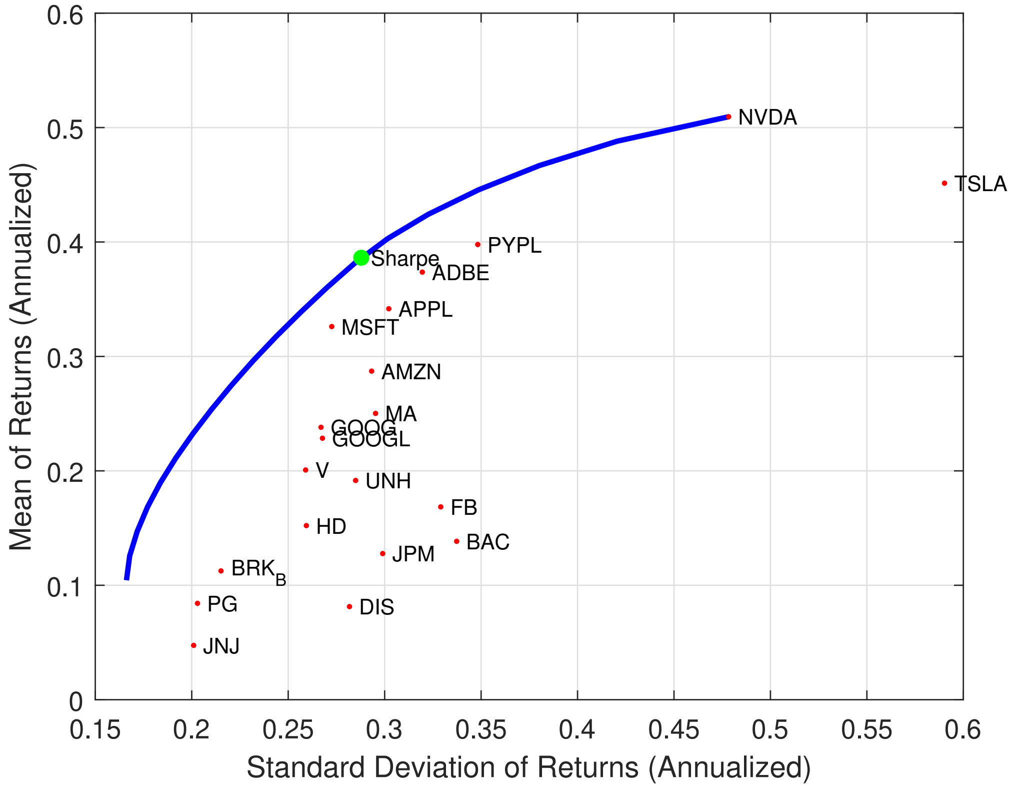


FIGURE 4.11: Efficient frontier with portfolio associated to the maximised Sharpe ratio.

as represented on figure 4.12. This new asset is highly profitable but also induces important risks.

The previous efficient frontier is drawn in light blue while the new one is in dark blue. This new frontier is clearly above the previous one, promising higher level of return. It is interesting to look at the portfolio weights for the same expected return and risk as previously done, a portfolio insuring 40% return and 20% of risk. Figure 4.13 includes the new efficient frontier with the targeted portfolio and the old efficient frontier with the previously target portfolio. For the same level of expected return it clearly appears that the risk is reduced by at least 15%, the same observation can be made about the portfolio that has targeted risk level. The simple addition of Bitcoin in the portfolio significantly impacts the returns and the risk. The associated weights for the portfolio can be found in table 4.8 and 4.9.

Asset	Weight
APPL	23.28%
ADBE	18.94%
MSFT	16.82%
NVDA	13.41%
PYPL	21.69%
TSLA	5.85%

TABLE 4.7: Weights associated to the portfolio with the maximised Sharpe ratio

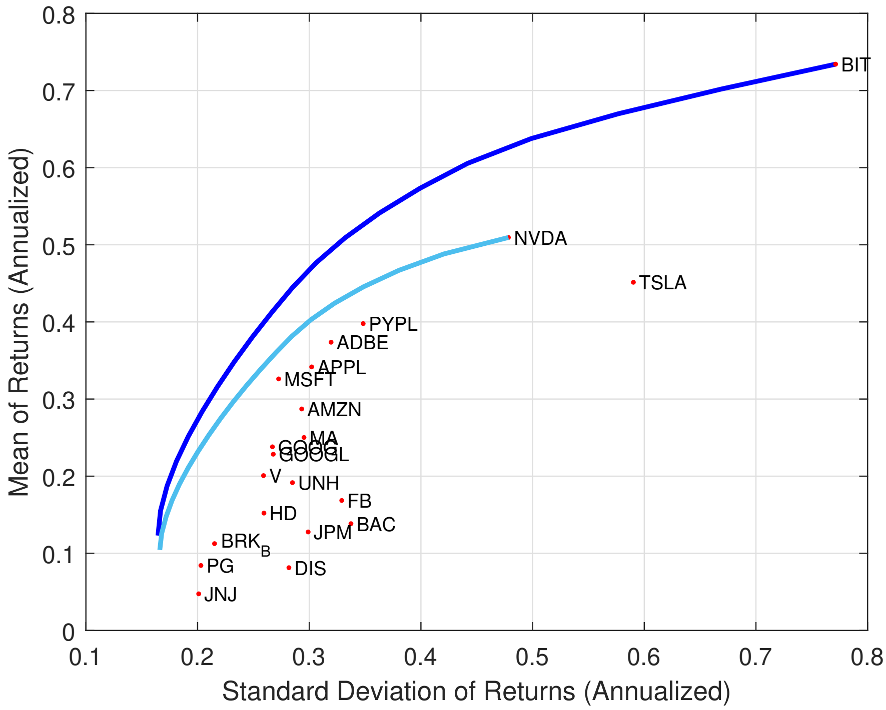


FIGURE 4.12: Efficient frontier with targeted portfolio and Bitcoin

The results from these two tables clearly expose the role of the introduction of Bitcoin in the portfolio analysis. For the two scenario, Bitcoin represents more than 10% of the portfolio composition. Globally, the same assets are represented with more or less the same distribution. Lastly, the portfolio associated with the maximised Sharpe ratio can be computed and compared to the previous one. Figure 4.14 presents the portfolio and table 4.10 the associated weights.

The new Sharpe portfolio has a risk that is barely higher for a return clearly more important. Almost a fifth of the portfolio is made of Bitcoin, which once again exacerbates the impact it can have on portfolio. One may

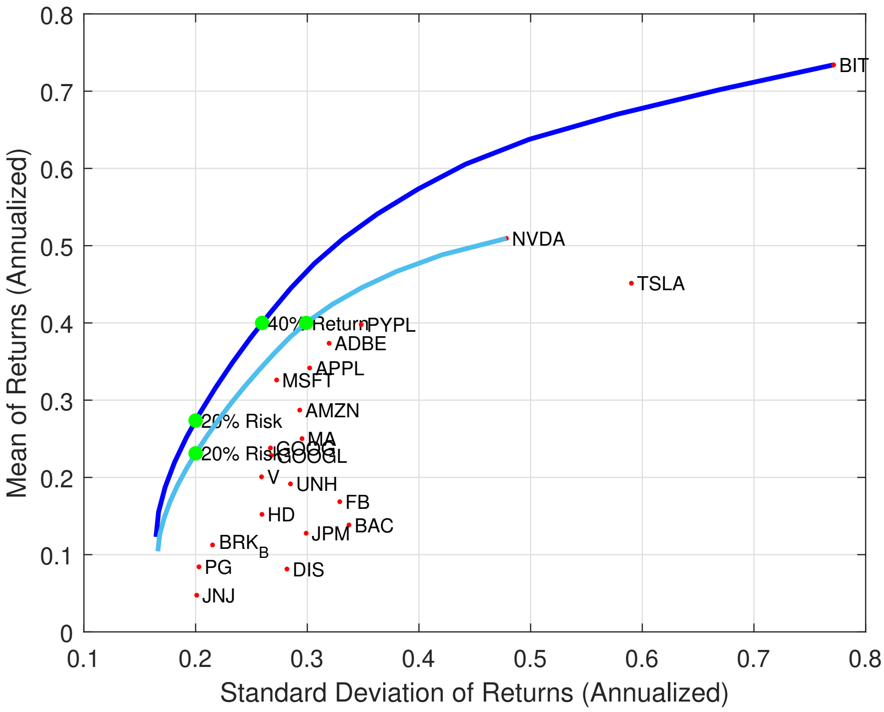


FIGURE 4.13: 1000 simulations of the portfolio for different weights

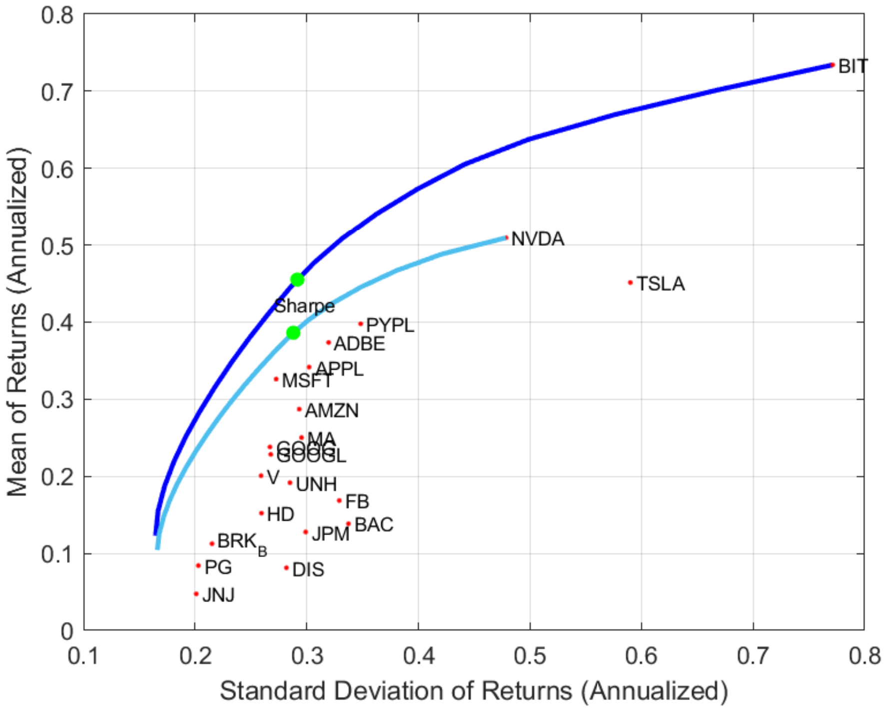


FIGURE 4.14: 1000 simulations of the portfolio for different weights

Asset	Weight
APPL	15.36%
ADBE	15.12%
AMZN	8.49%
MSFT	11.35%
NVDA	7.38%
PG	7.84%
PYPL	12.00%
TSLA	2.77%
UNH	3.56%
BIT	16.12%

TABLE 4.8: Weights associated to the portfolio with 40% of return

Asset	Weight
APPL	7.74%
ADBE	9.44%
AMZN	15.69%
BRK_B	9.26%
JNJ	6.58%
MSFT	0.25%
NVDA	1.68%
PG	25.2%
PYPL	6.85%
TSLA	2.04%
UNH	4.77%
BIT	10.51%

TABLE 4.9: Weights associated to the portfolio with 20% of risk

notice that for the most part the assets between the two portfolio are the same beside the stock MSFT that disappeared for the benefit of AMZN.

This last chapter presents the results of the thesis. Hedge capacities of Bitcoin have been exposed and discussed. Different GARCH models have been created and a more complex DCC-GARCH model helped drawing conclusions on Bitcoin capacities. Lastly, multiple portfolio have been created and the efficient frontier drawn. Targeted portfolio have been detailed and the Sharpe ratio have been computed in order to find the optimised portfolio. The impact of Bitcoin on portfolio and efficient frontier has also been shown.

Asset	**Weight**
APPL	17.11%
ADBE	19.52%
AMZN	2.14%
NVDA	12.40%
PYPL	15.29%
TSLA	3.57%
BIT	19.55%

TABLE 4.10: Weights associated to the portfolio with the maximised Sharpe ratio

Chapter 5

Conclusion

The goal of this master's thesis was to study the impact that Bitcoin has on high risk portfolio and in particular its capacity to act like a hedge. The effect of its introduction in a portfolio is also of interest.

In this work, simple method have been proposed to estimate the hedge capacity of Bitcoin in a first time. It helped to get a first idea of the expected results. A GARCH model have been constructed in order to capture the volatility of Bitcoin and helped to validate the more complex model built, the DCC-GARCH. The correlation between a portfolio of stocks and Bitcoin over a studied period of 5 years have thus been obtained. Lastly, the efficient frontier have been drawn for the portfolio that takes, or not, into account Bitcoin. The weight composition of targeted portfolio is provided. The optimal portfolio has been established thanks to the Sharpe ratio.

The results obtained throughout the thesis, indicate the possible hedge capacities of Bitcoin on a high risk portfolio made of stocks depending on the time period studied. Even though the correlation between the portfolio and Bitcoin is sometimes positive sometimes negative, it is close to zero. The impact Bitcoin has in portfolio optimisation and the efficient frontier has been made well apparent. However, as presented in the work, Bitcoin is a highly volatile asset, the results depend a lot on what assets are studied and the time period considered. Further investigations must be conducted in order to full comprehend the complexity of Bitcoin in hedging and portfolio optimisation.

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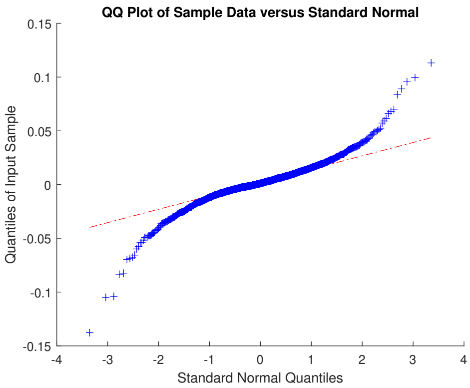
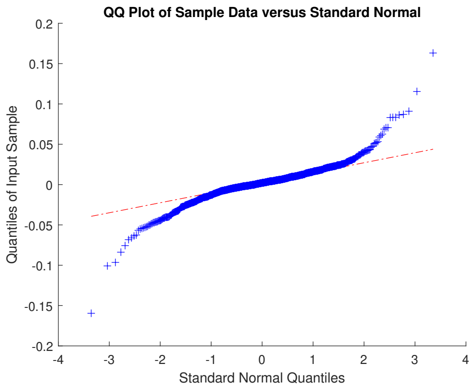
Appendix A

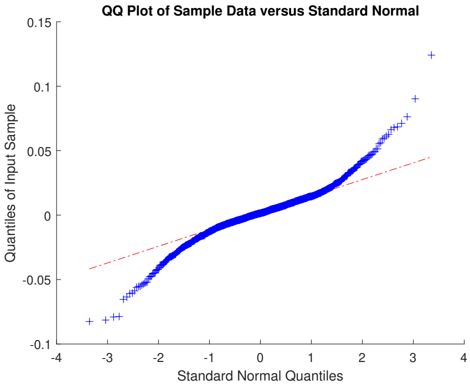
QQ-Plots



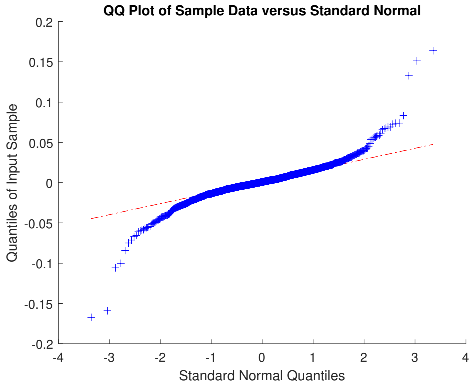
(A) QQ-Plot of APPL Returns



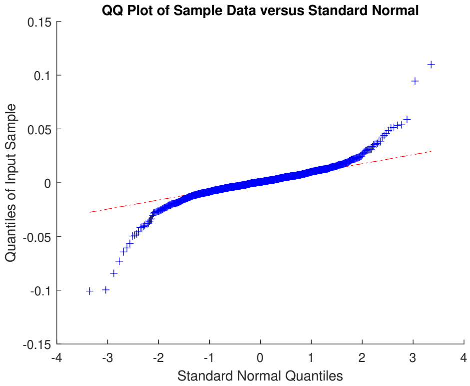
(B) QQ-Plot of ADBE Returns



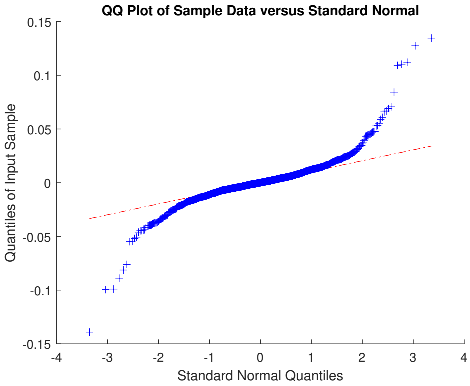
(C) QQ-Plot of AMZN Returns



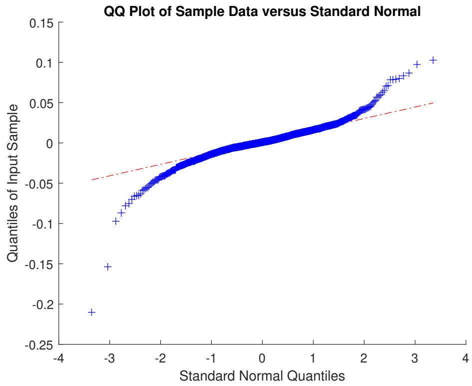
(D) QQ-Plot of BAC Returns



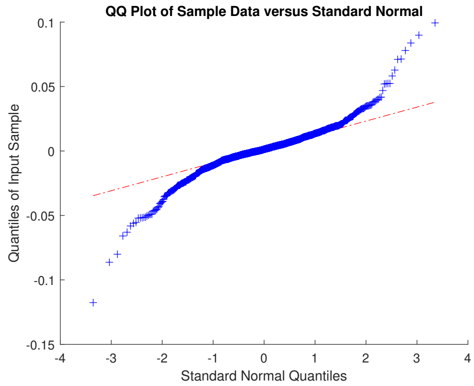
(E) QQ-Plot of BRK_B Returns



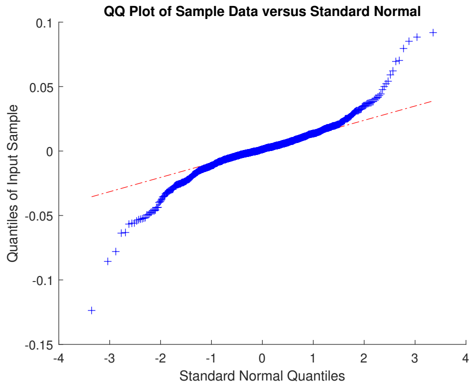
(F) QQ-Plot of DIS Returns



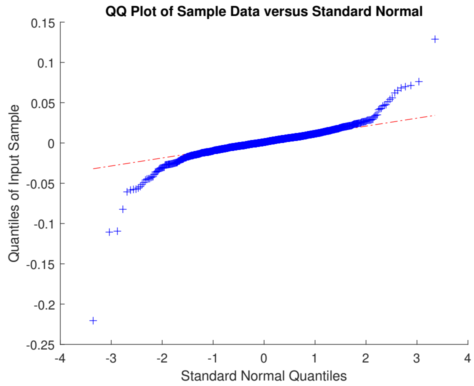
(G) QQ-Plot of FB Returns



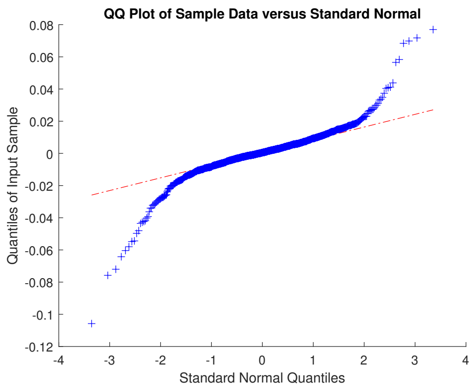
(H) QQ-Plot of GOOG Returns



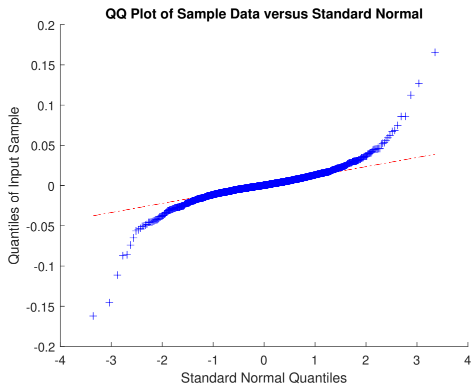
(I) QQ-Plot of GOOGL Returns



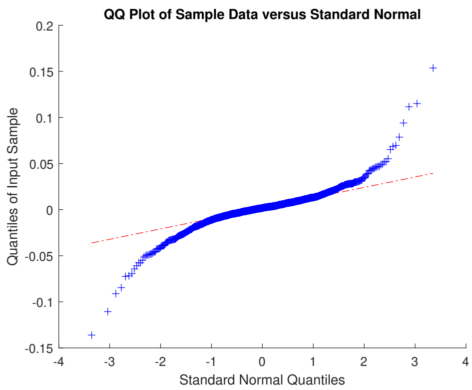
(J) QQ-Plot of HD Returns



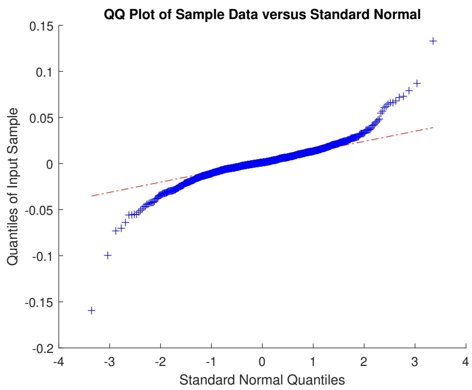
(K) QQ-Plot of JNJ Returns



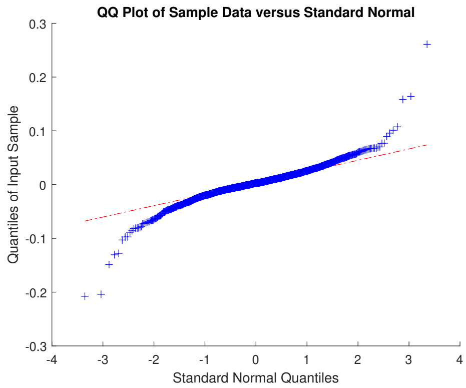
(L) QQ-Plot of JPM Returns



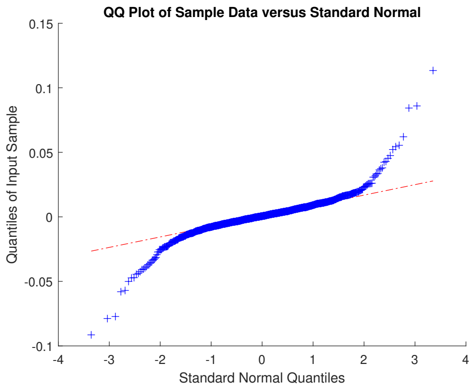
(M) QQ-Plot of MA Returns



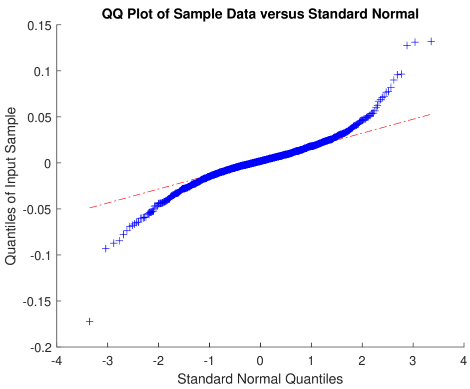
(N) QQ-Plot of MSFT Returns



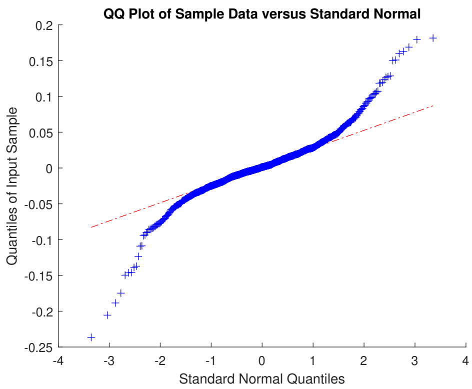
(O) QQ-Plot of NVDA Returns



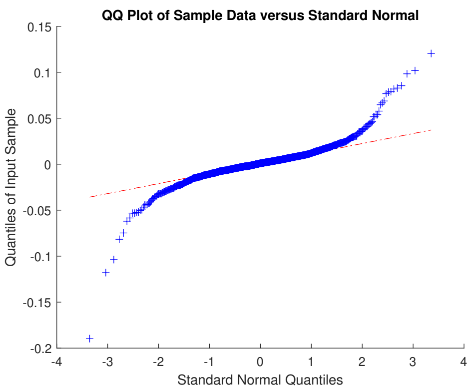
(P) QQ-Plot of PG Returns



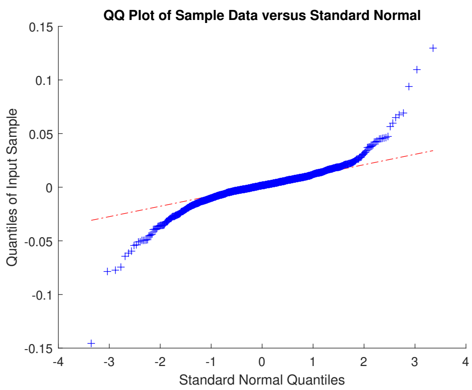
(Q) QQ-Plot of PYPL Returns



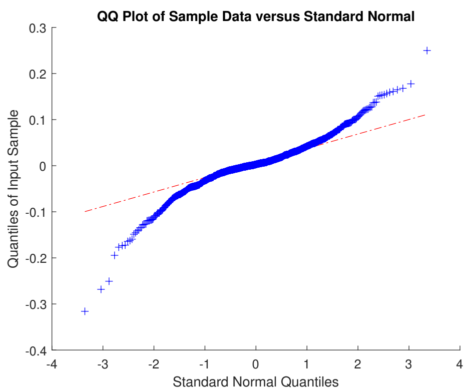
(R) QQ-Plot of TSLA Returns



(S) QQ-Plot of UNH Returns



(T) QQ-Plot of V Returns

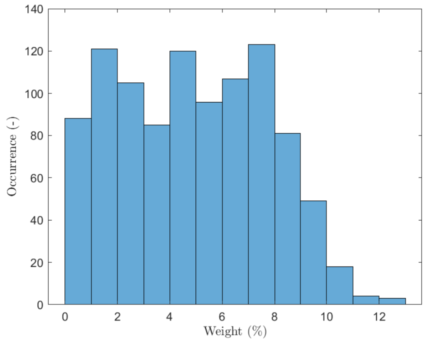


(U) QQ-Plot of BIT Returns

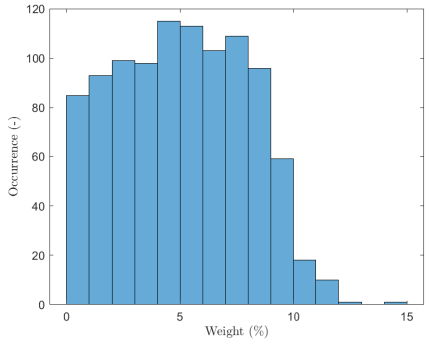
FIGURE A.1: QQ-Plot of the logarithmic returns of the assets

Appendix B

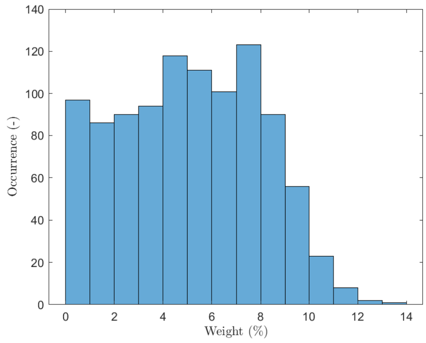
Histogram of the weights



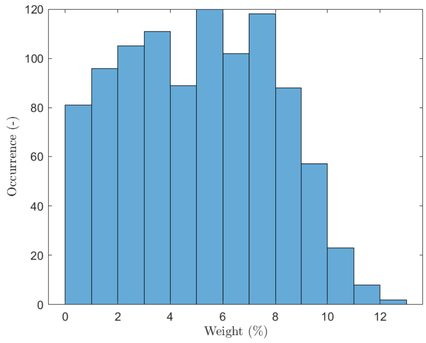
(A) Histogram of the weights associated with the APPL asset



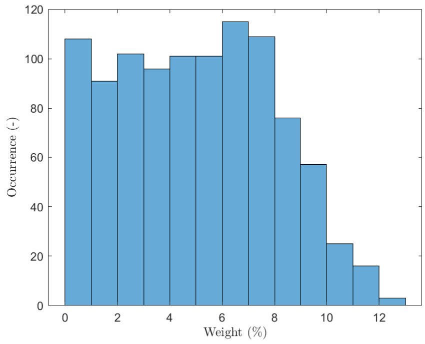
(B) Histogram of the weights associated with the ADBE asset



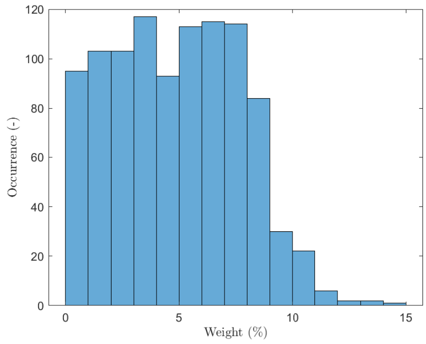
(C) Histogram of the weights associated with the AMZN asset



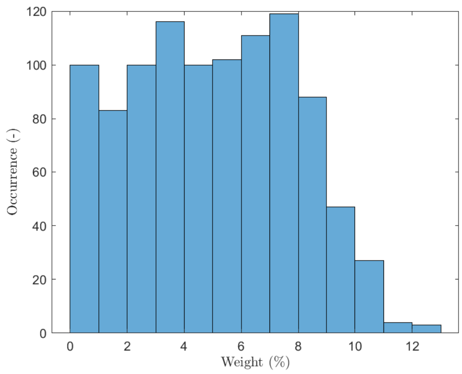
(D) Histogram of the weights associated with the BAC asset



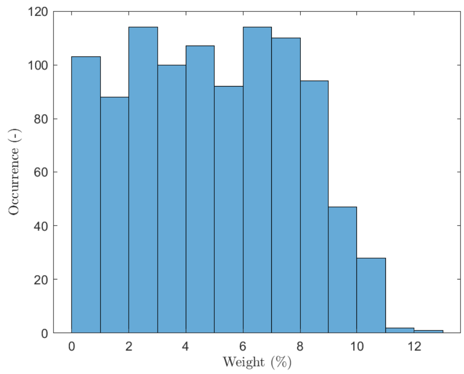
(E) Histogram of the weights associated with the BRK_B asset



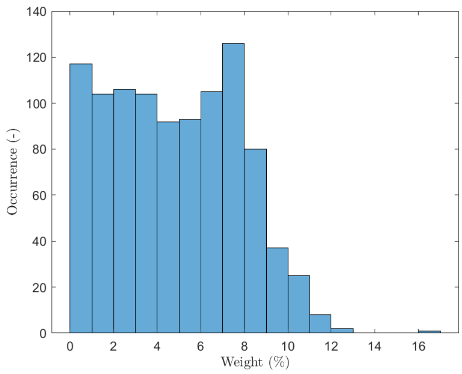
(F) Histogram of the weights associated with the DIS asset



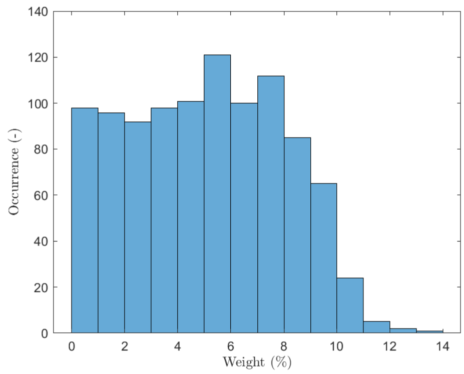
(G) Histogram of the weights associated with the FB asset



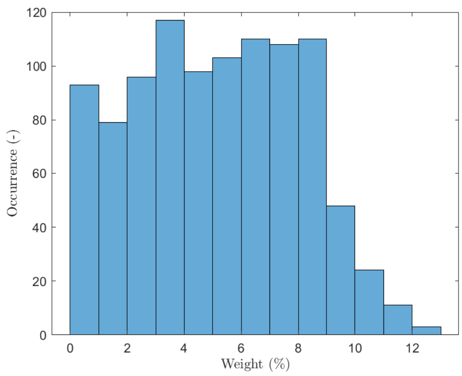
(H) Histogram of the weights associated with the GOOG asset



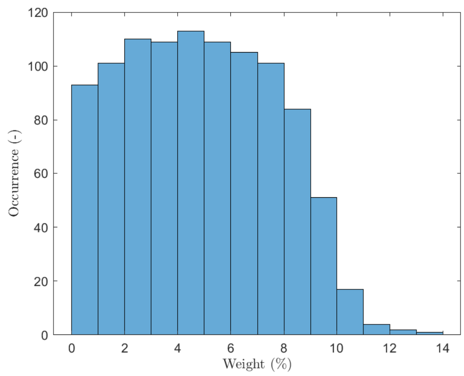
(I) Histogram of the weights associated with the GOOGL asset



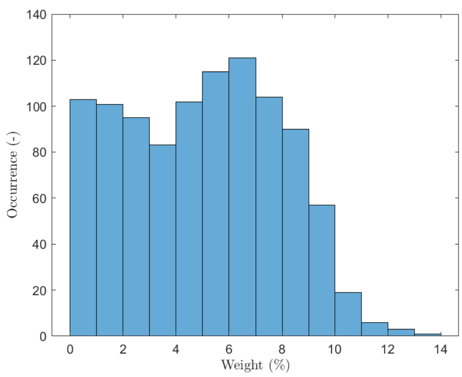
(J) Histogram of the weights associated with the HD asset



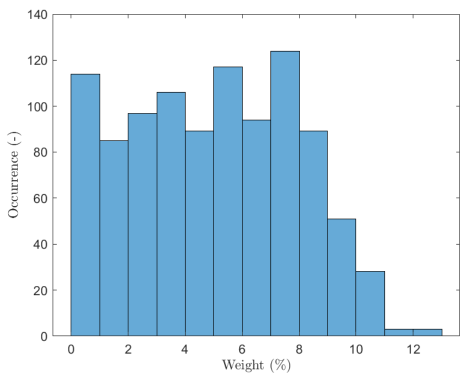
(K) Histogram of the weights associated with the APPL asset



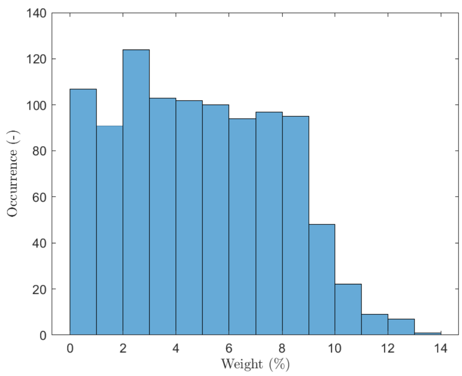
(L) Histogram of the weights associated with the JPM asset



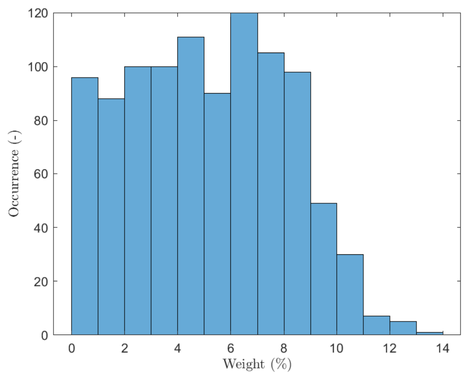
(M) Histogram of the weights associated with the MA asset



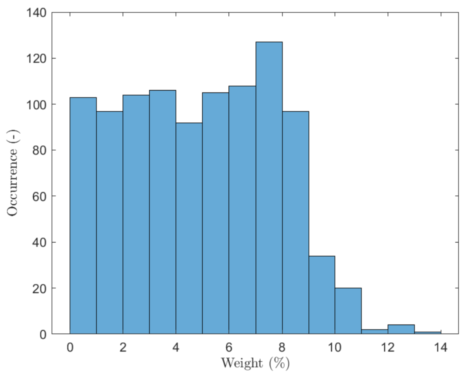
(N) Histogram of the weights associated with the MSFT asset



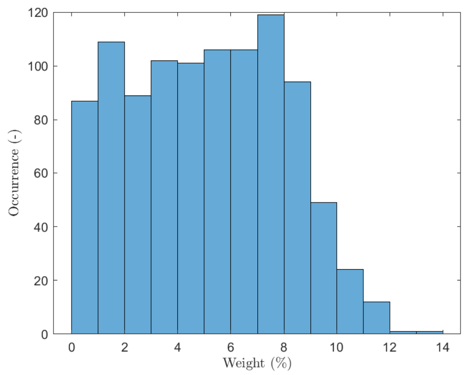
(O) Histogram of the weights associated with the NVDA asset



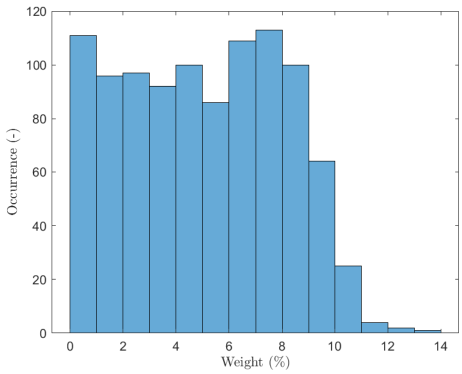
(P) Histogram of the weights associated with the PG asset



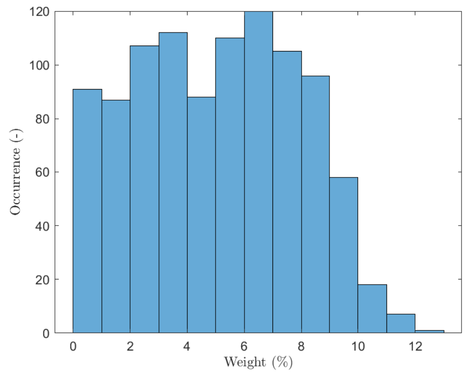
(Q) Histogram of the weights associated with the PYPL asset



(R) Histogram of the weights associated with the TSLA asset



(S) Histogram of the weights associated with the UNH asset



(T) Histogram of the weights associated with the V asset

FIGURE B.1: Histogram of the weights associated with the assets