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## **Three degrees of freedom weakly coupled resonators used for mass measurement.**

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# Three degrees of freedom weakly coupled resonators used for mass measurement

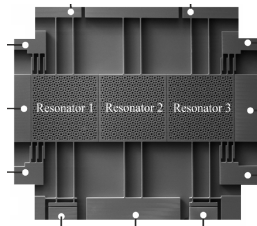
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## 1 Objectives

A three degrees of freedom (DOF) resonator was designed to measure stiffness perturbation. The task assigned to this master thesis was to study the existing chips and to create models in order to represent their response to tiny mass perturbation (virus, bacteria, ...). Based on those models, techniques to quantify mass perturbation should be proposed. The MEMS used in this project is shown in figure 1.

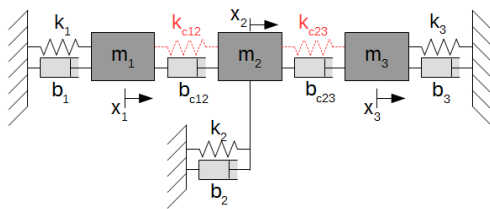


**Figure 1:** Picture by scanning electron microscope of the MEMS used in this project. Picture from Chun Zhao

## 2 MEMS Modelization

### 2.1 Mechanical Modelization

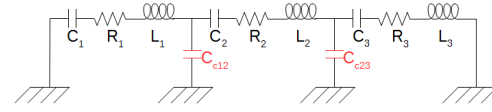
Four systems were characterized using the eigenvalue method. Those four systems are 2DOF and 3DOF, each of them with and without damping. The types of information obtained with those models are the resonant frequencies and the amplitude of mass movement in function of frequency. The mechanical modelization of the MEMS is shown in figure 2.



**Figure 2:** Model used to characterize the 3DOF MEMS

### 2.2 Electrical Modelization

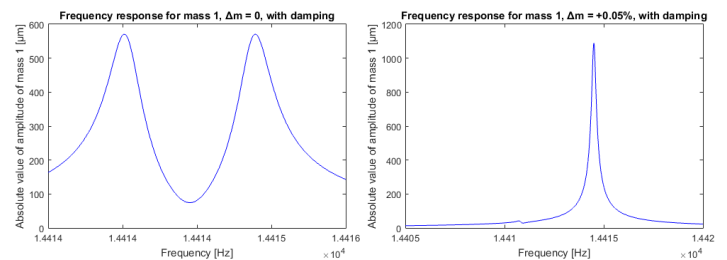
In order to simulate the behavior of the MEMS (frequency response and time domain simulations), an electrical model was derived from the mechanical model. Its schematics is shown in figure 3. Since the values of the components are not common values that can be bought, it was necessary to adapt them to build the circuit by a scaling process. A huge drawback of RLC circuits is that they cannot have as high a quality factor as MEMS. That is why quartz crystals were used instead of RLC components.



**Figure 3:** Electrical equivalent circuit of the MEMS.

## 3 Modelization Results

The 3DOF system has shown mode localization. Figure 4 shows the response curve of the first mass of the MEMS in its initial state (on the left) and for one proof mass perturbed by 0.05% of its mass (on the right). The equivalent circuit using quartz oscillators has also shown high similarity with the MEMS.



**Figure 4:** Frequency reponse of MEMS computed with (on the right) and without a mass perturbation (on the left) of 0.05% of one proof mass

## 4 Measurement Strategy

Based on modelization process results, three main strategies to measure mass variation were investigated :

- Resonant frequency shift : This could be easily measured by using a self-oscillating loop with the MEMS.
- Amplitude variation at the resonant frequency : by using the previous self-oscillating loop and a synchronous detector, the amplitude is obtained.
- Amplitude ratio variation at the resonant frequency : similar as the previous strategy, it required two synchronous detectors and a digital divider.

## 5 Conclusion

Based on the different models, it is clear that MEMS are used for their very high Q factor. The quartz model has shown that complex coupled mechanical systems can be realized by using smaller systems electrically coupled. It was also proved that 3DOF systems have a better sensitivity than 2DOF but still have low sensitivity around no perturbation. At the end of the project, the MEMS was turned into a sensor by adding a feedback loop.