A solution method for creating laminated wood panels with revalorized wood boards.

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A SOLUTION METHOD FOR CREATING LAMINATED WOOD PANELS WITH REVALORIZED WOOD BOARDS

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Abstract

The increasing demand for natural resources and the environmental commitments towards sustainable practices have led to a surge of interest in finding efficient solutions for the two-dimensional problem of creating laminated wood panels with revalorized wood boards. To address this challenge, sustainable and circular solutions are imperative. This thesis presents an exact solution to the problem, establishing its NP-hardness and justifying the need for an approximation method. This final solution method is developed in the form of a construction heuristic inspired by the concept of strip creation and draws inspiration from Best-Fit algorithms commonly used in Bin Packing.

Contents

List of abbreviations / Glossary

- BLC : Bottom-Left Corner
- CLT : Cross-Laminated Timber
- CSSP : Cutting and Skiving Stock Problem
- CSP : Cutting Stock Problem
- Glulam : Glued Laminated Wood
- ILP : Integer Linear Programming
- NP-hard : The complexity class of decision problems that are intrinsically harder than those that can be solved by a mathematical model in polynomial time.
- SCSP : Skiving and Cutting Stock Problem
- SSP : Skiving Stock Problem

1 Introduction

This first introductory section describes the problem after explaining why this topic is important in the current context. The section 1.3 explains why it is important to examine this topic at both managerial and academic levels. Subsequently, the expected benefits and added value of this thesis are exposed before describing how the dissertation is structured.

1.1 Context

High waste production has been a global problem for more than a century now. The increasing population and waste generation has led to rising global temperatures, i.e. the major challenge of environmental sustainability. Global warming, which has become the most important issue worldwide, has triggered sustainability as a critical goal of human activity and of the development of countries (Igarová, 2022). In order to limit the global warming increasing every year, everyone must reduce their culture and consumption of natural resources. From this perspective, the United Nations Framework Convention on Climate Change (UNFCCC) adopted the Paris Agreement, a new climate treaty in 2015 in Paris. This Agreement has been ratified by at least 55 countries involved in the UNFCCC and accounting for at least 55 per cent of the total greenhouse gas emissions. The main collective long-term goal of the Paris Agreement is to limit the increase in global average temperature to below 2°C above the pre-industrial levels. It also wants to continue to take action to limit the increase in temperature to 1.5°C. Moreover, all the Parties have committed to showing their contributions, communicating their ambitions and therefore being transparent and showing solidarity in order to limit global warming (Savaresi, 2016).

One of the phenomena that undeniably contributes to the depletion of natural resources and global warming is the ever-increasing demand for materials and raw materials such as wood. In addition, as explained by Braghiroli and Passarini (2020), Zbieć et al. (2022) and Pr. Tom De Mil from the Faculty of Gembloux Agro-Bio Tech at the University of Liège, more than 50 per cent of the yearly total mass of raw wood material harvested are planer shaving and mill residues such as sawdust, resulting from the first stage of manufacturing processes. These wastes are not necessarily considered economically important as the ideal uniform thickness of a wood element is difficult to obtain from these operation residues (Braghiroli and Passarini, 2020). Moreover, the production of waste is not only influenced by human activity such as production, but also by consumer behavior, maturity and lifestyle (Igarová, 2022). It means that, every year, hundreds of thousands of tons of raw wood waste are not only generated in the form of planer shaving and sawdust, but also in the form of beams, boards, pallets, cladding, doors, furniture, Oriented Strand Board (OSB) or Medium-Density Fibreboard (MDF) panels. The OSB is a type of engineered wood similar to particle board, formed by adding adhesives and then compressing layers of wood flakes in specific orientations, and the MDF s a type of board made from very small pieces of wood that have been pressed and stuck together. All these raw wood material wastes most of the time come from construction purposes and can therefore be related to timber. This type of wood is a natural buffer for carbon dioxide (CO2) as it stores this greenhouse gas. Unfortunately, this interesting propriety is not at all consider by everyone as more than 60 per cent of harvested timber is burned every year in Europe (Zbieć et al., 2022). Countries should thus pay more attention to promoting sustainable consumption and recycling behavior than only seeking to minimize waste generation (Igarová, 2022). Indeed, revalorizing this type of timber wood waste, instead of incinerating it, would have a positive impact on waste reduction efforts as well as keeping the carbon dioxide stored longer in these CO2 sinks. Moreover, this kind of timber waste also represents an economic advantage as it can be used to create wood-based panels, which have already been commercialized and are greatly demanded because of their desirable characteristics and properties (Braghiroli and Passarini, 2020).

Finally, the revalorization of these raw wood wastes, which are in the form of beams, boards,

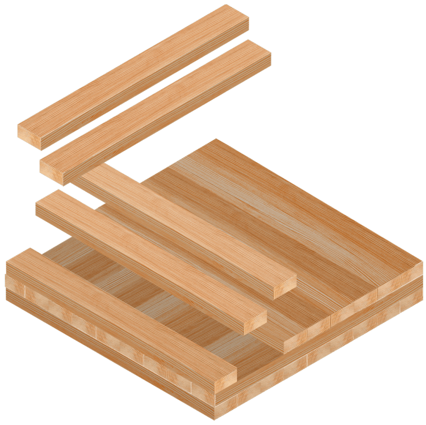

Figure 1: Arrangement of timbers to create CLT (Retrieved May 2, 2023, from `https://www.thinkwood.com/mass-timber/cross-laminated-timber-clt`).

pallets, cladding, doors, furniture, or even OSB or MDF panels; has several advantages both ecological and economic. From a sustainable point of view, this solution answers the urgent and critical need to reduce the consumption of raw materials and to limit the impact of humans on the environment. Indeed, by recycling such wood wastes and giving them a second life without burning them, this also allows for keeping the CO2 stored thanks to the wood properties. Moreover, from an economical point of view, these different raw wood waste materials could be considered as individual wood slats of different sizes that could be combined, assembled and glued to form Glued Laminated Wood (Glulam) and Cross-Laminated Timber (CLT) panels. A slat is a strip of wood or other thin material, while the Glulam is a structural material manufactured through the union of individual wood segments; and CLT is a wood product made of several layers of timber arranged crosswise and glued together as in Figure 1. These types of panels are already valued and used in the construction industry to replace concrete and steel for instance. Issa and Kmeid (2005) who studied the properties of Glulam, and Brandner et al. (2016) who published an overview of CLT and its development can provide more information to the readers about these types of wood panels. Dalimier (2022) also studied this topic last year in the framework of his master thesis at the University of Liège.

1.2 Problem statement

The aim of this research thesis is therefore to optimize the recycling of the different raw wood wastes that are in the form of beams, boards, pallets, cladding, doors, furniture, or even OSB or MDF panels, by considering them as wood slats of different sizes which can be assembled, combined and glued to create laminated wood based panels as Glulam and CLT panels. In other words, the aim is to generate two-dimensional (length x width) layouts composed of the different slats as shown in Figure 2, as we consider that all the slats have the same thickness. These layouts depict the positions of the revalorized wood slats that would produce one or more target panels. Thus, the layouts must logically correspond at least to the two dimensions, i.e. length by width, of the laminated wood panels that are to be created. It is possible and likely that the generated layouts will have dimensions in excess compare to the panels to be created. These excesses will then be cut off and called the surplus. In addition, it is necessary to join and glue the slats together to create the different panels, which involves making joints along the length of the panel. The joints method used between each slat along the length of the layer of CLT

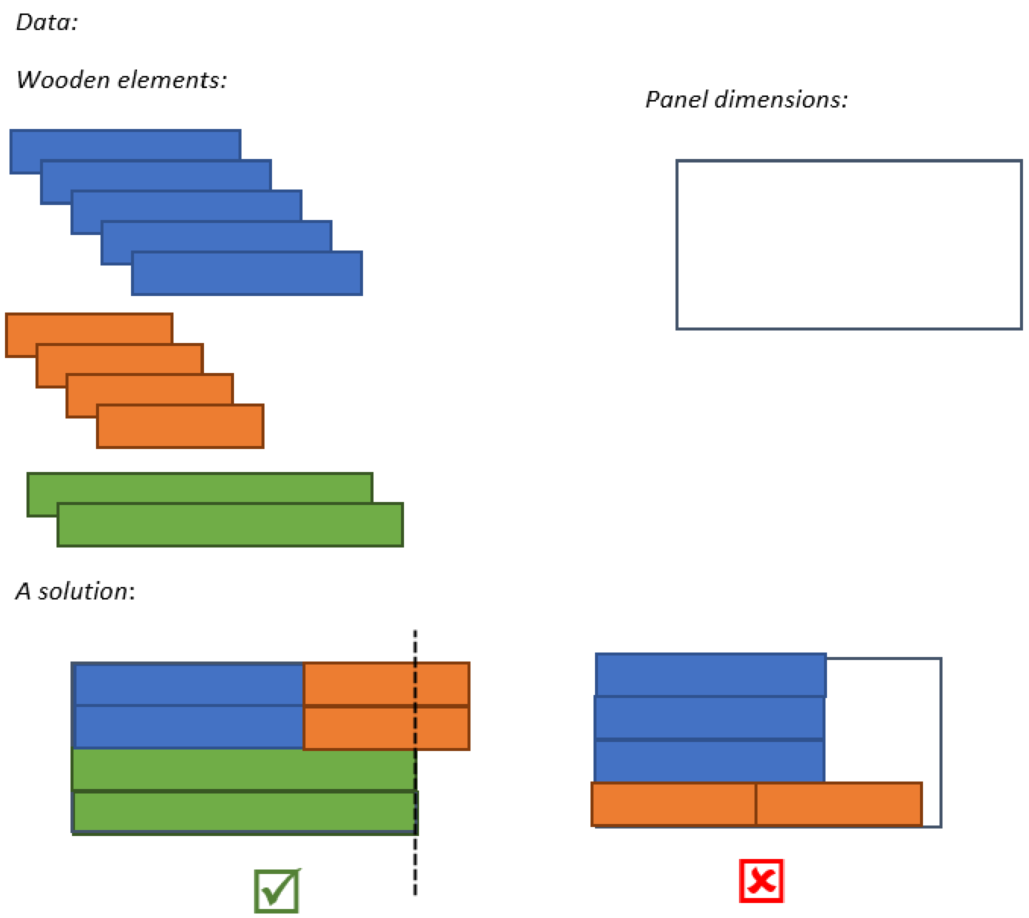

Figure 2: Example of a layout solution that creates one panel for a certain dataset (Documentation provided by Célia Paquay).

panels are "finger-joints". These joints are quite specific and are shown in Figure 3. Obviously, the more joints there are in a laminated wood panel, the more fragile the panel will be. Thus, as discussed with Pr. Tom De Mil, in the reality of the field, the panels with a lot of joints will be used as internal layers of CLT panels, as these layers will not be in direct contact with the external surrounding.

Knowing the different basic characteristics of the problem, several constraints can be taken into account and several objectives can be considered. Firstly, the characteristics of the panels to be created are important in determining the objective, and therefore the problem. Indeed, there will be several cases depending on the amount (fixed or unrestricted) and dimensions (homogeneous or heterogeneous) of panels to be created. If the dimensions of the panels are heterogeneous, we can also consider that each type of panel has an associated value.

Subsequently, other factors need to be taken into account, such as the importance of the wood surplus in the optimization of the solution and the number of joints for example. The main constraints related to this problem are the fact that the generated layouts should be at least as big as the dimensions of the panels to be created, and the fact that the slats cannot overlap in the generated layouts. Hence, there are a multitude of different configurations and objectives for this problem. Here is a non-exhaustive list of possible objectives:

- Maximize the number of created panels (with an unrestricted number of panels to be created of homogeneous dimensions)
- Maximize the total value of the different types of created panels (with a fixed or unrestricted number of panels to be created of heterogeneous dimensions and associated values)

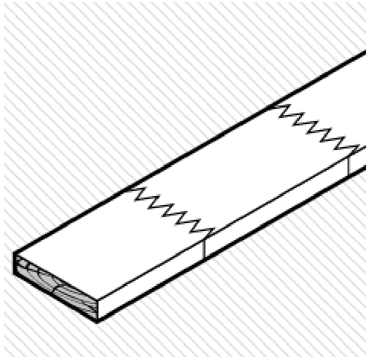

Figure 3: Glued joints between slats for the production of CLT panels : The individual panel slats are glued together in the longitudinal direction by a finger joint in order to transmit the forces. This way, a strip is created from the short individual slats. (VDMA woodworks, 2023)

- Minimize the total waste, i.e. sum of the surplus (in all cases)

It can be noted that minimizing the number of joints does not seem to be a relevant objective function and will be taken into account as a constraint if needed. Nevertheless, these different possibilities concerning the objective of the problem show the extent and richness of this optimization problem. Moreover, as mentioned, the dimensions of the panels and slats can vary from one case to another. It is therefore important in this research work to develop a solution method that can deal with different cases to make sure that this solution method is applicable in a maximum of configurations, regardless of the characteristics of the dimensions.

At first sight, a solution method for this optimization problem seems simple to implement with a mathematical model. However, identifying the layouts that make the best use of these wooden slats is a real challenge. Indeed, it will soon become apparent that the number of possible slats' layouts to be explored grows exponentially with the number of slats. This problem therefore falls into the category of NP-hard problems. This means, using the words of Garey and Johnson (2009), that "I can't find an efficient algorithm, but neither can all of these famous people." In other words, there is no known efficient way to find the optimal solution of such a problem. We then enter in a combinatorial optimization problem in which the set of feasible solutions is discrete or can be reduced to discrete, and in which the goal is to find the best solution (Žerovnik, 2015). The generation of these feasible solutions as well as the search for the optimal solution can be done thanks to heuristics and metaheuristics respectively. A heuristic is technique designed for solving a problem quicker when classic methods are too slow or fail to find any exact solution, while a metaheuristic is an optimization algorithm aimed at solving difficult optimization problems for which the most efficient classical method is unknown. For these types of solution methods, it is necessary to determine a trade-off between optimality and efficiency during their development and implementation. Indeed, if the optimal solution is unapproachable, it might be a lucid decision to sacrifice optimality and settle for a good feasible solution that can be computed efficiently (Hochbaum, 1997).

For our problem, it means that in a situation where there are a lot of slats, which will be the input data for the mathematical models; these models cannot provide the optimal solution, i.e. the best layouts, or event any solution in an acceptable and polynomial computing time. Hence, it is necessary to develop and implement a heuristic or a metaheuristic in order to provide a solution method that finds a good feasible solution to the problem in an efficient way. This solution method should be as flexible as possible as it should be able to deal with a maximum of configurations, including when there are a lot of slats and regardless of the dimensions' characteristics.

1.3 Research motivations

As already presented in Section 1.1, this topic presents both economic and ecological benefits and therefore addresses the critical issues of global warming and natural resource depletion. These issues affect everyone, including businesses. The solution method developed through this research work can therefore help companies to optimize the production of laminated wood panels by trying to find the best layouts, but also encourage them to recycle and be part of a more circular economy by revalorizing these wood wastes initially intended for incineration. This more sustainable vision of companies' activity is also part of the efforts towards reducing their carbon footprint, as well as of the Corporate Social Responsibility (CSR) of companies. According to Hopkins (2004), "the wider aim of social responsibility is to create higher and higher standards of living, while preserving the profitability of the corporation, for peoples both within and outside the corporation". Beyond preserving the profitability, the general solution method developed and provided through this research thesis could certainly have a positive impact on the profitability of companies. Indeed, in addition to making its products and services more attractive to consumers by being involved in some aspects of the CSR (Hopkins, 2004), the costs related to the raw materials needed to manufacture laminated wood panels, i.e. the wood wastes to be recycled, is lower compared to wood products made from virgin raw materials. These costs may even equal zero if the company recycles its own surplus raw materials that would have otherwise been junked or burned.

1.3.1 Managerial motivations

From a managerial point of view, this solution method for creating laminated wood panels with revalorized wood boards can easily be transposed to other managerial issues. Indeed, the general problem can be related to other topics than wood revalorization and the production of laminated wood panels. As a concrete example, we can talk about synthetic and organic grass football pitches, which are very expensive products in terms of acquisition and transport costs. Indeed, local amateur football clubs may be interested in reusing and recycling grass strips from wealthier professional football clubs, where the pitch has to be changed on a regular basis to meet the high-quality standards required and expected by all stakeholders. This could be a win-win situation, as professional clubs would be able to recover money from their used football pitches instead of scrapping them. This would allow them to enter a more circular, sustainable and environmentally friendly economy. On the other hand, local amateur clubs could benefit from better quality grass pitches at a more affordable price, which could help them improve their quality standards, optimize their spending and become part of a more circular economy.

This reuse and recycling of football pitches example can be enlarged to other sport disciplines which play on grass pitches. In addition, there could be a multitude of different possible second users. For example, outside playgrounds for kids, public parks and amusement parks; as well as companies and private individuals who own green areas and gardens. It becomes therefore quite complex to efficiently assign the different original grass strips to each customer. The grass strips could then be cut into several smaller grass slats which allow greater flexibility in optimizing layouts and minimizing the surplus for installations at the next users. Hence, the solution method initially developed for the problem of wood boards revalorization can be applied to other areas, which is very interesting from a managerial point of view.

It can also be mentioned in the managerial motivations that this research falls within the framework of the United Nations Sustainable Development Goals (SDGs). Indeed, a solution method for solving the studied problem will have a positive impact on the SDG No.12 related to Responsible Consumption and Production, as well as on the SDG No.15 related to Life on Land (United Nations, 2016).

1.3.2 Academic motivations

The main academic motivation for this research topic concerning the production of laminated wood panels with revalorized wood boards is that it is hardly addressed in the scientific literature in this specific form. This research topic can be likened to cutting and packing problems. Indeed, these problems can appear under various names in the literature, although they essentially follow the same logical structure which is to combine small items geometrically and assign them to large objects resulting in figures with possible residual pieces usually treated as trim loss (Dyckhoff, 1990). However, as stated earlier, the specific form of cutting and packing problems explored in this research work has never really been studied yet. Hence, from an academic point of view, it is very interesting to study something that represents a gap in the literature. Moreover, as this problem is NP-hard, experimental methods are explored in this thesis to find a first appropriate solution method for this complex optimization problem. Finally, as briefly mentioned before, it is also stimulating and fulfilling to study and work on a topic which, in addition to being related to purely theoretical subjects such as combinatorial analysis which have not yet been studied in this specific form, provides a solution to current global problems such as environmental issues..

1.4 Contributions

Even if a master thesis is not a peer reviewed scientific paper, this research work can hopefully contribute to the literature and try to fulfill the current gap as mentioned in Section 1.3.2. This work can also serve as basis and motivation for future research on the topic. Indeed, as explained throughout Section 1, this work could contribute to the implementation of a practical solution method concerning the optimization of the recycling and revalorization of wood boards. Those could serve as the basis to create laminated wood panels as well as cross-laminated timber panels that can replace concrete and steel in the construction industry. This practical solution method can also be applied to other similar issues and has economic and ecological benefits for those who apply it.

In a nutshell, the added value of this master thesis research work contributes firstly at the scientific level. It then adds value on an economic point of view, as the presented solution method solves a concrete and complex combinatorial optimization problem, related to the optimization of production and therefore the optimization of costs and surpluses. Lastly, this research thesis aims to contribute, in its own way, to the environmental cause by bringing a positive impact on both the issues of global warming and the over-exploitation of natural resources such as wood.

1.5 Approach

This dissertation is structured as follows: after this introduction Section 1, it consists of a literature review of other cutting and packing problems that come as close as possible to the specific form of the problem in Section 2. This literature review then ends with detailing the research objectives and the methodology used in this work respectively in Section 2.2 and Section 3.1 before moving on to a chapter dedicated to the development of the different solution methods. In this development Section 3, two mathematical models are first presented to solve a first simplified version of the studied problem in Section 3.2 and 3.3. Subsequently, the broader problem is considered without the assumption that all slats have the same width, and a third mathematical model capable of solving small instances of this complex problem is presented in Section 3.4. The limits of such a model found by testing justify a heuristic development presented in Section 3.5. These different sections also present the respective analyses of the different mathematical models and the heuristic one, all tested with real data. Thereafter, the different results obtained are discussed in Section 3.6. Finally, Section 4 includes a conclusion and the limitations and suggestions for future research. pagebreak

2 Theoretical framework

This part of the thesis aims at exploring the literature before clearly stating the research objective of this work.

2.1 Literature review

The specific problem of creating laminated wood panels with revalorized wood boards can be likened to cutting and packing problems, for which the reader can refer to Dyckhoff (1990) and Wäscher et al. (2007). This literature review section aims at identifying and exploring articles that deal with relevant ways of solving variants of the cutting and packaging problems closest to the problem studied in this research thesis.

The studied problem can be compared to the two-dimensional bin packing problem where the goal is to fill the largest number of bins exactly or with overflow, which is the definition of the "dual bin packing problem" given by Labbé et al. (1995). Ackermann and Diessel (2020) also studied a hierarchical approach for solving a packing problem with exact guillotine cuts in production of Glulam in sawmills. In addition to the link that can be made with our studied problem about the problem type, the general topic of this article is clearly related to this research thesis problem. Indeed, as already mentioned, the revalorized wood boards could be used to produce Glulam as well. However, the solution method provided in the article for the optimization of the pressing process in Glulam production is not extendable to our problem. Indeed, the objective of this article is to minimize the amount of waste required to complete the entire press, while minimizing the number of height changes between consecutive pressing steps as each change requires set up time. Here, the press has a minimal and maximal length as well as a minimal and maximal height. The dimensions of the pressing step, i.e. the ordered pieces plus the filling material, must be greater than or equal to the minimum length and height, but they cannot be greater than the maximal length and height. Hence, the only similarity between our problem and the one of Ackermann and Diessel (2020) is that no empty space is allowed in the production processes studied. Indeed, filling material considered as waste has to be added if the length and height of the ordered pieces do not reach the minimum size required. Similarly, no empty space is allowed in the production of our laminated wood panels as the production tries to avoid having any hole in the panels of our problem.

Cid-Garcia and Rios-Solis (2020) present a methodology to find optimal solutions to two-dimensional bin packing problems called Positions and Covering. This methodology consists of generating a set of valid positions that indicates all the possible ways of packing item into the different bins. This could be transposed to our problem by considering all the valid positions that the different slats can occupy into the different panels. In the same idea, Seizinger (2018) defined a coordinate system, or grid points, indicating where an item is placed into the different bins. This grid and coordinates system can again be transposed to our problem.

Blum and Schmid (2013) presented a hybrid evolutionary algorithm in order to solve two-dimensional bin packing problems. This algorithm is strongly based on a one-phase approach called the Improved Lowest Gap Fill proposed by Wong and Lee (2009). Very good results have been obtained using this hybrid evolutionary algorithm compared to current state-of-art approaches. Other hybrid algorithms consisting in a combination of metaheuristic algorithms (Genetic Algorithm, Simulated Annealing and Naïve Evolution) and heuristic packing routine or hill-climbing local search heuristic, have been compared by Hopper and Turton (2001). They found out that the different metaheuristics outperform the heuristic packing routine and the local search heuristic. More precisely, the Simulated Annealing showed the best results in achieving the best layout. However, its execution time growths quickly with the increase of the problem size. On the other hand, Genetic Algorithm and Naïve Evolution yield slightly worse results than the Simulated Annealing, but they need less computational time. Nevertheless, they conclude their comparison by stating that all of these hybrid algorithms are relevant in the context of industrial demands, which is the framework of our research problem. The choice between Simulated Annealing, Genetic Algorithm or Naïve Evolution mainly depends on the available execution time for solving the problem.

These different heuristics have been reviewed by Oliveira et al. (2016) in a survey that focus on the heuristic resolution methods for the two-dimensional rectangular strip packing problem. This problem is a specific cutting and packing problem where all items are characterized by a width and a height. Here a link with our research problem can be made again as the input data, i.e. the different slats to be revalorized, are also characterized by two dimensions, i.e. a length and a width. Through this review, they show that multiple search strategies, as constructive and improvement heuristics, can be implemented in order to solve this variant of cutting and packing problems. Among these constructive heuristics, the Best-Fit heuristic seems relevant to consider as a potential source of inspiration for the solution method developed for the creation of laminated wood panels with revalorized wood boards. An effective Best-Fit heuristic for the two-dimensional regular stock-cutting problem has been presented by Burke et al. (2004), and an efficient implementation of this heuristic for the rectangular strip packing problem is proposed by Imahori and Yagiura (2010), who also showed that the quality of the solution improves as the number of rectangles increases. Verstichel et al. (2013) improved the results of the original Best-Fit heuristic by adding new item orderings and item placement strategies.

Coming back to the variant called dual bin packing problem by Labbé et al. (1995) and mentioned at the beginning of this literature review, Peeters and Degraeve (2006) developed a branch-and-price algorithm for this problem that outperforms the different algorithms of Labbé et al. (1995, 2003). They also stated in their article that there is a counterpart of this kind of problem in the literature called Skiving Stock Problems (SSP), indeed discussed by Zak (2003). He defines it as a particular case of the set-packing problem. These SSPs, as well as different solution methods for it, have been studied in the recent years by Martinovic and Scheithauer (2016), Martinovic et al. (2020), Karaca and Samanlioglu (2022) and Korbacher et al. (2023). On the other hand, still as stated by Zak (2003) in his article, there is another class of combinatorial optimization problems called the cutting stock problems (CSP) which are similarly particular cases of the set-covering problem. Solution methods for this variant have been studied by Haessler and Sweeney (1991) for the one- and two-dimensional CSP, by Song et al. (2006) for the 1.5-dimensional CSP and by Wuttke and Heese (2018) for the two-dimensional CSP with sequence dependent setup times. Most recently, Fang et al. (2023) proposed an algorithm based on deep reinforcement learning that solved efficiently the one-dimensional CSP with the help of a deep neural network.

Do Nascimento et al. (2022) deals with the two-dimensional CSP where surpluses are called leftovers and are optimized so that they can be used to satisfy a future demand. This powerful idea seems promising and could be transposed to the problem studied through this research thesis. Indeed, this solution would have a strong economic and ecological impact for companies, which is totally in line with the motivations of this research work. Moreover, from a mathematical point of view, Do Nascimento et al. (2022) propose to consider the concept of strips that divide the plates in horizontal strips in which items can be assigned side by side. To address the topic of this thesis, the plates could be compared to the panels in our problem whereas the items could be the slats. In the same idea, Chen et al. (2019) studied the challenge faced in the paper and plastic film industry involving surplus pieces from prior cutting or production errors. These two-dimensional surpluses can potentially be reassembled by adhering them together, resulting in potential in-processing orders. This double problem of successive gluing and cutting seems to be a combination of SSPs and CSPs explored previously.

This double problem is called the Skiving and Cutting Stock Problem (SCSP) in the recent literature and is well suited for industry production issues. Thus, SCSP seem to be the closest variant to our studied problem, as my supervisor Célia Paquay said. Johnson et al. (1997) seem to be among the first to have studied the way of solving this kind of problem. Indeed, they examined the Cutting and Skiving Stock Problem (CSSP) within the paper industry as well. The CSSP involves a two-step process: first, large reels are cut into smaller finished rolls and auxiliary rolls. Thereafter, the auxiliary rolls are glued together widthwise to create additional finished rolls. Notably, the CSSP cutting step uses one reel, whereas the SCSP often utilizes multiple reels that are glued together. Additionally, the finished rolls produced by CSSP have glue joints running along the length direction, while those produced by SCSP have glue joints along the width direction. Due to the significant difference in roll length versus roll width, the total length of the glue joints in CSSP is much greater than the one in SCSP. As a result, the finished rolls from SCSP are more desirable than the ones from CSSP (Chen et al., 2019). Later, Arbib and Marinelli (2005) investigated a CSSP in a European plant that produced gear belts. They developed an optimization model for the production process involving two steps again, with reels initially cut into rectangular pieces and then stitched together as needed (Chen et al., 2019).

For the problem in paper and plastic film industries recently studied by Chen et al. (2019), several reels are glued together to form a "reels-pyramid", from which rolls are formed and cut. In this case, the reels and the rolls could be related respectively to the slats and the panels of our problem. However, the length of the reels is greater than the one of the rolls, which is the complete opposite situation in our studied problem. Wang et al. (2020) studied the two-dimensional SCSP with setup costs. This specificity is not interesting for our problem creating laminated wood panels. Nevertheless, in the same vein than Seizinger (2018) with the grid and coordinates system, Cid-Garcia and Rios-Solis (2020) with the set of valid positions and Do Nascimento et al. (2022) with the strips; the way Wang et al. (2020) address the SCSP based on column-and-row generation could be a source of inspiration in the development of the solution method for the studied problem.

2.2 Research objectives

As stated before, this research aims at developing a solution method for creating a maximum of glulam or CLT wood panels and thus reusing and revalorizing raw wood wastes. In the framework of this master thesis and under the guidance of Célia Paquay, the supervisor of this work, a simplified version of the problem has firstly been solved with mathematical models before developing another mathematical model to solve the global issue. The limits of this last model were then tested and determined before developing an efficient heuristic solution which can deal with a lot of input data. Moreover, an online meeting with Pr. Tom De Mil has been organized in December 2022. The goal was to clarify some aspects of the problem, as well as to obtain some guidance from an expert on the definition of the problem in order to get as close as possible to the reality of the field. Thus, hypothesises have been made jointly for the remaining of this research work to find a balance between realism, challenge and feasibility.

Firstly, we decided that it is preferable to set the objective of the problem as the maximization of the total number of produced panels in the framework of this research thesis. Thus, only on type of panel is produced and the goal is to produce a maximum of panels. The dimensions of the panels to be created are the following: 200 x 60 cm. These are existing panel sizes from real material shops. Furthermore, these dimensions have been validated by Pr. Tom De Mil as being standard and therefore suitable for this research work.

Secondly, the revalorized wood boards, i.e. slats, of less than 20 cm long are not considered. Indeed, there is very few short boards, or they are simply not considered for the creation of laminated wood panels. Thus, this hypothesis concerns mainly the creation of data sets for testing the different models, as in the real data there is a marginal chance of having slats measuring less than 20 cm. This hypothesis comes in the framework of my personal research thesis. Indeed, it might be interesting to consider short slats in future research, which study a dynamic version of the problem where the surplus can be reuse in the creation of future panels (Do Nascimento et al., 2022).

It has also been decided to apply a factor 5 to the data when the global problem is studied. It means that both the length and the width of the different slats and the panels to be created are divided by 5 and rounded to the nearest integer. As explained in details further in this research thesis, this reduces the complexity of the mathematical model that aims at solving the general problem. In addition, the data are more homogeneous, especially concerning the width of the different slats, which helps the developed heuristic to find better layouts given its characteristics.

Ultimately, the mains research objectives of this thesis are to prove that an exact method cannot find a solution in polynomial time and therefore to develop an approximation method capable of quickly finding a solution that optimizes the number of 200 x 60 cm panels created regardless of the number of input data.

3 Developments and Results

The present chapter is divided in six main sections. Whereas Section 3.1 states the research methodology, the four following parts present, detail and analyze respectively the three different mathematical models and the heuristic. They also aim at presenting the results obtained with these solution methods. Lastly, Section 3.6 discussed the lessons from the different tests that are drawn and already starts to summarize my research before leading the reader to the conclusion. As already stated in Section 2.2, the first two models deal with a simplified version of the problem studied where all the input data, i.e. the slats, have the same width; while the third mathematical model and the heuristic deal with the whole problem.

3.1 Research methodology

This study consists of creating two mathematical models with different objectives in order to solve a simplified version of the studied problem before considering the whole problem. Thus, the goal is to start exploring the problem and to find a solution method in the theoretical case where all slats would have the same width. These two models are detailed and analyzed in Section 3.2 and 3.3, where the associated results are also respectively presented. Subsequently, the global problem is considered without the assumption that all slats have the same width. As already mentioned, this combinatorial optimization problem becomes very complex at this point. Thus, a third mathematical model capable of solving small instances of this complex problem is developed and detailed in Section 3.4, where its associated results and limitations are also shown. However, it will appear that it mainly helps to determine the limits of such a mathematical model for the studied problem and to justify the development of a heuristic, which is presented and described in Section 3.6 before displaying the associated results and tests. Finally, the different results obtained are and discussed before drawing conclusions from it.

This "step by step" approach has been defined in agreement with my supervisor Célia Paquay as a consequence of the literature gap already mentioned in this thesis. From a theoretical point of view, an abductive approach is followed all over this research thesis since there is no existing theory concerning this specific form of the problem. Indeed, the mathematical models and the heuristic has been implemented based on my academic background and with the help of my supervisor. Naturally, it is also necessary to mention that this background sometimes had to be completed by some ideas from the existing literature. Therefore, this part of the thesis, including the development and the implementation of the three mathematical models and the heuristic as well as their respective analyses is both "from data to theory" and "from theory to data", which corresponds to an abductive approach (Dufays, 2022).

The data used for the third mathematical model and the heuristic are based on the data provided by Pr. Tom De Mil via Dalimier (2022) who measured a real sample of wood boards to be revalorized in the framework of his master thesis. These raw data are attached in Appendix 1. It is worth noting that a typing mistake in the wood board with the identity number 53 has been assumed to further test the research models. Indeed, the work mentions a width of 93 cm and length of 67.7 cm. Given the width of the other boards and the assumed fact that width should be smaller than the length, I considered the width of the wood board with the identity number 53 as equal to 9.3 cm instead of 93 cm. Moreover, as mentioned in Section 2.2, a factor 5 is applied to the different dimensions in order to reduce the complexity of the problem. The transformed data are attached in Appendix 2 and are called "Dataset 0". On the other hand, for the first two mathematical models dealing with the simplified version of the problem, datasets have been generated both randomly and arbitrarily. These datasets are presented in more details in Section 3.2 and 3.3.

Lastly, all models presented in this thesis have been coded and implemented on Atom version 1.59.0, a free and open-source text and source code editor. The programming language used to code is Julia version 1.8.3, and the solver used to solve the three different mathematical models is Gurobi version 9.5.0, an optimization software using branch and bound method. Concerning the hardware, all calculations have been run on an Asus brand laptop with an Intel® Core™ i3-7100U processor. Moreover, for the different mathematical models, a time limit of one hour has been set, again in agreement with my supervisor. Ultimately, "Visual Paradigm Online" tool has been used to create the Flowchart describing the developed constructive heuristic.

3.2 First mathematical model

This section describes the mathematical formulation of the first developed model before presenting the tests performed and the associated results. In addition to assuming that all the available wood boards to revalorized have the same width (i.e. 15 cm), the objective function of this model is to minimize the total surplus while producing five laminated wood panels. The dimensions shown in Table 1 represent the dimensions of each panel to be created knowing that there are enough slats to create all the panels.

Table 1: Panels to be created for the first mathematical model.

Panel ID nb	**Length**	**Width**
1	150	100
2	250	100
3	250	100
4	300	150
5	300	200

3.2.1 Description of the model

In order to solve this first model, the different panels to be created are divided into several strips of 15 cm as all the slats have the same width equal to 15 cm. This idea is inspired from Chen et al. (2019), Wang et al. (2020) and Do Nascimento et al. (2022), and is illustrated in Figure 4. In this illustrative example, the panel has a width of 70 cm while the strips have a width of 20 cm. It can be noticed that the fourth strip will have to be cut at least along the length of the panel that will be created to respect its dimensions. Below is the first mathematical model which solves the simplified problem and minimizes the total surplus obtained after the production of the five panels.

Assumptions

- All the slats have the same width.
- All the panels are created (enough slats to create the panels).

Constant

$w =$ width of the slats $= 15$ cm

Indices

$p = 1, \ldots, P$	for the panels
$s = 1, \ldots, S$	for the slats
$st = 1, \ldots, N_p$	for the strips (of panel p)

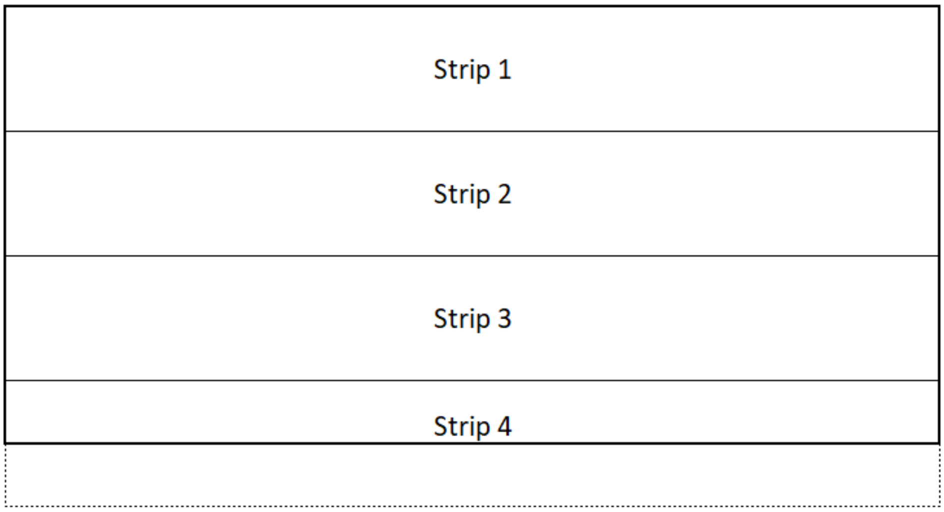

Figure 4: Decomposition of a panel into strips of the same width (Personal drawing on Excel).

Parameters

L_p = length of panel p
W_p = width of panel p
l_s = length of slat s
N_p = number of strips needed to cover panel p with $N_p = \left\lceil \frac{W_p}{w} \right\rceil$

Variables

$$X_{p,st,s} = \begin{cases} 1, & \text{if slat } s \text{ is used in strip } st \text{ of panel } p. \\ 0, & \text{otherwise.} \end{cases}$$

Objective function

$$\min \sum_{p=1}^{P} \left[\sum_{st=1}^{N_p} \left(\sum_{s=1}^{S} (X_{p,st,s} * l_s) - L_p \right) + L_p * (w * N_p - W_p) \right]$$

Constraints

1. A slat can only be used once:
   $\sum_{p=1}^{P} \sum_{st=1}^{N_p} X_{p,st,s} \leq 1$ for all s
2. The length of each strip must be longer or equal to the length of the panel:
   $\sum_{s=1}^{S} (X_{p,st,s} * l_s) \geq L_p$ for all p and st
3. Integrity constraint:
   $X_{p,st,s} \in \{0, 1\}$ for all p, s and st

This mathematical model is an Integer Linear Programming (ILP) model as the linear objective function is subjected to a set of linear constraints over integer variables. Indeed, the variable $X_{p,st,s}$ acts as a decision variable as it can only be equal to 1 if a certain slat s is used in a strip st of a panel p; or equal to 0 otherwise. The objective function is the computation of the total surplus that have has been cut throughout the production of the five panels. There can be a surplus along the length of the panels as already mentioned above and illustrated in Figure 5 as

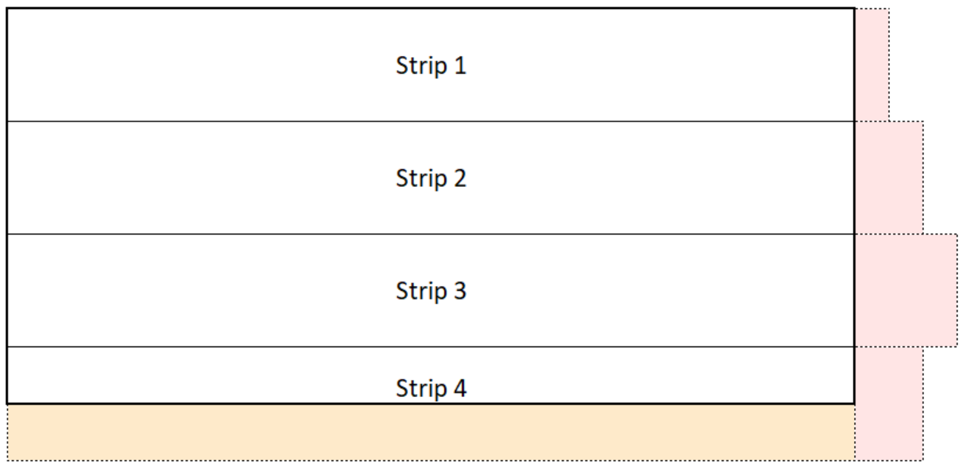

Figure 5: Potential surpluses to be cut throughout the production of the laminated wood panels (Personal drawing on Excel).

the "orange surplus", but also along the width of the panels depending on the composition of the different strips and thus the value of $X_{p,st,s}$. These potential surpluses along the width of the panels are illustrated in Figure 5 as the "red surplus". It can be noticed that in this simplified version of the problem, the "orange surplus" will remain constant as the width of the slats is constant, and the widths of the five panels to be created are already known and fixed. Hence, given the assumptions, this mathematical model can only minimize the "red surplus", while the "orange surplus" will always be constant. Its value then depends on w and on the dimensions of the panels to be created. Ultimately, it means that the total surplus to be minimized will always be greater or at least equal to the "orange surplus".

3.2.2 Tests and results

To test this mathematical model, a dataset has been created. As all the slats have the same width, a set of 150 slat ID numbers and associated lengths has been generated on Excel. The lengths have been generated randomly to be equal to a random integer between 50 and 200 cm. This dataset is attached in Appendix 3 and is called "Dataset 1". The result, i.e. the total surplus, obtained when the mathematical model is run with Dataset 1 is equal to 6520 cm^2, which is the optimal solution and corresponds to the constant "orange surplus" in Figure 5. The creation of the five panels implies that there is no surplus along the width of the panels. This comes from the fact that the randomly generated Dataset 1 used to test this model is very heterogeneous because of its generation method. In other words, the lengths of the slats have such different values that the model can find combinations of strips that are perfectly equal to the length of the panels to be created. Thus, to test the generation of "red surplus", another 150 slats more homogeneous dataset, called "Dataset 2", has been arbitrarily generated with only for different slat sizes. After running the mathematical model with Dataset 2, the total surplus is equal to 6904 cm^2, which means that surpluses along the width of the panels have been generated. Both tests and associated results are presented in Table 2.

This first mathematical model is a first step towards solving a very simplified version of the problem mentioned in this thesis before studying the whole problem. There is thus no need for other tests or further explorations for this model.

Table 2: Tests and associated results for the first mathematical model.

	Dataset 1	**Dataset 2**
Solution (cm^2)	6250	6904
Optimal solution?	Yes	Yes
Computing time (sec)	10	17

3.3 Second mathematical model

The second mathematical model developed in the framework of this master thesis is based on the first one with a different objective function. The goal here is to solve the simplified version of the problem with the objective determined with Pr. Tom De Mil, which is to maximize the total number of one type of panel to be created.

3.3.1 Description of the model

This second ILP model also assumes that all the slats have the same width. The same method is applied as the panels are divided into several strips (Chen et al., 2019; Wang et al., 2020; Do Nascimento et al., 2022). As a reminder, for this second model, all the panels to be created have the following same dimensions: 200 x 60 cm. Another small difference with the first model is that the width of the slats equals 18 cm in the description of the mathematical model. As the target panels have now a width of 60 cm, which is a multiple of 15, it was considered relevant to use a slat width that was not a multiple of the panel width, in order to directly test the creation of surplus over the length of the panels. The mathematical formulation of this second model is provided below.

Assumptions

- All the slats have the same width.
- There is only one type of panels to be created (200 x 60 cm).

Constant

w = width of the slats = 18 cm
L_p = length of a panel = 200 cm
W_p = width of a panel = 60 cm
N_p = number of strips needed to cover a panel $p = \lceil \frac{W_p}{w} \rceil = 4$

Indices

$p = 1, \dots, P$	for the panels	with $P = \lfloor \frac{S}{N_p} \rfloor$
$s = 1, \dots, S$	for the slats	
$st = 1, \dots, N_p$	for the strips (of panel p)	

Parameters

l_s = length of slat s

Variables

$$X_{p,st,s} = \begin{cases} 1, & \text{if slat } s \text{ is used in strip } st \text{ of panel } p. \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_p = \begin{cases} 1, & \text{if panel } p \text{ is created.} \\ 0, & \text{otherwise.} \end{cases}$$

Objective function

$$\max \sum_{p=1}^{P} Y_p$$

Constraints

1. A slat can only be used once:
   $\sum_{p=1}^{P} \sum_{st=1}^{N_p} X_{p,st,s} \leq 1$ for all s
2. The length of each strip must be longer or equal to the length of the panel:
   $\sum_{s=1}^{S} (X_{p,st,s} * l_s) \geq L_p * Y_p$ for all p and st
3. Integrity constraint:
   $X_{p,st,s} \in \{0, 1\}$ for all p, s and $st = 1, \ldots, N_p$
   $Y_p \in \{0, 1\}$ for all p

It can be noted that P is defined as the maximum number of panels that can be created if one slat per strip is sufficient. Indeed, a strip must be composed of at least one slat to be created. Thus, a panel cannot be created if no slat is assigned to its different strips. This trick aims at reducing the size of the model by fixing a maximum number of panels to be created respecting the dimensions of the slats. This maximum number of panels can be considered as an upper bound. Moreover, another binary decision variable Y_p is added compared to the previous model. This decision variable takes the value 1 if a panel p is created, or 0 otherwise. In this case, all the panels have the same dimensions. This variable relates thus more to the number of panels that could be created and is used in the second constraint in order to assign slats to strips of panel p only if this panel p is created.

3.3.2 Tests and results

It was concluded in the first model that Dataset 1 was very heterogeneous and Dataset 2 very homogeneous with only four different lengths for 150 slats. Thus, to make further analysis of this second model, it was decided to create another arbitrary 150 slats dataset called "Dataset 3", which is more balanced between homogeneity and heterogeneity. This second model has thus been tested with this Dataset 3 and the width of the slats equals 18 cm. The obtained solution equals 21, which means that 21 panels are created. Figure 6 represents the layout of the first panel created in this non-optimal solution obtained after three minutes of computation before reaching the time limit of one hour without any improvement. It can be noticed that there are surpluses both along the length and the width of the created panel. This second mathematical model has also been tested with Dataset 1, Dataset 2 and the real data provided by Pr. Tom De Mil. A slats' width of 18 cm was thus considered in order to test the creation of surplus along the length of the panels. The different results are displayed in Table 3.

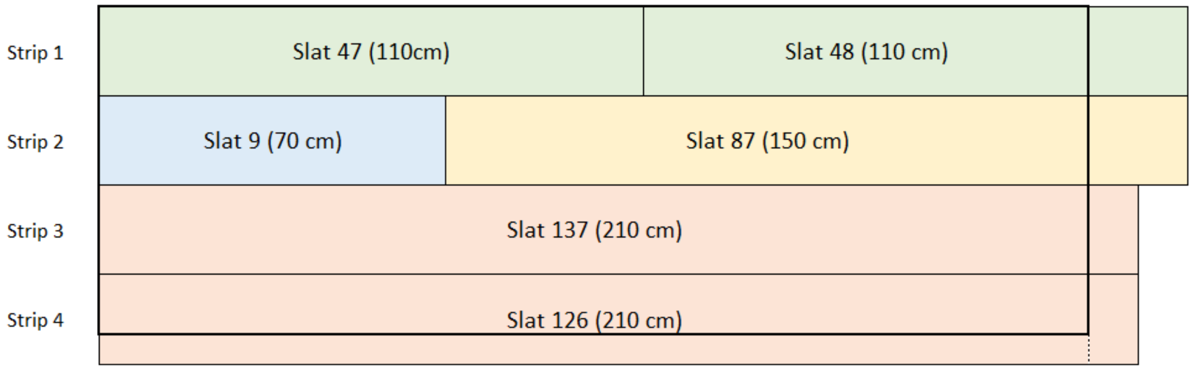

Figure 6: Layout of the first panel created in the solution obtained after running the second model with Dataset 3 and a slats' width of 18 cm (Personal drawing on Excel).

Table 3: Tests and associated results for the second mathematical model with a slats' width of 18 cm.

	Dataset 1	**Dataset 2**	**Dataset 3**	**Real Data**
Number of slats	150	150	150	142
Type of data	Heterogeneous	Homogeneous	Balanced	Real
Solution	18	18	21	13
Best bound	18	18	23	13
Optimal solution?	Yes	Yes	No	Yes
Total waste (cm^2)	128070	99900	113454	42990
Computing time (sec)	273	1370	3600	2

It can be noticed that there is a big difference between the computing times. The test with the real data only takes 2 sec to find the optimal solution, which could be explained by the smaller optimal number of panels created. As already mentioned, the test with Dataset 3 reaches the time limit without finding the optimal solution. This could be explained by the solution and upper bound that are greater than the ones for the other three tests. It seems logical that the test with Dataset 1, which is heterogeneous, finds the optimal solution quicker than the second test with Dataset 2, which is homogeneous. Indeed, with Dataset 2, the mathematical model has to explore more configurations as it is not as apparent than with heterogeneous slats. However, there is less waste generated with the homogeneous dataset than with the heterogeneous one, which seems illogical. An explanation could be that as the model needs to explore more with the homogeneous dataset to find the optimal solution, the optimal layout optimizes the surpluses more than the solution yielded with Dataset 1. Indeed, with heterogeneous slats, the model needs less time to find a solution equal to the optimal which could yield a solution with less optimized surpluses. Even though these results seem a bit paradoxical, the objective of the model is to maximize the number of panels created, not to minimize the surpluses. Ultimately, as for the first model, it does not seem very interesting to make other tests or to explore further limits of this second model as it only deals with a simplified version of the problem.

3.4 Third mathematical model

The next step of this research work was to consider the problem as a whole. This means that the slats can have different width dimension. This mathematical model is therefore much more constrained than the previous two because the arrangement of the slats becomes more complex to explore.

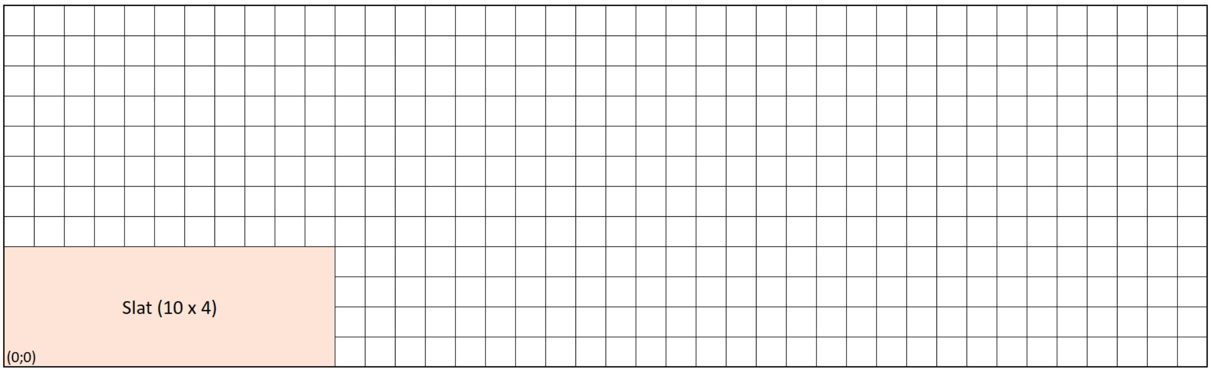

Figure 7: A panel (200 x 60 cm) gridded in squared zones of 5 cm sides (Personal drawing on Excel).

3.4.1 Description of the model

This model has been developed mixing two ideas. The first one is to construct a matrix of all valid positions of each slat. The objective is then to fill it in with "1" if a slat occupies certain positions, and with "0" otherwise (Cid-Garcia and Rios-Solis, 2020). The second idea is to grid the panels, where the coordinates represent the Bottom Left Corner (BLC) of each position (Seizinger, 2018). Indeed, both ideas are used since the panels to be produced are gridded in 5 cm square zones, and a new binary variable Z_s, p, i, j is equal to 1 if the position is occupied by a slat, or 0 otherwise. The length of these 5 cm square zones was determined together with Pr. Tom De Mil and my supervisor Célia Paquay in order to simplify the model while remaining as realistic as possible. As it is easier to work with units, all dimensions of the slats and panels were divided by five and rounded to the nearest integer, i.e. a factor 5 is applied to the different dimensions. Concretely, it means that a slat that is 50 cm long and 20 cm wide now has a length of 10 units and a width of 4 units. It also occupies a total of 40 square areas on a panel to be created. This is shown in Figure 7 when the BLC of this example slat is placed on the coordinates (0;0).

The mathematical model must now explore every potential combination of every possible position for all slats unlike the previous two models using strips. Thus, a first usefulness of applying a factor 5 to the dimensions can be seen here as it transforms the 12,000 possible positions, keeping the centimeter as a unit (200 x 60 cm), to be explored on a panel for each slat into 480 positions (40 x 12 zones of 5 cm sides). Despite this, the model includes a lot of integer variables and constraints which are not linear anymore and is therefore an Integer Programming (IP) model. The resulting size of the model makes it much more time consuming to solve. The mathematical formulation of this third model is presented below.

Assumptions

- The slats can have different widths.
- There is only one type of panels to be created (200 x 60 cm).
- Consider all possible positions that a slat can occupied within a panel.
- The coordinate (i, j) corresponds to the BLC of a panel squared zone.

Constant

L_p = length of a panel = 200 cm
W_p = width of a panel = 60 cm

Indices

$p = 1, \ldots, P$	for the panels	with $P = \lfloor \frac{\sum_{s=1}^{S}(w_s * l_s)}{L_p * W_p} \rfloor$
$s = 1, \ldots, S$	for the slats	
$i = 0, \ldots, I$	for the position along the length of a panel	with $I = L_p - 1$
$j = 0, \ldots, J$	for the position along the width of a panel	with $J = W_p - 1$

Parameters

l_s = length of slat s
w_s = width of slats s

Variables

$$X_{s,p,i,j} = \begin{cases} 1, & \text{if the BLC of slat } s \text{ is positioned on coordinate } (i,j) \text{ of panel } p. \\ 0, & \text{otherwise.} \end{cases}$$

$$Z_{s,p,i,j} = \begin{cases} 1, & \text{if the position } (i,j) \text{ of panel } p \text{ is occupied by slat } s. \\ 0, & \text{otherwise.} \end{cases}$$

$$Y_p = \begin{cases} 1, & \text{if panel } p \text{ is created.} \\ 0, & \text{otherwise.} \end{cases}$$

Objective function

$$\max \sum_{p=1}^{P} Y_p$$

Constraints

1. A slat can only be used once:
   $\sum_{p=1}^{P} \sum_{i=0}^{I} \sum_{j=0}^{J} X_{s,p,i,j} \leq 1$ for all s
2. All positions of a panel must be occupied in order to create the panel:
   $\sum_{s=1}^{S} \sum_{i=0}^{I} \sum_{j=0}^{J} Z_{s,p,i,j} \geq (L_p * W_p) * Y_p$ for all p
3. A position can only be occupied by one slat (no overlap allowed):
   $\sum_{s=1}^{S} Z_{p,s,i,j} \leq 1$ for all p, i and j
4. If a position (i,j) is occupied by the BLC of a slat s, the following "dimensions of the slat" positions of the panel are also occupied by slat s:
   $X_{s,p,i,j} * min\{l_s; L_p - 1\} * min\{w_s; W_p - j\} \leq \sum_{k=i}^{min\{L_p; i+l_s\}} \sum_{l=j}^{min\{W_p; j+w_s\}} Z_{s,p,k,l}$

   for all p, s, i and j
5. If the BLC of slat s occupies the position (i,j), the following "dimensions of the slat" positions of the panel are also occupied:
   $\sum_{p=1}^{P} \sum_{i=0}^{I} \sum_{j=0}^{J} Z_{s,p,i,j} \leq \sum_{p=1}^{P} \sum_{i=0}^{I} \sum_{j=0}^{J} X_{s,p,i,j} * min\{l_s; L_p - i\} * min\{w_s; W_p - j\}$

   for all s
6. Integrity constraint:

   $X_{p,s,i,j} \in \{0,1\}$ for all p, s, i and j
   $Z_{p,s,i,j} \in \{0,1\}$ for all p, s, i and j
   $Y_p \in \{0,1\}$ for all p

As in the previous model, P is defined to reduce the size of the model by only exploring the potential maximum number of panels that could be created given the total area covered by all the slats. It can also be noticed that I and J are respectively defined as $L_p - 1$ and $W_p - 1$. This is simply explained by the fact that the first coordinate of a panel is $(0, 0)$ instead of $(1, 1)$.

3.4.2 Tests and results

Firstly, all the dimensions from Dataset 0, based on the real data provided by Pr. Tom De Mil, were divided by 5 and rounded to the nearest integer. The model was then run with this Dataset 0 and gives the best objective of 0 and the best bound of 9 in a computing time of one hour, which is the time limit. Hence, the model does not find any solution in one hour with this dataset. It means that the size of the model is too big to find a solution in reasonable computing time, which is quite limiting. Thus, the model cannot be used in real situation.

Therefore, it might be interesting to determine the limits of this third model to support the relevance of developing a heuristic solution method. In order to do so, the model has been tested with growing numbers of slats until it does not find any solution in one hour. In other words, N slats are randomly selected from Dataset 0 to form a new smaller dataset. These random selections following by their associated test are done three times with the same number of random slats equal to N before repeating the process with $N + 1$ slats until no solution is found in one hour. Initial N has been arbitrarily set to 15, because with less than 15 slats, it is in most of the cases impossible to create at least one panel given the dimensions of the real data. All the tests are compiled in Table 4.

Table 4: Tests and associated results for the third mathematical model with random sample of slats from Dataset 0.

Test	Nb slats	Best Solution	Best Bound	Time (sec)
1	15	0	0	0
2	15	0	0	0
3	15	1	1	140
4	16	1	1	209
5	16	0	0	0
6	16	0	0	0
7	17	0	0	98
8	17	1	1	1551
9	17	1	1	229
10	18	1	1	156
11	18	1	1	617
12	18	1	1	307
13	19	1	1	1487
14	19	1	1	265
15	19	1	1	343
16	20	1	1	1100
17	20	1	1	617
18	20	1	1	293
19	21	1	1	623
20	21	1	1	144
21	21	1	1	555
22	22	1	1	88
23	22	1	1	329

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Table 4 – continued from previous page

Test	Nb slats	Best Solution	Best Bound	Time (sec)
24	22	1	1	277
25	23	1	1	115
26	23	1	1	852
27	23	1	1	466
28	24	1	1	297
29	24	1	1	132
30	24	1	1	106
31	25	1	1	400
32	25	1	1	189
33	25	1	1	1160
34	26	1	1	639
35	26	1	1	594
36	26	1	1	989
37	27	1	1	432
38	27	1	1	2829
39	27	1	1	1264
40	28	1	1	1273
41	28	1	1	158
42	28	1	1	165
43	29	1	1	825
44	29	1	1	191
45	29	1	2	3600
46	30	1	1	272
47	30	0	2	3600
48	30	1	1	747
49	31	1	2	3600
50	31	0	2	3600
51	31	0	2	3600
52	32	2	2	2657
53	32	0	2	3600
54	32	2	2	1639
55	33	2	2	1001
56	33	0	2	3600
57	33	2	2	1575
58	34	0	2	3600
59	34	0	2	3600
60	34	2	2	1255
61	35	0	2	3600
62	35	2	2	1041
63	35	0	2	3600
64	36	0	2	3600
65	36	2	2	3385
66	36	2	2	1925
67	37	0	2	3600
68	37	0	2	3600
69	37	0	2	3600
70	38	2	2	1431
71	38	0	2	3600

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Table 4 – continued from previous page

Test	Nb slats	Best Solution	Best Bound	Time (sec)
72	38	0	2	3600
73	39	0	2	3600
74	39	0	2	3600
75	39	0	2	3600

Firstly, we can notice that with 15, 16 and 17 slats, the best solution is sometimes zero which can be explained in two ways. The first one is illustrated with Tests 1, 2, 5 and 6. Here the computing time is also equal to zero which may mean that before even starting to compute, the total area of the slats forming the sample does not allow to create a panel. This is due to the random generation of sample data for the test. Therefore, only slats with relatively small dimensions could be selected from Dataset 0. On the other hand, with Test 7, it can be noticed that the best solution equaling zero is obtained with a computing time of 98 sec. This may mean that unlike the first case, before starting to compute, the total area of the slats composing the sample for the test is greater than the area of a panel to be created, and therefore the upper bound was equal to 1 at the beginning of the test. However, after 98 sec of branch and bound, the solver finds that the best bound is equal to 0 and therefore the best solution is also equal to 0. This means that the best possible layout does not allow to create a panel because of the surpluses that must be cut and that reduce the total area of the slats when combined.

After a series of tests that all find the optimal solution in less than an hour, the first test to reach the time limit is Test 45 with 29 slats. After that, it can be noticed that from 30 slats onwards, the mathematical model does not manage to find the optimal solution before the set limit in more than half of the tests. Moreover, from 37 to 39 slats, all the tests reached the time limit with a best solution equal to 0 except Test 70. Therefore, it seems that the model is able to find a solution within an hour with a number of slats equaling 37 or less. Hereafter, the mathematical model needs more than an hour to find a solution greater than 0 with the computer equipment used for the tests in this master thesis framework. Hence, 37 slats can be defined as the limit for the third mathematical model in this research process.

All the tests and associated results have been represented in Figure 8, which presents the computing time per number of slats in the form of a scatter chart. Despite an inconsistency in the computation times for the same number of slats probably due to the random nature of the generated data samples, we can observe that from 30 slats, most of the tests reach the time limit of one hour. This strongly reflects the limit of this mathematical model which considers the problem in its entirety. Although it allows to optimize the production process of laminated wood panels, its combinatorial characteristic makes it unusable with more than 37 slats. This is very few and therefore makes this mathematical model useless for planning large-scale laminated wood panels production.

3.5 Heuristic

After having determined the limits of the previous mathematical model which considers the problem as a whole, it seems relevant to develop a heuristic solution for a larger number of input data. Furthermore, the aim of this heuristic solution is to propose the most agile method possible for this research work.

First, a constructive heuristic method based on the BLC method was explored Chazelle (1983); Burke et al. (2006). The main difficulty was not to avoid leaving any holes in the panel construction, which differs from the Bin Packing problems for which this method is effective. It was therefore quite difficult to build an initial solution considering all the slats. Under the advice of my supervisor Célia Paquay, I am thus opting for a constructive heuristic based on the

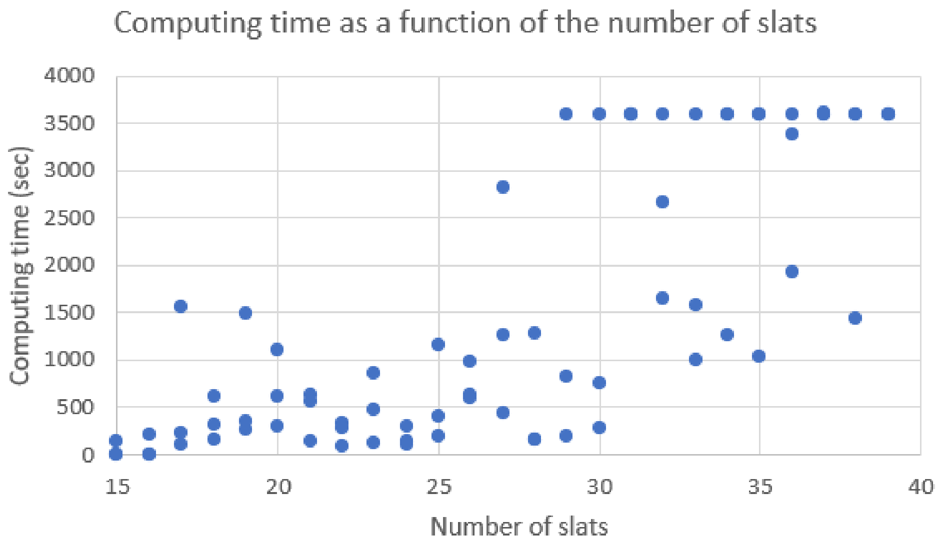

Figure 8: Computing time in seconds as a function of the number of slats from Dataset 0 for the tests of the third mathematical model (Personal generation on Excel)

formation of strips with slats of the same width, which joins the idea implemented in the first two mathematical models and inspired by Chen et al. (2019), Wang et al. (2020) and Do Nascimento et al. (2022). There is here again a major interest in applying a factor of 5 to the real data. This factor indeed makes the widths of the different slats more uniform since they are divided by 5 and rounded to the nearest integer. There will therefore be more slats with the same width and it will be possible to create more strips that are at least the length of the panels.

3.5.1 Description of the heuristic

The developed heuristic is a constructive heuristic as it builds a solution following an arbitrarily determined procedure. In general, the slats are assigned in a decreasing way with respect to both their length and their width. This means that the algorithm will first try to create a strip of the width corresponding to the maximum width of the slats. Then, in each strip, the slats are assigned in decreasing order of their length. The goal of this method is to keep the shortest and narrowest slats for the end to retain maximum flexibility. However, some subtleties are implemented through the algorithm to make it agile. Indeed, to create a good initial solution following the descending order assignment procedure, a "Best-Fit" approach is followed, inspired by the works of Burke et al. (2004), Imahori and Yagiura (2010) and Verstichel et al. (2013). The algorithm looks for the slat length or strip width that is most suitable at strategic points in the procedure. Each time a strip is about to be completed, i.e. to reach the length of the panel to be created, the algorithm looks for the slat to be assigned that will generate the least surplus. In the same idea, each time the last strip of a panel is about to be generated to complete the panel along its width, a strip with the most adequate width is chosen according to the remaining availability. Finally, after having generated a solution, the algorithm will explore the different surpluses again from the most important to the least important. The algorithm tries to swap one of the slats composing the strip with a slat not yet assigned, i.e. being part of *Rest*, in order to reduce the surplus generated by the strip in question. These steps are detailed with the presentation of the algorithm hereunder.

The diagrammatic method of Flowcharts has been used to represent the algorithm. The flowchart has been designed on "Visual Paradigm Online" and is presented in Figure 9. This flowchart is also provided as a .png file with this research thesis to enable the reader to zoom in and thus read and examine the algorithm in a more pleasant way. The first part of the algorithm is in fact a kind of giant loop that builds a panel before starting the same construction process for the next panel and so on until there are not enough slats to finish the panel under construction. Once a solution is obtained, highlighted in blue on the flowchart, the algorithm moves on the second part where it tries to improve the solution by reducing the surplus generated by exchanging slats of the obtained solution with remaining slats still available.

To begin with, the first part of the algorithm starts by creating a subgroup called "*Sgroup*" with all the slats that have the highest width called "*max*". If the total length covered by the slats forming this subgroup is greater than or equal to the length of the panel, the strip creation starts. If not, all the slats in *Sgroup* are assigned to the "*Rest*" and will not be used anymore in this first part of the algorithm. *Sgroup* then takes all the available slats with a width equal to "*max - 1*" until *Sgroup* allows the creation of strips to begin. The slats are assigned in descending order of their length. That is, the longest slat belonging to *Sgroup* will be assigned first in the strip under construction until the longest slat to be assigned allows the strip to be completed. In this case, the algorithm does not directly assign this slat but looks for the slat that will generate the smallest surplus. This is therefore the first strategic place in the algorithm where the idea of "Best-Fit" is applied.

Then, the strip creation process is repeated in order to build the panel and create the last strip fitting the completion of the panel. Here, the algorithm first looks if the current strip width completes the panel without creating any surplus along the length of the panel. If it does, the strip is then created and the loop starts again from the beginning to create the next panel. If there is a surplus with the current width of the strip, the algorithm first looks if it is possible to create a strip with the exact missing width with the available slats. If not, it looks to create a strip with a width of 1, then, if not, it will look if it is possible to create a strip with a width of 2 and so on in increasing order. If a smaller width strip than the one needed is created, the search process starts again. The goal is to be as flexible as possible in the creation of the last strips to complete a panel that generates as little surplus as possible. This is the second time the algorithm follows the "Best-Fit" logic. This completes the loop that makes up the first part of the algorithm that is designed to build the panels.

Once there are not enough slats of the same width to create strips of the length of the panels, or the maximum possible number of panels created is reached, this first loop stops, and a solution is obtained. It is worth noting that this second condition will rarely be reached since this maximum number of panels is based on the dimensions of the slats and therefore the total area that could theoretically be covered in case no surplus is created which would be exceptional.

The second part of the algorithm, which comes after the solution highlighted in blue on the flowchart, aims to adjust this solution. The first step serves to group all the slats that could have been assigned to the construction of a last unfinished panel in the subgroup called "*Rest*". Then, all the surplus of each strip of the created panels are considered from the biggest to the smallest to try to reduce them. Indeed, the algorithm first looks if the shortest slat from the *Rest* subgroup that has the same width as the strip to be improved also has a length smaller than the longest slat currently assigned to the analyzed strip. However, the length of this potential new slat must still be sufficient to complete the strip. In other words, no matter which slats compose the strip, its total length must be greater or equal to the length of the panel. If these two conditions are met, the longest slat of the strip is replaced by this new slat. After examining each surplus and possibly interchanging slats to reduce it, the algorithm stops. The possibly modified solution obtained after this second part is the final solution provided by this constructive heuristic method. Although the first objective of the method is to create a maximum of panels, this second part of the process still adds value to the solution firstly obtained as it serves to improve the quality of the solution. Indeed, as mentioned in the context and motivations of this research work, it is also important to waste and generate as little surplus as possible. Moreover, seeking to reduce the surplus allows to keep a maximum of material in the *Rest* which can be used thereafter in future production processes, or in possible future research concerning the problem in a dynamic

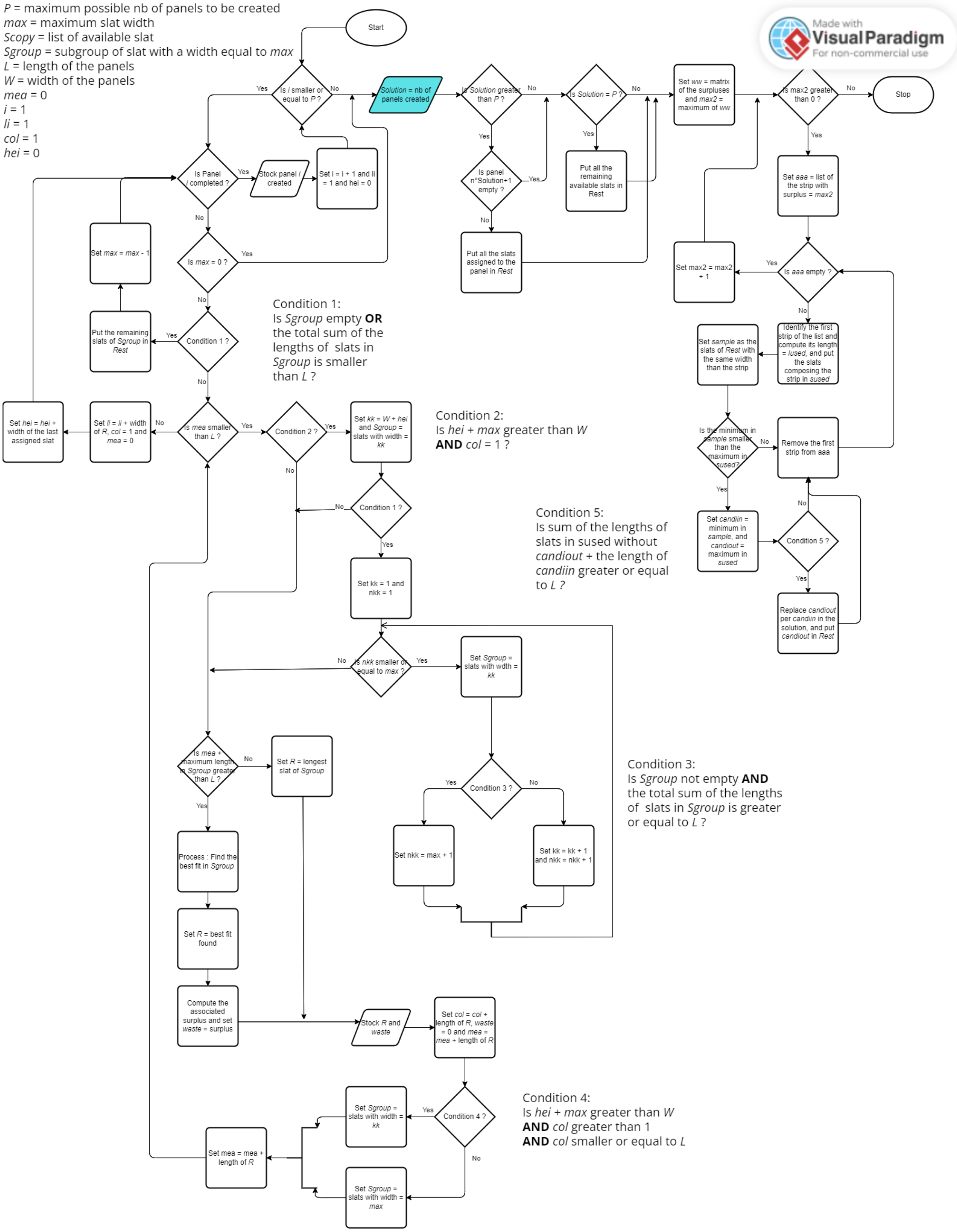

Figure 9: Flowchart description of the developed algorithm (Personal drawing on Visual Paradigm Online).

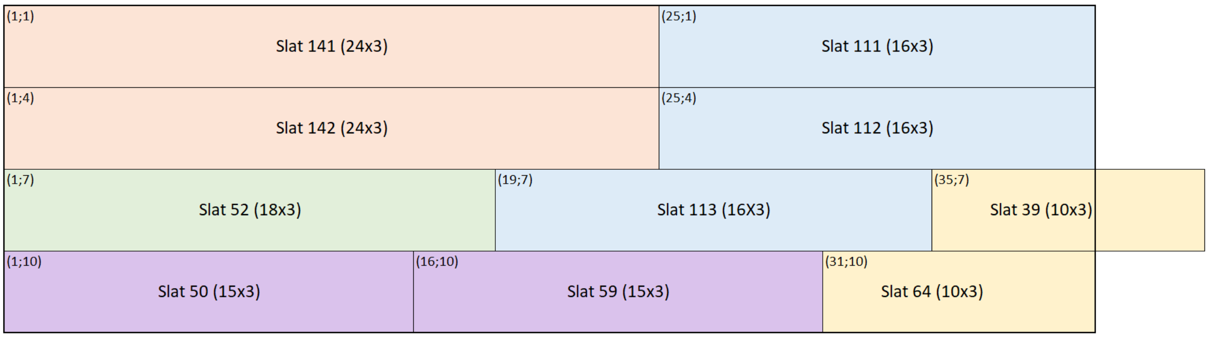

Figure 10: Layout of the first panel created with the constructive heuristic with Dataset 0 (Personal drawing on Excel).

form where the rests are taken into account in the optimization process.

3.5.2 Tests and results

The first test performed with this algorithm uses Dataset 0 mentioned earlier in this work. This test is called "Test 0" in Table 5 presenting all the tests and associated results performed with this heuristic solution. It can be noticed that this method finds a solution in 0.004 sec with Dataset 0 composed of 142 slats. Unlike the third mathematical model which is not able to deal with datasets of more than 37 slats in less than one hour, this solution method differs in performance. The development of such a heuristic solution method makes thus perfect sense. Figure 10 is an illustration of the first panel built with Test 0 this heuristic method. It is worth noting that only strips with a width of 3 are created to form this panel. This indeed does not generate any surplus along the length of the panel. However, a surplus needs to be cut along the width.

Then, the other tests follow the same idea as the tests performed on the third mathematical model. It means that the goal is to explore the computation time and the quality of the associated solution obtained for an increasing number of input data still based on Dataset 0. The procedure is therefore similar when testing three datasets of a certain number of slats by randomly selecting them from Dataset 0 until the desired number of slats is obtained for testing. I arbitrarily chose to start at 5000 slats and to increase by 5000 slats every three tests. It can be noticed that, in Dataset 0, the widths of the slats are all between 1 and 4 included. The width of the panels which is 12 is thus a multiple of all the slats. This results in no surplus along the length of the panels in all the tests performed. This is why only the total waste appears in Table 5. In fact, this equals the waste along the width of the panels generated by the creation of the strips. Moreover, the last column of this table entitled "Waste ratio" expresses the waste rate of the material used to produce the panels. It is calculated on the basis of the theoretical total surface that the slats assigned to the panels produced could cover in relation to the total surface of these panels. This ratio notably hovers around 9% with this solution method, which seems quite appropriate given the industry's waste rate mentioned in Section 1.

Regarding the analysis of the computation time as a function to the number of slats represented in Figure 11, it is worth noting that it does not increase in an exponential way as it is the case with the third mathematical model. Indeed, the curve drawn on this graph is a power trend-line, i.e. a curved line which is used to show an increase at a specific rate. It can be noted that the R-squared value is 0.998, which indicates a nearly perfect fit of the line to the data. Elsewhere, the maximum number of slats tested in this test series is 100,000 slats. The corresponding computation time is a little more than 330 sec, which is quite reasonable considering the number of input data. Moreover, having more than 100,000 slats in reality seems unlikely. Hence, I did not test beyond this number of slats to reach the hour of computation for example.

Table 5: Tests and associated results for the constructive heuristic with random sample of slats from Dataset 0.

Test	Nb of slats	Solution	Time (sec)	Total waste (cm^2)	Waste ratio
0	142	8	0.004	8100	0.0778
1	5000	302	0.877	358350	0.09
2	5000	305	0.945	359000	0.0893
3	5000	303	0.9	370600	0.0925
4	10000	603	3.037	721650	0.0907
5	10000	604	2.996	736725	0.0923
6	10000	607	2.922	730975	0.0912
7	15000	910	6.131	1067650	0.0891
8	15000	913	6.363	1080375	0.0898
9	15000	909	6.011	1092475	0.091
10	20000	1215	14.892	1431925	0.0894
11	20000	1220	12.634	1453200	0.0903
12	20000	1216	12.634	1428225	0.0892
13	25000	1523	20.263	1789700	0.0892
14	25000	1523	17.647	1832350	0.0911
15	25000	1516	17.911	1812075	0.0906
16	30000	1824	26.044	2121450	0.0884
17	30000	1821	26.452	2179375	0.0907
18	30000	1824	24.675	2143700	0.0892
19	35000	2127	33.939	2520000	0.0899
20	35000	2126	35.375	2499525	0.0892
21	35000	2121	33.875	2537225	0.0907
22	40000	2433	44.757	2880575	0.0898
23	40000	2437	46.024	2899275	0.0902
24	40000	2438	45.906	2904850	0.0903
25	45000	2736	57.799	3285075	0.091
26	45000	2736	58.473	3251300	0.0901
27	45000	2740	58.845	3272975	0.0905
28	50000	3043	72.931	3584150	0.0894
29	50000	3037	72.725	3621700	0.0904
30	50000	3033	74.398	3613125	0.0903
31	55000	3341	92.762	3971950	0.0901
32	55000	3341	92.172	3978925	0.0903
33	55000	3342	93.217	3947975	0.0896
34	60000	3649	110.573	4345475	0.0903
35	60000	3651	113.42	4339175	0.0901
36	60000	3644	101.733	4345075	0.0904
37	65000	3965	120.356	4706925	0.09
38	65000	3956	122.935	4699475	0.0901
39	65000	3953	125.066	4695525	0.0901
40	70000	4252	148.281	5034550	0.0898
41	70000	4253	147.022	5044175	0.0899
42	70000	4258	149.216	5001250	0.0892
43	75000	4575	174.795	5384800	0.0893
44	75000	4559	161.985	5416225	0.0901
45	75000	4562	164.652	5424125	0.0901

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Table 5 – continued from previous page

Test	Nb of slats	Solution	Time (sec)	Total waste (cm^2)	Waste ratio
46	80000	4870	189.868	5764625	0.0898
47	80000	4870	194.322	5839375	0.0908
48	80000	4869	195.252	5781325	0.09
49	85000	5168	227.198	6087900	0.0894
50	85000	5169	228.689	6107225	0.0896
51	85000	5167	230.663	6137250	0.0901
52	90000	5466	250.109	6480150	0.0899
53	90000	5476	245.81	6459125	0.0895
54	90000	5484	249.02	6441300	0.0892
55	95000	5775	282.087	6869375	0.0902
56	95000	5786	289.001	6837125	0.0896
57	95000	5784	293.531	6874575	0.0901
58	100000	6088	325.723	7214300	0.0899
59	100000	6076	327.72	7268425	0.0907
60	100000	6079	332.767	7196050	0.0898

3.6 Discussion

As explained, the first mathematical model has a different objective from the other solutions developed, since it minimizes surplus while creating a fixed number of panels. This first step in the research was intended to be a first approach to the problem and a first implementation before making things more complex by getting closer to the target problem. The results are therefore not very comparable with what follows, although it is worth noting that this model deals easily with datasets of 150 slats, taking 10 and 17 sec respectively to find the optimal solution in the two tests carried out.

The second mathematical model, deals with a simplified version of the problem where all slats have the same width. Given the tests carried out, the calculation time is highly dependent on the input data. However, generally speaking, this model is efficient in finding the optimal solution or a very good solution, in one hour's time for a number of slats less than or equal to 150. Although the problem is not treated in its full complexity, this model could possibly be used in reality if the slats are very homogeneous and the user pre-sorts them by width. Indeed, once enough slats of the same width are collected, this model could find an optimal solution, rather than using the construction heuristic, whose yielded solution is not necessarily optimal.

The third model deals with the problem as a whole, taking into account the slats' two dimensions. In practice, the wood boards to be revalorized do not necessarily come from the same source and therefore have no standardized dimensions. However, the complexity of the mathematical model and its resulting size do not allow it to provide an optimal solution in a reasonable time for a quantity of slats greater than 37. Further, given the combinatorial nature of the problem and the fact that it is NP-hard; for a high number of slats, it is impossible to solve this model in polynomial time. Above all, this model justifies the hypothesis that the problem under study is NP-hard and therefore cannot be solved by an exact solution method. This also justifies the development of an approximate solution method.

The construction heuristic developed within the framework of this research thesis provides a solution to the problem studied yielding an average of 9% waste, regardless of the number of slats and panels created. This waste rate for the production of laminated wood panels seems very good compared to conventional industry rates when working with pure raw materials, where sawmills can generate up to 50% waste in the form of sawdust, wood shavings and heat. In addition, this solution method is capable of finding a solution in 330 sec for 100,000 slats. This even exceeds

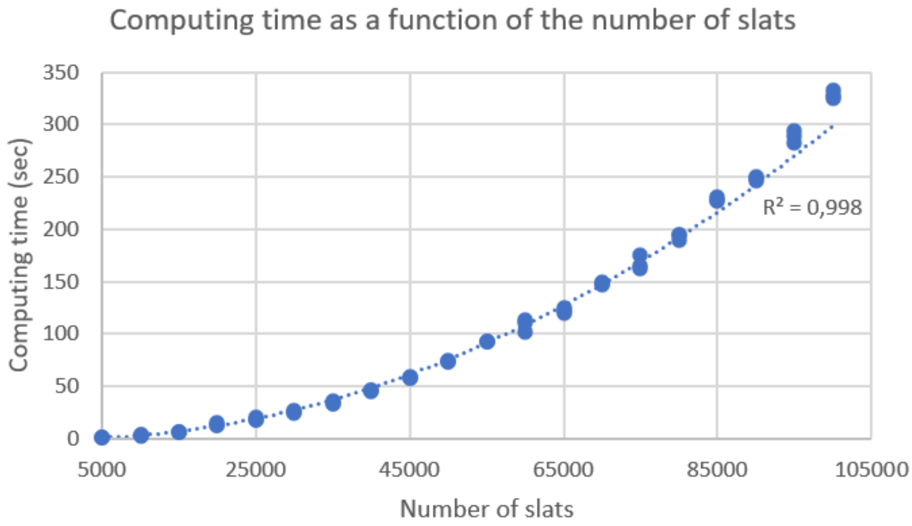

Figure 11: Computing time in seconds as a function of the number of slats from Dataset 0 for the tests of the constructive heuristic (Personal generation on Excel).

reality, given that most laminated wood panel producers do not wait to accumulate more than 100,000 slats before starting producing. Moreover, it is not in their interest to wait, given that the waste ratio remains constant regardless of the number of slats as well as the number of panels to be produced. Thus, it is not possible to achieve economies of scale in terms of waste generated. Hence, it seems unlikely that anyone would actually need to solve the problem with 100,000 slats or more. On the other hand, it might be interesting to test this heuristic with even more slats as part of future research to determine its real limits.

4 Conclusion

The major thrust of this thesis is to examine the development of a solution method for creating laminated wood panels with revalorized wood boards. More concretely, the aim is to find an efficient way of generating two-dimensional layouts of slats that form the target panels. A step by step approach is adopted to firstly tackle a simplified version of the problem before considering the whole parameters. The testing of the mathematical model developed to deal with the global problem proves the problem to be NP-hard as an exact method cannot yield the optimal solution in polynomial time. On the other hand, the development of an approximation method through a constructive heuristic proves, by testing with transformed real data, to be a very suitable solution method for the studied problem.

In conclusion, this research work answers the research question of my master thesis by finding an efficient solution method for creating laminated wood panels with revalorized wood boards, after proving through the testing of the third mathematical model that it was impossible to solve this problem with an exact method. This solution method is an approximation method in the form of a constructive heuristic based on the idea of strips creation and inspired by Best-Fit algorithms used in Bin Packing. This constructive heuristic is able to deal with a lot of input data and yield a very good solution in a relatively very short computing time.

4.1 Limitations and suggestions for future research

This sub-section summarizes the limitations encountered during the course of this research work, as well as suggestions for future research on the topic. Indeed, the problem is very complex from a combinatorial analysis point of view and represents a gap in the current literature. There are many different ways of approaching the problem. It is therefore very interesting to give initial indications of the various possibilities for further study.

4.1.1 Limitations

During the progress of this research work, several limitations became apparent. The first one inevitably concerns the study of the two-dimensional problem. Indeed, the thicknesses of the wood boards to be revalorized as well as the thicknesses of the panels to be created are not considered.

Another limitation is that some of the layouts provided by the third mathematical model may have special and messy joint configurations as illustrated in Figure 12. As discussed with Prof. De Mil, this kind of joint configuration is not suitable for an outer layer of a CLT panel. However, it is potentially suitable for an inner layer not in direct contact with the external environment.

Several limitations are notable regarding the development of the heuristic. The first one concerns the difficulty to create an initial solution. Indeed, as described in this research work, I had to make a choice of method, i.e. the creation of strips, to build a solution efficiently. However, this choice limits the optimization of the solution. Slats are only used for strips of the same width, which limits potential layouts and ignores combinations of slats of different widths. In addition, as part of the development and implementation of a metaheuristic, it seems difficult to find a first initial solution, and secondly, to determine the exploration neighborhood.

The second limitation is simply due to the development of the heuristic itself. Indeed, in its second part, the algorithm explores all the surpluses with the aim of reducing them by exchanging slats. However, only the longest slat belonging to the analyzed strip is considered. Similarly, for the potential new slat, the algorithm only looks at the shortest one still available. Even though this method can be effective, it is clearly possible to miss slat combinations that could create less surplus than simply swapping the longest slat in the strip with the shortest still available. In addition, the strips' length must greater or equal to the panels' length. This condition implies that the shortest slat available may not be suitable as it could not be long enough to reach the

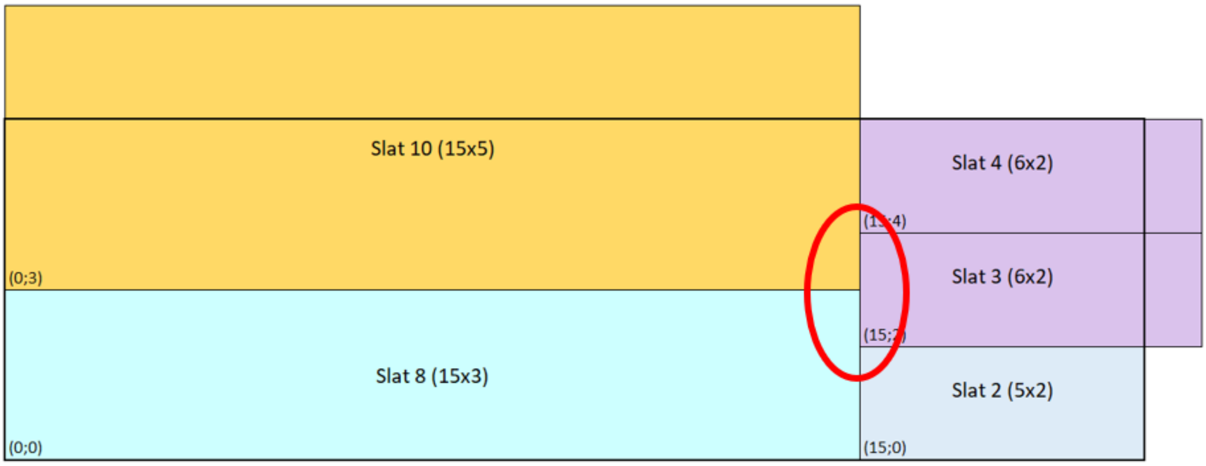

Figure 12: Layout of the second panel created with the third mathematical model with a sandbox dataset used for testing while implementing (Personal drawing on Excel).

length of the panel, whereas the second-shortest slat, for example, could be suitable. At the moment, the algorithm does not explore the different possible combinations of slats to reduce the surplus as much as possible.

The final limitation of the heuristic concerns the transformation performed on the real base data provided by Prof. Tom De Mil. Applying a factor of 5 and rounding the various dimensions to the nearest integer results in more homogeneous slat widths. Therefore, more strips of the same width that measure at least the length of a panel can be created. This also means more panels can be created, all other things being equal. The proposed solution method is therefore less efficient if the actual dimensions of the slats are unchanged. However, it is also unrealistic to work with slats of totally different widths when creating CLT or Glulam panels. In reality, the slats are adjusted and potentially pre-cut to standardize dimensions. This limitation therefore remains purely theoretical in terms of performance of the construction heuristic developed through this research work.

4.1.2 Suggestions for future research

Firstly, the three-dimensional problem could be explored by considering the thicknesses of the wood boards to be revalorized as well as the thicknesses the panels to be created. This would study the problem as realistically as possible.

Secondly, a more in-depth study of the granularity of widths could be of interest, as it seems to play a major role in the complexity of the problem and therefore in the development of a suitable solution. Different slat widths may also influence the physical quality of the proposed solution, with possibly too many joints in the same area of the panel, making it less robust.

It might also be interesting to test the heuristic more extensively to find out its real limits after an hour's computation for example. This would enable us to compare this solution method with other future methods being studied and developed. In addition, this construction heuristic could also be improved by modifying the second part to explore more combinations of slats in order to reduce surplus. The constructive procedure could also be changed to improve performance.

The problem could also be studied in a more dynamic version where the surpluses are used directly to create the target panels. Indeed, a future solution method could choose to voluntarily create a surplus when creating a panel, to reuse it in the creation of the next panels. This way, the production process is more optimized as it tries to do the best usage of every available material. Hence, with a large number of slats, an efficient method that reuses the surplus in the creation of the solution will automatically generate a better solution than with the construction heuristic proposed in this research work.

Ultimately, it might be worth exploring variants of the problem to better suit the actual situation in the field. One variant is to create panels with holes to fit a window, for example, rather than creating a solid panel and then cutting it to fit the window. Finally, sensitivity analysis of joints is also an interesting option. Indeed, limiting the number of joints, i.e. the number of boards to cover a panel, allows to create stronger panels which can be a major requirement depending on the use for which the panels are produced.

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Appendices

Appendix 1 : Raw Real Data provided by Pr. Tom De Mil via Dalimier (2022).

Salt ID	Length (cm)	Width (cm)	Thickness (mm)
1	82	8.8	18
2	82	8.8	18
3	82	8.8	18
4	82	8.8	18
5	82	8.8	18
6	82	8.8	18
7	82	8.8	18
8	69.9	9.6	18
9	78.6	8.5	19
10	61.5	13.3	19
11	60.6	13.3	19
12	61	12.9	18
13	65.5	13.3	19
14	59	13.2	18
15	64	13.3	18
16	64.6	13.2	18
17	59	13.3	19
18	67.3	13.3	18
19	61.2	13.3	19
20	60.1	13.4	18
21	63	11.4	19
22	50.5	10.2	20
23	69.4	9.6	18
24	48	10.3	19
25	50.6	10.3	20
26	50.6	10.3	20
27	50.6	10.2	19
28	63.5	13.2	18
29	64	13.3	18
30	59.5	13.2	18
31	59	13.2	18
32	65.5	13.4	18
33	59	13.3	18
34	69.7	9.6	18
35	68	9.7	18
36	64.8	13.2	17
37	90	9.3	20
38	86	8.7	19
39	51.8	12.9	18
40	58.8	13.2	17
41	69	9	16
42	68.5	8.9	16
43	69.8	9.6	18
44	59.2	13.5	18

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Real data – continued from previous page

Salt ID	Length (cm)	Width (cm)	Thickness (mm)
45	81	7.9	18
46	50.5	10.3	19
47	66	13.1	18
48	66	13.2	18
49	50.5	10.3	20
50	72.8	13.8	18
51	72	13.9	18
52	88.5	13.3	19
53	67.7	9.3	20
54	47.8	10.3	19
55	72	14	18
56	50.5	10.2	20
57	69.6	9.6	18
58	48	10.5	19
59	74.5	13.8	18
60	88.4	10.4	19
61	69.4	9.6	18
62	73.3	13.9	18
63	72.2	13.9	18
64	47.9	13.9	18
65	69	9	15
66	88.5	10.2	20
67	69.7	9.7	18
68	69.6	9.7	18
69	69	9	16
70	50.4	10.3	20
71	50.5	10.2	20
72	69.4	9.6	18
73	51.4	8.5	19
74	46.3	10.3	19
75	50.5	10.2	20
76	73.4	14	18
77	69	8.9	15
78	50.5	10.3	19
79	70.5	9.7	20
80	50	10.2	19
81	74.3	9.4	23
82	69	9	16
83	69	9	16
84	69	9	16
85	69.2	9.6	18
86	69.4	9.6	18
87	69.8	9.6	18
88	70.8	9.6	20
89	50.5	10.3	19
90	48.9	10.3	19
91	71.8	13.9	18
92	72.2	13.9	18

Continued on next page

Real data – continued from previous page

Salt ID	Length (cm)	Width (cm)	Thickness (mm)
93	73.5	13.9	18
94	92.3	10.3	20
95	88.6	10.3	20
96	50.7	10.1	19
97	50.7	10.1	19
98	50.7	10.1	19
99	68.9	9	15
100	69.2	9.6	17
101	69.5	9.6	18
102	88.3	10.3	20
103	48	10.2	19
104	59.8	10	19
105	48.8	10.2	19
106	46.5	10.2	20
107	66.4	15.1	22
108	65.3	13.2	17
109	120	11.4	20
110	93.5	18.2	22
111	79.9	13.1	22
112	80	13.5	22
113	79.8	13.5	23
114	166	8.6	19
115	120	9.7	20
116	120	9.5	19
117	57.5	13.3	18
118	60.6	13.3	18
119	128.5	6.5	16
120	128.3	6.5	16
121	128.2	6.5	16
122	132.4	6.5	16
123	130	6.5	16
124	130	6.5	16
125	125.6	6.5	16
126	127.6	6.5	16
127	128	6.5	16
128	126.5	6.5	16
129	130.3	6.5	16
130	129.6	6.5	16
131	128.5	6.5	16
132	129.8	6.5	16
133	130.3	6.5	16
134	133.2	6.5	16
135	128.6	6.5	16
136	130.8	6.5	16
137	128.1	6.5	16
138	73.1	6.5	16
139	87	10.3	27
140	126.1	10.3	27

Continued on next page

Real data – continued from previous page

Salt ID	Length (cm)	Width (cm)	Thickness (mm)
141	119.7	13.5	21
142	120.3	13.5	21

Appendix 2 : " Dataset 0 " composed of the real data transformed by applying a factor of 5 to the dimensions and rounding them off to the nearest integer.

Salt ID	Length (cm)	Width (cm)
1	16	2
2	16	2
3	16	2
4	16	2
5	16	2
6	16	2
7	16	2
8	14	2
9	16	2
10	12	3
11	12	3
12	12	3
13	13	3
14	12	3
15	13	3
16	13	3
17	12	3
18	13	3
19	12	3
20	12	3
21	13	2
22	10	2
23	14	2
24	10	2
25	10	2
26	10	2
27	10	2
28	13	3
29	13	3
30	12	3
31	12	3
32	13	3
33	12	3
34	14	2
35	14	2
36	13	3
37	18	2
38	17	2
39	10	3
40	12	3
41	14	2
42	14	2
43	14	2
44	12	3
45	16	2
46	10	2

Continued on next page

Dataset 0 – continued from previous page

Salt ID	Length (cm)	Width (cm)
47	13	3
48	13	3
49	10	2
50	15	3
51	14	3
52	18	3
53	14	2
54	10	2
55	14	3
56	10	2
57	14	2
58	10	2
59	15	3
60	18	2
61	14	2
62	15	3
63	14	3
64	10	3
65	14	2
66	18	2
67	14	2
68	14	2
69	14	2
70	10	2
71	10	2
72	14	2
73	10	2
74	9	2
75	10	2
76	15	3
77	14	2
78	10	2
79	14	2
80	10	2
81	15	2
82	14	2
83	14	2
84	14	2
85	14	2
86	14	2
87	14	2
88	14	2
89	10	2
90	10	2
91	14	3
92	14	3
93	15	3
94	18	2

Continued on next page

Dataset 0 – continued from previous page

Salt ID	Length (cm)	Width (cm)
95	18	2
96	10	2
97	10	2
98	10	2
99	14	2
100	14	2
101	14	2
102	18	2
103	10	2
104	12	2
105	10	2
106	9	2
107	13	3
108	13	3
109	24	2
110	19	4
111	16	3
112	16	3
113	16	3
114	33	2
115	24	2
116	24	2
117	12	3
118	12	3
119	26	1
120	26	1
121	26	1
122	26	1
123	26	1
124	26	1
125	25	1
126	26	1
127	26	1
128	25	1
129	26	1
130	26	1
131	26	1
132	26	1
133	26	1
134	27	1
135	26	1
136	26	1
137	26	1
138	15	1
139	17	2
140	25	2
141	24	3
142	24	3

Appendix 3 : " Dataset 1 " composed of slats that have the same width, and a length randomly generated to be equal to a random integer between 50 and 200 cm.

Salt ID	Length (cm)
1	76
2	106
3	183
4	117
5	94
6	150
7	130
8	67
9	179
10	178
11	132
12	106
13	165
14	60
15	110
16	197
17	163
18	138
19	177
20	134
21	169
22	105
23	87
24	99
25	54
26	158
27	129
28	157
29	125
30	146
31	122
32	107
33	179
34	94
35	169
36	166
37	117
38	81
39	189
40	183
41	175
42	190
43	146
44	54
45	173

Continued on next page

Dataset 1 – continued from previous page

Salt ID	Length (cm)
46	149
47	163
48	70
49	135
50	180
51	162
52	196
53	182
54	60
55	64
56	109
57	113
58	185
59	112
60	184
61	162
62	177
63	179
64	162
65	196
66	191
67	135
68	200
69	97
70	80
71	78
72	79
73	143
74	80
75	186
76	64
77	103
78	177
79	135
80	68
81	157
82	154
83	76
84	50
85	188
86	177
87	67
88	178
89	154
90	129
91	157
92	162
93	86

Continued on next page

Dataset 1 – continued from previous page

Salt ID	Length (cm)
94	183
95	59
96	186
97	190
98	79
99	96
100	138
101	70
102	172
103	189
104	148
105	179
106	111
107	111
108	86
109	185
110	83
111	119
112	162
113	113
114	73
115	182
116	76
117	69
118	74
119	99
120	135
121	106
122	108
123	56
124	189
125	166
126	101
127	161
128	171
129	117
130	130
131	50
132	114
133	75
134	183
135	150
136	68
137	73
138	113
139	54
140	144
141	183

Continued on next page

Dataset 1 – continued from previous page

Salt ID	Length (cm)
142	193
143	71
144	79
145	185
146	76
147	121
148	74
149	148
150	131

Appendix 4 : " Dataset 2 " arbitrarily generated manually.

Salt ID	Length (cm)
1	70
2	70
3	70
4	70
5	70
6	70
7	70
8	70
9	70
10	70
11	70
12	70
13	70
14	70
15	70
16	70
17	70
18	70
19	70
20	70
21	70
22	70
23	70
24	70
25	70
26	70
27	70
28	70
29	70
30	70
31	110
32	110
33	110
34	110
35	110
36	110
37	110
38	110
39	110
40	110
41	110
42	110
43	110
44	110
45	110
46	110
47	110

Continued on next page

Dataset 2 – continued from previous page

Salt ID	Length (cm)
48	110
49	110
50	110
51	110
52	110
53	110
54	110
55	110
56	110
57	110
58	110
59	110
60	110
61	110
62	110
63	110
64	110
65	110
66	110
67	110
68	110
69	110
70	110
71	110
72	110
73	110
74	110
75	110
76	110
77	110
78	110
79	110
80	110
81	135
82	135
83	135
84	135
85	135
86	135
87	135
88	135
89	135
90	135
91	135
92	135
93	135
94	135
95	135

Continued on next page

Dataset 2 – continued from previous page

Salt ID	Length (cm)
96	135
97	135
98	135
99	135
100	135
101	135
102	135
103	135
104	135
105	135
106	135
107	135
108	135
109	135
110	135
111	135
112	135
113	135
114	135
115	135
116	135
117	135
118	135
119	135
120	135
121	163
122	163
123	163
124	163
125	163
126	163
127	163
128	163
129	163
130	163
131	163
132	163
133	163
134	163
135	163
136	163
137	163
138	163
139	163
140	163
141	163
142	163
143	163

Continued on next page

Dataset 2 – continued from previous page

Salt ID	Length (cm)
144	163
145	163
146	163
147	163
148	163
149	163
150	163

Appendix 5 : " Dataset 3 " arbitrarily generated manually.

Salt ID	Length (cm)
1	70
2	70
3	70
4	70
5	70
6	70
7	70
8	70
9	70
10	70
11	75
12	75
13	75
14	75
15	75
16	92
17	92
18	100
19	100
20	100
21	100
22	100
23	100
24	100
25	100
26	100
27	100
28	100
29	100
30	100
31	100
32	100
33	100
34	100
35	100
36	100
37	100
38	100
39	100
40	110
41	110
42	110
43	110
44	110
45	110
46	110
47	110

Continued on next page

Dataset 3 – continued from previous page

Salt ID	Length (cm)
48	110
49	110
50	110
51	110
52	110
53	110
54	110
55	110
56	110
57	110
58	110
59	110
60	110
61	110
62	110
63	110
64	110
65	110
66	110
67	110
68	110
69	110
70	110
71	150
72	150
73	150
74	150
75	150
76	150
77	150
78	150
79	150
80	150
81	150
82	150
83	150
84	150
85	150
86	150
87	150
88	150
89	150
90	150
91	150
92	150
93	150
94	150
95	150

Continued on next page

Dataset 3 – continued from previous page

Salt ID	Length (cm)
96	150
97	150
98	150
99	150
100	150
101	150
102	150
103	150
104	150
105	150
106	150
107	150
108	164
109	170
110	170
111	170
112	170
113	170
114	170
115	170
116	170
117	170
118	170
119	170
120	170
121	170
122	170
123	170
124	170
125	170
126	210
127	210
128	210
129	210
130	210
131	210
132	210
133	210
134	210
135	210
136	210
137	210
138	210
139	210
140	210
141	210
142	210
143	210

Continued on next page

Dataset 3 – continued from previous page

Salt ID	Length (cm)
144	210
145	210
146	210
147	210
148	210
149	210
150	210

List of resource persons

- Pr. Célia Paquay, from the HEC Management School Faculty of the University of Liège.
- Pr. Tom De Mil, from the Gembloux Agro-Bio Tech Faculty of the University of Liège.

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Executive summary

The two-dimensional problem of creating laminated wood panels with revalorized wood boards can be likened to Cutting and Packing problems. This problem falls into the domain of combinatorial analysis and proves to be NP-hard. The problem has received increase interest in recent years as the demand for natural resources continues to grow and will continue to do the same over the coming decades. Moreover, the revalorization of materials such as wood is a solution in line with the different environmental commitments. It is therefore necessary to provide sustainable and more circular solutions. Good optimization tools are therefore needed to solve the problem studied.

In this thesis, I create and test an exact solution to solve the two-dimensional problem of creating laminated wood panels with revalorized wood boards, thanks to which I prove this problem to be NP-hard. Therefore, it justifies the development of an approximation method. The final solution method proposed takes the form of a construction heuristic based on the idea of strips creation and inspired by Best-Fit algorithms used in Bin Packing. Through extensive testing, the efficiency and effectiveness of this solution method are demonstrated, particularly when dealing with a large volume of data. In comparison to exact methods, which may struggle with scalability, the construction heuristic proves to be a valuable optimization tool for solving the studied problem.

By providing a sustainable and circular approach to the creation of laminated wood panels with revalorized wood boards, this research thesis contributes to addressing the resource scarcity and environmental challenges of the future. The findings of this thesis offer practical insights for the development of optimized processes in the wood industry and underline the importance of employing efficient approximation methods to tackle complex combinatorial problems in the pursuit of sustainable solutions.

Keyword: Optimization, CLT, Glulam, Revalorization, Cutting and Packing Problem, NP-hard, Combinatorial analysis, Mathematical model, Constructive algorithm, Best-fit

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