Master thesis : Numerical Simulation of Transport Ship's Dynamics when Launching Wind Turbine Floating Fundament

Auteur : Mundekkatt, Mohamed Asib
Promoteur(s) : 14957; 18536
Faculté : Faculté des Sciences appliquées
Diplôme : Master : ingénieur civil mécanicien, à finalité spécialisée en "Advanced Ship Design"
Année académique : 2021-2022
URI/URL : http://hdl.handle.net/2268.2/16051

Traditio et Innovatio

With the support of the Erasmus+ Programme of the European Union

Numerical Simulation of Transport Ship's Dynamics When Launching Wind Turbine Floating Fundament

submitted on 26 July 2022

by

MUNDEKKATT Mohamed | Erich-Schlesinger-Straße 19| 18059 | mohamed.mundekkatt@uni-rostock.de

Student ID No.: 221 200 006

First Reviewer:

Prof. Dr.-Eng./Hiroshima Univ. Patrick Kaeding

Chair of Ship Structures

University of Rostock

Albert-Einstein-Str. 2

18059 Rostock

Germany

Second Reviewer:

Dipl.-Ing. Mathias Vieth

Special Field Ship Theory

Neptun Ship Design GmbH

Kurt-Dunkelmann-Str. 4

18057 Rostock

Germany

EMship+ Advanced Design

Master Thesis

Declaration of Authorship

I **Mohamed Asib Mundekkatt** declare that this thesis and the work presented in it are my own and have been generated by me as the result of my own original research.

Where I have consulted the published work of others, this is always clearly attributed.

Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.

I have acknowledged all main sources of help.

Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma.

I cede copyright of the thesis in favour of the University of Rostock.

Date: Signature

26th July 2022

[This page is intentionally left blank]

ABSTRACT

The installation procedure is one of the most critical and important stages of the design and installation of offshore wind turbines (OWT). Among the various steps of installation, the launching process is the most critical operation. As a cost-effective way, the launching technique is preferred, rather than lifting it directly using a heavy lift vessel. In this Master Thesis, an evaluation of logistics concepts for transporting and installing offshore floating wind components has been performed.

To ensure safe and effective launching, the entire operation has to be mathematically modeled and numerically simulated and the results to be scrutinized thoroughly to verify the operation complies with the class rules and current regulations. As part of this thesis, different phases of launching have been identified and corresponding equations of motions with necessary constraints are developed. The ship's coupled motions (pitch & heave motion) are mathematically modeled and a time-dependent numerical simulation of the operation is made to assess the dynamic trim, velocities, and accelerations experienced by the vessel during the launching of the wind turbine fundament (SPAR). A computer code has been developed to solve the mathematical models and generate required plots to analyze the results. The required added mass coefficients including coupled ones are calculated by integrating the sectional added mass with the help of Lewis conformal mapping. Whereas damping coefficients are obtained from the critical damping and restoring coefficients from respective empirical formulas.

All results are verified against current rules and regulations. The maximum values of the pitch angle, pitch angular velocity, pitch angular acceleration, heave displacement, heave velocity and heave accelerations are within the allowable limits. This study gives insight into the ship's dynamics at the critical stage of the launching sequence. Further studies are required to check the structural integrity of the ship during the launching operation.

Keywords: Launching, SPAR, Mathematical Modelling, Hydrodynamic Coefficient, Added Mass, Coupled Pitch-Heave Motion, Strip Theory, Lewis Conformal Mapping.

[This page is intentionally left blank]

CONTENTS

LIST OF FIGURES

LIST OF TABLES

[This page is intentionally left blank]

1 INTRODUCTION

1.1 Aims & Objectives

SPAR-type platforms for floating offshore wind turbines are economically suitable for wind farm deployment. SPAR cylinders are fabricated in the shipyard / workshops / suppliers' berths and transferred to the transport ship with the vessel's skidding system, then the SPAR cylinders are transported to the operation site and slipped into the water by trimming the vessel. The slipping process is called launch and it is the most critical stage during the installation of SPAR-type platforms. Before the launching, the SPAR has to be fixed to the main deck of the vessel with proper sea fastening and lashing arrangements, vessel has to be ballasted in such a way to attain predefined trim by aft. Once the vessel achieves predetermined trim, the sea fastening and lashing arrangements can be removed so that the SPAR will start sliding by its own weight until the tipping point, then starts rotating about the tipping point and finally separates from the vessel and plunges into sea.

The vessel can be out of control after the SPAR is released, it can experience high pitching moments and slamming which can cause dangerous scenarios for both ship and crew. Therefore, it is important to make a critical evaluation and feasibility study before the launch of the SPAR at the sea. In order to have a successful launching operation, the pitching motion of the ship should be investigated thoroughly in terms of the stability of the ship, as well as the maximum pitch angle, angular velocity and angular accelerations to ensure that all these parameters are within the acceptable limits proposed by classification society. Numerical simulations of the pitch motion, angular velocity and angular accelerations in time series are the best representation to convey the results.

Model tests are required to calculate the pitch motions, loads on the ships, angular velocity and angular accelerations of the ship in time domain, however the present paper explores only numerical simulation results.

This novel approach would greatly reduce costs. However, such a launching operation from a transport vessel had never been performed before, hence a complete thorough investigation is needed before the actual operation execution.

The launching operation can only be performed in calm water and all analysis are made without considering the effect of sea waves.

The slip of the SPAR cylinder over the skid beams can change the vessel draft, trim and restoring characteristics of the ship instantaneously, which affects the ship motion directly.

Hence a dynamic analysis to be made for a detailed investigation and also to ensure the safe operation.

1.2 Structure of the Report

The Master Thesis consists of the following structure:

- **Chapter 2** is comprised of literature review on some important theories and concepts used to prepare this Thesis like added mass determination, uncoupled motions – free oscillations & forced oscillations, coupled pitch heave motions and rules/regulations for the launching operations.
- **Chapter 3** is mainly to discuss about the characteristics dimensions of Ship-SPAR Launching system, launching conditions, different launching stages and some import parameters to be taken care during launching.
- **Chapter 4** presents mathematical model to describe the ship dynamics during launching of SPAR from the transport vessel. Equation of motion and relevant constraints are developed for each phase of the motion.
- **Chapter 5** explains about the calculation of added mass coefficients using Strip Theory with the help of Lewis Conformal Mapping technique.
- **Chapter 6** gives the dynamic simulation results for Ship's uncoupled pitch, uncoupled heave and coupled pitch-heave motions during phase V of the SPAR launching. All the important parameters such as pitch angle, pitch angular velocity, pitch angular acceleration, heave displacement, heave velocity and heave accelerations are plotted in time domain and its maximum values are checked with current rules and regulations.
- **Chapter 7** summarizes the entire works with some major findings.
- **Chapter 8** covers some suggestions and recommendations put forward related to the topic for future works.

2 LITERATURE REVIEW

2.1 Added Mass Determination

As ship accelerate or decelerate in water, the fluid around the hull moves along with it which creates an additional hydrodynamic force on the ship hull called added mass. In order to form mathematical models for the ship motions, the added mass components have to be determined. In actual scenario, for a particular type of ships, the hydrodynamic coefficients required to form mathematical models are obtained by doing experiments. As part of this Thesis, these coefficients are calculated based on the theoretical methods called Strip Theory with the help of Lewis Transformation method.

When the ship moves at a steady speed through the water, no force exists as per the potential flow theory. However, when the ship starts to accelerate or decelerate, a hydrodynamic force would be experienced which is proportional to the acceleration and direction will be opposite to the direction of motion.

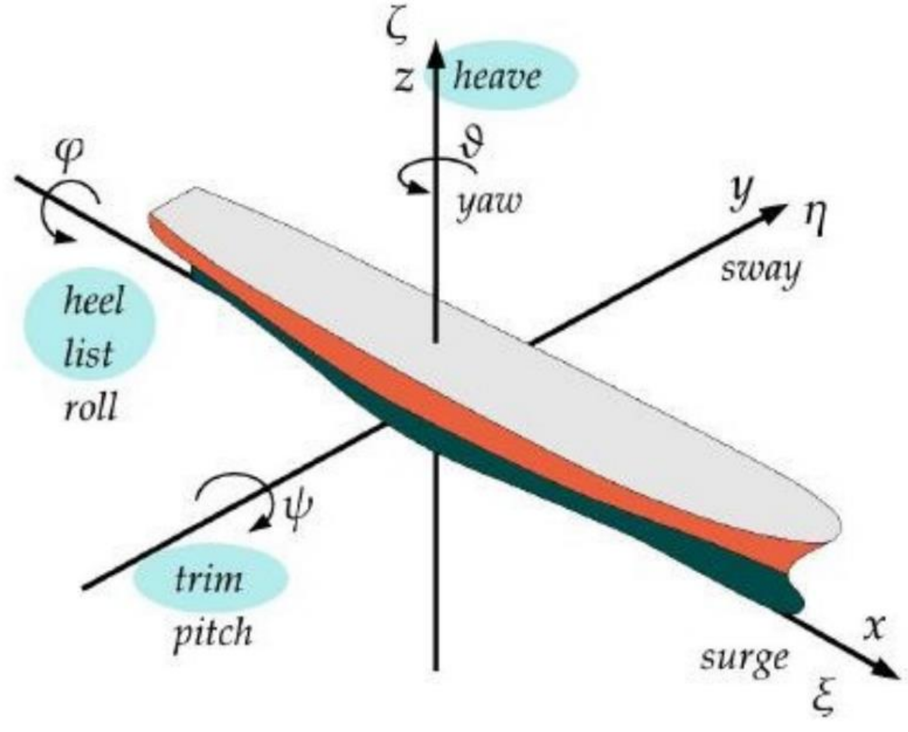

Figure 1: Ship motion with 6 degrees of freedom (Prof. Dr.-Ing. habil. Nikolai Kornev, 2012) [**8**]

The ship has got six degrees of freedom named as surge(ξ), sway (η), heave(ζ), roll (φ), yaw(ϑ) and pitch (ψ) as mentioned in Table 1. The entire launching operations are supposed to be carried out in calm water condition and the ship's forward speed has been considered as zero.

Table 1: Ship's Degrees of freedom

DOF	Description
1	Surge - motion in X-direction
2	Sway - motion in Y-direction
3	Heave - motion in Z-direction
4	Roll - rotation about X-axis
5	Pitch - rotation about Y-axis
6	Yaw - rotation about Z-axis

Considering SPAR and vessel as separate rigid bodies, ship's system inertia matrix including added mass can be written as below:

$$M = M_v + M_a \tag{1}$$

Where:

M_v = Mass and inertia matrix of the vessel

M_a = Added mass and added inertia moment matrix of the vessel

$$M_v = \begin{bmatrix} m_v & 0 & 0 & 0 & m_v z_g & -m_v y_g \\ 0 & m_v & 0 & -m_v z_g & 0 & -m_v x_g \\ 0 & 0 & m_v & m_v y_g & m_v x_g & 0 \\ 0 & -m_v z_g & m_v y_g & I_{xx} & -I_{xy} & -I_{xz} \\ -m_v z_g & 0 & -m_v x_g & -I_{yx} & I_{yy} & -I_{yz} \\ -m_v y_g & m_v x_g & 0 & -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \tag{2}$$

Assuming that m_{ij} is a component in the i^{th} direction caused by acceleration in direction j. Added mass with 36 components can be written as below[**5**]:

$$M_a = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{12} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \tag{3}$$

Due to axis symmetry of ship (hull is symmetric on port-starboard (xy plane)), it can be stated that vertical motions due to heave and pitch make no transversal force:

$$m_{32} = m_{34} = m_{36} = m_{52} = m_{54} = m_{56} = 0 \tag{4}$$

Due to the symmetry of added mass matrix, it can be stated that $m_{ij} = m_{ji}$, hence:

$$m_{23} = m_{43} = m_{63} = m_{25} = m_{45} = m_{65} = 0 \tag{5}$$

In the case of longitudinal motion (i=1) caused by acceleration in j=2, 4, 6:

$$m_{12} = m_{14} = m_{16} = 0 \tag{6}$$

Due to symmetry of added mass matrix:

$$m_{21} = m_{41} = m_{61} = 0 \tag{7}$$

For a ship moving in six degrees of freedom, 36 components of added mass can be reduced to 18 as follows [**5**]:

$$M_a = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 & m_{15} & 0 \\ 0 & m_{22} & 0 & m_{24} & 0 & m_{26} \\ m_{31} & 0 & m_{33} & 0 & m_{35} & 0 \\ 0 & m_{42} & 0 & m_{44} & 0 & m_{46} \\ m_{51} & 0 & m_{53} & 0 & m_{55} & 0 \\ 0 & m_{62} & 0 & m_{62} & 0 & m_{66} \end{bmatrix} \tag{8}$$

For further simplification only coupled motion of pitch, heave and surge motion can be considered, only added masses related to this coupled motion will be non-zero. Now the added mass matrix can be further reduced to 9.

$$M_a = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 & m_{15} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ m_{31} & 0 & m_{33} & 0 & m_{35} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ m_{51} & 0 & m_{53} & 0 & m_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

The effect of added mass in the longitudinal axis (surge motion) caused by acceleration due to pitch and heave motion can be removed from the matrix equation (9) by taking into consideration that only coupled pitch and heave motion is significant, hence for further calculations only coupled pitch-heave motion is contemplated. Now the simplified added mass matrix can be derived as in equation (10).

$$M_a = \begin{bmatrix} m_{33} & m_{35} \\ m_{53} & m_{55} \end{bmatrix} \tag{10}$$

2.1.1 Method of Equivalent Ellipsoid

As part of this Thesis, the calculation of the added mass coefficients is one of the important coefficients for solving equations of motions. Calculating those coefficients for special types of vessels are more difficult where its shapes are not simple geometry. Added mass coefficients are normally found from experimental results and numerical methods such as strip theory. However, these methods are costly and difficult at the same time. Moreover, the numerical methods cannot model the full three-dimensional body and can produce significant errors that can cause a large deviation in the values. So, extracting the added mass coefficients using empirical formulas considering ships to an equivalent ellipsoid will be way easy and a good approximation in the early design phase.

In order to estimate values of m_{ij}, the ship can be assumed as a 3D body equivalent to an ellipse as in Figure 2.

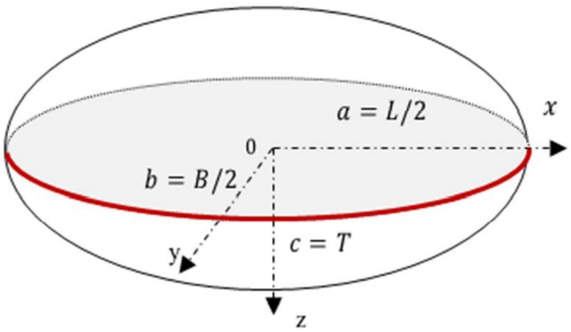

Figure 2: Ship assumed as an Ellipsoid (Do Thanh Sen & Tran Canh Vinh, 2016) [**5**]

Based on the theory of hydrostatics,

$$m_{11} = m_v k_{11} \tag{11}$$

$$m_{33} = m_v k_{33} \tag{12}$$

$$m_{55} = m_v I_{yy} \tag{13}$$

For simplifying the calculation and estimation of added mass coefficients, each component can be represented by a corresponding k_{ij} called hydrodynamic coefficient. [**5**]

$$k_{11} = \frac{A_0}{2-A_0} \tag{14}$$

$$k_{33} = \frac{C_0}{2-C_0} \tag{15}$$

$$k_{55} = \frac{\left(L^2-4T^2\right)^2(A_0-C_0)}{2(c^4-a^4)+(C_0-A_0)(4T^2+L^2)^2} \tag{16}$$

Where:

$$A_0 = \frac{2(1-e^2)}{e^3}\left[\frac{1}{2}ln\frac{(1+e)}{(1-e)} - e\right] \tag{17}$$

$$B_0 = C_0 = \frac{1}{e^2} - \frac{(1- e^2)}{2e^3}ln\frac{(1+e)}{(1-e)} \tag{18}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} \tag{19}$$

Mass moment of Inertia of the displaced water is approximately mass moment of inertia of the equivalent ellipsoid:

$$I_{yy} = \frac{1}{120}\pi\rho LBT\,(4T^2 + L^2) \tag{20}$$

This is an approximation method only to find the added mass coefficient m_{ij} where i=j. However, this method cannot be used to estimate the added mass coefficient m_{ij} where $i \neq j$ This empirical method cannot be used to determine m_{24} m_{26} m_{35} m_{44} m_{15} and m_{51}.

2.1.2 Strip Theory Method: Lewis Conformal Mapping

Strip Theory was introduced by Korvin-Kroukovsky and Jacobs in 1957. Later in 1969, the theory was modified by Tasai. According to the theory, Ship can be subdivided into a finite number of 2D slices with cross sections closer to the hull form sections [**5**].

Strip theory can be used to find the forces acting on the oscillating ships under following assumptions;

- The fluid is incompressible, inviscid and irrotational.
- Effect due to surface tension can be ignored.
- The motion amplitudes and velocities are small enough to neglect the nonlinear terms in the free-surface condition and kinematic boundary condition on the ship frame.
- Two-dimensional character of the flow around each ship frame.

Also, the main assumption is that flow around each ship frame does not depend on the flow at other cross sections.

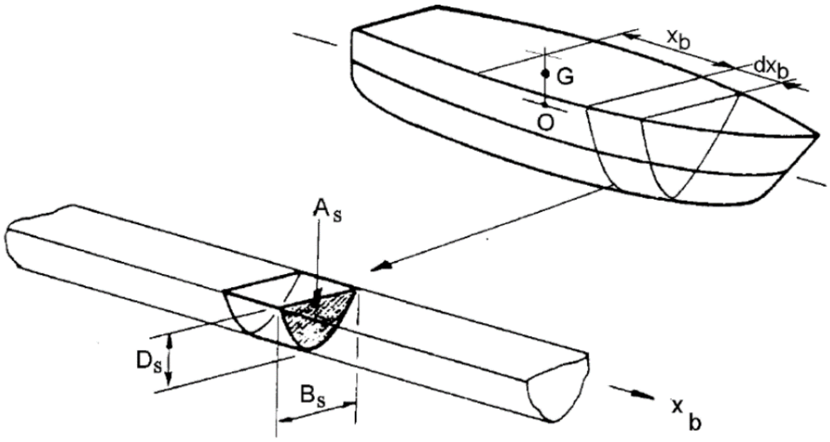

Figure 3: Ship hull subdivided into different cross sections (Prof. Dr.-Ing. habil. Nikolai Kornev, 2012) **[8]**

As per Strip Theory, ship will be subdivided into different cross sections and each frame is treated hydrodynamically as different segments. So here in this Thesis, strip theory can be used to find out the hydrodynamic coefficients of heave motions and coupled motion along with pitch motions.

The forces acting on each frame are added to get the overall force on the hull.[**8**]

$$F_i = \int_0^L \delta\, F_i dx \tag{21}$$

The pitch moment is calculated by multiplying with corresponding forces and lever.

$$M_5 = -\int_0^L x\delta\, F_3 dx \tag{22}$$

Using the Strip Theory, the added mass can be calculated by integrating each 2D section over the length of the ship.

The following equations represent the added mass coefficients [**5**]:

$$m_{22} = \int_{L_1}^{L_2} m_{22}(x)dx \tag{23}$$

$$m_{33} = \int_{L_1}^{L_2} m_{33}(x)dx \tag{24}$$

$$m_{24} = \int_{L_1}^{L_2} m_{24}(x)dx \tag{25}$$

$$m_{44} = \int_{L_1}^{L_2} m_{44}(x)dx \tag{26}$$

$$m_{26} = \int_{L_1}^{L_2} m_{22}(x)xdx \tag{27}$$

$$m_{35} = \int_{L_1}^{L_2} m_{33}(x)xdx \tag{28}$$

$$m_{46} = \int_{L_1}^{L_2} m_{24}(x)xdx \tag{29}$$

$$m_{66} = \int_{L_1}^{L_2} m_{22}(x)x^2dx \tag{30}$$

Where m_{ij} is added mass of a 2D cross section at location X_s.

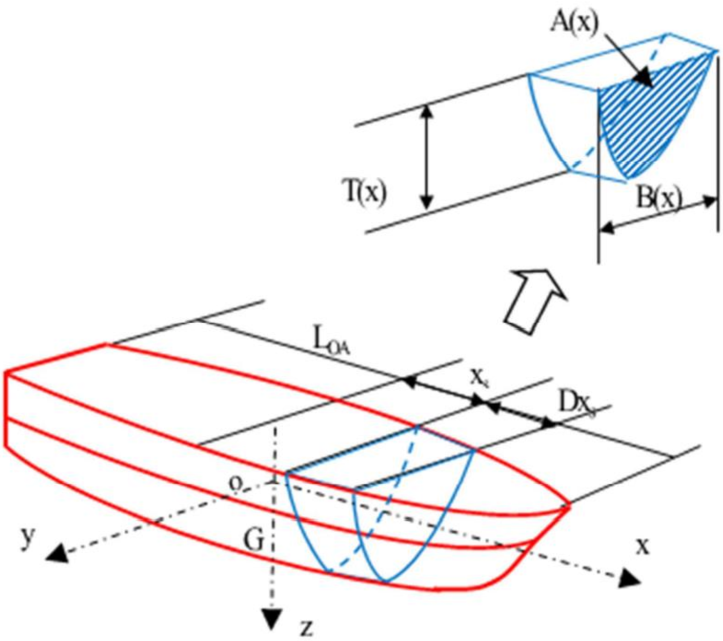

Figure 4: Hull subdivided as 2D slices (Do Thanh Sen & Tran Canh Vinh, 2016) [**5**]

The ship hull forms are complex and each of the ship sections has got various shapes. For numerical calculation, Lewis conformal mapping technique is the best approximation method to transform complex ship sections to simple sections. By using this method, the hull can be mapped to the unit semi-circle (ζ plane) which can be written as in detail as below;[**5**]

$$\zeta = \mathrm{y} + \mathrm{iz} = \mathrm{i}a_0\left(\sigma + \frac{p}{\sigma} + \frac{q}{\sigma^3}\right) \tag{31}$$

Equation for unit semi-circle can be written as;

$$\sigma = e^{i\emptyset} = cos\Theta + isin\Theta \tag{32}$$

Where:

$$\mathrm{i} = -1 \tag{33}$$

$$a_0 = \frac{T(x)}{1+p+q} \tag{34}$$

The parameters of the cross sections are written below form,

$$y = [(1+p)sin\Theta - bsin3\Theta]\frac{B(x)}{2(1+p+q)} \tag{35}$$

$$z = -[(1-p)cos\Theta + bcos3\Theta]\frac{B(x)}{2(1+p+q)} \tag{36}$$

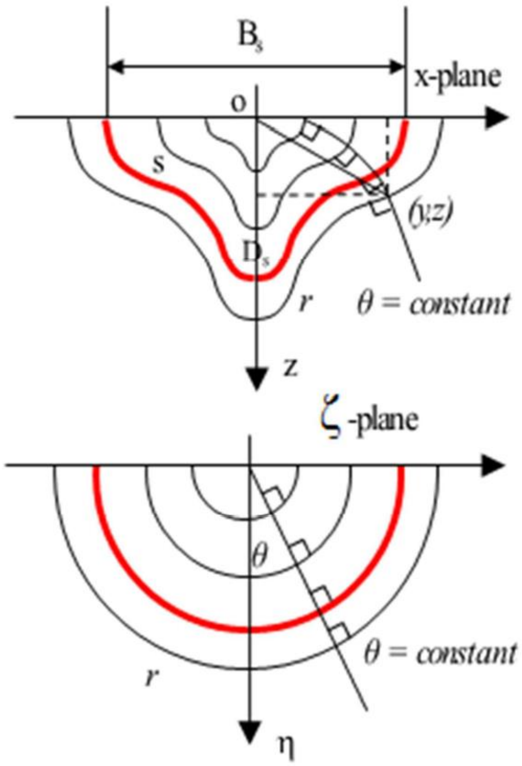

Figure 5: Representation of Lewis Conformal Mapping plane relationship (Do Thanh Sen & Tran Canh Vinh, 2016) **[5]**

Where;

B(x) = breadth of the cross section

T(x) = draft of the cross section

Parameter p & q are described with the help of other two ratios named H(x) and β(x)

$$H(x) = \frac{B(x)}{2T(x)} = \frac{(1+p+q)}{(1-p+q)} \tag{37}$$

$$\beta(x) = \frac{A(x)}{B(x)T(x)} = \frac{\Pi(1+p^2-3q^2)}{4(1+q)^2-p^2)} \tag{38}$$

Parameter Θ doesn't have a any physical meaning and it lies in between $\frac{\Pi}{2} \geq \Theta \geq -\frac{\Pi}{2}$

$$q = \frac{\frac{3}{4}\Pi + \sqrt{(\frac{\Pi}{4})^2 - \frac{\Pi}{2}\alpha(1-\gamma^2)}}{\Pi + \alpha(1-\gamma^2)} - 1 \tag{39}$$

$$p = (q+1)q \tag{40}$$

$$\alpha = \beta - \frac{\Pi}{4} \tag{41}$$

$$\gamma = \frac{H-1}{H+1} \tag{42}$$

The added mass component for heave motion of each section is found using below formulas:

$$m_{33}(x) = \frac{\rho\Pi B(x)^2((1+p)^2+3q^2)}{8(1+p+q)^2} = \frac{\rho\Pi B(x)^2}{8} k_{33}(x) \tag{43}$$

The total added mass components for heave motion and coupled pitch-heave motions can be found using below formulas[**5**]:

$$m_{33}(x) = \mu_1(\lambda = \frac{L}{B})\frac{\rho\Pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)dx \tag{44}$$

$$m_{35}(x) = -\mu_2(\lambda = \frac{L}{B})\frac{\rho\Pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)xdx \tag{45}$$

$$m_{35}(x) = -\mu_2(\lambda = \frac{L}{B})\frac{\rho\Pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)xdx \tag{46}$$

Here $\mu_1(\lambda)$, $\mu_2(\lambda)$ are added mass and added moment of inertia corrections related to the motion of fluid along x-axis due to elongation of the body.

$$\mu_1(\lambda) = \frac{\lambda}{\sqrt{(1+\lambda^2)}}(1 - 0.425\frac{\lambda}{\sqrt{(1+\lambda^2)}}) \tag{47}$$

$$\mu_2(\lambda) = k_{66}(\lambda, q)q(1 + \frac{\lambda}{\lambda^2}) \tag{48}$$

The formulas for finding added mass components for coupled pitch-heave motions can be summarised as below[**5**];

$$m_{55} = k_{55}I_{55} \tag{49}$$

$$m_{33}(x) = \mu_1(\lambda = \frac{L}{B})\frac{\rho \Pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)dx \tag{50}$$

$$m_{35}(x),\ m_{53}(x) = -\mu_2(\lambda = \frac{L}{B})\frac{\rho \Pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)xdx \tag{51}$$

2.2 Uncoupled Motions - Free Oscillations & Forced Oscillations with Small Amplitudes

Before understanding the uncoupled forced oscillation motion, it is imperative that to know the theory behind free oscillations. It gives an insight to the period of motion and eigen frequencies. The equation of motion for free oscillation can be written as; [**8**]

$$(m + A_{33})\ddot{\zeta} + B_{33}\dot{\zeta} + \rho g A_{wp}\zeta = 0 \tag{52}$$

$$\left(I_{yy} + A_{55}\right)\ddot{\psi} + B_{55}\dot{\psi} + \rho g \nabla_0 \mathrm{G}M_L\psi = 0 \tag{53}$$

The same above equations in the normalized forms are as follows;

$$\ddot{\zeta} + 2\nu_\zeta\dot{\zeta} + \omega_\zeta{}^2\zeta = 0 \tag{54}$$

$$\ddot{\psi} + 2\nu_\psi\dot{\psi} + \omega_\zeta{}^2\psi = 0 \tag{55}$$

Where:

Damping coefficients

$$\nu_\zeta = \frac{B_{33}}{2(m + A_{33})} \tag{56}$$

$$\nu_\psi = \frac{B_{55}}{2(I_{yy} + A_{55})} \tag{57}$$

Eigen frequencies of non damped oscillation

$$\omega_\zeta = \sqrt{\frac{\rho g A_{wp}}{(m + A_{33})}} \tag{58}$$

$$\omega_\psi = \sqrt{\frac{\rho g \nabla_0 \mathrm{GM}_L}{(I_{yy} + A_{55})}} \tag{59}$$

Above Figure 6 shows the resulting pitching periods, based on the pitch period formula for undamped oscillation with the variable of LBP up to 300 m is incorporated.

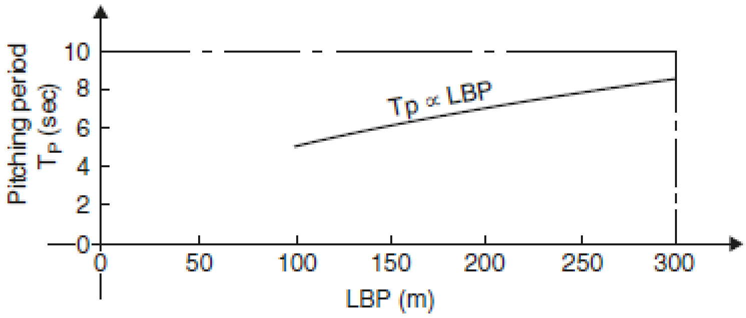

Figure 6: Pitching period vs LBP of the vessel (Captain D.R. Derrett, revised by Dr C.B. Barrass M.Sc C.Eng FRINA CNI, 2006) **[7**]

Considering the launching operation to be carried out at calm water condition, the ship assumed to have zero forward speed. Once the SPAR gets separated from the ship, it will undergo oscillation motions mainly pitch and heave motions due to the restoring hydrostatic force and moments.

Since SPAR rotates around tipping point during the last phase of the launch, uncoupled pitch motion equation can be formed by equating equation 52 to the tipping moment produced by SPAR.

$$\left(I_{yy} + A_{55}\right)\ddot{\psi} + B_{55}\dot{\psi} + \rho g \nabla_0 \mathrm{GM}_L \psi = m_s l = M \tag{60}$$

Similarly, due to separation of the SPAR from the ship during the last phase of the launch, a weight reduction can occur to the system. Hence, uncoupled pitch motion equation can be formed by equating equation 53 to the weight of the SPAR detached.

$$(m + A_{33})\ddot{\zeta} + B_{33}\dot{\zeta} + \rho g A_{wp}\zeta = m_s g = F \tag{61}$$

2.3 Coupled Pitch-Heave Motion

Among the six degrees of freedom of the vessel motions, the coupled heave- pitch motions are most important as compared to the other motions in the event of SPAR launching, since these coupled motions have direct impact on green water on deck, slamming effect, propeller raising out of water. Hence, the simulation of coupled heave - pitch motion is very important for launching operations.

The coupled heave-pitch motions of the vessel can be represented by two coupled second order linear ordinary differential equations as shown below[**6**]:

$$(m_v + A_{33})\ \ddot{Z}_v + B_{33}\ \dot{Z}_v + C_{33}\ Z_v + (A_{35}\text{-}m_v x_G)\ddot{\Theta}_v + B_{35}\ \dot{\Theta}_v + C_{35}\ \Theta_v = F \quad (62)$$

$$(I_{yy} + A_{55})\ \ddot{\Theta}_v + B_{55}\ \dot{\Theta}_v + C_{55}\ \Theta_v + (A_{53}\text{-}m_v x_G)\ddot{Z}_v + B_{53}\ \dot{Z}_v + C_{53}\ Z_v = M \quad (63)$$

Where:

Z = Heave displacement

ϴ = Pitch angle

x_G = Longitudinal coordinate of the center of gravity of the vessel

m_v = Mass displacement of the vessel

I_{yy} = Pitch moment of inertia;

A_{33} and A_{55} are the added mass and added moment of inertia, respectively.

B_{33} and B_{55} are the damping coefficients, respectively.

C_{33} and C_{55} are the restoring force coefficients respectively.

A_{ij}, B_{ij}, and C_{ij} (i, j = 3, 5) are the coupled coefficients;

F and M are the force and moment due to separation of the SPAR from the vessel respectively.

Normalizing the second derivative terms in the above equation, the normalized differential equations of coupled heave-pitch motions can be written as follows [**6**]:

$$\ddot{Z}_v + b_{33}\ \dot{Z}_v + b_{35}\ \dot{\Theta}_v + c_{33}\ Z_v + c_{35}\ \Theta_v = F' \quad (64)$$

$$\ddot{\Theta}_v + b_{55}\ \dot{\Theta}_v + b_{53}\ \dot{Z}_v + c_{55}\ \Theta_v + + c_{53}\ Z_v = M' \quad (65)$$

Where:

$$\begin{pmatrix} b_{33} & b_{35} \\ b_{53} & b_{55} \end{pmatrix} = \begin{pmatrix} m_v + A_{33} & A_{35} - m_v x_G \\ A_{53} - m_v x_G & I_{yy} + A_{55} \end{pmatrix}^{-1} \begin{pmatrix} N_{33} & N_{35} \\ N_{53} & N_{55} \end{pmatrix} \tag{66}$$

$$\begin{pmatrix} c_{33} & c_{35} \\ c_{53} & c_{55} \end{pmatrix} = \begin{pmatrix} m_v + A_{33} & A_{35} - m_v x_G \\ A_{53} - m_v x_G & I_{yy} + A_{55} \end{pmatrix}^{-1} \begin{pmatrix} C_{33} & C_{35} \\ C_{53} & C_{55} \end{pmatrix} \tag{67}$$

$$\begin{pmatrix} F' \\ M' \end{pmatrix} = \begin{pmatrix} m_v + A_{33} & A_{35} - m_v x_G \\ A_{53} - m_v x_G & I_{yy} + A_{55} \end{pmatrix}^{-1} \begin{pmatrix} F \\ M \end{pmatrix} \tag{68}$$

The governing equations for motion equations are solved numerically by using Runge-Kutta method.

The required hydrodynamic coefficients are obtained from integration of sectional added-mass, damping and restoring coefficients, derived from conformal mapping method.

For a floating body with lateral symmetry in shape and weight distribution, the six coupled equations of motion here can be reduced to two sets of equations, where the first set consisting of surge, heave, and pitch motions are important, especially the coupled heave-pitch motions.

2.4 Rules and Regulations

All rules and regulations taken here for the study are based on the jacket launching from a barge, considering the SPAR launch operation from the transport vessel is analogous to the jacket launching from a barge. The following limits and requirements are found relevant to this Master Thesis[**9**];

- Launch Trim Angle: The maximum trim angle to initiate launching as per DNV class rules [**9**] should not exceed 4 degrees while satisfying all other stability parameters.
- Bottom clearances during and after SPAR launch: Greater than 10% of water depth or 5m during launch & self-upending of the SPAR.
- Stability during and after the launch as per DNV class rules:
- Minimum GM requirements:
    - During launch: Minimum static stability range should be not less than 15 + (10/GM) degrees or 20 degrees, whichever is higher, with GM in meters.
    - After launch: Transverse (GM_T) and longitudinal (GM_L) should be minimum 0.5m. The combined system of SPAR and transport vessel should possess a positive Metacentric height during the launch operation.
- Model test of the launch to be performed to validate the numerical analysis.

3 PHYSICAL MODELS

Here in this Thesis Paper, physical model refers to the SPAR and Ship which are taking part in the launching process and subsequent dynamics.

3.1 Ship-SPAR Launching System

Launching system consit of SPAR cylinder and Transport Vessel from where SPAR would be launched. The details of Transport Vessel and SPAR used in this Thesis for the numerical analysis are given in this section.

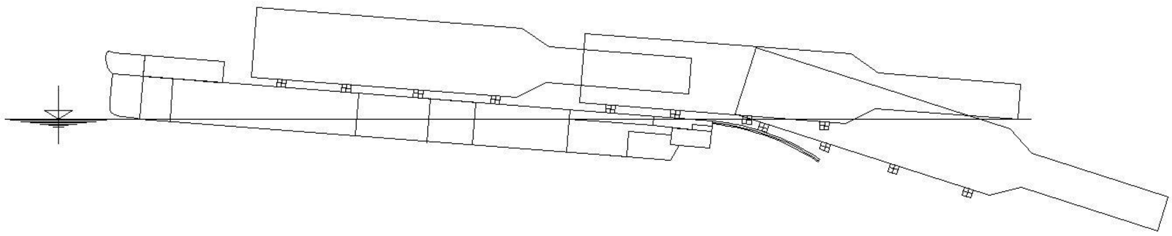

Figure 7: Illustration of SPAR launching from the Transport Vessel during self upending phase

3.1.1 Main Dimensions of the Transport Vessel

Main particulars and class of the vessel have been mentioned below:

Class: ABS ✠ A1, Offshore Support Vessel (Supply, Wind IMR, SPS)

Table 2: Main Dimensions of the Transport Vessel

LOA	178.34 m	Length, overall
L_{pp}	171.8 m	Length, between perpendiculars
B	36.5 m	Breadth, moulded
D	11.5 m	Depth
T_{des}	5.5 m	Draught, design
DWT_{des}	9,300 t	Deadweight, design

Ship's structure includes the constructive extension of the runway by means of a 20 m longer tail section (roughly up to frame #-36) and 3 m contour at the end. These structures could support the whole loads exerted by the SPAR in the final stage of the launch operation.

3.1.2 Characteristics Dimensions of the SPAR

SPAR is cylindrical in shape with 10m diameter at the top and 20m diameter at the bottom. Overall length of the SPAR is 130m. It is supported by 6 blocks. Centre of buoyancy lies at 55.5m and centre of gravity of the SPAR at 53.33 m from the bigger diametric end.

Table 3: Details of SPAR

Mass of the SPAR	3800 t
Mass of the bracing	575 t
Total mass of the SPAR including the bracing	4750 t

Three different heights of the buoy to the deck are designed for placing the SPAR cylinder which are 2m, 1.5m & 1m.

3.2 SPAR Pre-Launch Parameters

3.2.1 Launching Conditions

To start the launching process, the vessel has to be ballasted to trim the vessel by aft 4 degree with a midship draft of 7.37 meters.

An interesting fact is that as the angle of trim and initial draft increases, the time taken for the SPAR to slide down from its initial position and leave from the ship is getting shorter. In general, large trim angle and initial draft implies more rapid launching.

3.2.2 Launching Stages

The entire launch process is comprised of four stages:

1. Ballasting stage: The vessel is ballasted to attain the predefined trim and draft.
2. Sliding stage: The SPAR cylinder slides on the vessel's main deck due to its own weight without any rotation.
3. Tipping stage: The SPAR cylinder slides until the tipping point, rotates about it and slides through the tail section to the sea.
4. Self-righting stage: The SPAR cylinder once separated from the vessel, oscillates and tries to upright itself with the help of reserve buoyancy.

3.2.3 Launching Problems

There are few launching operational concerns and which are listed as follows,

- Excessive friction between SPARs and the sliding ways.
- Insufficient longitudinal structural strength of the ship to overcome the bending moment developed during the launching operation.
- High angular acceleration produced when the SPAR separates from the ship can cause loss of foot stability of the ship crew and cause serious problems. Also, the slamming at the forward end of the ship can cause structural failure.
- Instability of the vessel during the launching.
- Overturning of the SPAR unit or ship, and even total loss of the structure.

3.2.4 Friction coefficients

The friction force between the SPAR surface and the skid beams is one of the important parameters for the launching. Here the value for the kinematic friction has been taken as 0.2.
The force equation of the SPAR sliding over the skid beams during trimming of the vessel will give following results,

$$\mu_K = tan(\Theta) - \frac{a_x}{g \; Cos(\Theta)} \tag{69}$$

Where ϴ is the trim angle, g is the acceleration due to gravity and a_x is the longitudinal acceleration of the SPAR.
Lower the coefficient of friction and greater the mass of the bracings attached to the SPAR; rapid launch can be achieved. It has a minor effect on launch behaviour; however, it largely affects the duration of the launch.

In order to have low frictional resistance, an arrangement of wheels under the trestles which are guided on the rails can be advised for placing the SPAR. This special arrangement also significantly reduces the slipway running time and limits wear and tear on the structure due to excessive slipping over a period of time.
For a least necessary trim, the static coefficient of friction has to be less than or equal to ship's trim angle. Here in this case, angle of repose would be the least possible trim angle the ship can have during the launching operation. Angle of Repose can be defined as the maximum angle at which the SPAR will stay on the skid beam without sliding. Once the ship exceeds the angle of repose, the SPAR will start sliding on the skid way beam by overcoming the static friction. In short, the Angle of Repose can be considered as the least necessary trim required for the launching operation.

4 MATHEMATICAL MODEL

The mathematical model of the launching of the SPAR cylinder from the Offshore wind turbine transport vessel has been developed in this paper. As a cost-effective way, launching technique for placing the SPAR cylinder onto its location is preferred, rather than lifting it directly using a heavy lift vessel. The launching process takes only a few minutes; however, the entire process is quite critical and dynamic. A time domain numerical simulation is required to analyse the launch process.

4.1 Coordinate System

Defining coordinate systems for launching operation and direction is import before formulating the equation of motions. The numerical model involves coordinate systems as illustrated in Figure 8.

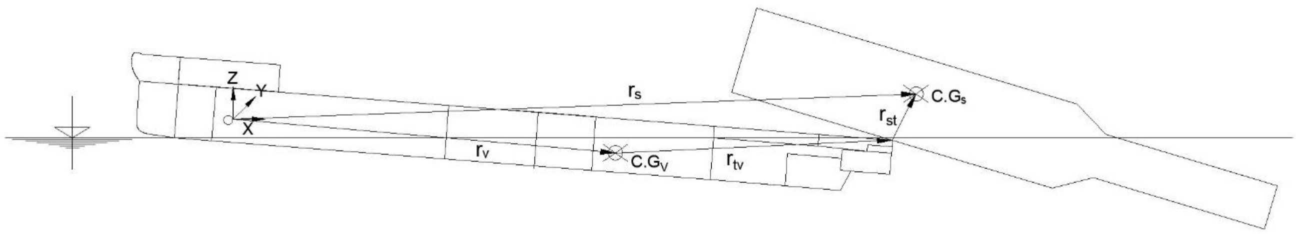

Figure 8: SPAR cylinder rotating about tipping point, The global coordinate system and relative position of the SPAR and vessel CG’s

Where:

$\vec{r}_s$ = Position vector of the SPAR

$\vec{r}_v$ = Position vector of the vessel

$\vec{r}_{tv}$ = Relative position vector of the vessel from tipping point

$\vec{r}_{st}$ = Relative position vector of the SPAR from tipping point

Geometry of the SPAR & transport vessel

Positive Z axis is upward and the X axis lies in the water-plane with positive direction pointing toward the aft end of the vessel.

4.2 Launching Phases

A calm sea-state is a necessary condition to execute the launching. Depending on the relative motions of the SPAR and vessel, five phases of motion are possible:
Phase I: SPAR is sliding on the skid beam placed on the vessel deck due to hydraulic jack pushing or winch pulling.
Phase II: SPAR is sliding on the skid beams due to its own weight.
Phase III: SPAR is only rotating about the tipping point.
Phase IV: SPAR is rotating about tipping point and sliding on tilting beam simultaneously.
Phase V: SPAR and vessel have separated from each other.

When the initial trim angle exceeds the dynamic friction coefficient angle, after cutting sea fastening, phase II will start to occur, depending on the friction in the beginning of the sliding, an initial pull/push by winch jack may be required. If that is not the case, phase I can occur and SPAR will slide on the skid beam with constant velocity of the winch. There are two conditions required for the SPAR to start rotating about the tipping point: 1) SPAR's centre of gravity passes the tipping point and 2) reaction moment is negative. As per the second condition, the centre of gravity of the SPAR should move a few meters past the tipping point and then start rotating. If these two conditions are met, then phase III will start to occur. SPAR stops at tipping point and starts to only rotate about the tipping point until the SPAR exceeds the static friction and coefficient of angle.
From this point onwards, SPAR rotates and slides simultaneously. Therefore, phase IV can only occur after Phase II & III.

4.3 Equation of Motion

Equations governing the motion are Newton-Euler equations of motion. In addition to the equation of motions, constraints are needed to mathematically model the launch process. These constraints differ in each phase of the launch. The whole equations including equations of motion and constraint equations will give a system of different algebraic equations, where the unknowns will be the acceleration and reaction forces.
The newton-Euler equation of motion by definition state that rate of change of system momentum equals the forces acting on the system. Consider the SPAR and vessel as two separate systems of rigid bodies, with common reaction forces acting on both of the bodies. Considering the SPAR is placed on the main deck of the vessel, the centre of gravity of the

SPAR lies just above the centre of gravity of the vessel in the same vertical plane. Hence the launch operation can be taken as a two-dimensional problem. In this context, SPAR and vessel can be treated as two separate rigid bodies and their position of centre of gravity can be represented by three coordinates namely X, Y & Ө as below [**3**].

$$m_s\ddot{X}_s = P_c{}^X + P_w{}^X + F^X{}_{F,s} \tag{70}$$

$$m_s\ddot{Z}_s = P_c{}^Z + P_w{}^Z + F^Z{}_{F,s} - W_s \tag{71}$$

$$I_s\ddot{\Theta}_s = M_c{}^Y + M^Y{}_{F,s} + (P_c{}^X + P_w{}^X)\ x\ (Z_c - Z_s) + (P_c{}^Z + P_w{}^Z)\ x\ (X_c - X_s) \tag{72}$$

$$m_v\ddot{X}_v = -P_c{}^X - P_w{}^X + F^X{}_{F,v} \tag{73}$$

$$m_v\ddot{Z}_v = -P_c{}^Z - P_w{}^Z + F^Z{}_{F,v} - W_v \tag{74}$$

$$I_v\ddot{\Theta}_v = -M_c{}^Y + M^Y{}_{F,v} + (P_c{}^X + P_w{}^X)\ x\ (Z_c - Z_v) + (P_c{}^Z + P_w{}^Z)\ x\ (X_c - X_v) \tag{75}$$

Above system of equations (70 to 75) can be numbered as (i)
Where:
m = Mass
I = Moment of inertia about centre of gravity of body
X, Z, Ө: Coordinates and components of position vectors.
P, F: Forces.
M = Moment.
W = Weight.
Subscripts s, v represents SPAR and vessel respectively.
Subscripts c, w, F represent contact, winch and fluid (hydrodynamic and hydrostatic) forces respectively.
Double dot denotes the second derivative with respect to the time.
In the 6 equations of motions, there are eleven unknowns, which all are six accelerations, three contact forces and two winch forces. In order to solve the equation of motions, one more equation is needed, hence one constraint equation is necessary to fulfil the system of equations.

4.3.1 Constraint Equations / Relations

Constraint equations are geometrical / force relationships that are connected with the SPAR and vessel motions. These constraint equations are applicable in all phases of the launching motion except in the last phase (5th phase), where the SPAR would be separated from the vessel and all the reaction forces would be zero. Hence in the last phase, six accelerations are the only unknown values and those six equations of motion would be sufficient to solve the unknowns.

In phases 1-4 where the SPAR and the vessels are connected, hence the position of their centre of gravities are also connected. Using the relation from the Figure 8, differentiate the position vectors with respect to the time gives velocity and differentiating again the velocity vector with respect to the time yields acceleration constraint vector [**3**].

$$\vec{r}_s = \vec{r}_v + \vec{r}_{tv} + \vec{r}_{st} \tag{76}$$

$$\vec{V}_s = \vec{V}_v + (\vec{\dot{\Theta}}_v\ X\ \vec{r}_{tv}) + (\vec{\dot{\Theta}}_s\ X\ \vec{r}_{st}) + \vec{\dot{r}}_{st} \tag{77}$$

$$\begin{gathered} \vec{a}_s = \vec{a}_v + (\overrightarrow{\ddot{\Theta}_v}\ X\ \vec{r}_{tv}) + (\vec{\dot{\Theta}}_v\ X\ (\vec{\dot{\Theta}}_v\ X\ \vec{r}_{tv})) + (\overrightarrow{\ddot{\Theta}_s}\ X\ \vec{r}_{st}) + (\vec{\dot{\Theta}}_s\ X\ (\vec{\dot{\Theta}}_s\ X\ \vec{r}_{st})) \\ + (2\vec{\dot{\Theta}}_s\ X\ \vec{\dot{r}}_{st}) + \overrightarrow{\ddot{r}_{st}} \end{gathered} \tag{78}$$

Above constraint equations (76 to 78) can be numbered as (ii)

Where:

r = position vector

V = Velocity vector

a = Acceleration vector

Dot denotes the first derivative with respect to the time and subscript ‘t’ denotes tipping point. Since the acceleration constraint vector represent the acceleration in two axes and that can be separately re-write in the scalar forms as follows [**3**];

$$\begin{gathered} \ddot{X}_s = \ddot{X}_v + (Z_t - Z_v)\,\ddot{\Theta}_v - (X_t - X_v)\dot{\Theta}_v^{\,2} + (Z_s - Z_t)\,\ddot{\Theta}_s - (X_s - X_t)\dot{\Theta}_s^{\,2} - \\ 2V_r\dot{\Theta}_s\,Sin(\Theta_s) + \dot{V}_r\,Cos(\Theta_s) \end{gathered} \tag{79}$$

$$\begin{gathered} \ddot{Z}_s = \ddot{Z}_v - (X_t - X_v)\,\ddot{\Theta}_v - (Z_t - Z_v)\dot{\Theta}_v^{\,2} - (X_s - X_t)\,\ddot{\Theta}_s - (Z_s - Z_t)\dot{\Theta}_s^{\,2} - \\ 2V_r\dot{\Theta}_s\,Cos(\Theta_s) - \dot{V}_r\,Sin(\Theta_s) \end{gathered} \tag{80}$$

Above acceleration equations (79 & 80) can be numbered as (iii)

Where:

V_r = SPAR relative sliding velocity on launch skids / extension platform.

$\dot{V}_r$ = SPAR relative sliding acceleration on launch skids / extension platform.

Equation (iii) is valid throughout phases 1 to 4. One more unknown has been introduced from the constraint equation, that is SPAR relative sliding acceleration ($\dot{V}_r$).

Since there are 12 unknowns, in addition to the equation (iii), four more equations are needed to construct a system of equations and solve for the 12 unknowns.

In each phase, these additional equations will be different.

4.3.2 Constraint Equations during Phase I

During Phase I, the SPAR is sliding on the skid beams because of the initial hydraulic jack push / winch pull, hence the relative velocity of the SPAR on the skid beam will be constant and equal to the velocity of the winch. Since the relative velocity is constant, the relative acceleration will be zero, which implies;

$$\dot{V}_r = 0 \tag{81}$$

Above condition can be numbered as equation (81). However, for the summary of equation of motion, same has been taken as (iv)

Considering the fact that the winch force is acting parallel to the vessel main deck, contact forces are formed from normal reaction and friction force which are directly related by the dynamic friction coefficient and finally the vessel and the SPAR rotate together, the following equations are developed.

$$P_w{}^X \, Sin(\Theta_s) + P_w{}^Z \, Cos(\Theta_s) = 0 \tag{82}$$

Above equation (82) can be numbered as (v)

$$P_c{}^X \, (Cos(\Theta_s) + \mu_d \, Sin(\Theta_s)) - P_c{}^Z \, (Sin(\Theta_s) - \mu_d \, Cos(\Theta_s)) = 0 \tag{83}$$

Above equation (83) can be numbered as (vi)

$$\ddot{\Theta}_v = \ddot{\Theta}_s \tag{84}$$

Above equation (84) can be numbered as (vii)

Where:

μ_d : Dynamic friction coefficient between SPAR and launch skid beams.

4.3.3 Constraint Equations during Phase II

During Phase II, SPAR is sliding on skid beams due to its own weight, which means that the angle barge has exceeded the dynamic friction coefficient ($\Theta_v > \mu_d$)

This indicates that the relative sliding velocity exceeds the winch velocity; hence equation (iv) is not valid in this phase. Also, the winch force can be taken as null. All the equations except these two will be the same as phase I and will be applicable in phase II.

$$P_w{}^X = 0 \tag{85}$$

$$P_w{}^z = 0 \tag{86}$$

Above equations (85 & 86) can be numbered as (viii)

4.3.4 Constraint Equations during Phase III

In Phase III, SPAR is rotating about tipping point without sliding. Therefore equation (iv) and (viii) are valid. Since there is no sliding, friction force relation (vi) is not valid in this phase. In addition to that, the tipping point does not resist the moment, hence the moment reaction at the tipping point will be zero.

$$M_c{}^Y = 0 \tag{87}$$

Above condition can be numbered as equation (87). However, for the summary of equation of motion, same has been taken as (ix)

4.3.5 Constraint Equations during Phase IV

All additional constraints in this phase are the same as phase III. Whereas equation (vi) is valid due to sliding at the extension platform and equation (iv) is invalid.

The 12 equations required in each of the first four phases to develop a set of system of differential-algebraic equations that are tabulated as below Table 4.

Table 4: Equation of motion & constraints for each phase

Phase	Motion Equations	Constraints	Total no. of motion equations & constraints
I	(i)	(iii), (iv), (v), (vi), (vii)	12
II	(i)	(iii), (v), (vi), (vii), (viii)	12
III	(i)	(iii), (iv), (vii), (viii), (ix)	12
IV	(i)	(iii), (vi), (vii), (viii), (ix)	12
V	(i)	-	6

4.3.6 Fluid Forces Acting on SPAR

SPAR is cylindrical in shape; therefore, we can derive the fluid forces acting on a typical submerged cylinder.

Various fluid forces acting on the SPAR can be listed as below;

- Buoyancy Force
- Added Mass Force
- Drag Force
- Water Entry/Exit Forces

Three components of fluid force acting on SPAR are calculated by summing up the above-mentioned forces on the SPAR and the same have been formulated as below [**3**].

$$F^{X}{}_{F,s} = (F^{X}{}_{D,s} + F^{X}{}_{A,s} + F^{X}{}_{E,s}) \tag{88}$$

$$F^{Z}{}_{F,s} = (F^{Z}{}_{B,s} + F^{Z}{}_{D,s} + F^{Z}{}_{A,s} + F^{Z}{}_{E,s}) \tag{89}$$

$$M^{Y}{}_{F,s} = (M^{Y}{}_{B,s} + M^{Y}{}_{D,s} + M^{Y}{}_{A,s} + M^{Y}{}_{E,s}) \tag{90}$$

4.3.7 Fluid Forces Acting on the Vessel

Fluid forces acting on the vessel are the same as on the SPAR excluding water entry/exit forces. So, various fluid forces acting on the vessel can be listed as below [**3**];

$$F^{X}{}_{F,v} = (F^{X}{}_{D,v} + F^{X}{}_{A,v}) \tag{91}$$

$$F^{Z}{}_{F,v} = (F^{Z}{}_{B,v} + F^{Z}{}_{D,v} + F^{Z}{}_{A,v}) \tag{92}$$

$$M^{Y}{}_{F,v} = (M^{Y}{}_{B,v} + M^{Y}{}_{D,v} + M^{Y}{}_{A,v}) \tag{93}$$

4.3.8 Solving System of Equation

In phases I to IV, motions of the two bodies are mathematically modelled by 12 equations and 12 unknowns. In phase V, only six equations of motions without any constraints defines the phase completely.

Since these unknowns are accelerations and some reactions, the all set of systems of equations is a differential-algebraic one.

Therefore, the system has to be separated with accelerations as unknowns. Which results in a system 7 second order nonlinear differential equations and it can be solved using standard time integration techniques with help of computer code which determines the motion phase.

Assembling all the equations of motions and constraints can be written in the matrix form [**3**]:

$$\begin{bmatrix} M_{aa} & M_{af} \\ M_{fa} & M_{ff} \end{bmatrix} \begin{pmatrix} \ddot{q} \\ R \end{pmatrix} = \begin{pmatrix} F_a \\ F_f \end{pmatrix} \tag{94}$$

Where:

$$\ddot{q} = \left[\ddot{X}_v \;\; \ddot{Z}_v \;\; \ddot{\Theta}_v \;\; \ddot{X}_s \;\; \ddot{Z}_s \;\; \ddot{\Theta}_s \;\; \dot{V}_r\right]^T \tag{95}$$

$$R = \left[{P_c}^X \;\; {P_c}^Z \;\; {M_c}^Y \;\; {P_w}^X \;\; {P_w}^Z\right]^T \tag{96}$$

The SPAR launching from the transport vessel has been taken as similar to jacket launching operation from the barge and all mathematical modelling for various phases are derived under this assumption. The method followed is based on the numerical modelling for jacket launching from transportation barge [**3**].

5 CALCULATION OF ADDED MASS USING LEWIS CONFORMAL MAPPING

In order to calculate added mass coefficient using strip theory, the ship's hull is divided longitudinally as 42 stations. At a draft of 7.37m, respective sectional data of H(x) & β(x) are determined with the help of Strip Theory Method: Lewis Conformal Mapping.

Table 5: Calculation of sectional data H(x) & β(x)

Station Index	Station	dx	x	H(x)	β(x)
1	-3.00	0.00	67.05	10.78	0.61
2	-2.29	0.71	66.34	11.15	0.67
3	-1.50	0.79	65.55	9.77	0.61
4	0.00	1.50	64.05	10.22	0.71
5	4.58	4.58	59.48	8.37	0.82
6	9.15	4.58	54.90	6.34	0.86
7	13.73	4.58	50.33	5.04	0.89
8	18.30	4.58	45.75	4.18	0.91
9	22.88	4.58	41.18	3.56	0.92
10	27.45	4.58	36.60	3.11	0.92
11	32.03	4.58	32.03	2.86	0.88
12	36.60	4.58	27.45	2.66	0.84
13	41.18	4.58	22.88	2.48	0.94
14	45.75	4.58	18.30	2.48	0.97
15	50.33	4.58	13.73	2.48	0.98
16	54.90	4.58	9.15	2.48	0.98
17	64.05	9.15	0.00	2.48	0.98
18	68.63	4.58	-4.58	2.48	0.98
19	73.20	4.58	-9.15	2.48	0.98
20	77.78	4.58	-13.73	2.48	0.98
21	82.35	4.57	-18.30	2.48	0.98
22	86.93	4.58	-22.88	2.48	0.98
23	91.50	4.58	-27.45	2.48	0.98
24	96.08	4.58	-32.03	2.48	0.98
25	100.65	4.58	-36.60	2.47	0.94
26	105.23	4.57	-41.18	2.45	0.93
27	109.80	4.58	-45.75	2.43	0.91

28	114.38	4.58	-50.33	2.40	0.90
29	118.95	4.58	-54.90	2.35	0.87
30	123.53	4.58	-59.48	2.29	0.85
31	128.10	4.57	-64.05	2.19	0.83
32	132.68	4.58	-68.63	2.07	0.81
33	137.25	4.57	-73.20	1.92	0.80
34	141.83	4.57	-77.78	1.73	0.80
35	146.40	4.58	-82.35	1.53	0.78
36	150.98	4.57	-86.93	1.30	0.75
37	155.55	4.58	-91.50	1.05	0.70
38	160.13	4.57	-96.08	0.78	0.65
39	164.70	4.57	-100.65	0.48	0.57
40	169.28	4.58	-105.23	0.20	0.95
41	172.31	3.03	-108.26	0.17	0.83
42	175.34	3.03	-111.29	0.16	0.83

Necessary inputs and required added mass coefficients obtained using strip theory are mentioned in the below Table 6.

Table 6: Added mass for coupled pitch-heave motion during phase V

L	171.8 m	Length of the ship
B	36.5 m	Breadth of the ship
T	7.37 m	Draft of the ship
m	3.68E+07 kg	Mass displacement of ship
λ (L/B)	12.10	Length to breadth ratio
$\mu_1(\lambda)$	0.57	Correction for the added mass
$\mu_2(\lambda)$	-0.03	Correction for the added moment of inertia
A_{33}	3.26E+07 kg	Added mass due to heave motion
$A_{35} = A_{53}$	5.64E+07 N.s^2	Added mass due to coupled heave-pitch motion

6 DYNAMIC SIMULATION

The proposed coupled mathematical model was used to perform dynamic simulations of the vessel. In the coupled analysis, the vessel motion is solved through equations of motions. The purpose of this study is to investigate the behavior of the vessel and the effect of the SPAR launching on the vessel and its subsequent consequences.

During the launching operation, the ship draft, displacement and trim is changing instantaneously, it is evident that heave oscillations cause the pitch motion and vice versa, which means both heave and pitch oscillations are coupled.

6.1 Uncoupled Pitch Motion During Phase V

Among the five phases of the SPAR launching, phase V would be critical and need to analyse this phase thoroughly to understand the dynamics of the ship.

As part of the ship dynamic analysis, our aim is to check the uncoupled pitch motion when the SPAR detaches from the ship.

Uncoupled pitch motion during phase V can be represented using below equation[**8**];

$$\left(I_{yy} + A_{55}\right)\ddot{\psi} + B_{55}\dot{\psi} + C_{55}\psi = M \tag{97}$$

$$\left(I_{yy} + A_{55}\right)\ddot{\psi} + B_{55}\dot{\psi} + \rho g \nabla_0 \mathrm{G}M_L\psi = M \tag{98}$$

Where:

$$\ddot{\psi} = \frac{d^2\psi}{dt^2} \tag{99}$$

$$\dot{\psi} = \frac{d\psi}{dt} \tag{100}$$

In order to solve the 2nd degree differential equation, the computation of hydrodynamic coefficients such as added mass, damping coefficients and restoring coefficients are required. The 2nd degree differential equations are solved using python programming language with the help of Scipy.integrate package using function ODEINT. The results obtained using ODEINT functions are again checked with Runge-Kutta numerical solution.

By using ellipsoid method, added mass for uncoupled pitch motion can be found.

Ship main particulars and density of seawater required for the calculation are tabulated as below.

Initial conditions have been set to solve the mathematical model. Simulation results are based on the following initial conditions included in the Table 7.

Table 7: Hydrodynamic coefficients for uncoupled pitch motion during phase V

L	171.8 m	Length of the ship
B	36.5 m	Breadth of the ship
T	7.37 m	Draft of the ship
ρ_sw	1025 kg/m^3	Density of sea water
I_yy	3.69E+10 kg.m^2	Mass moment of inertia of Ship about transverse axis
A_{55}	4.21E+10 kg.m^2	Added mass moment of inertia of Ship about transverse axis
I_yy + A_{55}	7.90E+10 kg.m^2	Total mass moment of inertia of Ship about transverse axis
C_{55}	7.52E+10 N/m	Hydrostatic coefficient for pitch motion
B_{55}	1.54E+09 N.s/m	Damping coefficient for pitch motion
ψ_0	4 deg	Initial trim angle
ψ_0	0.0698 rad	Initial trim angle
$\dot{\psi}_0$	0 rad/s	Initial angular velocity
ω_{55}	0.98 rad/s	Frequency for undamped pitch oscillation
T_{55}	6.44 s	Time period for undamped pitch oscillation

Following results are obtained for uncoupled pitch motion. Pitch angle in degree, angular velocity(rad/s) & angular acceleration (rad/s^2) plots in time domain are as given below.

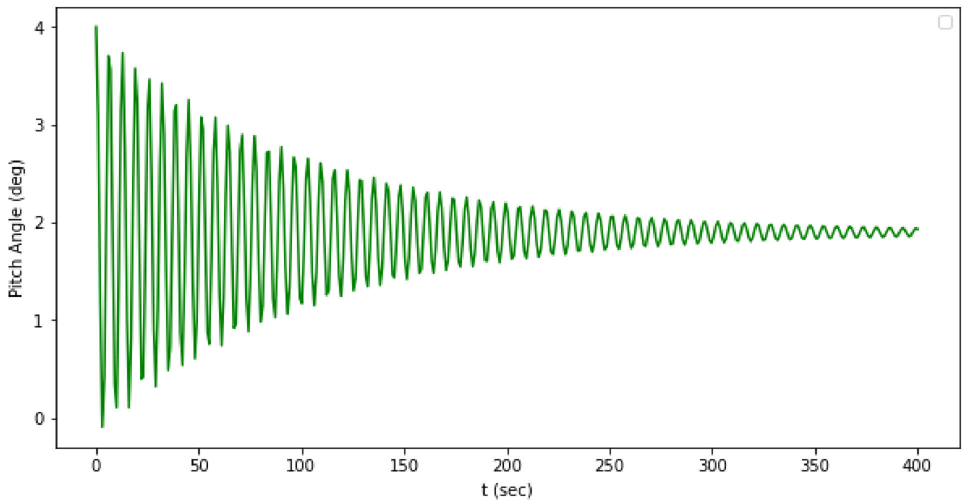

Figure 9: Pitch angle in time domain for uncoupled pitch motion during phase V

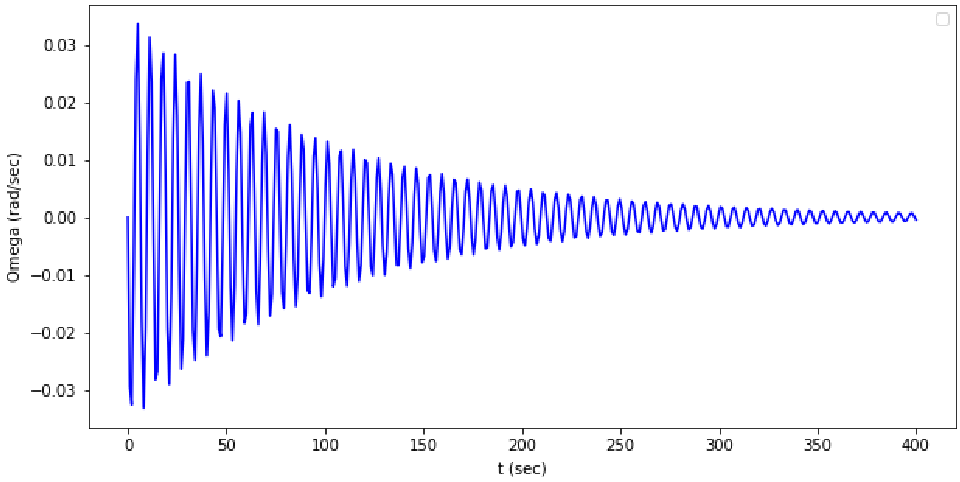

Figure 10: Pitch anglular velocity in time domain for uncoupled pitch motion during phase V

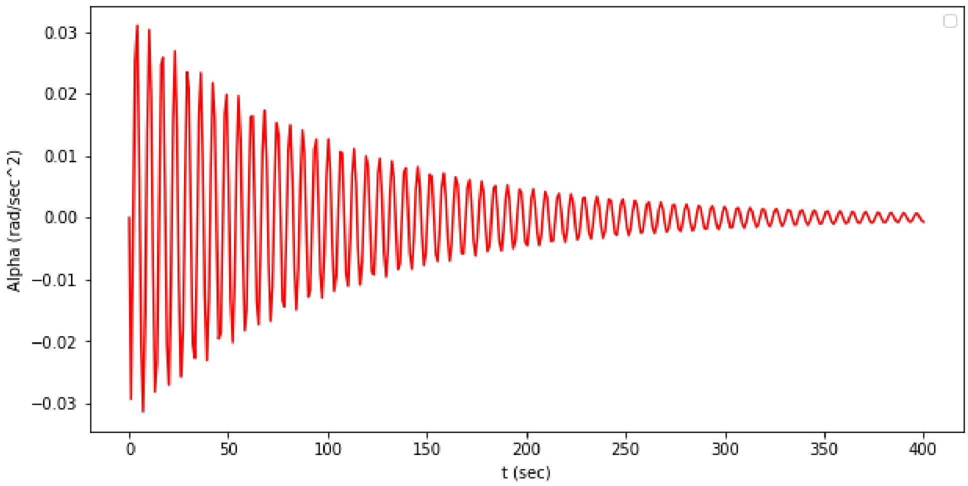

Figure 11: Pitch anglular acceleration in time domain for uncoupled pitch motion during phase V

Also, maximum values from the above plots are tabulated in Table 8.

Table 8: Maximum pitch angle, pitch angular velocity and pitch acceleration for uncoupled pitch motion during phase V

ψ_{max}	4 deg	Maximum pitch angle during uncoupled pitch motion
ψ_{max}	0.0698 rad	Maximum pitch angle during uncoupled pitch motion
$\dot{\psi}_{max}$	3.35E-02 rad/s	Maximum pitch velocity during uncoupled pitch motion
$\ddot{\psi}_{max}$	3.14E-02 rad/s^2	Maximum pitch acceleration during uncoupled pitch motion

6.2 Uncoupled Heave Motion During Phase V

Uncoupled heave motion during phase V can be represented using below equation[**8**];

$$(m + A_{33})\ddot{\zeta} + B_{33}\dot{\zeta} + C_{33}\zeta = F \tag{101}$$

$$(m + A_{33})\ddot{\zeta} + B_{33}\dot{\zeta} + \rho g A_{wp}\zeta = F \tag{102}$$

Table 9: Hydrodynamic coefficients for uncoupled heave motion during phase V

L	171.8 m	Length of the ship
B	36.5 m	Breadth of the ship
T	7.37 m	Draft of the ship
ρ_sw	1025 kg/m^3	Density of sea water
m	3.68E+07 kg	Mass displacement of ship
A_{33}	3.26E+07 kg	Added mass due to heave motion
m + A_{33}	6.94E+07 kg	Total mass displacement due to heave motion
C_{33}	4.60E+09 N/m	Hydrostatic coefficient for heave motion
B_{33}	1.13E+06 N.s/m	Damping coefficient for heave motion
ζ_0	0 m	Initial heave
$\dot{\zeta}_0$	0 m/s	Initial Heave velocity
ω_{33}	0.81 rad/s	Frequency for undamped heave oscillation
T_{55}	7.71 s	Time period for undamped heave oscillation

Following results are obtained for uncoupled heave motion. Heave displacement in meters, heave velocity(m/s) & heave acceleration (m/s^2) plots in time domain are as given below.

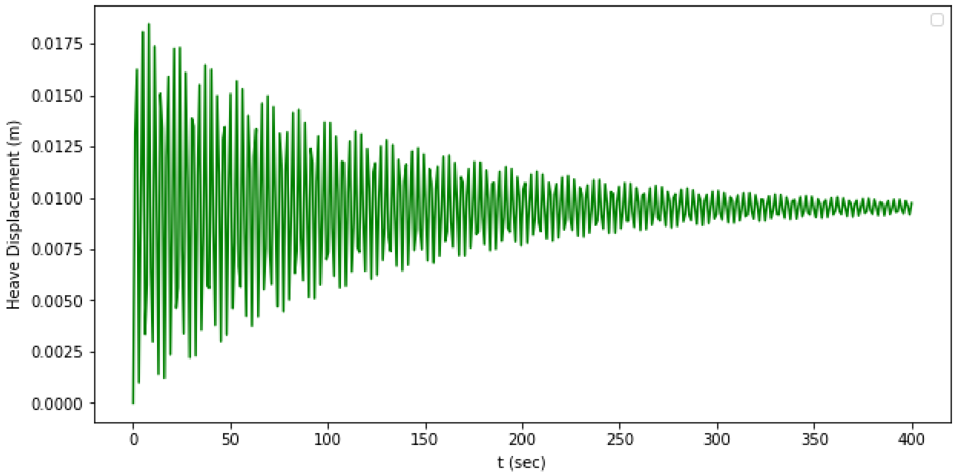

Figure 12: Heave displacement in time domain for uncoupled heave motion during phase V

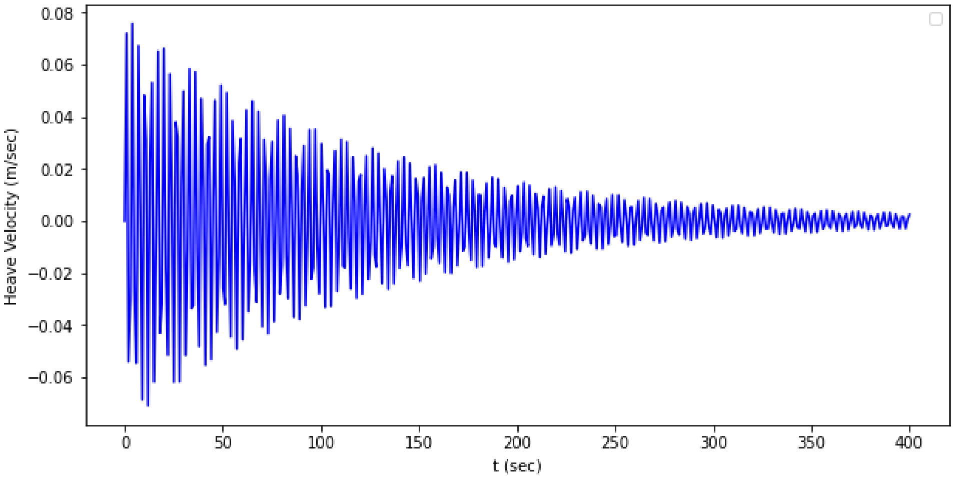

Figure 13: Heave velocity in time domain for uncoupled heave motion during phase V

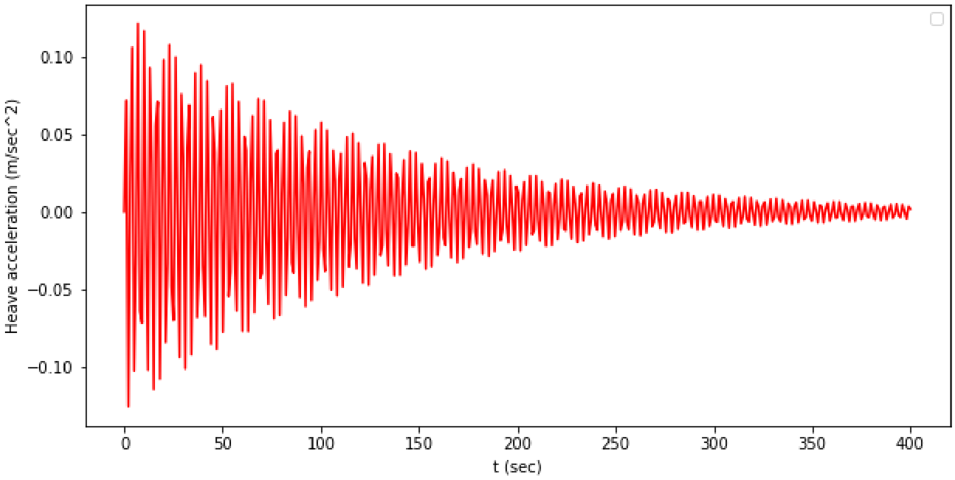

Figure 14: Heave acceleration in time domain for uncoupled heave motion during phase V

Also, maximum values from the above plots are tabulated in Table 10.

Table 10: Maximum heave displacement, heave velocity and heave acceleration for uncoupled heave motion during phase V

ζ_{max}	1.85E-02 m	Maximum heave displacement during uncoupled heave motion
$\dot{\zeta}_{max}$	7.58E-02 m/s	Maximum heave velocity during uncoupled heave motion
$\ddot{\zeta}_{max}$	1.26E-01 m/s^2	Maximum heave acceleration during uncoupled heave motion

6.3 Coupled Pitch-Heave Motion During Phase V

The linearized decoupled equation of motions for pitch and heave are as equations 62 & 63.

Using Green's theorem, following symmetry conditions can be derived for the zero forward speed case.

$$m_{35} = m_{53} \quad (103)$$

Table 11: Hydrodynamic coefficients for coupled pitch-heave motion during phase V

I_yy	3.69E+10 kg.m^2	Mass moment of inertia of Ship about transverse axis
A_{55}	4.21E+10 kg.m^2	Added mass moment of inertia of Ship about transverse axis
I_yy + A_{55}	7.90E+10 kg.m^2	Total mass moment of inertia of Ship about transverse axis
C_{55}	7.52E+10 N/m	Hydrostatic coefficient for pitch motion
B_{55}	1.54E+09 N.s/m	Damping coefficient for pitch motion
m	3.68E+07 kg	Mass displacement of ship
A_{33}	3.26E+07 kg	Added mass due to heave motion
m + A_{33}	6.94E+07 kg	Total mass displacement due to heave motion
C_{33}	4.60E+09 N/m	Hydrostatic coefficient for heave motion
B_{33}	1.13E+06 N.s/m	Damping coefficient for heave motion
$A_{35} = A_{53}$	5.64E+07 N.s^2	Added mass due to coupled heave-pitch motion
B_{35}	6.16E+08 N.s	Damping coefficient due to coupled heave-pitch motion
B_{53}	4.52E+05 N.s	Damping coefficient due to coupled pitch-heave motion
C_{35}	4.51E+09 N	Restoring coefficient due to coupled heave-pitch motion
C_{53}	2.76E+08 N	Restoring coefficient due to coupled pitch-heave motion

Following results are obtained for coupled pitch-heave motion. Pitch angle in degree, angular velocity(rad/s), angular acceleration (rad/s^2), heave displacement in meters, heave velocity(m/s) & heave acceleration (m/s^2) plots in time domain are as given below.

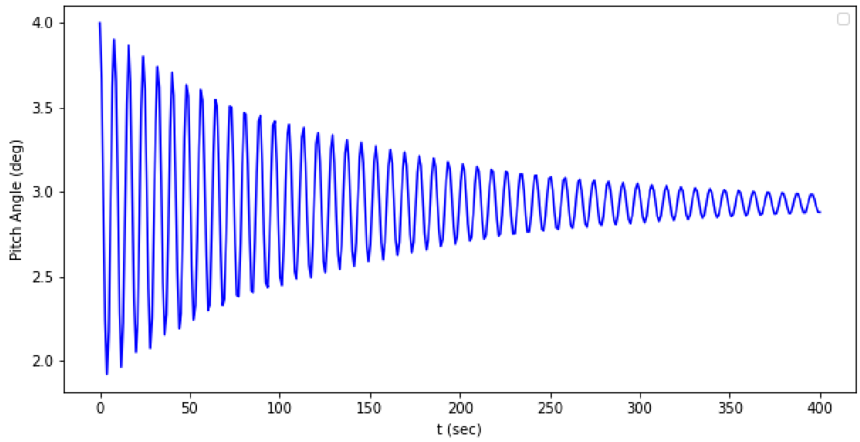

Figure 15: Pitch angle in time domain for coupled pitch-heave motion during phase V

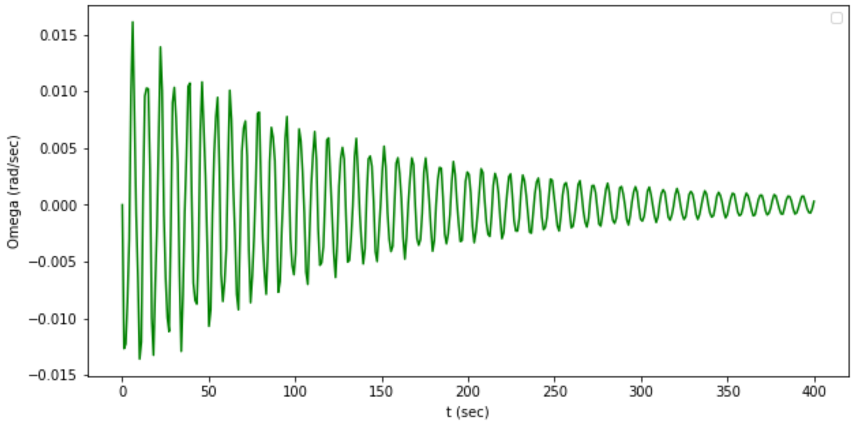

Figure 16: Pitch anglular velocity in time domain for coupled pitch-heave motion during phase V

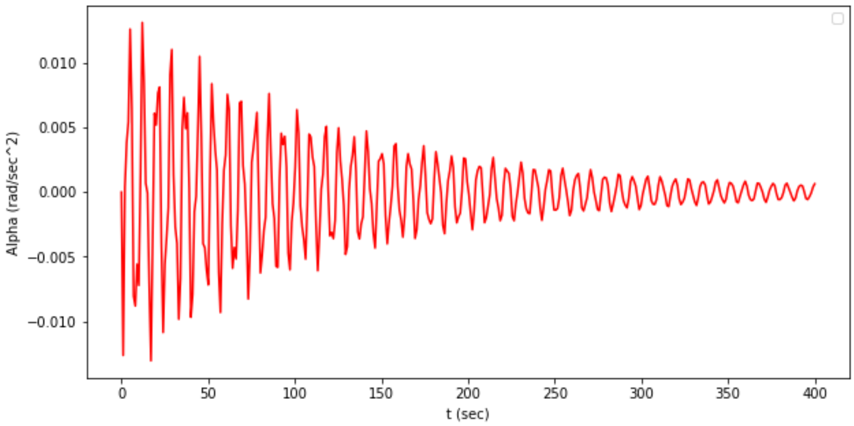

Figure 17: Pitch angular velocity in time domain for coupled pitch-heave motion during phase V

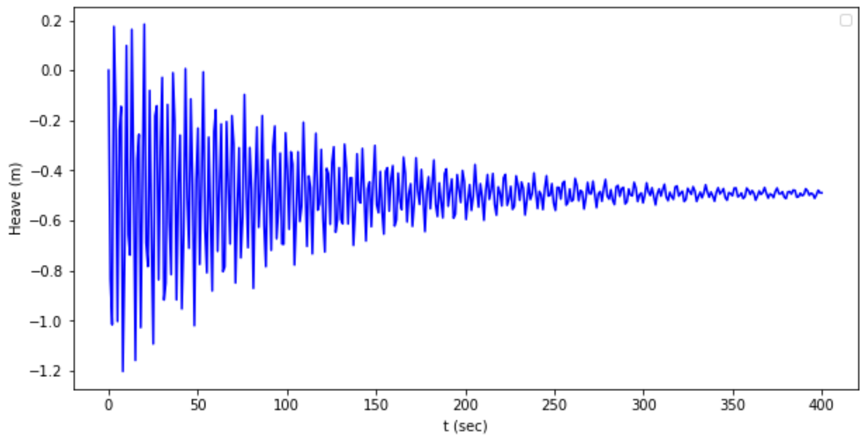

Figure 18: Heave displacement in time domain for coupled pitch-heave motion during phase V

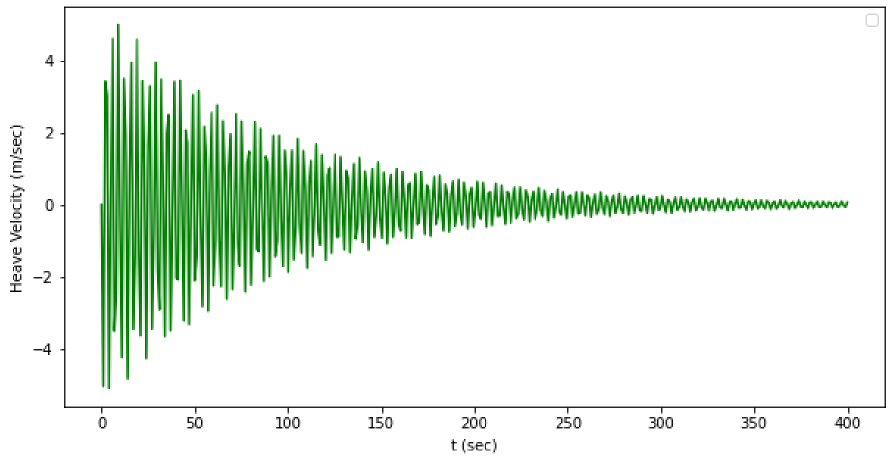

Figure 19: Heave velocity in time domain for coupled pitch-heave motion during phase V

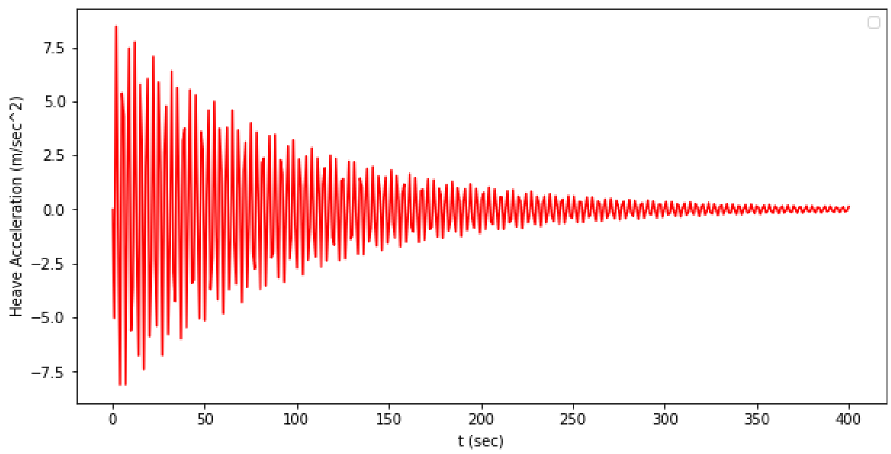

Figure 20: Heave acceleration in time domain for coupled pitch-heave motion during phase V

Also, maximum values from the above plots are tabulated in Table **12**.

Table 12: Maximum pitch angle, pitch angular velocity, pitch acceleration heave displacement, heave velocity and heave acceleration for coupled pitch-heave motion during phase V

$\psi_{\max\ \_coup}$	4 deg	Maximum pitch angle during coupled pitch-heave motion
$\psi_{\max\ \_coup}$	0.0698 rad	Maximum pitch angle during coupled pitch-heave motion
$\dot{\psi}_{\max\ \_coup}$	1.61E-02 rad/s	Maximum pitch velocity during coupled pitch-heave motion
$\ddot{\psi}_{\max\ \_coup}$	1.31E-02 rad/s^2	Maximum pitch acceleration during coupled pitch-heave motion
$\zeta_{\max\ \_coup}$	1.20 m	Maximum heave displacement during coupled pitch-heave motion
$\dot{\zeta}_{\max\ \_coup}$	5.10 m/s	Maximum heave velocity during coupled pitch-heave motion
$\ddot{\zeta}_{\max\ \_coup}$	8.45 m/s^2	Maximum heave acceleration during coupled pitch-heave motion

7 CONCLUSION

Here in this Thesis, coupled pitch and heave partial differential motion equations interact with each other and the solution of one Partial Differential Equation (PDE) influences the solution of the other PDE. Hence the solution of uncoupled pitch and heave motion gives independent results whereas coupled motion gives a more realistic solution.

After computing all the hydrodynamic coefficients for the given hull form, numerical simulation of the equations of motions are performed by inputting initial conditions.

This paper presents a mathematical modeling followed by numerical simulation of Ship's dynamics when launching SPAR from the transport vessel into water. The following conclusions are made:

- The mathematical model developed in this paper can represent the entire process of the SPAR launching from initial phase to final phase. However, numerical simulations are carried out for the final critical stage (phase V) and can gives the results of the ship's coupled pitch-heave motion parameters such as pitch angle, pitch angular velocity, pitch angular acceleration, heave displacement, heave velocity and heave acceleration in time domain.
- All added mass components except m_{11} and m_{55} can be solved using strip theory method and lewis conformal mapping, whereas m_{11} and m_{55} can be calculated using ellipsoid method using empirical formulas.
- The results obtained from numerical simulation of uncoupled pitch motion, uncoupled heave motion and coupled pitch-heave motion are within allowed limits proposed by class rules and regulations.
- One of the findings of the analysis is that heave displacement values during coupled pitch-heave motion vary significantly as compared to the values from uncoupled heave motion.
- According to the guidelines from the classification societies, the launching analysis should be validated by model test or using different commercial software. Model tests are mandatory to verify the results obtained by numerical simulations.

8 FUTURE SCOPE OF WORKS

In this Master Thesis, the coupled motion of the ship is considered as only pitch and heave (2 degrees of freedom). Future study can consider the effect of the ship's other degree of freedoms for a better result. In addition to that, following few recommendations can be suggested for future scope of works:

- A complete strength calculation of the transport vessel can be done with the following parameters to verify the values are within the allowable limits.
  - Tipping point / rocker arm reactions.
  - Longitudinal bending moment and shear forces experienced by the vessel during launching.
  - Loads & stresses on skid beam rails, sea fastening, SPAR and transport vessel.
- The results obtained from numerical simulations can be compared with the commercial software and model test methods.
- Sensitivity studies on following parameters can be carried out,
  - Hydrodynamic coefficients (SPAR and transport vessel).
  - Dynamic friction coefficient.
  - Initial trim angle.
  - SPAR's initial centre of gravity position on the vessel and mass.
  - Initial draft.
  - Other initial conditions required for the launch analysis.
- The detailed design and strength calculation of arrangements with wheels under the trestles, which are guided on rails in the main deck.

9 ACKNOWLEDGMENT

I would like to express my deepest gratitude to my thesis supervisor Dipl.-Ing. Mathias Vieth for his valuable suggestions, immense support and guidance during the thesis period. Also, I would like to thank Harald Arndt, Head of Research & Development for giving me an opportunity to work on this Master Thesis and providing necessary support during the internship period.

I would like to extend my sincere thanks to the Dr.-Ing. Patrick Kaeding, Dr.-Ing. Thomas Lindemann and Dipl.-Ing. Gunnar Kistner, my supervisors at the University of Rostock for their selfless assistance and guidance to complete this thesis.

Many thanks to the Prof. Philippe Rigo (coordinator of EMSHIP+ program, University of Liege) for giving me an opportunity to be part of this prestigious study program and his constant support throughout the EMSHIP+ journey. Also, special mention to Christine Reynders for their valuable assistance and great support during this study program. I wish to convey my extreme thanks to all professors who taught me in the EMSHIP+ program.

Last but not least I would like to thank my family for believing in me and supporting me throughout my life. My special thanks to my friends and colleagues who made this academic journey memorable.

Mohamed Asib Mundekkatt

10 REFERENCES

[1]. M Saeful M H1, Juswan1, and D Paroka1 (2020). "NC3 CPP Jacket Launching Analysis by Using Numerical Simulation (Case Study of NC3 Gas Field, SK316 Block Bintulu Sarawak Malaysia)"

[2]. Claudio A. Rodrıguez, Mario Moura, Paulo T. T. Esperanca¸ Jacques Raigorodsky (2014). "An Experimental Approach for the Offshore Launching of Jack-Ups"

[3]. Nikzad Nourpanah and Moharram Dolatshahi Pirooz (2008). "Numerical Modeling of Launching Offshore Jackets from Transportation Barge & the Significance of Water Entry Forces on Horizontal Jacket Members," Journal of Faculty of Engineering, Vol. 42, No. 6. December 2008, PP. 809-821.

[4]. Baigang Huang, Jianjun Jiang and Zaojian Zou (2021). "Online Prediction of Ship Coupled Heave-Pitch Motions in Irregular Waves Based on a Coarse-and-Fine Tuning Fixed-Grid Wavelet Network"

[5]. Do Thanh Sen & Tran Canh Vinh (2016). "Determination of Added Mass and Inertia Moment of Marine Ships Moving in 6 Degrees of Freedom", International Journal of Transportation Engineering and Technology. Vol. 2, No. 1.

[6]. Hassan Ghassemi & Ehsan Yari (2011). "The Added Mass Coefficient computation of sphere, ellipsoid and marine propellers using Boundary Element Method," POLISH MARITIME RESEARCH 1(68) 2011 Vol 18; pp. 17-26 10.2478/v10012-011-0003-1.

[7]. Captain D.R. Derrett, revised by Dr C.B. Barrass M.Sc C.Eng FRINA CNI (2006). "Ship Stability for Masters and Mates," Sixth edition – Consolidated 2006.

[8]. Prof. Dr.-Ing. habil. Nikolai Kornev (2012). "Ship dynamics in waves (Ship Theory II)"

[9]Technical Standard Committee (2016). "Guidelines for Steel Jacket Transportation & Installation"

[10]. Shaoyang Qiu, Hongxiang Ren and Haijiang Li (2020). Key Laboratory of Marine Dynamic Simulation and Control for Ministry of Communications, "Computational Model for Simulation of Lifeboat Free-Fall during Its Launching from Ship in Rough Seas"