Integrated Portfolio Management: Risk Parity, Momentum and Black-Litterman Strategies. An examination of synergies and effectiveness.

Auteur : El Khawli, Bassel
Promoteur(s) : Bodson, Laurent
Faculté : HEC-Ecole de gestion de l'Université de Liège
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INTEGRATED PORTFOLIO MANAGEMENT: RISK PARITY, MOMENTUM AND BLACK-LITTERMAN STRATEGIES. AN EXAMINATION OF SYNERGIES AND EFFECTIVENESS.

Jury:
Supervisor:
Laurent BODSON
Reader:
Pierre-François WEYDERS

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Bassel EL KHAWLI
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CONTENTS

LIST OF ABBREVIATIONS / GLOSSARY

APT	Arbitrage Pricing Theory
BL	Black-Litterman
BPT	Behavioral Portfolio Theory
CAGR	Compound Annual Growth Rate
CAPM	Capital Asset Pricing Model
CVaR	Conditional Value at Risk
DD	Drawdown
ECB	European Central Bank
EMA	Exponential Moving Average
EMH	Efficient Market Hypothesis
EW	Equal Weight
GA	Genetic Algorithms
IP	Integrated Portfolio
K	Kurtosis
M2	Modigliani-Modigliani Measure
MPT	Modern Portfolio Theory
MVO	Mean-Variance Optimization
MVU	Mean-Variance Utility
PMPT	Post-Modern Portfolio Theory
PSO	Particle Swarm Optimization
RFR	Risk-Free Rate
RP	Risk Parity
S	Skewness
SR	Sharpe Ratio
UPR	Upside Potential Ratio
VaR	Value at Risk

1. INTRODUCTION

The evolution of financial markets has given rise to a variety of portfolio management strategies, each tailored to meet the demands of different investor profiles and market conditions. Among these strategies, Risk Parity, Momentum, and the Black-Litterman model have garnered significant attention due to their distinct approaches to risk and return optimization. This thesis explores the potential benefits of integrating these strategies into a cohesive portfolio management framework, with particular attention to their performance under varying interest rate environments.

The integration of these strategies is motivated by the pursuit of enhanced portfolio diversification and improved risk-adjusted returns. Risk Parity emphasizes equal risk contribution from each asset, ensuring balanced exposure; Momentum capitalizes on the continuation of asset price trends, exploiting short-term market inefficiencies; and the Black-Litterman model offers a sophisticated method of incorporating market equilibrium and, optionally, investor views into portfolio construction. In this thesis, the Black-Litterman model is employed without the incorporation of explicit investor views, focusing instead on implied returns derived from market data. By combining these strategies, this research seeks to identify synergies that can lead to a more robust and resilient investment portfolio.

This study is structured to provide a comprehensive analysis of each strategy's theoretical foundation, followed by an empirical evaluation of their combined performance over a 20-year period. The analysis spans various interest rate regimes, offering insights into how these strategies adapt to changing economic conditions. This approach not only highlights the effectiveness of the integrated strategies but also addresses a critical gap in existing literature, providing practical insights into their application in contemporary portfolio management.

2. LITERATURE REVIEW

2.1 Introduction to Portfolio Management

2.1.1 *Overview of Portfolio Management*

Portfolio management is the art and science of making decisions about investment mix and policy, matching investments to objectives, asset allocation, and balancing risk against performance. This process is essential for both individual and institutional investors aiming to achieve their financial goals while managing risk (Fabozzi et al., 2002). Historically, portfolio management evolved from simple diversification strategies to sophisticated models that consider various risk factors and return expectations (Leković, 2021). Key developments in this field include the introduction of Modern Portfolio Theory (MPT) by Harry Markowitz in 1952. This evolution highlights the increasing complexity and sophistication in investment strategies over time.

2.1.2 *Historical Development of Portfolio Management*

The development of portfolio management can be traced back to the early 20th century when diversification was first recognized as a means to reduce investment risk. However, it was not until Harry Markowitz introduced Modern Portfolio Theory (MPT) in 1952 that portfolio management became a systematic and quantitative discipline (Markowitz, 1991). MPT laid the foundation for a more scientific approach to portfolio construction, emphasizing the trade-off between risk and return (Markowitz, 1999). While MPT was revolutionary, its reliance on historical data and assumptions about investor behavior and market efficiency have been points of critique (Fabozzi et al., 2002).

2.1.3 *Key Concepts in Portfolio Management*

Key concepts in portfolio management include risk and return, diversification, and the efficient frontier. Risk and return are fundamental to investment decisions, as investors seek to maximize returns while minimizing risk (Elton & Gruber, 1997). Diversification involves spreading investments across various assets to reduce risk, based on the principle that a diversified portfolio will, on average, yield higher returns and pose lower risk than any individual investment within the portfolio (Mangram, 2013). The efficient frontier represents the set of optimal portfolios that offer the highest expected return for a defined level of risk (Cochrane, 2005). These concepts form the backbone of traditional portfolio management but are often challenged by market anomalies and investor behavior.

2.1.4 *Modern Portfolio Theory (MPT)*

Modern Portfolio Theory (MPT), introduced by Harry Markowitz, revolutionized the field of portfolio management by providing a mathematical framework for constructing optimal portfolios (Markowitz, 1991). MPT uses the following mathematical formulation:

- **Expected Return (μ)**: The weighted sum of the expected returns of the individual assets in the portfolio (Cochrane, 2005).
- **Portfolio Variance (σ^2)**: The measure of portfolio risk, calculated using the variances and covariances of the returns of the assets (Markowitz, 1991).

The set of optimal portfolios that offer the highest expected return for a given level of risk is known as the efficient frontier. Investors can choose portfolios on this frontier based on their risk tolerance (Mangram, 2013).

While MPT revolutionized portfolio management, it is not without its critiques. One major criticism is its reliance on historical data to predict future returns and risks, which may not always be accurate (Fabozzi et al., 2002). Additionally, MPT assumes that investors are rational and markets are efficient, which behavioral finance has challenged (Mangram, 2013). Several extensions have been proposed to address these limitations, including the Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Theory (APT):

- **Capital Asset Pricing Model (CAPM)**: Introduced by Sharpe, Lintner, and Mossin, CAPM extends MPT by introducing the concept of a market portfolio and the risk-free rate to determine the expected return on an asset (Elton and Gruber, 1997).
- **Arbitrage Pricing Theory (APT):** Proposed by Ross, APT offers a multi-factor model for asset pricing, considering various macroeconomic factors that could affect returns (Leković, 2021).

2.1.5 *Behavioral Finance and Portfolio Management*

Behavioral finance incorporates psychological insights into financial theory, challenging the assumption that investors are always rational (Fabozzi et al., 2002). It explains why and how markets might be inefficient and why investors might make irrational decisions (Mangram, 2013). Integrating behavioral finance into MPT involves considering factors such as overconfidence, loss aversion, and mental accounting. This integration helps in designing portfolios that align better with actual investor behavior and preferences (Fabozzi et al., 2002). The recognition of these behavioral factors marks a significant advancement in portfolio management, providing a more holistic view of investor behavior and market dynamics.

While the traditional and modern theories of portfolio management have laid a strong foundation for investment strategies, they also have notable limitations. The reliance on historical data and the assumption of rational behavior often do not hold true in real-world scenarios. Behavioral finance provides a more nuanced understanding of investor behavior, but it also introduces complexity in modeling and predicting market outcomes. Future research should focus on integrating these diverse perspectives to develop more robust and adaptive portfolio management strategies.

2.2 Overview of Traditional and Advanced Portfolio Strategies

2.2.1 Introduction to Portfolio Strategies

Portfolio strategies are crucial in the field of finance as they provide a structured approach to managing investments to achieve specific financial objectives. Historically, these strategies have evolved over time, addressing the increasing complexity of financial markets and the diverse needs of investors (Beyhaghi & Hawley, 2013). As financial markets developed, so did the strategies to optimize investment returns while managing associated risks.

2.2.2 Traditional Portfolio Strategies

Modern Portfolio Theory (MPT), introduced by Harry Markowitz in 1952, fundamentally changed how investors approach portfolio management by emphasizing diversification to optimize risk and return (Mangram, 2013). The Capital Asset Pricing Model (CAPM), further developed by Sharpe (1992) and others, introduced a method to price risk and determine expected returns based on an asset's systematic risk relative to the market (Elbannan, 2014). The Efficient Market Hypothesis (EMH), proposed by Fama, suggests that asset prices fully reflect all available information, making it impossible to consistently achieve higher returns without taking on more risk (Fama & French, 2004).

These traditional strategies primarily focus on the trade-off between risk and return, advocating that through diversification, investors can construct an efficient frontier of optimal portfolios that offer the highest expected return for a given level of risk (Elton & Gruber, 1997). Despite their widespread use, these models have been criticized for their assumptions about market efficiency and investor rationality.

2.2.3 Advanced Portfolio Strategies

Post-Modern Portfolio Theory (PMPT) builds on MPT by addressing its limitations, particularly its assumption of normally distributed returns and its focus on downside risk (Galloppo, 2010). Multi-Factor Models, such as the Fama-French three-factor model, incorporate multiple sources of risk and return, providing a more comprehensive understanding of asset pricing and portfolio construction (Fama & French, 2004).

Behavioral Portfolio Theory (BPT) considers the psychological factors influencing investor behavior, acknowledging that investors do not always act rationally and that their decisions can be influenced by biases and heuristics (Blume & Friend, 1973). This theory challenges the traditional assumptions of rationality and efficient markets by incorporating elements of human behavior into financial models.

Risk management techniques have also evolved, with strategies such as Value at Risk (VaR) and Conditional Value at Risk (CVaR) becoming integral in assessing potential losses and managing portfolio risk in volatile markets (Meucci, 2005). These advanced risk management techniques provide more accurate measures of potential losses in extreme market conditions, enhancing the ability of portfolio managers to safeguard investments.

2.2.4 *Strategic Asset Allocation*

Strategic asset allocation involves setting a long-term investment strategy based on the investor's risk tolerance, investment horizon, and financial goals. This approach contrasts with tactical asset allocation, which involves short-term adjustments to the asset mix based on market conditions (Brennan et al., 1997). Effective strategic asset allocation requires a deep understanding of the fundamental drivers of asset returns and a disciplined approach to maintaining the desired asset mix over time (Sharpe, 1992).

2.2.5 *Performance Evaluation of Portfolio Strategies*

Evaluating the performance of portfolio strategies involves comparing the returns achieved against benchmarks and assessing the risk taken to achieve those returns (Risk-adjusted performance). Traditional performance metrics such as the Sharpe ratio, Treynor ratio, and Jensen's alpha remain widely used, but newer metrics that account for downside risk and other factors are gaining prominence (Schulmerich et al., 2015). A comparative analysis of traditional and advanced strategies reveals that while traditional methods provide a solid foundation, advanced strategies offer enhanced risk management and better alignment with investor behavior (French, 2003).

The transition from traditional to advanced portfolio strategies reflects an ongoing effort to address the limitations of early financial models. Traditional strategies like MPT and CAPM provide foundational concepts that are still relevant, but their assumptions about market efficiency and investor rationality are often challenged by real-world anomalies and behavioral biases. Advanced strategies, including PMPT and behavioral finance, offer more nuanced approaches that incorporate psychological insights and multiple risk factors. These developments enhance the robustness of portfolio management practices but also introduce new complexities in modeling and implementation.

2.3 Risk Parity Strategy

2.3.1 *Introduction to Risk Parity*

The concept of risk parity has garnered significant attention in the asset allocation landscape, particularly following the financial crisis of 2008. This approach to portfolio construction focuses on balancing the risk contributions of different asset classes rather than their capital weights. The central idea is to allocate investments such that each asset class contributes equally to the overall risk of the portfolio (Kazemi, 2013), aiming for a more stable and diversified portfolio. Unlike traditional asset allocation methods, risk parity does not rely heavily on expected returns, which are notoriously difficult to estimate accurately (Braga, 2016). This shift in focus from capital weights to risk contributions represents a significant evolution in portfolio management strategies.

2.3.2 *Basics of Risk Parity*

The risk parity strategy seeks to create a well-diversified portfolio where each asset class has the same marginal contribution to the total risk of the portfolio. This is achieved by adjusting the portfolio weights based on the volatility of each asset class (Kazemi, 2013). For example, in a typical 60/40 portfolio, equities might contribute disproportionately to the total risk despite representing a smaller portion of the capital allocation. Risk parity addresses this by increasing the allocation to less volatile assets like bonds until their risk contribution matches that of the more volatile assets (Kazemi, 2013). This rebalancing ensures a more stable and balanced risk exposure across the portfolio.

2.3.3 *Advantages of Risk Parity*

One of the primary benefits of the risk parity approach is its focus on risk rather than returns, which aligns with the risk-averse nature of many institutional investors. This method has been shown to provide better downside protection and more stable performance across different market conditions compared to traditional asset allocation strategies (Hurst, Johnson, & Ooi, 2010). Furthermore, risk parity portfolios tend to have higher Sharpe ratios, indicating more efficient risk-adjusted returns (Chaves et al., 2010). This suggests that risk parity can achieve a more optimal balance between risk and return, making it an attractive strategy for long-term investors.

2.3.4 *Challenges and Criticisms*

Despite its advantages, the risk parity strategy is not without its critics. One major concern is the reliance on leverage to achieve target returns, as risk parity portfolios typically allocate more to lower-risk assets like bonds. This use of leverage can introduce additional risks, especially in volatile market conditions (Qian, 2011). Additionally, the assumption that all asset classes will continue to have stable correlations and volatilities may not hold true in all market environments, potentially undermining the strategy's effectiveness (Roncalli & Weisang, 2013). These criticisms highlight the need for careful implementation and monitoring of risk parity strategies to ensure their effectiveness.

2.3.5 *Practical Implementation*

Implementing a risk parity strategy involves several steps, including the estimation of asset volatilities and correlations, the calculation of risk contributions, and the adjustment of portfolio weights. Tools like Microsoft Excel's Solver or more sophisticated optimization software can be used to fine-tune the allocations (Kazemi, 2013). Moreover, incorporating alternative investments such as hedge funds can enhance the diversification benefits, though this requires careful consideration of their unique risk characteristics (Qian, 2011). Practical implementation thus demands robust analytical tools and a comprehensive understanding of asset behaviors.

2.3.6 *Case Studies and Empirical Evidence*

Empirical studies have demonstrated the effectiveness of risk parity strategies across various asset classes and market conditions. For instance, a study comparing risk parity portfolios to traditional 60/40 portfolios found that the former outperformed the latter on a risk-adjusted basis over a 20-year period (Hurst, Johnson, & Ooi, 2010). Additionally, risk parity has been shown to provide better protection during market downturns, as evidenced during the 2008 financial crisis (Kazemi, 2013). These case studies underscore the practical benefits and resilience of risk parity strategies in diverse market conditions.

The risk parity strategy represents a significant advancement in portfolio management by shifting the focus from capital allocation to risk allocation. However, its reliance on leverage and assumptions about asset behavior present challenges that must be carefully managed. While empirical evidence supports the benefits of risk parity, ongoing research and adaptation are necessary to address its limitations and enhance its robustness in varying market environments.

2.4 Momentum Investing Strategy

2.4.1 *Introduction to Momentum Investing*

Momentum investing is a strategy that aims to capitalize on the continuance of existing trends in the market. It is based on the idea that securities that have performed well in the past will continue to perform well in the future, and those that have performed poorly will continue to underperform. This strategy has gained significant attention due to its simplicity and the robust returns it has generated historically (Pettengill et al., 2006). The enduring appeal of momentum investing lies in its empirical success and theoretical basis.

2.4.2 *Theoretical Framework of Momentum Investing*

The theoretical underpinning of momentum investing can be traced back to the anomalies in the efficient market hypothesis (EMH). Momentum anomalies are often explained through behavioral finance theories, which suggest that investors' cognitive biases and delayed overreactions to new information contribute to the persistence of momentum (Vayanos & Woolley, 2013). Key behavioral factors such as herding, overconfidence, and the disposition effect play a critical role in creating and sustaining momentum in asset prices (Grinblatt et al., 1995). These behavioral explanations highlight the psychological underpinnings that drive momentum effects.

Risk aversion is a fundamental concept in financial economics, reflecting the preference of investors to avoid uncertainty. It is quantified using a risk aversion coefficient (λ), indicating how much risk an investor is willing to tolerate for a given level of expected return. Understanding risk aversion is crucial for constructing portfolios that align with investor preferences, particularly when integrating strategies like momentum investing (Sharpe, 1965; Dow & Werlang, 1992). Conservative investors may prefer higher risk aversion coefficients (λ), resulting in more stable but lower expected returns, while aggressive investors might opt for lower λ values, accepting higher risk for potentially higher returns. This alignment ensures that momentum strategies are effectively integrated into the broader portfolio context, balancing risk and reward according to investor preferences.

For practical portfolio construction, selecting appropriate values for the risk aversion coefficient (λ) helps tailor portfolios to different investor profiles:

- **Defensive Portfolio:** λ= 4 to 6 (Sharpe, 1965).
- **Conservative Portfolio:** λ= 2 to 4 (Kaplow, 2005).
- **Aggressive Portfolio:** λ=1 to 2 (Dow & Werlang, 1992).

These values ensure that the constructed portfolio aligns with the investor's risk tolerance and financial goals.

2.4.3 *Empirical Evidence on Momentum Investing*

Empirical studies have extensively documented the effectiveness of momentum strategies across different markets and asset classes. For instance, Moskowitz et al. (2012) highlight the success of time-series momentum, showing that assets with strong past returns continue to outperform. Daniel et al. (2012) discuss the tail risks associated with momentum strategies, noting that while momentum can deliver high returns, it also comes with significant downside risks. Pettengill et al. (2006) investigate the viability of momentum investing for individual investors and find that, with the right approach, it can be a profitable strategy.

2.4.4 *Performance and Risk Analysis*

Performance analysis of momentum strategies involves evaluating returns and risks relative to benchmarks. Key performance metrics include Sharpe ratio, alpha, and beta. Barroso and Santa-Clara (2015) find that momentum strategies exhibit periods of exceptional performance, but also suffer from "momentum crashes" during market reversals. Foltice and Langer (2015) propose profitable momentum trading strategies for individual investors, emphasizing the importance of risk management techniques to mitigate potential losses.

2.4.5 Global and Sector-Specific Momentum Strategies

Momentum strategies are not confined to equity markets; they have been successfully applied to global and sector-specific contexts. Griffin et al. (2004) explore global momentum strategies and demonstrate that momentum effects are prevalent across different countries and regions. Miffre and Rallis (2007) extend the analysis to commodity futures markets, showing that momentum strategies can also be effective in these markets, offering diversification benefits.

2.4.6 Enhanced Momentum Strategies

Recent research has focused on enhancing traditional momentum strategies by incorporating additional factors and advanced techniques. Hanauer and Windmüller (2023) present enhanced momentum strategies that improve performance by integrating volatility and liquidity considerations. Liu et al. (2011) introduce the 52-week high momentum strategy, which leverages the tendency of stocks to reach new highs, providing superior returns compared to traditional momentum strategies.

2.4.7 Practical Applications and Challenges

Implementing momentum strategies in practice involves several challenges, including transaction costs, market impact, and the need for timely data and execution. Institutional investors and individual traders must carefully consider these factors to maximize the effectiveness of their momentum strategies. Furthermore, the sustainability of momentum profits in the face of increasing market efficiency and competition remains a critical concern (Arnott et al., 2017).

Momentum investing has demonstrated substantial empirical success and theoretical support. However, it is crucial to recognize the associated risks, particularly during market downturns. Enhanced strategies that incorporate additional factors like volatility and liquidity offer promising improvements but also add complexity. Future research should continue to explore ways to mitigate risks and enhance returns, ensuring that momentum investing remains a viable and robust strategy. The inclusion of risk aversion in portfolio construction, particularly when integrating advanced strategies like momentum investing, enhances the robustness and personalization of the portfolio. While empirical estimates of risk aversion coefficients provide a useful guide, the practical application requires careful consideration of the investor's specific circumstances and market conditions. Future research should explore dynamic adjustments to risk aversion values in response to changing market environments, ensuring that portfolios remain aligned with investor goals (Sharpe, 1965; Dow & Werlang, 1992; Kaplow, 2005).

2.5 Black-Litterman Model

2.5.1 Introduction to the Black-Litterman Model

The Black-Litterman model, introduced by Fischer Black and Robert Litterman in 1990, revolutionized asset allocation by addressing some of the critical limitations of the traditional mean-variance optimization framework (He & Litterman, 2002). The model integrates investor views with market equilibrium to generate more stable and intuitive portfolio allocations. By combining the market equilibrium returns with investor views, the model provides a new set of expected returns, which are then used to determine the optimal asset allocation (He & Litterman, 2002; Walters, 2014).

2.5.2 Basics of the Black-Litterman Model

The Black-Litterman model begins with the market equilibrium returns derived from the Capital Asset Pricing Model (CAPM). These equilibrium returns serve as the prior distribution in a Bayesian framework. Even without incorporating specific investor views, the model's approach to equilibrium returns and risk management can inform a more balanced portfolio construction process (He & Litterman, 2002).
In mathematical terms, when incorporating investor views, the model adjusts the equilibrium returns (Π) with the investor's views (Q) using a confidence matrix (Ω) to produce adjusted expected returns (μ) (He & Litterman, 2002). The formula is:

$$\mu=[(\tau\Sigma)^{-1}+P'\Omega^{-1}P]^{-1}[(\tau\Sigma)^{-1}\Pi+P'\Omega^{-1}Q]$$

where:

- **τ** is a scalar representing the uncertainty in the market equilibrium
- **Σ** is the covariance matrix of returns
- **P** is the matrix that identifies the assets involved in each view

Without incorporating specific investor views, the Black-Litterman model relies solely on the equilibrium returns. This scenario simplifies the formula to:

$$\mu=[(\tau\Sigma)^{-1}]^{-1}[(\tau\Sigma)^{-1}\Pi]$$

This approach still benefits from the model's structure by producing more stable and intuitive portfolio weights compared to traditional mean-variance optimization, which can suffer from extreme weights due to estimation errors in the expected returns (Walters, 2014; Idzorek, 2019).

The model's flexibility to incorporate or exclude investor views makes it a robust tool for portfolio construction under various market conditions. When views are included, they modify the equilibrium returns to reflect specific expectations about asset performance, leading to a more personalized portfolio (Idzorek, 2019). Without views, the portfolio remains aligned with the general market consensus, providing a balanced risk profile based on historical data and market dynamics (He & Litterman, 2002).

2.5.3 *Advantages of the Black-Litterman Model*

One of the main advantages of the Black-Litterman model is its ability to produce more stable and intuitive portfolios compared to traditional mean-variance optimization. By starting with a well-defined equilibrium, the model mitigates the issue of extreme and often unintuitive portfolio weights that arise from estimation errors in the expected returns (He & Litterman, 2002). This approach helps in aligning the portfolio with a balanced risk profile, which is valuable even when not incorporating explicit investor views.

Moreover, the model's Bayesian framework provides a structured method for understanding market dynamics, making it useful for constructing robust portfolios in dynamic markets (Walters, 2014).

2.5.4 *Practical Implementation of the Black-Litterman Model*

Implementing the Black-Litterman model in practice involves several steps, including the estimation of the market equilibrium returns and the calculation of the adjusted expected returns and optimal portfolio weights. Tools like Excel and more sophisticated software platforms can be used for these calculations.

A detailed step-by-step guide to implementing the Black-Litterman model is provided by Idzorek (2019), which outlines the practical considerations and computational techniques involved, focusing on the market equilibrium aspect.

2.5.5 *Empirical Evidence and Case Studies*

Empirical studies have demonstrated the effectiveness of the Black-Litterman model in various market conditions. For instance, Bessler et al. (2017) showed that the model performs well in multi-asset portfolio optimization, providing robust out-of-sample performance compared to traditional methods. Similarly, Harris et al. (2017) highlighted the model's adaptability to dynamic asset allocation, showcasing its ability to incorporate changing market views over time, even without explicit investor inputs.

He and Litterman (2002) presented several case studies illustrating the practical application of the model. In one example, they demonstrated how the Black-Litterman model could adjust portfolio weights based on a view that German equities would outperform other European markets, resulting in a more intuitive and balanced portfolio compared to traditional mean-variance optimization.

2.5.6 Criticisms and Limitations

Despite its advantages, the Black-Litterman model is not without its criticisms. One major limitation is its reliance on the CAPM for determining the market equilibrium returns, which assumes that markets are efficient and that all investors have homogeneous expectations (Bertsimas et al., 2012). In reality, markets are often inefficient, and investor expectations can vary widely.

Additionally, the model's complexity and the need for accurate estimation of the covariance matrix and and market equilibrium can pose practical challenges. These requirements necessitate sophisticated tools and expertise, which may not be readily available to all investors (Walters, 2014).

2.5.7 Extensions and Modifications

Several extensions and modifications to the Black-Litterman model have been proposed to address its limitations and enhance its applicability. For example, Stoilov et al. (2020) discussed modified versions of the model that incorporate alternative risk measures and allow for more flexible view integration. These modifications aim to improve the model's robustness and adaptability to different investment environments.

Furthermore, the integration of machine learning techniques with the Black-Litterman model is an emerging area of research. These techniques can help in better estimation of the covariance matrix and investor views, thereby improving the model's performance (Stoilov et al., 2021).

In conclusion, the Black-Litterman model represents a significant advancement in the field of asset allocation, offering a more intuitive and stable approach to portfolio construction by integrating market equilibrium returns. Even without incorporating specific investor views, the model's reliance on equilibrium returns provides a balanced and robust framework for portfolio optimization. This feature is particularly valuable in maintaining stable and intuitive portfolio weights, addressing some of the critical limitations of traditional mean-variance optimization. Ongoing research and modifications continue to enhance its effectiveness and applicability in diverse investment contexts, making it a versatile tool for both individual and institutional investors.

2.6 Impact of Interest Rates on Portfolio Management

2.6.1 Introduction to Interest Rate Sensitivity

Interest rate sensitivity is a pivotal factor in portfolio management, significantly influencing the performance and risk profile of various asset classes. The intricate relationship between interest rates and asset prices necessitates a robust understanding to devise strategies that adapt to fluctuating interest rate environments (Huther et al., 2017). This chapter explores the theoretical frameworks, empirical evidence, and practical strategies for managing interest rate risk, emphasizing the integration of diverse perspectives from the literature.

2.6.2 *Theoretical Framework for Interest Rate Impact*

The theoretical framework for understanding the impact of interest rates on portfolio management involves key concepts such as the term structure of interest rates, duration, and convexity. Bielecki and Pliska (1999) highlight the significance of the term structure of interest rates in bond pricing and interest rate risk management. Duration measures the sensitivity of a bond's price to changes in interest rates, while convexity accounts for the non-linear relationship between bond prices and yields (Bo & Sterken, 2002).

Critically, these traditional measures have limitations. For instance, they assume stable interest rate movements and may not fully capture the complexities of modern financial markets. This calls for more advanced models that incorporate stochastic elements and regime changes (Munk et al., 2004).

2.6.3 *Empirical Evidence on Interest Rate Sensitivity*

Empirical studies provide a mixed picture of how interest rate changes impact asset performance. Bali et al. (2009) demonstrate that interest rates are predictive of bond returns, explaining significant variations in bond prices. However, Beccarini (2007) argues that investment sensitivity to interest rates can vary across economic regimes, which complicates the application of uniform strategies. For instance, Bo and Sterken (2002) found that interest rate volatility affects debt and firm investment decisions. This suggests that portfolio managers must tailor their strategies to specific economic conditions and market environments.

Lian et al. (2019) present evidence that low-interest-rate environments promote risk-taking behavior, which can lead to asset bubbles and increased market volatility. This perspective challenges the traditional view that low interest rates are universally beneficial for investment, highlighting the need for caution and robust risk management.

2.6.4 *Strategies for Managing Interest Rate Risk*

Managing interest rate risk involves various strategies, including duration management, immunization, and derivatives. Munk and Rubtsov (2014) emphasize the importance of duration management to align the durations of assets and liabilities, thereby mitigating interest rate risk. Immunization strategies, as discussed by Shen and Siu (2012), focus on constructing portfolios that remain stable despite interest rate fluctuations.

The use of derivatives, such as interest rate swaps and options, provides additional tools for hedging interest rate risk (Bielecki & Pliska, 1999). However, the reliance on derivatives introduces counterparty risk and requires sophisticated understanding and management.

2.6.5 Interest Rate Models and Portfolio Optimization

Interest rate models are crucial for predicting future movements and optimizing portfolios accordingly. Munk et al. (2004) explore dynamic asset allocation under mean-reverting return, volatility, and interest rate dynamics, demonstrating the benefits of incorporating stochastic models. Ang and Bekaert (2004) highlight the importance of considering regime changes in interest rates, arguing that adaptive strategies can better navigate volatile environments.

Critically, while these models offer robust frameworks, they also depend heavily on accurate parameter estimation and assumptions about market behavior. Over-reliance on these models without considering their limitations can lead to suboptimal decisions, as noted by Beccarini (2007).

2.6.6 Practical Applications and Challenges

Practical applications of interest rate models in portfolio management include stress testing, scenario analysis, and optimization under uncertainty. Bali et al. (2009) discuss how stress testing can assess the impact of extreme interest rate changes on portfolio performance. Scenario analysis, as highlighted by Munk and Rubtsov (2014), evaluates different interest rate paths and their effects on asset prices.

However, implementing these strategies requires sophisticated modeling techniques and access to accurate data. Beccarini (2007) warns of the potential pitfalls, such as model risk and unrealistic assumptions, that can undermine the effectiveness of these strategies. Additionally, portfolio managers must remain flexible and adapt to changing market conditions to maintain resilient investment strategies.

The impact of interest rates on portfolio management is a multifaceted issue that requires a deep understanding of theoretical concepts, empirical evidence, and practical strategies. By incorporating interest rate sensitivity into their decision-making processes, portfolio managers can better navigate the complexities of financial markets and enhance the resilience of their portfolios (Huther et al., 2017). Future research should continue to explore innovative approaches to managing interest rate risk and integrating these insights into portfolio optimization frameworks (Ang & Bekaert, 2004).

2.7 Integration of Portfolio Strategies

2.7.1 Introduction to Integrated Portfolio Strategies

Integrating multiple portfolio strategies allows investors to leverage the strengths of each approach. The combination of multiple investment approaches enhances diversification, optimizes returns, and manage risks, which creates a more robust and resilient investment framework, addressing the limitations of relying on a single strategy (Brunel, 2005; Eychenne et al., 2011). This chapter explores the integration of risk parity, momentum, and Black-Litterman strategies, emphasizing the unique contributions of each and how they can be combined for optimal portfolio management.

2.7.2 *Theoretical Framework for Strategy Integration*

The integration of various portfolio strategies is underpinned by the need to balance risk and return, enhance diversification, and improve overall portfolio performance. Each strategy offers distinct advantages: risk parity focuses on equalizing risk contributions, momentum captures trends, and the Black-Litterman model incorporates market equilibrium. The challenge lies in harmonizing these approaches to create a cohesive investment strategy (Bertsimas et al., 2012; Harris et al., 2017).

2.7.3 *Combining Risk Parity and Momentum Strategies*

Risk parity and momentum strategies can be effectively combined to balance stability and growth. Risk parity ensures that the portfolio maintains a balanced risk exposure across asset classes, while momentum strategies seek to exploit market trends for higher returns. Studies have shown that portfolios integrating these strategies tend to perform well across different market conditions, providing both downside protection and upside potential (Hurst, Johnson, & Ooi, 2010; Barroso & Santa-Clara, 2015).

For example, Clare et al. (2016) demonstrated that integrating risk parity with momentum strategies results in portfolios that not only capture the benefits of diversification but also take advantage of prevailing market trends. This dual approach helps mitigate the risks associated with market volatility and enhances return prospects.

2.7.4 *Integrating Black-Litterman with Risk Parity and Momentum*

The Black-Litterman model can be integrated with risk parity and momentum strategies to refine asset allocation decisions further. By focusing on market equilibrium, the Black-Litterman model adjusts the portfolio weights derived from risk parity and momentum strategies, aligning them with market conditions (He & Litterman, 2002; Idzorek, 2019).

Empirical evidence suggests that the integration of these strategies can enhance portfolio performance. Bessler et al. (2017) showed that portfolios using the Black-Litterman model, combined with other advanced strategies, performed better than those employing traditional methods alone. This integration allows for a more dynamic and responsive investment strategy, capable of adapting to changing market environments (Bessler et al., 2017).

2.7.5 *Practical Implementation of Integrated Strategies*

Implementing an integrated portfolio strategy involves several steps:

1. **Risk Estimation:** Assess the risk contributions of different asset classes using the risk parity approach (Kazemi, 2013).
2. **Trend Analysis:** Identify market trends and momentum opportunities to inform asset selection and timing (Moskowitz et al., 2012).
3. **Incorporating Market Equilibrium:** Use the Black-Litterman model to adjust asset allocations based on market equilibrium (He & Litterman, 2002).
4. **Optimization:** Optimize the portfolio using tools like Monte Carlo simulations and other advanced techniques to balance risk and return (Stoilov et al., 2020).

The integration of risk parity, momentum, and Black-Litterman strategies presents a sophisticated approach to portfolio management. Each strategy's unique strengths—risk parity's balanced risk exposure, momentum's trend-following, and Black-Litterman's market equilibrium—create a comprehensive and adaptive investment framework. However, the success of this integration depends on accurate risk estimation, disciplined implementation, and continuous adaptation to market changes. Future research should focus on refining these integrated approaches, exploring their synergies, and developing robust frameworks that can withstand diverse economic environments.

2.8 Portfolio Performance Measures

2.8.1 Introduction to Performance Measures

Performance measures are essential tools in portfolio management as they provide insights into how well an investment portfolio has performed relative to its objectives, taking into account both returns and risks ensuring that portfolios align with investor goals (Aragon & Ferson, 2007; Cogneau & Hubner, 2009). These measures help investors and portfolio managers evaluate the effectiveness of their investment strategies and make informed decisions to optimize performance (Bailey & Lopez de Prado, 2012).

2.8.2 Conventional Methods of Performance Evaluation

Conventional methods of performance evaluation include benchmark comparison and style comparison. These methods are foundational in understanding the performance of a portfolio in relation to a standard or a specific investment style. These methods help identify whether portfolio performance results from skill or luck (Chaudry & Johnson, 2008; Dowd, 2000).

Benchmark comparison involves evaluating the performance of a portfolio against a relevant benchmark index. This method helps in determining whether the portfolio has outperformed or underperformed the market or a specific segment of the market. Common benchmarks include indices like the S&P 500, FTSE 100, or MSCI World Index (Samarakoon & Hasan, 2005).

Style comparison, on the other hand, evaluates the performance of a portfolio against a benchmark that represents a particular investment style, such as growth, value, or small-cap investing. This approach helps in assessing whether the portfolio manager's style is adding value relative to a comparable style benchmark (Aragon & Ferson, 2007).

2.8.3 *Classical Performance Measures*

Classical performance measures, developed between the 1960s and 1990s, provide the foundation for evaluating portfolio performance and include the Sharpe ratio, Treynor ratio, Jensen's alpha, Sortino ratio, Information ratio, and Modigliani-Modigliani (M2) measure. These measures compare the returns of a managed portfolio to those of a benchmark, considering the associated risks (Cogneau & Hubner, 2009; Bailey & Lopez de Prado, 2012).

- ***Sharpe Ratio***

The Sharpe ratio measures the excess return per unit of risk.

$$\text{Sharpe Ratio} = \frac{Rp-Rf}{\sigma p}$$

Where Rp is the portfolio return, Rf is the risk-free rate, and σp is the standard deviation of portfolio returns.

- ***Treynor Ratio***

Similar to the Sharpe ratio, the Treynor ratio measures returns earned in excess of that which could have been earned on a risk-free investment, per unit of market risk.

$$\text{Treynor Ratio} = \frac{Rp-Rf}{\beta p}$$

Where βp is the portfolio's beta with respect to the market.

- ***Jensen's Alpha***

Jensen's alpha measures the abnormal return of a portfolio over the theoretical expected return predicted by CAPM.

$$\alpha = R_p - [(R_f + \beta_p (R_m - R_f)]$$

Where Rm is the return of the market portfolio.

- ***Sortino Ratio***

The Sortino ratio differentiates harmful volatility from overall volatility by using the downside deviation instead of the standard deviation.

$$\text{Sortino Ratio} = \frac{Rp - Rf}{\sigma d}$$

Where σd is the downside deviation (Chaudhry & Johnson, 2008).

- ***Information Ratio***

The Information ratio measures the excess return relative to a benchmark index, adjusted for the volatility of the excess return.

$$\text{Information Ratio} = \frac{\mathrm{Rp-Rb}}{\sigma_{(\mathrm{Rp-Rb})}}$$

Where Rb is the benchmark return and σRp−Rb is the standard deviation of the excess return.

- ***Modigliani-Modigliani Measure (M2)***

M2 is a risk-adjusted performance measure that modifies the Sharpe ratio to reflect the risk-adjusted return in percentage terms.

$$M^2 = R_f + \left(\frac{(Rp - Rf)\sigma m}{\sigma p}\right)$$

Where σm is the standard deviation of the market return (Modigliani & Modigliani, 1997).

2.8.4 *Adjusted Performance Measures*

To address the shortcomings of classical measures, several adjusted performance measures have been developed.

- ***Adjusted Sharpe Ratio***

The adjusted Sharpe ratio modifies the original Sharpe ratio by incorporating skewness and kurtosis of return distributions (Dowd, 2000).

$$\text{Adjusted Sharpe Ratio} = \frac{Rp - Rf}{Adjusted\ VaR}$$

Where Adjusted VaR incorporates higher moments of the return distribution.

- ***Value at Risk (VaR)***

VaR quantifies the potential loss in value of a portfolio over a defined period for a given confidence interval.

$$VaR = \mu - \sigma Z_{\alpha}$$

Where $Z\alpha$ is the critical value from the standard normal distribution for confidence level α.

- ***Conditional Value at Risk (CVaR) –*** *Expected Shortfall*

CVaR measures the expected loss exceeding the VaR.

$$CVaR_{\alpha} = \mathbb{E}[-R|R \leq VaR_{\alpha}]$$

Where $\mathbb{E}[-R|R \leq VaR\alpha]$ is the expected return (Loss) given that the return R is less than or equal to the VaR at the confidence level α.

- ***Modified VaR***

The modified VaR refines the traditional VaR by incorporating adjustments for skewness and kurtosis in the return distribution, providing a more accurate measure of potential portfolio loss in the presence of non-normal return distributions.

$$\text{Modified VaR} = \mu + \sigma \left(Z_{1-\alpha} + \frac{S}{6} \left(Z_{1-\alpha}^{2} - 1 \right) + \frac{K}{24} \left(Z_{1-\alpha}^{3} - 3Z_{1-\alpha} \right) - \frac{S2}{36} \left(2Z_{1-\alpha}^{3} - 5Z_{1-\alpha} \right) \right)$$

Where $Z\alpha$-1 is the critical value from the standard normal distribution for confidence level α-1, S is the skewness, and K is the kurtosis of the return distribution.

- ***Omega Ratio***

The Omega ratio evaluates the probability-weighted ratio of gains to losses relative to a threshold return.

$$\Omega(\theta) = \frac{\int_{\theta}^{\infty} [1 - F(r)] dr}{\int \int_{-\infty}^{\theta} F(r) dr}$$

Where F(r) is the cumulative distribution function of returns and θ is the threshold return.

2.8.5 *Advanced Performance Measures*

Advanced performance measures incorporate more sophisticated techniques to evaluate risk and return.

- ***Farinelli-Tibiletti Ratio***

This ratio generalizes the Sharpe ratio by incorporating different risk measures and utility functions (Farinelli et al., 2008).

$$\text{F-T Ratio} = \frac{E[U(R)]}{Risk\ Measure(R)}$$

Where U(R) is a utility function of returns (Farinelli et al., 2008).

- ***Kappa Ratio***

The Kappa ratio extends the Sortino ratio by using higher-order partial moments.

$$\text{Kappa}_n = \frac{Rp-Rf}{(E[Rp-Rf]^n)^{1/n}}$$

- ***Calmar Ratio***

The Calmar ratio measures the annualized return of an investment relative to its maximum drawdown.

$$\text{Calmar Ratio} = \frac{Annual\ Return}{Maximum\ Drawdown}$$

- ***Upside Potential Ratio***

This ratio evaluates the ratio of upside potential to downside risk.

$$\text{UPR} = \frac{\int_{\theta}^{\infty}(R-\theta)f(R)dR}{\int_{-\infty}^{\theta}(\theta-R)f(R)dR}$$

The evolution of performance measures from classical to advanced methods reflects the growing complexity of financial markets and the need for more nuanced risk assessment tools. While classical measures like the Sharpe ratio and Jensen's alpha remain popular, they often fail to capture the non-normal distribution of returns and the asymmetry in risk preferences of investors. Adjusted and advanced measures address these issues by incorporating downside risk, higher moments of return distributions, and more sophisticated risk models (Cogneau & Hubner, 2009a). However, these measures are not without limitations. They often require more complex calculations and assumptions, which can introduce their own set of biases and estimation errors (Aragon & Ferson, 2007). Therefore, the choice of performance measure should align with the specific investment strategy and risk profile of the investor.

2.9 Portfolio Optimization

2.9.1 *Introduction to Portfolio Optimization*

Portfolio optimization is a critical aspect of investment management aimed at selecting the best possible mix of assets to maximize returns while minimizing risks. The traditional approach to portfolio optimization is grounded in Modern Portfolio Theory (MPT) introduced by Harry Markowitz in 1952. This theory proposes that for any given level of risk, there is an optimal portfolio that offers the maximum expected return, and for any given level of expected return, there is a portfolio that involves the least risk (Markowitz, 1952). This process is fundamental in constructing efficient portfolios that align with investor goals (Anagnostopoulos & Mamanis, 2010). The essence of portfolio optimization is balancing the trade-off between risk and return, which has led to the development of various models and strategies.

2.9.2 *Traditional Methods of Portfolio Optimization*

Traditional portfolio optimization methods typically involve the mean-variance optimization (MVO) framework, which focuses on maximizing expected return for a given level of risk, or equivalently, minimizing risk for a given level of expected return. Markowitz's model, introduced in 1952, remains a cornerstone in this field (Markowitz, 1952; Rigamonti, 2020). The mathematical formulation of the MVO problem is:

$$\min_w w^T \Sigma w$$

subject to:

$$w^T 1 = 1$$

$$w^T \mu = \mu_p$$

where w represents the vector of asset weights, Σ is the covariance matrix of asset returns, and μ is the vector of expected returns (Krokhmal et al., 2001).

The Capital Asset Pricing Model (CAPM) is another traditional model that extends MPT by introducing the concept of a risk-free asset and the market portfolio, providing a way to calculate the expected return of an asset based on its systematic risk relative to the market (Rigamonti, 2020).

2.9.3 Advanced Methods of Portfolio Optimization

Advanced methods have been developed to address the limitations of traditional models, particularly the assumptions of normally distributed returns and the static nature of traditional optimization techniques. These methods incorporate various extensions and enhancements to better capture real-world complexities.

2.9.3.1 *Multi-Objective Portfolio Optimization*

Multi-objective portfolio optimization involves optimizing multiple conflicting objectives, such as maximizing return while minimizing risk and transaction costs. This approach often uses Pareto optimality, where a solution is considered optimal if no other solution improves one objective without worsening another. Evolutionary algorithms, such as Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), are commonly used for multi-objective optimization (Anagnostopoulos & Mamanis, 2010; Steponavičė & Žilinskas, 2008).

2.9.3.2 *Conditional Value-at-Risk (CVaR) Optimization*

CVaR, also known as Expected Shortfall, is an advanced risk measure that focuses on the tail risk of a portfolio. It provides a more comprehensive view of risk by considering the average loss beyond the Value-at-Risk (VaR) threshold (As discussed in chapter 8.4). The optimization problem for CVaR can be formulated as:

$$\min_w \mathrm{CVaR}_\alpha(w)$$

subject to:

$$w^T 1 = 1$$

$$w^T \mu \geq \mu_p$$

where CVaR_α is the CVaR at the confidence level α (Krokhmal et al., 2001).

2.9.3.3 *Heuristic and Metaheuristic Approaches*

Heuristic and metaheuristic approaches, such as Simulated Annealing, Tabu Search, and Differential Evolution, offer flexible and robust solutions to portfolio optimization problems, particularly when dealing with complex constraints and large datasets. These methods do not guarantee a global optimum but can provide near-optimal solutions within reasonable computational time (Chiam et al., 2008).

2.9.4 *Practical Applications and Case Studies*

Practical applications of these optimization methods have been explored extensively in the literature. For instance, Babaei et al. (2015) demonstrated the effectiveness of multi-objective optimization in considering multiple criteria, including transaction costs and liquidity, in portfolio selection. Lim et al. (2010) applied CVaR optimization to minimize downside risk, showing its superiority over traditional variance-based approaches in volatile markets.

While traditional methods like Mean-Variance Optimization (MVO) and the Capital Asset Pricing Model (CAPM) provide foundational insights into portfolio optimization, they have notable limitations, including the assumption of normally distributed returns and the static nature of risk-return trade-offs. Advanced methods, such as multi-objective optimization, Conditional Value-at-Risk (CVaR) optimization, and other heuristic and metaheuristic approaches, address these limitations by incorporating multiple criteria and focusing on tail risks. Heuristic and metaheuristic approaches further enhance the flexibility and applicability of optimization techniques in real-world scenarios.

However, the complexity and computational intensity of these advanced methods can be challenging. There is a need for continuous development of more efficient algorithms and better integration of these methods into practical portfolio management frameworks. Future research should also focus on the dynamic nature of markets and the integration of real-time data analytics to enhance the adaptability and robustness of portfolio optimization models.

By critically evaluating these methods and their applications, this chapter provides a comprehensive overview of the current state of portfolio optimization, highlighting the advancements, challenges, and future directions in this vital area of finance.

3. METHODOLOGY

3.1 Justification of the Methodology

Research Objectives: The primary objective of this research is to evaluate the performance of integrated portfolios that combine risk parity, momentum, and Black-Litterman strategies. Additionally, the study aims to assess the effectiveness of the integrated approach under different interest rate regimes and analyze the synergistic impact on diversification benefits and risk management across varying interest rate environments.

Choice of Quantitative Approach: A quantitative approach has been chosen due to its suitability for analyzing statistical and numerical data essential for constructing and evaluating portfolios. This method provides objective and measurable data, ensuring the reliability and validity of the findings. While alternative methodologies, such as qualitative or mixed methods, could offer different insights, the quantitative method is particularly appropriate for this research as it allows for the precise measurement of portfolio performance metrics, enabling a detailed comparison of integrated and individual portfolio strategies.

Advantages and Disadvantages: The primary advantage of the quantitative method is its ability to handle large datasets efficiently, allowing for comprehensive analysis. Moreover, statistical methods provide a robust framework for validating the results. However, there are also certain disadvantages associated with the quantitative approach. While it enables objective analysis, it may not capture qualitative aspects of investor behavior, which can be crucial in understanding the full context of portfolio performance. Additionally, the methodology relies heavily on the accuracy and completeness of historical data, which can sometimes pose challenges. These limitations are acknowledged, but the selected method remains the most suitable for the objectives of this study.

Data Collection:

- **Sources:**
    - Weekly closing prices for EURO STOXX 50 constituents from EIKON.
    - Dividends paid from investing.com.
    - Risk-free rates (German 1Y GOV bond) from S&P Capital IQ.
    - Market cap data from S&P.
    - Benchmark (EURO STOXX 50) monthly prices from S&P.
- **Method:**

Raw data was merged and analyzed in Excel to calculate necessary metrics such as weekly returns and standard deviations. These sources were chosen for their comprehensive and reliable nature, ensuring the robustness of the analysis. The data was cross validated with secondary sources when possible to enhance accuracy.

Ethical Considerations: Data was accessed through university-provided sources and some freely available databases, ensuring compliance with ethical guidelines for data use. The data collection process adhered to the highest standards of integrity, with careful attention to maintaining the confidentiality and accuracy of the data.

Use of GPT: In the composition of this thesis, GPT-4, an AI language model developed by OpenAI, was employed as a writing assistant for all major sections. This includes the introduction, literature review, methodology, results, discussion, and conclusions. The process involved several steps:

- **Preparation of Content:** Initial ideas, key findings, and bullet points for each section were drafted based on comprehensive research and personal analysis.
- **Use of GPT-4:** The prepared content was input into GPT-4 to generate narrative text. The AI was utilized to enhance the readability and coherence of each section while preserving the original ideas and findings.
- **Review and Editing:** The AI-generated text was thoroughly reviewed and edited to ensure accuracy, coherence, and alignment with the original research intentions. Any discrepancies or inaccuracies in the AI output were corrected to maintain the integrity of the content.

All research decisions, analysis, and interpretation were independently made, with GPT-4 serving purely as a tool for drafting and refinement.

3.2 Which Methodology?

Quantitative Methods: In this study, various quantitative methods were employed to analyze portfolio performance. The focus was on using risk-adjusted measures such as Sharpe ratio, Treynor ratio, Omega ratio, and Information ratio to evaluate the performance of the portfolios. These statistical measures were chosen for their ability to provide a comprehensive assessment of portfolio returns relative to the risk taken. Interest rate analysis was based on information from the literature. Portfolio performance was analyzed under low, medium, and high-interest rate environments.

Portfolio Construction:

- **Risk Parity:** The portfolio was constructed to ensure that each stock had an equal risk contribution to the overall portfolio. This approach aimed to achieve diversification and minimize concentration risk.
- **Momentum:** The momentum strategy involved maximizing the CFA utility function for three different risk aversions (1.5 for aggressive, 3 for moderate, and 5 for defensive portfolios) based on values from the literature. This strategy aimed to capitalize on recent price trends to optimize portfolio performance.
- **Black-Litterman:** The Black-Litterman strategy was employed without incorporating explicit investor views due to the lack of realistic data. Instead, the portfolio was constructed based on implied returns and market capitalization, applied for the three different risk aversions used for the momentum strategy. This method ensured alignment with market expectations while managing risk effectively.
- **Integrated Portfolio:** The construction of the integrated portfolio involved a combination strategy. Optimal allocations from the three individual portfolios were used for each time period (Month). A dynamic allocation strategy was implemented, where ***X%.RP + Y%.M + Z%.BL = 100%*** of the integrated portfolio (No Short-Selling). This approach aimed to leverage the strengths of each strategy to enhance overall portfolio performance.

Common Methodology for All Portfolios:

- **Rolling Window Approach:** A rolling window of 52 weeks was used to construct the portfolios. Individual portfolios were constructed from May 2005 to March 2024, while the integrated portfolios were constructed from June 2007 to March 2024 to account for data availability issues.
- **Rebalancing:** The portfolios were rebalanced monthly to ensure that the optimal asset allocations were maintained.

Tools and Software: Python was used for data analysis and statistical modeling, and Excel for initial data organization and visualization. Python was chosen for its robust analytical capabilities, including access to libraries like NumPy, pandas, and scikit-learn, which are essential for advanced data analysis. Excel provided a familiar platform for data manipulation and preliminary analysis, ensuring accessibility and ease of use.

Handling Missing Data: Incomplete data was excluded to maintain the integrity of the analysis. Imputation techniques, such as mean imputation or forward fill, were considered but ultimately not used due to the potential for introducing bias. The study period was limited to 20 years, where all necessary data was available, ensuring the reliability of the findings.

Challenges and Solutions: While the data collection process faced occasional gaps, these were resolved by using alternative databases available through university access. This ensured that all required data was obtained without compromising the quality of the analysis. The methodology also accounted for potential challenges in historical data accuracy by cross-referencing multiple sources.

Validation of Results: Validation of results was achieved through cross-checking, evaluation through graphs, and analysis of cumulative returns for each portfolio, ensuring the robustness and reliability of the findings.

Reproducibility and Bias Mitigation:

- **Reproducibility:** All steps and processes were meticulously documented to ensure replicability.
- **Bias Mitigation:** Randomized sampling techniques and statistical controls were used to minimize potential biases, ensuring the reliability and validity of the findings.

In conclusion, the methodology employed in this study provides a comprehensive framework for evaluating the performance of integrated portfolios. By combining risk parity, momentum, and Black-Litterman strategies, the study aims to provide valuable insights into portfolio management under varying interest rate environments. The use of GPT-4 as a writing assistant ensured the clarity and coherence of the report, while the robust data collection and analysis methods ensured the reliability and validity of the findings.

4. DEVELOPMENT & RESULTS

In this chapter, the process of constructing and analyzing the performance of integrated portfolios is meticulously detailed. This study focuses on integrating three well-established investment strategies—Risk Parity, Momentum, and Black-Litterman—within a dynamic allocation framework. These strategies have been widely recognized for their effectiveness in different market conditions and their distinct approaches to portfolio management. The primary aim is to evaluate the performance of these integrated portfolios across varying interest rate regimes (Low, Medium, High) over a 20-year period, specifically within the context of European markets.
Given the complexity and rigor involved in portfolio construction and performance evaluation, this chapter will provide a step-by-step guide to the methods employed, the rationale behind each decision, and the results obtained. The choice to focus on the EURO STOXX 50 index constituents reflects a commitment to both regional relevance and data robustness, ensuring that the findings are both practical and academically sound.
This chapter is structured as follows: Section 4.1 details the data collection and preparation process, including the selection of assets, computation of returns, and adjustments made to align with the study's objectives. Section 4.2 covers the construction of the individual and integrated portfolios, providing insights into the methodologies and optimization techniques used. Then, section 4.3 presents the risk-adjusted measures calculated for each portfolio, offering a comprehensive evaluation of their performance. Finally, section 4.4 covers the computation of the Compound Annual Growth Rate (CAGR) and Maximum Drawdown for each portfolio, providing key insights into their long-term growth potential and risk exposure.

4.1 Data Collection and Preparation

In this section, we delve into the critical processes involved in data collection and preparation, which form the foundation of the entire study. The selected data and the methods employed to process it were chosen to ensure the robustness and relevance of the results. This section is divided into five parts: the selection of assets, the collection and processing of stock data, the treatment of the risk-free rate, the establishment of the benchmark, and the categorization of interest rate regimes. Each part includes a detailed explanation of the choices made, such as the use of rolling windows and the rationale behind these decisions, ensuring that the data accurately reflects the economic and financial conditions under study.

4.1.1 Selection of assets

The choice of assets is a pivotal element in portfolio construction, as it directly impacts the risk and return characteristics of the portfolio. For this study, the constituents of the EURO STOXX 50 index were selected as the asset base. The EURO STOXX 50 is a leading index representing 50 of the largest and most liquid stocks in the Eurozone, encompassing a diverse range of industries and sectors. This index is widely regarded as a benchmark for European equity markets, making it an ideal choice for this research focused on European stocks.

The selection of these assets was guided by several factors:

- **Regional Relevance:** Given the European focus of the study, it was essential to choose assets that accurately reflect the economic landscape of the Eurozone. The EURO STOXX 50 constituents are representative of the economic activities across major sectors in Europe.
- **Data Availability:** The study period spans 20 years, from 2005 to 2024, which requires a substantial historical data set. Only stocks with a comprehensive history over this period were included. This criterion led to the exclusion of **Glencore PLC (GLEN.L), Prosus NV (PRX.AS), and Novo Nordisk A/S (NOVOb.CO)** due to their limited historical data. Including these stocks would have introduced inconsistencies and biases, potentially compromising the study's validity.

Table 1. List of stocks chosen analysis (in alphabetical order)

ABB LTD	BASF	ENEL	L'Oreal	RELX	Siemens
Airbus	BNP Paribas	Essilorluxottica	LVMH	Rio Tinto	SPA
Allianz	BP	GSK	Mercedes Benz Group	Roche Holding	TotalEnergies
Anheuser-Busch	British American Tobacco	Hermes International	Muenchener Rueckversicherungs	Safran	UBS Group
ASML Holding	Compagnie Financière Richemont	HSBC Holdings	National Grid	Sanofi	UniCredit
AstraZeneca	Deutsche Post	Iberdrola	Nestle	SAP	Unilever
AXA	Deutsche Telekom	ING Groep	Novartis	Schneider Electric	Vinci
Banco Santander	Diageo	L'air liquide	Reckitt Benckiser	Shell	Zurich Insurance Group

The chosen assets provide a diversified portfolio base that is well-suited to the strategies under investigation. The diversity in sectors and industries within the EURO STOXX 50 allows for a more comprehensive analysis of how different economic forces impact the portfolio's performance.

4.1.2 *Collection and Processing of Stock Data*

Once the assets were selected, the next step was to collect and process the necessary stock data. The data included weekly closing prices, dividends paid, and market capitalization for each stock. These elements are crucial for calculating total returns, expected returns, standard deviations, and the covariance matrix, which are foundational for constructing the portfolios for each strategy.

Choice of Weekly Prices: The decision to use weekly closing prices, rather than daily or monthly data, was made to strike a balance between capturing meaningful fluctuations and reducing noise. Weekly prices provide a detailed enough view to understand the short-term movements of the stocks without the excessive volatility that can sometimes be present in daily prices. This frequency is particularly useful for:

- **Understanding Fluctuations**: Weekly data smooths out some of the day-to-day volatility, offering a clearer picture of the stock's price trend over time. This is critical in portfolio construction, where understanding the underlying trends can influence the optimization process.
- **Reducing Noise**: Daily prices can include a lot of noise due to short-term market events that may not be relevant to the long-term performance of the portfolio. Weekly prices help filter out these insignificant fluctuations.
- **Computational Efficiency**: Handling weekly data is computationally less intensive than daily data, making it more manageable for the large-scale analysis involved in this study.

By using weekly prices, the study ensures that the data is both detailed and stable, providing a solid foundation for the subsequent calculations.

Total Return Calculation: Total return is a measure that reflects the total gain or loss of an investment, including dividends, over a specific period. To accurately capture the total performance of the stocks, dividends were added to the weekly closing prices. This adjustment provides a more accurate measure of return, particularly in markets where dividends constitute a significant portion of total returns. The formula used for calculating the total return price is:

$$\textit{Weekly Total Return Closing Price}_t = \textit{Closing Price}_t + \textit{Dividends Paid}_t$$

Weekly Returns: Weekly returns were then calculated to measure the percentage change in price from one week to the next. This measure is essential for understanding the stock's price movement over time, which directly impacts the portfolio's performance. The weekly return was calculated using the following formula:

$$\textit{Weekly Return}_t = \frac{\textit{Weekly Price}_t - \textit{Weekly Price}_{t-1}}{\textit{Weekly Price}_{t-1}}$$

Rolling Window Application: A rolling window was applied to the weekly returns to compute the various statistical measures, including standard deviation (stdev) for individual stocks, which serves as a proxy for the stock’s volatility, and the covariance matrix between them. A rolling window is a method where calculations are based on a moving subset of the data. In this case, a 52-week rolling window was used, meaning that at each point in time, the standard deviation and covariance matrix were calculated using the previous 52 weeks of data.

- **Standard Deviation Calculation:** The standard deviation for each stock's returns over the 52-week window was calculated using the following formula:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N}(R_i - \bar{R})^2}$$

Where:
σ is the standard deviation
N is the number of observations (52 weeks)
R_i is the return for week i
$\bar{R}$ is the mean return over the 52 weeks

- **Covariance Matrix Calculation:** The covariance between two stocks i and j was calculated using the 52-week rolling window:

$$Cov(R_i,R_j) = \frac{1}{N-1}\sum_{i=1}^{N}(R_{ik} - \overline{R_i})(R_{jk} - \overline{R_j})$$

Where:
$Cov(R_i,R_j)$ is the covariance between the returns of stock i and stock j
R_{ik} and R_{jk} are the returns of stocks i and j for week k
$\overline{R_i}$ and $\overline{R_j}$ are the mean returns of stocks i and j over the 52 weeks

The choice of a rolling window provides several advantages:
- **Capturing Recent Volatility:** Volatility is not static; it changes over time. The rolling window allows for capturing the most recent market conditions, reflecting more current volatility levels.
- **Smoothing:** By using a rolling window, short-term fluctuations are smoothed out, providing a more stable measure of volatility and covariance that is less influenced by temporary market shocks.
- **Alignment with the Study Period:** A 52-week window aligns well with the annual performance cycles, ensuring that the standard deviation and covariance matrix reflect a full year's worth of data.

These calculated standard deviations and covariance matrices were essential for optimizing the portfolios, particularly in strategies where risk minimization correlation between assets is a key objective.

4.1.3 Risk-Free Rate Data

The risk-free rate (RFR) plays a crucial role in portfolio theory, serving as the benchmark for evaluating the performance of risky assets. For this study, the German 1-Year Government Bond yield was chosen as the risk-free rate. This bond was selected for several reasons:

- **Short-Term Nature:** The 1-year maturity aligns well with the monthly rebalancing strategy employed in the portfolios, ensuring that the RFR is representative of the time horizon of the investment decisions.
- **Germany's Economic Stability:** Germany is known for its strong credit rating and economic stability, making its government bonds a reliable measure of the risk-free rate in the Eurozone.

Data Collection and Conversion: The RFR data was collected as weekly yields, which were annualized percentages. These rates were then converted to monthly rates using the formula:

$$\text{Monthly RFR} = \left(1 + \frac{RFR\ (Annualized)}{100}\right)^{\frac{1}{12}} - 1$$

This conversion ensures that the RFR aligns with the monthly rebalancing of the portfolios. However, to account for the changing nature of interest rates over time, a rolling window approach was again employed. A 12-month rolling average was calculated by taking the first observation of each month over the preceding 12 months and averaging them. The rationale behind this approach includes:

- **Reflecting Recent Rate Trends:** Interest rates fluctuate based on economic conditions. Using a rolling average allows the RFR to reflect these changes more accurately, ensuring that the portfolio performance is evaluated against a rate that accounts for recent trends.
- **Alignment with Investment Decisions:** The choice of taking the first observation of each month, rather than the last, is particularly significant. In practice, when making investment decisions at the start of each month, the RFR available to the investor would be the rate observed at that time. By using the first observation, the study ensures that the RFR used in the evaluation reflects the rate that could have realistically been chosen at the moment of portfolio rebalancing. This approach simulates the real-world scenario where investment decisions are based on the most current data available, providing a direct comparison between the actual investment choice and the alternative risk-free option that was accessible when that choice was made.

This rolling average of the RFR provides a more nuanced measure that aligns with the timing and decision-making process of portfolio management.

4.1.4 Benchmark Data

The EURO STOXX 50 index was selected as the benchmark for this study. A benchmark serves as a standard against which the performance of a portfolio can be measured. The choice of the EURO STOXX 50 was guided by several factors:

- **Representation of the Market:** The EURO STOXX 50 is a widely recognized benchmark that represents the performance of the 50 largest companies in the Eurozone. It is an appropriate benchmark for evaluating portfolios composed of the same or similar assets.

- **Alignment with Study Focus:** Since the study focuses on European equities, using the EURO STOXX 50 ensures consistency between the portfolio and the benchmark, allowing for a more accurate performance comparison.

Benchmark Returns Calculation: The benchmark's performance was measured by calculating its monthly returns. Daily closing prices of the index were collected, and the monthly return was calculated as follows:

$$\textit{Monthly Return} = \frac{Price\ of\ First\ Day\ of\ Month_{t+1} - Price\ of\ First\ Day\ of\ Month_t}{Price\ of\ First\ Day\ of\ Month_t}$$

This method was chosen to align the benchmark returns with the rebalancing strategy of the portfolios, ensuring that the comparison between the portfolio returns and the benchmark returns is valid. Additionally, the benchmark’s risk-adjusted measures, were calculated to provide further insights into the relative performance of the constructed portfolios.
The benchmark plays a dual role in this study: first, as a standard for comparison, and second, as a component in the calculation of certain risk-adjusted measures, such as the Information Ratio. This ensures that the performance evaluation is comprehensive and considers the benchmark's market context.

4.1.5 Categorization of Interest Rate Regimes

To analyze the impact of varying interest rate environments on portfolio performance, the ECB's main refinancing rate was categorized into three distinct regimes: low, medium, and high. This categorization was based on historical ECB data, spanning from May 2004 to April 2024.
Thresholds Used for Categorization:

- **Low Interest Rates (< 1%)**: Periods where the ECB maintained a refinancing rate below 1% were classified as low-interest rate regimes. This threshold reflects periods of monetary easing, often associated with economic downturns or attempts to stimulate economic activity.
- **Medium Interest Rates (1% - 3%):** Rates within the range of 1% to 3% were categorized as medium. These periods are typically characterized by balanced economic conditions, where monetary policy is neither highly restrictive nor excessively stimulative.
- **High Interest Rates (> 3%)**: Periods with rates exceeding 3% were classified as high-interest rate regimes. Such rates usually indicate a tightening monetary policy aimed at controlling inflation or cooling an overheated economy.

These thresholds were chosen based on economic theory and historical precedent, where rates below 1% are commonly seen during significant economic downturns, while rates above 3% are often used to curb inflation during periods of rapid economic growth. The ECB's rate decisions were reviewed across the given period, and each month's rate was categorized accordingly.
This categorization allows for a structured analysis of how different interest rate environments affect portfolio performance, which will be further discussed in the subsequent sections.

4.2 Portfolio Construction

This section provides an in-depth explanation of the construction process for the portfolios using the three distinct strategies—Risk Parity, Momentum, and Black-Litterman—followed by the integration of these strategies. We will explore the methodologies used to compute expected returns, the reasoning behind each strategy's approach, and the optimization techniques used to determine the optimal portfolio weights. The distinction between expected returns and actual returns will be clarified to ensure a comprehensive understanding of their roles within the context of portfolio construction.

4.2.1 *Risk Parity Portfolio*

The Risk Parity strategy is centered on the concept of equalizing the risk contributions of the assets within a portfolio. Unlike strategies that aim to maximize returns, Risk Parity focuses on balancing the overall risk, thereby reducing the portfolio's exposure to any single asset's volatility.

Expected Returns Calculation: In the context of the Risk Parity strategy, expected returns were calculated to reflect the relative change in an asset's performance over a consistent annual time frame. The goal was to ensure that the calculation method aligned with the strategy's emphasis on stability and risk balancing.

- **Methodology:** To calculate the expected returns, the newest weekly return before month T was compared to the last observation from the same month in the previous year. This approach captures the asset's relative performance change over a one-year period.
- **Simplified Formula**: The expected return for each stock was calculated as:

$$\text{Expected Return}_T = \frac{Newest\ Weekly\ Return\ Before\ Month\ (T) - Last\ Observation\ of\ Month\ (T-1)\ From\ a\ Year\ Ago}{Last\ Observation\ of\ Month\ (T-1)\ From\ a\ Year\ Ago}$$

In other words, the expected return is derived by comparing the most recent weekly return before month T to the corresponding value from a year earlier, providing a relative measure of performance change over a consistent period.

- **Reasoning:** This method ensures that the expected returns reflect recent performance trends while maintaining a consistent comparison period. It aligns with the Risk Parity strategy's focus on balancing risk, as it emphasizes stability and consistency in performance over time.

Optimization Process: With the expected returns and the 52-week rolling standard deviations, the optimization process in Python determined the portfolio weights that equalized the risk contributions of each asset. Here, the vector of portfolio weights **w** represents the allocation to each asset, where w= $(w_1, w_2, \ldots, w_n)$, and n is the number of assets. The optimization aimed to minimize the overall portfolio risk while ensuring that no single asset disproportionately influenced the portfolio's risk profile. The weights **w** are subject to the constraint:

$$\sum_{i=1}^{n} w_i = 1$$

This constraint ensures that the entire portfolio is fully invested, and the optimization process adjusts **w** to achieve the desired risk parity.

Actual Returns Calculation: After determining the optimal weights for each month, the actual returns of the Risk Parity portfolio were calculated. These returns represent the realized performance of the portfolio after holding it for the entire month. The monthly actual returns for each stock were computed as:

$$\textit{Actual Return}_T = \frac{First\ Price\ of\ Month\ (T+1) - First\ Price\ of\ Month\ (T)}{First\ Price\ of\ Month\ (T)}$$

These returns were then weighted by the optimal allocations to obtain the overall portfolio return:

$$\textit{Portfolio Return}_T = \sum_{i=1}^{n} Weight_i\ \ x\ \ Actual\ Return_i$$

Result: The process culminated in the construction of the Risk Parity (RP) portfolio, characterized by equalized risk contributions from each asset.

4.2.2 *Momentum Portfolios*

The Momentum strategy leverages the principle that assets with strong recent performance are likely to continue performing well in the near future. This strategy is driven by the persistence in asset price trends. Three Momentum portfolios were constructed, each corresponding to a different level of risk aversion: 1.5 (aggressive), 3 (moderate), and 5 (defensive).

Expected Returns Calculation: Expected returns are central to the Momentum strategy, as they reflect the recent performance trend of each asset, which the strategy aims to exploit.

- **Methodology:** The expected return for each stock was calculated as the average of the previous 52 weekly returns before month T. This method captures the asset's momentum over the past year, which is essential for the strategy's objective of capitalizing on short-term trends.
- **Simplified Formula:** The expected return was calculated as:

$$\textit{Expected Return}_T\ = \frac{\sum Previous\ 52\ Weekly\ Returns\ Before\ Month\ (T)}{52}$$

This formula takes the average of the past year's weekly returns to estimate how the asset is likely to perform in the near future.

- **Reasoning:** Using a simple average of the past 52 weekly returns provides a straightforward measure of the asset's momentum. This method assumes that the trend observed over the past year will continue into the future, making it a suitable choice for a momentum-based strategy. The focus on recent performance aligns with the strategy's goal of capturing and exploiting trends.

Risk Aversion and CFA Utility Function: The Momentum strategy was further refined by incorporating risk aversion coefficients into the optimization process. The risk aversion coefficient represents the investor's tolerance for risk, with higher values indicating a more conservative approach.

- **Risk Aversion Levels**:
    - **Aggressive Portfolio (1.5):** Assumes a lower aversion to risk, focusing on maximizing returns even if it involves higher volatility.
    - **Moderate Portfolio (3):** Balances risk and return, reflecting a typical investor's risk tolerance.
    - **Defensive Portfolio (5):** Prioritizes minimizing risk, even at the cost of potentially lower returns.
- **CFA Utility Function:** The expected returns and standard deviations were used in the following utility function, which was maximized to determine the optimal portfolio weights:

$$U = \mathbb{E}[R_p] - \left(\frac{A}{2} \times \sigma_p^2\right)$$

where U is the utility, E[Rp] is the expected portfolio return, A is the risk aversion coefficient, and σ_p^2 is the portfolio variance.

- **Reasoning**: The CFA utility function balances the trade-off between risk and return based on the investor's risk aversion. By maximizing this function, the optimization process seeks to find the portfolio weights **w** that offer the highest expected return for a given level of risk, subject to the constraint that the weights sum to 1.

Actual Returns Calculation: Once the optimal weights were determined, the actual returns of the Momentum portfolios were calculated in the same manner as the Risk Parity portfolio, by applying the weights to the actual monthly returns of each stock.
Result: This process resulted in the construction of three Momentum portfolios: M-1.5, M-3, and M-5, each tailored to a different level of risk aversion.

4.2.3 Black-Litterman Portfolios

The Black-Litterman model combines equilibrium market returns with subjective views (or implied returns) to generate expected returns for portfolio optimization. In this study, a hybrid approach was employed, combining historical compounded returns with a market adjustment term. This approach allowed for insights into individual asset performance while maintaining consistency with broader market dynamics, aligning the portfolio with market equilibrium.
Expected Returns Calculation: In the Black-Litterman portfolios, the expected returns were calculated using a compounded return method, which reflects the growth of each asset over the past year. This method was chosen to provide a realistic measure of asset performance, accounting for actual growth or decline over the study period.

- **Methodology**: The expected return was calculated by comparing the most recent weekly return before month T to the 52nd weekly return before month T. This method captures the compounded growth of the asset over the previous year, offering a realistic view of its performance.

- **Simplified Formula**: The expected return was calculated using the following formula:

$$\text{Expected Return}_T = \left(\frac{Newest\ Weekly\ Return\ Before\ T}{52nd\ Weekly\ Return\ Before\ T}\right) - 1$$

This formula measures the growth rate of the asset over the year leading up to month T, which is then used as the expected return in the optimization process.

Reasoning for the Hybrid Approach: The Black-Litterman model traditionally uses implied returns based on market equilibrium. However, in this study, a hybrid approach was adopted to enhance the model's flexibility and applicability to real-world scenarios. The reasoning behind this approach includes:

- **Compounded Returns**: The compounded return method provides a realistic measure of the asset's performance over the last year, capturing actual growth or decline. This method is straightforward and aligns with practical approaches to estimating expected returns, offering a direct reflection of historical performance.
- **Market Adjustment Term:** To ensure that the portfolio remains aligned with market equilibrium, a market adjustment term was incorporated into the utility function. This adjustment penalizes portfolios that deviate significantly from market weights, thereby balancing individual asset performance with the broader market dynamics.
- **Rationale for the Hybrid Method:** The hybrid approach combines the benefits of using historical performance data (compounded returns) with the stability and diversification benefits of market-consistent portfolios. By incorporating both elements, this method accounts for individual asset performance while respecting broader market conditions, leading to more robust portfolio construction.

Market Adjustment: To align the portfolio more closely with market equilibrium, a market adjustment term was added to the utility function. This term penalizes portfolios that deviate significantly from market weights, ensuring that the portfolio benefits from market consensus while still reflecting individual asset performance.

- **Market Adjustment Formula:**

$$\text{Market Adjustment} = \text{Risk Aversion x} \sum (Weights - Market\ Weights)^2$$

Optimization Process: The utility function, which was maximized to determine the optimal portfolio weights, incorporated both the expected returns and the market adjustment:

$$U = \mathbb{E}\left[R_p\right] - 0.5\ x\ \sigma_p^2 - Market\ Adjustement$$

where σ_p^2 is the portfolio variance.

Actual Returns Calculation: As with the other strategies, the actual returns of the Black-Litterman portfolios were calculated by applying the optimized weights to the actual monthly returns of each stock.

Result: This process resulted in the construction of three Black-Litterman portfolios: BL-1.5, BL-3, and BL-5.

4.2.4 Integrated Portfolios

The integrated portfolios combine the strengths of the three individual strategies—Risk Parity, Momentum, and Black-Litterman—using a dynamic allocation approach. Four different optimization objectives were used: maximizing returns, maximizing the Sharpe ratio, maximizing mean-variance utility, and minimizing volatility. An additional portfolio was constructed using equal weights for all three strategies.

Expected Returns Calculation Using EMA: The expected returns for each integrated portfolio were calculated using an Exponential Moving Average (EMA) with a 12-month rolling window.

- **EMA Methodology**: The EMA gives more weight to recent returns, which is particularly useful in a dynamic allocation strategy where recent performance trends are more relevant.

$$EMA_t = R_{t-1} \; x \; \alpha + EMA_{t-1} \; x \; (1 - \alpha)$$

Where $\alpha = \frac{2}{n+1}$ and n = 12 months.

- **Initial EMA Calculation**: For the first month (EMA_0), where previous EMA values are unavailable, the simple average of the previous 12 months' returns was used:

$$EMA_0 = \frac{\sum Monthly\ Returns\ of\ Previous\ 12\ months}{12}$$

- **Reasoning**: The use of EMA allows the expected returns to adapt more quickly to recent market conditions, which is crucial for optimizing the integrated portfolios. The EMA method is preferred in dynamic portfolio strategies because it places greater emphasis on the most recent data, which is more relevant in a rapidly changing market environment.

Optimization Objectives: The following four optimization objectives were employed to determine the optimal allocation of the integrated portfolios:

1. **Maximizing Returns**: The objective function aimed to find the allocation that maximizes the combination's overall return.

$$max \sum_{i=1}^{n} w_i \; x \; \mathbb{E}[R_i]$$

2. **Maximizing Sharpe Ratio**: This objective sought to maximize the Sharpe ratio, which measures the portfolio's return relative to its risk:

$$max \frac{\sum_{i=1}^{n} w_i \, x \, \mathbb{E}[R_i] - R_f}{\sigma_p}$$

3. **Maximizing Mean-Variance Utility**: This function maximized the mean-variance utility, balancing return against risk based on the investor's risk aversion:

$$max \sum_{i=1}^{n} w \; x \; \mathbb{E}[R_i] - 0.5 \; x \; A \; x \; \sigma_p^2$$

4. **Minimizing Volatility**: This objective minimized the portfolio's volatility:

$$min \; \sigma_p$$

Actual Returns Calculation: After obtaining the optimal allocations for each integrated portfolio, the actual monthly returns were calculated by applying these weights **w** to the actual returns of each strategy.

An additional portfolio with equal weights for the three strategies was also constructed, and its actual returns, monthly, were calculated as:

$$Portfolio\ Return_t = 0.333 \times RP\ Return_t + 0.333 \times M\ Return_t + 0.333 \times BL\ Return_t$$

Result: This process resulted in the construction of 15 integrated portfolios: five strategies (Max Sharpe, Max Returns, Max Utility, Min Volatility, Equal Weights) each applied across three risk aversion levels.

4.3 Risk-Adjusted Measures Calculation

This section focuses on applying a set of nine risk-adjusted measures to evaluate the performance of the constructed portfolios. These measures, which include the Sharpe Ratio, Sortino Ratio, Treynor Ratio, Information Ratio, Omega Ratio, Value at Risk (VaR), Conditional Value at Risk (CVaR), Modified VaR, and Adjusted Sharpe Ratio, provide a comprehensive assessment of the portfolios' performance relative to the risk they assume.

4.3.1 *Overview of Risk-Adjusted Measures*

While the theoretical foundation and formulas for these measures were discussed in the literature review, this section concentrates on how they were implemented to assess the portfolios constructed in this study. The risk-adjusted measures were selected to give a holistic view of each portfolio's risk-return profile, considering both the portfolios' absolute performance and their performance relative to the benchmark, the EURO STOXX 50.

4.3.2 Implementation of Risk-Adjusted Measures

The application of these risk-adjusted measures was conducted using Python, utilizing the actual monthly returns of each portfolio along with the calculated risk-free rate.

- **Data Inputs**: The analysis relied on the actual monthly returns generated for each portfolio, along with the corresponding risk-free rate. The rolling standard deviations and beta values (where applicable) were derived from the historical data, and these were used as inputs in the calculations.
- **Risk Measures and Rolling Windows**: For measures like VaR, CVaR, and Modified VaR, the risk assessments were performed using a rolling window approach, which allowed for a dynamic and evolving analysis of the portfolio's risk profile over time. This approach is crucial for capturing the shifting market conditions and the portfolios' responsiveness to these changes.
- **Benchmark Comparison**: The EURO STOXX 50 served as the benchmark for relative performance evaluation. The Information Ratio, along with other relative measures, utilized the benchmark's returns to assess how well each portfolio performed compared to a widely recognized market standard. This comparison was critical for understanding the value added by the strategies implemented in this study.
- **Purpose of Each Measure**: Each risk-adjusted measure was selected based on its ability to provide insights into different aspects of the portfolios' performance:
    - **The Sharpe and Sortino Ratios** were primarily used to assess risk-adjusted returns, with the Sharpe Ratio considering total volatility and the Sortino Ratio focusing on downside risk.
    - **The Treynor Ratio** evaluated the performance relative to market risk (beta), providing a perspective on how well the portfolios performed per unit of market risk taken.
    - **The Information Ratio** offered insights into the portfolios' excess returns over the benchmark, helping to assess the effectiveness of the strategies in generating returns beyond those of the market index.
    - **The Omega Ratio, VaR, CVaR, and Modified VaR** provided a deeper understanding of downside risk and the tail-end risks that the portfolios might face, offering a more comprehensive view of potential losses under adverse conditions.
    - **The Adjusted Sharpe Ratio** enhanced the traditional Sharpe Ratio by incorporating skewness and kurtosis, offering a more refined measure of risk-adjusted performance for portfolios with non-normal return distributions. This adjustment makes the measure particularly useful in assessing portfolios that exhibit significant asymmetry in their return profiles.

4.3.3 Presentation of Results

The results derived from these risk-adjusted measures are presented in the following sections, focusing on comparing the performance of the 22 constructed portfolios and the EURO STOXX 50 benchmark.

- **Visualization**: The results will be illustrated through tables and graphs, making it easier to compare the performance across different strategies. This visual representation helps to identify which portfolios offer the best balance between risk and return and how they stack up against the benchmark.
- **Implications**: The implications of these risk-adjusted measures will be explored in the subsequent discussion chapter. This analysis will focus on what these results mean for portfolio management, particularly in the context of varying market conditions and different risk preferences.

Note: Specific numerical results and visual representations will be provided in the final section of the report, where they will be discussed in the context of the overall study findings.

4.4 CAGR and Maximum Drawdown

4.4.1 *Compound Annual Growth Rate (CAGR) Calculations Methodology*

The Compound Annual Growth Rate (CAGR) is calculated to assess the average annual growth rate of the portfolios over the study period. The formula used for calculating CAGR is:

$$CAGR = \left(\frac{V_f}{V_i}\right)^{\frac{1}{n}} - 1$$

Where:

- V_f is the final value of the portfolio at the end of the period.
- V_i is the initial value of the portfolio at the beginning of the period.
- n is the number of years over which the growth is measured.

Data Used:

- The initial and final values of each portfolio were obtained from the cumulative return data spanning the study period (June 2007 to March 2024).
- The time period n corresponds to the total duration of the study, which is approximately 16.75 years.

4.4.2 *Maximum Drawdown Calculation Methodology:*

Maximum Drawdown (Max DD) represents the largest observed loss from a peak to a trough of a portfolio's value, measured as a percentage of the peak value. It provides insight into the risk of significant losses during the investment period. The formula for Maximum Drawdown is:

$$Max\ DD = \frac{Trough\ Value - Peak\ Value}{Peak\ Value}$$

Where:
The Peak Value is the highest value reached by the portfolio before the drawdown begins.
The Trough Value is the lowest value reached by the portfolio during the drawdown period.

Understanding Multiple Drawdowns:

- Throughout the study period, a portfolio may experience multiple drawdowns—periods where the portfolio's value declines from a peak. The Maximum Drawdown specifically refers to the largest single decline (in percentage terms) observed from the highest peak to the lowest trough before a new peak is established.
- For each portfolio, the Maximum Drawdown is calculated by identifying the largest percentage loss from any peak to any subsequent trough throughout the entire study period.

Data Used:

- The peak and trough values were extracted from the daily portfolio values over the study period. The Maximum Drawdown was determined by identifying the largest drawdown event, i.e., the most significant decline from a peak to a trough.

4.5 Results Presentation

In this section, we systematically present the key findings from the study, focusing on cumulative returns, risk-adjusted measures, and the impact of interest rate regimes on portfolio performance. The results are presented with the goal of offering a clear and comprehensive view of how each portfolio performed, setting the stage for a more in-depth discussion in the next chapter.

4.5.1 *Cumulative Returns*

Cumulative returns illustrate the total growth of an investment over time, starting from an initial investment of 1 unit. This metric is crucial for comparing the long-term performance of different strategies. We begin by presenting the cumulative returns of individual portfolios and then proceed to the integrated portfolios, each compared against the benchmark.

4.5.1.1 *Individual Portfolios (RP, M, BL) and Benchmark*

The cumulative returns for the individual portfolios—Risk Parity (RP), Momentum (M), Black-Litterman (BL), and the Benchmark (EURO STOXX 50)—are shown across three different levels of risk aversion. These figures provide a comprehensive view of how each strategy has performed over time, starting from May 2005 to March 2024.

Each graph represents the growth of an initial investment of 1 unit in each portfolio. By comparing these cumulative returns, we gain insights into the consistency and effectiveness of each strategy across different market conditions.

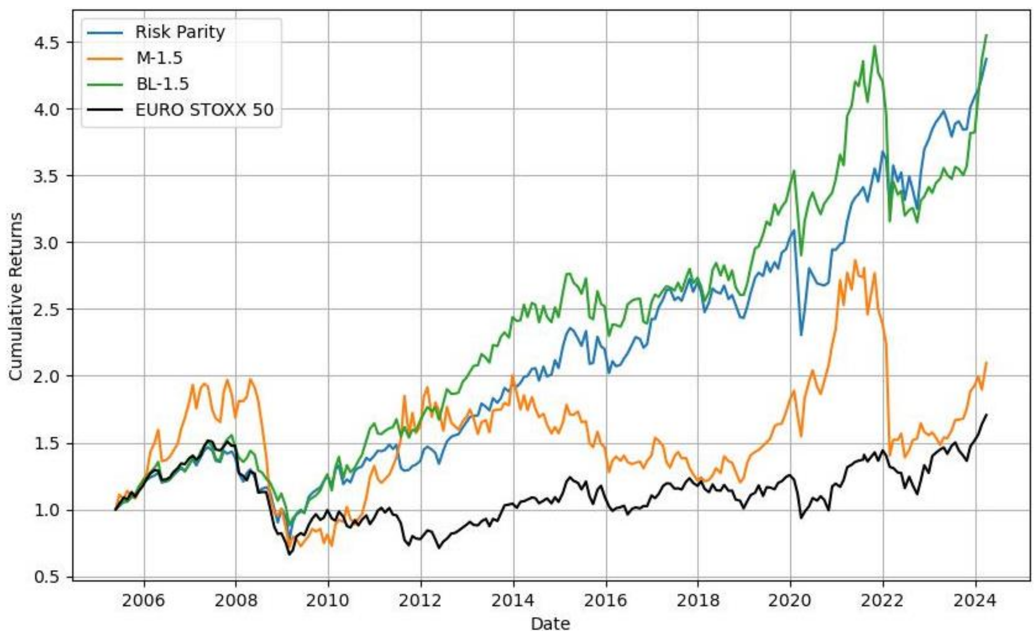


Figure 1: Cumulative Returns for RP, M-1.5, BL-1.5, and Benchmark.

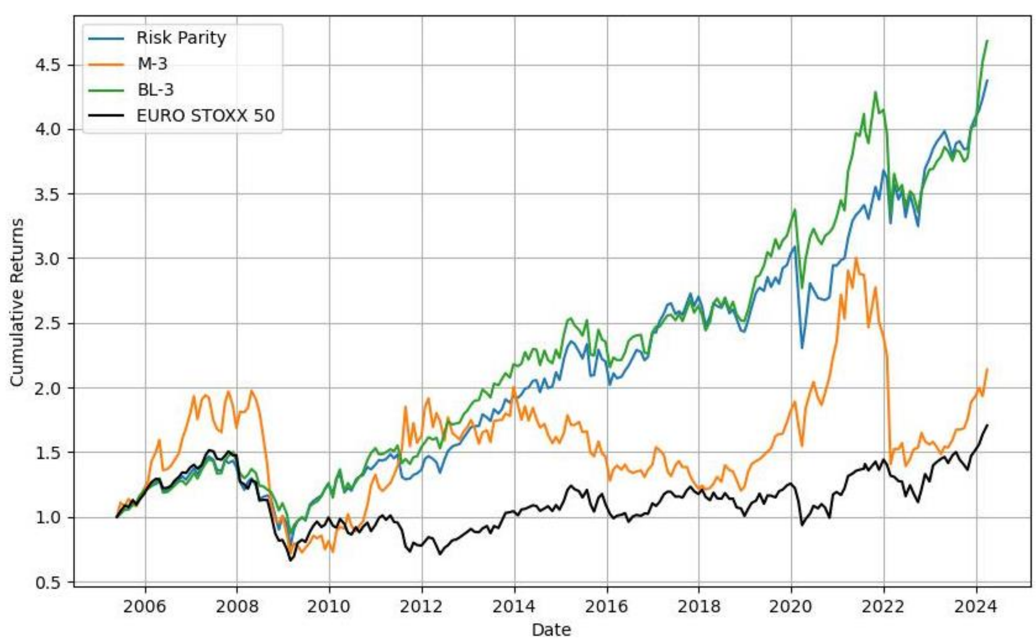


Figure 2: Cumulative Returns for RP, M-3, BL-3, and Benchmark.

Figure 3: Cumulative Returns for RP, M-5, BL-5, and Benchmark.

These figures reveal the comparative performance of each strategy, helping us to identify which portfolios have consistently outperformed or underperformed the benchmark over nearly two decades.

4.5.1.2 *Integrated Portfolios (MAX R, MAX SR, MAX MVU, MIN V, EW) and Benchmark*

The integrated portfolios represent a combination of the three strategies—Momentum, Black-Litterman, and Risk Parity—optimized for different objectives: maximum return (MAX R), maximum Sharpe ratio (MAX SR), maximum mean-variance utility (MVU), minimum volatility (MIN V), and equal weight (EW). These portfolios are designed to explore whether combining different approaches can lead to improved performance.

The cumulative returns for the integrated portfolios are compared against the benchmark, with the initial investment set to 1 unit, covering the period from June 2007 to March 2024. This comparison allows us to evaluate the potential benefits of strategy integration.

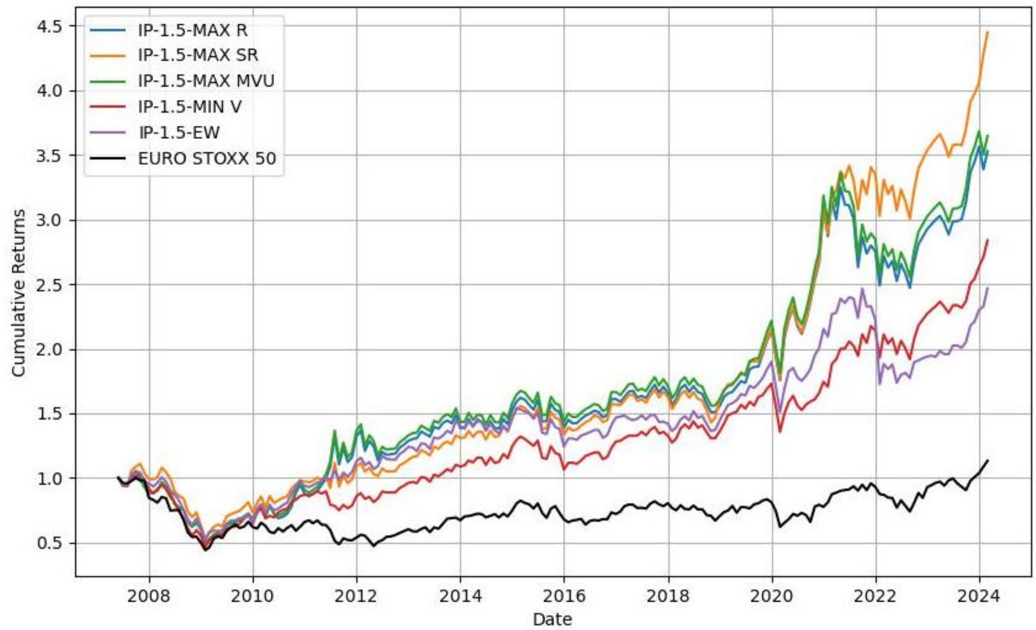


Figure 4: Cumulative Returns for IP-1.5 and Benchmark.

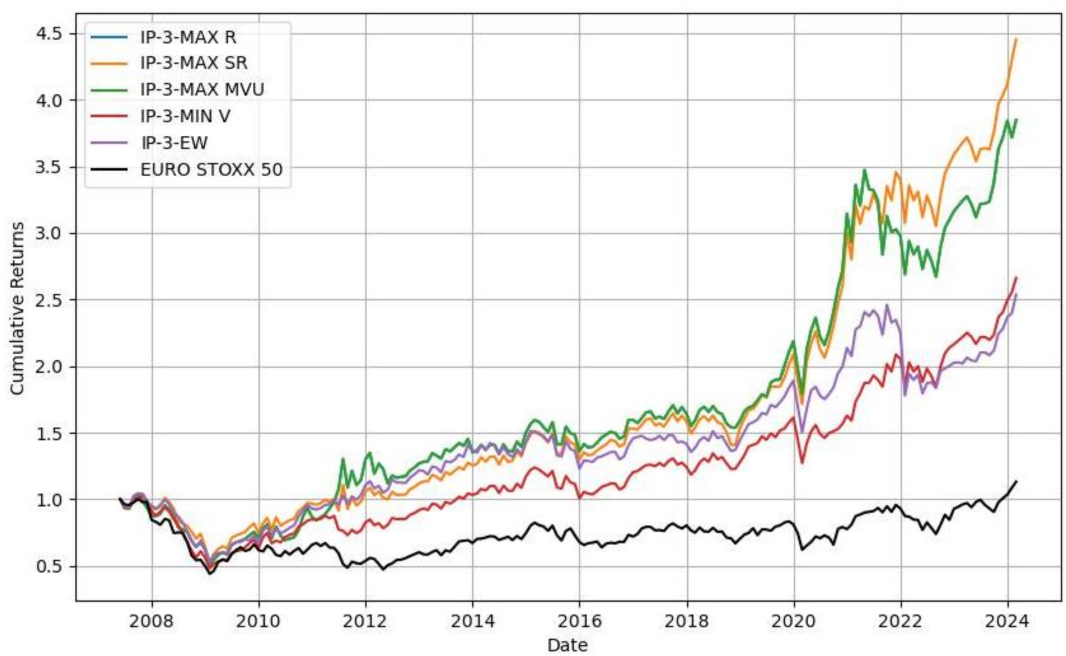


Figure 5: Cumulative Returns for IP-3 and Benchmark

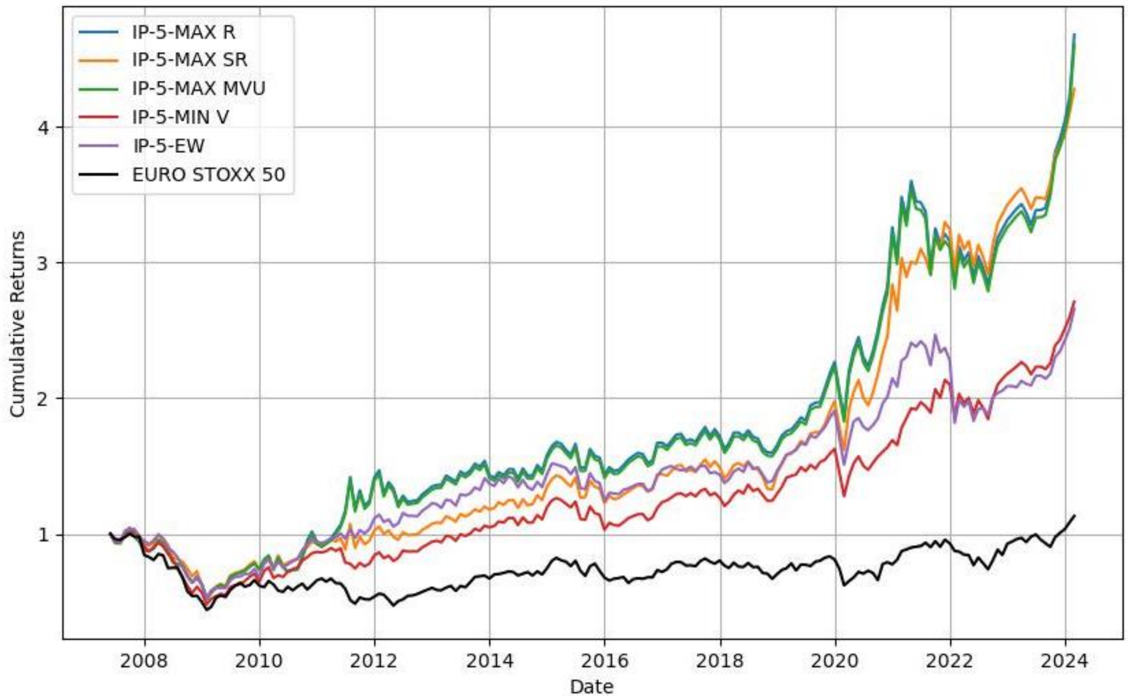


Figure 6: Cumulative Returns for IP-5 and Benchmark.

These figures provide insights into the effectiveness of the integrated portfolios, helping us understand if and how the combination of strategies results in superior performance compared to individual approaches. The results presented here will be revisited in the discussion chapter, where we will analyze the performance of the best-integrated portfolio in direct comparison with individual portfolios, ensuring a fair evaluation by aligning their starting dates.

4.5.2 *Risk-Adjusted Measures*

In this subsection, we summarize the key risk-adjusted performance measures for all portfolios, providing a deeper understanding of the trade-off between risk and return. These measures were already introduced and explained in detail in Section 4.4.2, so here we focus on presenting the results. The following table includes the Sharpe Ratio, Treynor Ratio, Sortino Ratio, Information Ratio, and Omega Ratio for each portfolio, allowing for a detailed comparison of performance adjusted for risk.

Table 2: Risk-Adjusted Measures for All Portfolios

	SHARPE RATIO	TREYNOR RATIO	SORTINO RATIO	INFORMATION RATIO	OMEGA RATIO	VAR	CVAR	MODIFIED VAR	ADJUSTED SHARPE RATIO
RISK PARITY	0.1275	0.0095	0.1850	0.1580	1.4176	-0.0723	-0.1126	0.0793	0.0771
M-1.5	0.0386	0.0067	0.0538	0.0375	1.1065	-0.1210	-0.1757	0.1158	0.0252
M-3	0.0401	0.0069	0.0561	0.0390	1.1109	-0.1215	-0.1757	0.1173	0.0259
M-5	0.0492	0.0083	0.0692	0.0480	1.1378	-0.1204	-0.1822	0.1172	0.0317
BL-1.5	0.1304	0.0130	0.1882	0.1258	1.4030	-0.0708	-0.1073	0.0758	0.0809
BL-3	0.1434	0.0130	0.2073	0.1414	1.4518	-0.0661	-0.1004	0.0725	0.0876
BL-5	0.1464	0.0124	0.2111	0.1491	1.4643	-0.0645	-0.0947	0.0711	0.0893
IP-1.5-MAX R	0.1188	0.0150	0.1815	0.1164	1.3603	-0.0905	-0.1174	0.1075	0.0659
IP-1.5-MAX SR	0.1492	0.0153	0.2321	0.1556	1.4822	-0.0837	-0.1146	0.1026	0.0817
IP-1.5-MAX MVU	0.1217	0.0152	0.1861	0.1195	1.3714	-0.0902	-0.1174	0.1074	0.0674
IP-1.5-MIN V	0.1204	0.0098	0.1719	0.1359	1.3623	-0.0722	-0.1061	0.0790	0.0727
IP-1.5-EW	0.1011	0.0098	0.1444	0.1031	1.3087	-0.0768	-0.1136	0.0791	0.0638
IP-3-MAX R	0.1259	0.0158	0.1948	0.1237	1.3916	-0.0902	-0.1182	0.1086	0.0691
IP-3-MAX SR	0.1512	0.0156	0.2381	0.1563	1.4927	-0.0821	-0.1135	0.1017	0.0821
IP-3-MAX MVU	0.1259	0.0158	0.1948	0.1237	1.3916	-0.0902	-0.1182	0.1086	0.0691
IP-3-MIN V	0.1160	0.0094	0.1656	0.1286	1.3448	-0.0703	-0.0985	0.0770	0.0698
IP-3-EW	0.1053	0.0098	0.1516	0.1086	1.3225	-0.0751	-0.1120	0.0789	0.0655
IP-5-MAX R	0.1416	0.0176	0.2236	0.1399	1.4498	-0.0899	-0.1162	0.1116	0.0763
IP-5-MAX SR	0.1478	0.0151	0.2321	0.1534	1.4829	-0.0822	-0.1104	0.1015	0.0805
IP-5-MAX MVU	0.1406	0.0174	0.2213	0.1388	1.4452	-0.0897	-0.1140	0.1109	0.0759
IP-5-MIN V	0.1180	0.0095	0.1676	0.1316	1.3508	-0.0706	-0.0998	0.0766	0.0718
IP-5-EW	0.1108	0.0101	0.1600	0.1158	1.3421	-0.0742	-0.1113	0.0786	0.0686
EURO STOXX 50	0.0286	0.0015	0.0400		1.0757	-0.0843	-0.1149	0.0871	0.0172

Source: S&P Capital IQ, own calculations

The table offers a clear snapshot of how each portfolio performed when considering the risks taken to achieve those returns. This comparison is crucial for identifying which strategies are most efficient in generating returns relative to their risk levels.

4.5.3 *Interest Rate Regimes*

Interest rate regimes are crucial factors in understanding how different portfolios perform under varying economic conditions. In this section, we present the identified periods of different interest rate regimes and set the stage for analyzing their impact on portfolio performance.
The following table outlines the identified periods corresponding to high, medium, and low-interest rate regimes throughout the study period:

Table 3: Identified Interest Rate Regimes

INTEREST RATE REGIME	START DATE	END DATE
HIGH	Jun 2007	Nov 2008
MEDIUM	Dec 2008	Jun 2012
LOW	Jul 2012	Feb 2023
HIGH	Mar 2023	Mar 2024

Source: European Central Bank

This table provides a clear summary of when each interest rate regime occurred during the study period. These periods will be crucial in the subsequent Discussion chapter, where we will analyze how different portfolios performed under each regime and discuss the implications for strategic asset allocation.

4.5.4 *CAGR and Maximum Drawdown*

In this section, we present the results of the Compound Annual Growth Rate (CAGR) and Maximum Drawdown (Max DD) for each of the integrated portfolios over the study period. These metrics are crucial for understanding both the long-term growth potential and the risk exposure of the portfolios

Table 4: CAGR and Max Drawdown

	CAGR	MAX DRAWDOWN
RISK PARITY	0.08109	-0.47439
M-1.5	0.03988	-0.63672
M-3	0.04099	-0.63672
M-5	0.04875	-0.63310
BL-1.5	0.08336	-0.43415
BL-3	0.08497	-0.41957
BL-5	0.08460	-0.41852
IP-1.5-MAX R	0.07778	-0.53143
IP-1.5-MAX SR	0.09270	-0.52896
IP-1.5-MAX MVU	0.07989	-0.53143
IP-1.5-MIN V	0.06393	-0.54981
IP-1.5-EW	0.05510	-0.50700
IP-3-MAX R	0.08332	-0.51291
IP-3-MAX SR	0.09276	-0.49216
IP-3-MAX MVU	0.08332	-0.51292
IP-3-MIN V	0.05986	-0.53179
IP-3-EW	0.05686	-0.50241
IP-5-MAX R	0.09600	-0.49868
IP-5-MAX SR	0.09020	-0.49632
IP-5-MAX MVU	0.09495	-0.49866
IP-5-MIN V	0.06105	-0.52903
IP-5-EW	0.05980	-0.50049
EURP STOXX 50	0.00738	-0.55984

Own calculations[1]

4.5.5 *Summary of Results*

This section has presented a comprehensive overview of the performance of various portfolios, highlighting the cumulative returns, risk-adjusted measures, and their sensitivity to interest rate regimes.

Key points include:

- **Cumulative Returns**: The figures show how each portfolio has grown over time, with particular attention to how integrated portfolios compare to individual strategies.
- **Risk-Adjusted Measures**: The table summarizes how efficiently each portfolio has delivered returns relative to the risks taken, offering insights into their overall effectiveness.
- **Interest Rate Sensitivity**: The analysis of interest rate regimes provides additional context for understanding the performance dynamics under different economic conditions.

[1] Values are own calculations based on portfolio returns computed from the study

- **CAGR and Max Drawdown**: By examining the CAGR and Max Drawdown together, we can assess how well each portfolio manages the balance between achieving high returns and minimizing potential losses.

These results lay the groundwork for the discussion chapter, where we will explore the implications of these findings in greater detail, particularly focusing on the comparative performance of the best-integrated portfolio against the individual portfolios using aligned starting dates to ensure a fair and rigorous comparison.

5. DISCUSSION

5.1 Overview of Integrated Portfolios

In this chapter, we analyze the performance of integrated portfolios that combine Risk Parity, Momentum, and Black-Litterman strategies. These portfolios have been optimized for different objectives across three levels of risk aversion: low (1.5), moderate (3), and high (5). The portfolios under consideration include:

- **Maximizing Returns (MAX R)**: Portfolios focused on achieving the highest possible returns.
- **Maximizing Sharpe Ratio (MAX SR)**: Portfolios optimized to achieve the highest risk-adjusted returns, prioritizing the balance between risk and return.
- **Maximizing Mean-Variance Utility (MAX MVU)**: Portfolios aiming to balance returns against risk, optimizing the overall utility for investors with different levels of risk aversion.
- **Minimizing Volatility (MIN V)**: Portfolios designed to minimize risk by reducing volatility.
- **Equally Weighted (EW)**: Baseline portfolios where the three strategies are combined with equal weighting.

These portfolios span from June 2007 to March 2024, covering significant market events and various interest rate regimes. The analysis focuses on cumulative returns, the impact of risk aversion, performance across different market conditions, and the influence of varying interest rate regimes, as well as detailed analysis of risk-adjusted measures and comparisons with individual strategies.

5.2 Detailed Analysis of Cumulative Returns

5.2.1 IP-1.5 Portfolios

The **IP-1.5 portfolios** encompass all five portfolio types (MAX R, MAX SR, MAX MVU, MIN V, and EW). The performance of these portfolios has been tracked across the entire study period, with a particular focus on their behavior during significant market events.

- **Performance Highlights**:
    - The cumulative returns of **IP-1.5-MAX SR**, **IP-1.5-MAX MVU**, and **IP-1.5-MAX R** portfolios initially followed similar trajectories but began to diverge after 2020 (please refer to Figure 4). The Sharpe Ratio portfolio eventually outperformed the others during the post-pandemic recovery phase starting mid-2021. However, all portfolios, including MIN V and EW, saw improved performance during this period.
    - The **IP-1.5-MIN V** portfolio underperformed throughout much of the study, though it did exhibit some improvement post-2022. Notably, MIN V began to show a slight edge over EW towards the end of the study, suggesting that a focus on minimizing volatility might provide marginal benefits during periods of market instability.

This section shows that while portfolios optimized for returns, Sharpe Ratio, and mean-variance utility performed well post-2020, the dominance of the Sharpe Ratio portfolio highlights the importance of risk-adjusted returns in a recovering market. The slight outperformance of MIN V over EW at the end suggests that volatility minimization may offer benefits in specific conditions.

5.2.2 *IP-3 Portfolios*

The **IP-3 portfolios** represent moderate risk aversion and include all five portfolio types, demonstrating significant distinctions in performance, particularly during the COVID-19 pandemic and its aftermath.

- **Performance Highlights**:
  - The **IP-3-MAX SR** portfolio emerges as the strongest performer from mid-2020 onwards, outperforming the MAX MVU and MAX R portfolios after a period of similar performance (please refer to Figure 5). This reflects the benefits of risk-adjusted optimization in a volatile market.
  - **IP-3-MIN V** consistently underperformed until around 2022, when it showed marginal improvement, though it remained below the EW portfolio. This suggests that a strategy focused solely on minimizing volatility may not be sufficient to achieve higher returns in a moderate risk aversion context.

These findings suggest that a moderate level of risk aversion benefits significantly from strategies that balance risk and return. The superior performance of the Sharpe Ratio portfolio emphasizes the value of risk-adjusted returns, particularly during turbulent market conditions.

5.2.3 *IP-5 Portfolios*

The **IP-5 portfolios**, targeting high-risk aversion investors, display distinct performance characteristics, especially during the post-COVID-19 recovery.

- **Performance Highlights**:
  - **IP-5-MAX SR** portfolio initially demonstrates the highest cumulative returns, particularly after 2020, indicating the effectiveness of maximizing the Sharpe Ratio in a high-risk aversion context (please refer to Figure 6). However, in the last three months of the study (beginning of 2024), **IP-5-MAX MVU** and **IP-5-MAX R** portfolios outperformed the Sharpe Ratio portfolio, finishing with almost 4.8 relative to 4.4 for the SR portfolio. This highlights that, while SR was the best performer for most of the study, the momentum shifted towards the end, favoring the portfolios optimized for returns and mean-variance utility.
  - The **IP-5-MIN V** portfolio consistently underperformed, yet it is important to note that in 2022, the EW portfolio dropped to the level of MIN V, and the two portfolios began to overlap in a zigzag pattern. Ultimately, MIN V held a very slight advantage by the end of the study.

These results indicate that while the Sharpe Ratio portfolio performed exceptionally well throughout most of the study, portfolios optimized for returns and mean-variance utility took the lead in the final phase. This shift underscores the dynamic nature of market conditions and the need to consider multiple optimization strategies, especially in high-risk aversion scenarios.

5.3 Analysis of Risk-Adjusted Measures

To further understand the performance of the integrated portfolios, a detailed analysis of risk-adjusted measures is conducted. These measures include Sharpe Ratio, Treynor Ratio, Sortino Ratio, Information Ratio, Omega Ratio, Value at Risk (VaR), Conditional VaR (CVaR), Modified VaR, and Adjusted Sharpe Ratio.

- **Sharpe Ratio**: The **IP-5-MAX SR** portfolio consistently shows the highest Sharpe Ratio within its risk aversion level, highlighting its strong risk-adjusted returns. However, across different risk aversion levels, the Sharpe Ratio remains the most favorable portfolio type, demonstrating superior risk management and return balance (please refer to Table 2).
- **Treynor Ratio**: While the Sharpe Ratio portfolio performed well in most measures, it did not consistently lead in Treynor Ratio across all risk aversion levels. In fact, the **MAX MVU** and **MAX R** portfolios performed better in this metric, particularly in the high-risk aversion category (IP-5).
- **Sortino Ratio**: The Sharpe Ratio portfolio did excel in the Sortino Ratio, especially at higher risk aversion levels, further solidifying its standing as a portfolio that effectively balances downside risk with returns.
- **Information Ratio**: This measure also favored the Sharpe Ratio portfolios across different risk aversion levels, indicating their consistent outperformance relative to the benchmark (EURO STOXX 50).
- **Omega Ratio**: The Omega Ratio values highlight the performance of the MAX SR and MAX MVU portfolios in the low and moderate risk aversion categories (IP-1.5 and IP-3). However, in the high-risk aversion category (IP-5), the MAX R portfolio outperformed MVU, indicating a shift in effectiveness towards maximizing returns as risk tolerance increased.
- **VaR and CVaR**: The Sharpe Ratio portfolios maintain competitive VaR and CVaR metrics, particularly when compared to portfolios focused on volatility minimization. This suggests effective risk management while pursuing higher returns.
- **Adjusted Sharpe Ratio**: The Adjusted Sharpe Ratio further confirms the strong performance of the Sharpe Ratio portfolios, with consistent outperformance across risk aversion levels, taking into account the skewness (S) and kurtosis (K) of returns.

This section underscores the importance of considering multiple risk-adjusted measures to evaluate portfolio performance comprehensively. The Sharpe Ratio portfolios generally perform well across these metrics, though other strategies, such as MAX MVU and MAX R, show strengths in specific areas.

5.4 Market Conditions and Portfolio Performance

The performance of integrated portfolios across various market conditions offers critical insights into their resilience and adaptability. Key periods of analysis include the 2008 Global Financial Crisis, the European Debt Crisis, and the COVID-19 pandemic.

1. **2008 Global Financial Crisis**:
    - The 2008 crisis was marked by severe market downturns and liquidity shortages. Portfolios optimized for returns and mean-variance utility (MAX R and MAX MVU) demonstrated resilience during this period, recovering faster and outperforming the Sharpe Ratio portfolio until after 2020. The Sharpe Ratio portfolio's delayed outperformance suggests that while it was effective during the recovery, the benefits of risk-adjusted returns became more apparent as market conditions stabilized (please refer to Figures 4,5,6).
    - From a financial perspective, the crisis highlighted the importance of liquidity and risk management, making portfolios that balance returns and risk particularly valuable during such downturns.
2. **European Debt Crisis (2010-2012)**:
    - The European Debt Crisis led to significant instability in the Eurozone, with sovereign debt issues threatening the region's financial stability. During this period, all integrated portfolios outperformed the benchmark, demonstrating their robustness in navigating regional economic instability. The MAX R and MAX MVU portfolios showed particular strength during this period, followed by the MAX SR that slightly outperformed the remaining portfolios and the benchmark (please refer to Figures 4,5,6).
    - The crisis underscored the need for diversification and careful risk assessment, particularly in portfolios exposed to regional economic risks.
3. **COVID-19 Pandemic (2020-2021)**:
    - The COVID-19 pandemic resulted in unprecedented market volatility and economic disruption. The initial pandemic shock led to significant drawdowns across all portfolios, but the recovery was most pronounced in the MAX SR portfolios, particularly from 2021 onwards. This suggests that portfolios focusing on risk-adjusted returns were better positioned to capitalize on the market rebound (please refer to Figures 4,5,6). It is worth noting that MAX R and MAX MVU were also slightly behind.
    - The pandemic emphasized the importance of flexibility and adaptability in portfolio management, as well as the value of strategies that can mitigate downside risks while capturing upside potential during recovery phases.

These analyses reveal that integrated portfolios' performance can vary significantly depending on the prevailing market conditions. Portfolios optimized for risk-adjusted returns, such as those focusing on the Sharpe Ratio, tend to perform well during recovery periods following market disruptions. In contrast, portfolios optimized for returns or mean-variance utility may offer better performance during stable or growing market conditions.

5.5 Interest Rate Regimes and Portfolio Performance

The analysis of portfolio performance across different interest rate regimes provides further insights into how economic environments influence investment outcomes. The study period includes high, medium, and low-interest rate regimes, defined as follows:

- **High Interest Rates (June 2007 - November 2008, March 2023 - March 2024)**:
    - During these periods, portfolios like IP-5-MAX SR demonstrated resilience, reflecting their ability to manage risk effectively even when borrowing costs were higher, and economic growth was more constrained (please refer to Table 3 for the interest rate regime overlays).
    - Higher interest rates often lead to increased borrowing costs and reduced corporate profits, making risk management crucial for maintaining portfolio performance.
- **Medium Interest Rates (December 2008 - June 2012)**:
    - Portfolios performed well during this period, with MAX R and MVU portfolios showing robustness. The MAX SR's focus on risk-adjusted returns again proved beneficial, as evidenced by the superior performance of IP-3-MAX SR during this regime.
    - Medium interest rates typically reflect balanced economic conditions, where neither inflationary pressures nor recession risks dominate, allowing for a more measured approach to portfolio optimization.
- **Low Interest Rates (July 2012 - February 2023)**:
    - The extended period of low interest rates post-2012 favored portfolios optimized for returns, particularly the MAX R and MVU portfolios. The Sharpe Ratio portfolios also performed well, underscoring the importance of managing risk even in low borrowing cost environments.
    - Low interest rates are often associated with expansive monetary policies aimed at stimulating growth, which can benefit portfolios that capitalize on growth opportunities while managing associated risks.

This section highlights the importance of aligning portfolio strategies with the prevailing interest rate environment. Portfolios that adapt to these conditions, particularly those focusing on risk-adjusted returns, tend to perform well across various economic environments (Complimentary Figure 11 in Appendix section).

5.6 Analysis of CAGR and Maximum Drawdown

In this section, we provide a detailed analysis of the Compound Annual Growth Rate (CAGR) and Maximum Drawdown for the integrated portfolios. These metrics offer crucial insights into the long-term performance and risk characteristics of each strategy, helping to evaluate their effectiveness over the study period.

5.6.1 *Compound Annual Growth Rate (CAGR)*

Performance Insights:

- Portfolios optimized for maximizing returns (MAX R) and mean-variance utility (MAX MVU) exhibit higher CAGR values, indicating their effectiveness in achieving consistent growth over the study period.

- The Sharpe Ratio (MAX SR) portfolios also demonstrate strong CAGR performance, particularly in the higher risk aversion category (IP-5), underscoring their ability to balance risk and reward effectively.

5.6.2 *Maximum Drawdown*

Performance Insights:

- Portfolios focusing on minimizing volatility MIN V tend to have lower maximum drawdowns, reflecting their conservative nature and emphasis on risk aversion.
- The MAX SR portfolios, while not always having the lowest drawdowns, show competitive drawdown figures, suggesting that they manage downside risk effectively while still pursuing higher returns.

These insights, drawn from the comparison of CAGR and Maximum Drawdown metrics, complement the earlier analysis of risk-adjusted measures and cumulative returns. Together, they provide a comprehensive view of the performance and risk characteristics of the integrated portfolios, reinforcing the overall findings of this study. (Complimentary Figure 10 in the Appendix providing a visual representation of the relationship between CAGR and Maximum Drawdown for various portfolio strategies, offering a comparative perspective in their risk-return profiles).

5.7 Identifying the Optimal Integrated Portfolio

Based on the extensive analysis conducted, it is clear that the Sharpe Ratio MAX SR portfolios have consistently delivered strong performance, particularly in higher risk aversion scenarios (IP-5). However, it is important to recognize that the MAX SR portfolio was not always the top performer throughout the study. For example, during the early phase of the study and towards the end (beginning of 2024), portfolios optimized for returns MAX R and mean-variance utility MAX MVU outperformed in the high-risk aversion category.

This observation highlights that while the Sharpe Ratio portfolio offers a robust balance between risk and return, there are periods where other strategies may provide superior results, particularly in stable or bullish market conditions. Therefore, while the MAX SR strategy may be optimal in many scenarios, it is crucial for investors to remain aware of changing market conditions and adjust their strategies accordingly, based on their risk tolerance and investment goals.

This nuanced understanding of portfolio performance underscores the importance of selecting an investment strategy that aligns with both the investor's objectives and the prevailing market environment. As we move forward, the next section will compare the best-performing integrated portfolio against the individual strategies (Risk Parity, Momentum, and Black-Litterman) to determine whether the integrated approach provides a superior investment strategy.

5.8 Evaluating the Performance of the Optimal Integrated Ratio Portfolio Against Individual Strategies

In this section, we focus on evaluating whether the integrated Sharpe Ratio (SR) portfolios provide superior performance compared to the individual strategies—Risk Parity (RP), Momentum (M), and Black-Litterman (BL)—across different risk aversion levels (1.5, 3, and 5). It is important to note that this analysis spans the period from June 2007 to March 2024. This time frame differs from previous results that started in 2006 for the individual strategies due to the effect of constructing the integrated portfolios using a rolling period of individual returns, which inherently results in losing the first year of data (2006).

Figure 7: Cumulative Returns for IP-1.5-MAX SR, RP, M-1.5, BL-1.5 and Benchmark

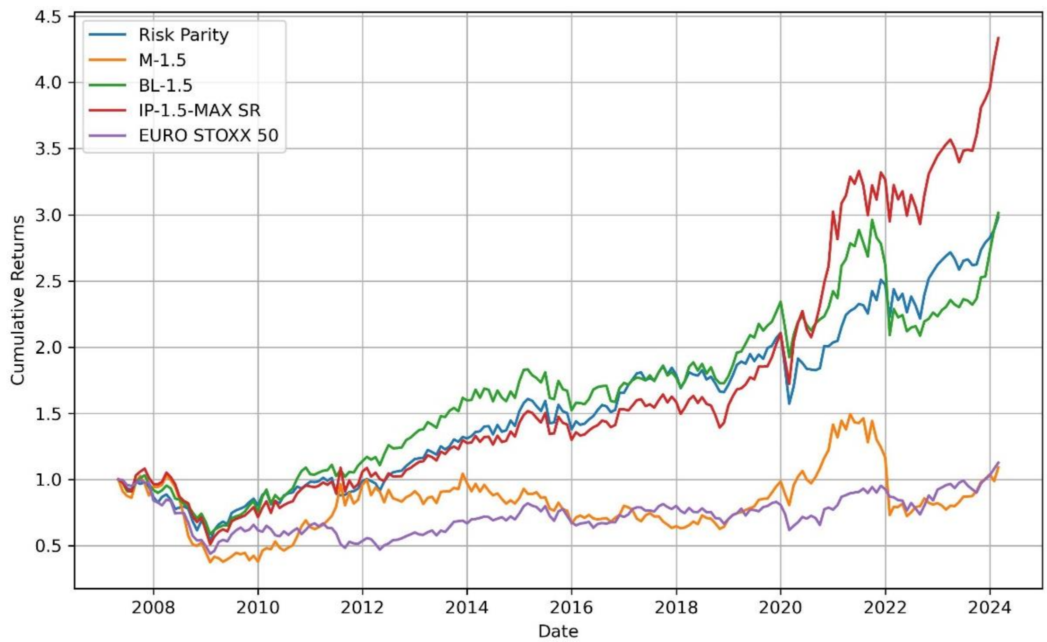


Figure 8: Cumulative Returns for IP-3-MAX SR, RP, M-3, BL-3 and Benchmark

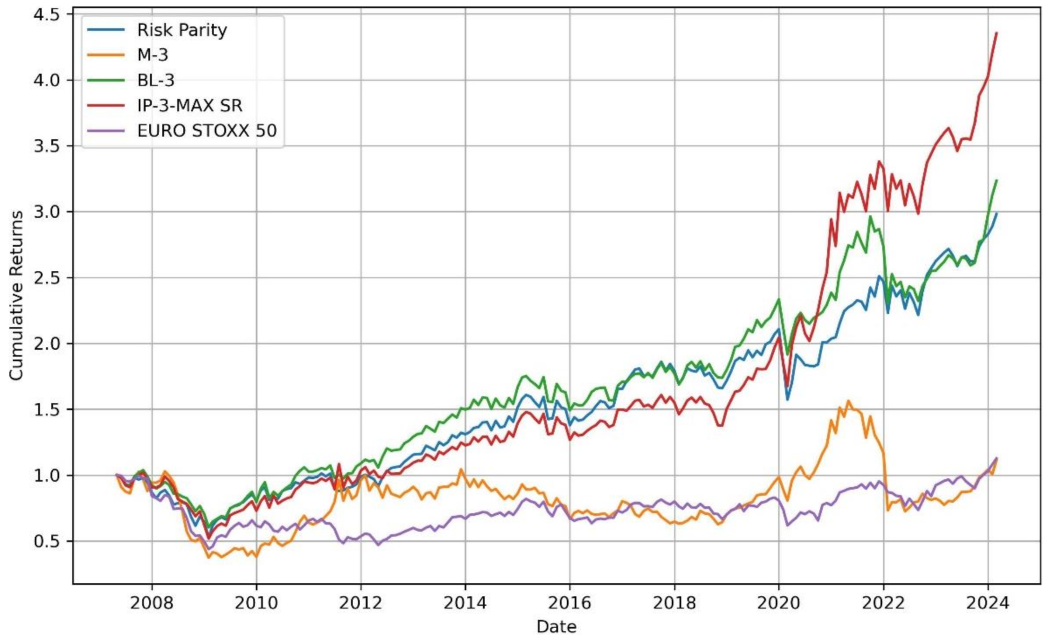
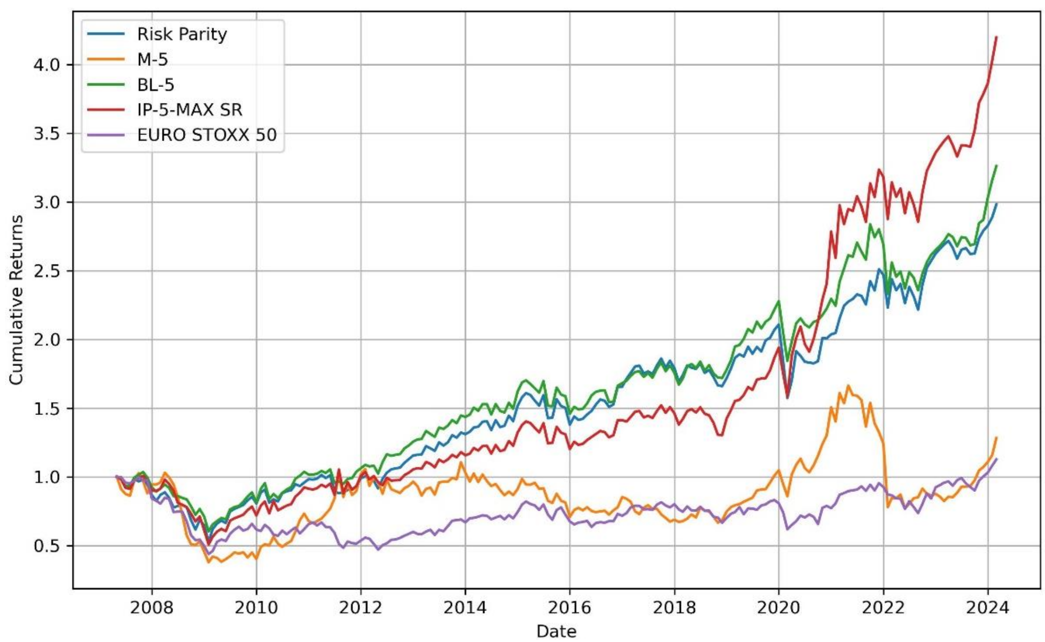


Figure 9: Cumulative Returns for IP-5-MAX SR, RP, M-5, BL-5 and Benchmark



5.9 Cumulative Returns Comparison

IP-1.5-MAX SR vs. RP, M, BL Strategies:
In Figure 7, the cumulative returns of the IP-1.5-MAX SR portfolio are compared against the RP, M-1.5, and BL-1.5 portfolios from June 2007 to March 2024. Initially, the SR portfolio tracks closely with RP and BL-1.5, while the Momentum (M-1.5) strategy lags behind. However, from 2019 onwards, the SR portfolio begins to outperform all three strategies significantly. Notably, the BL and RP strategies, which finish almost equal, both show strong performances but are eventually surpassed by the SR portfolio, particularly during the period of market recovery following the 2020 crisis.

- **Performance Trends:** The SR portfolio begins to outpace both RP and BL strategies after 2019, indicating that the SR portfolio's focus on maximizing risk-adjusted returns is particularly effective in volatile market conditions. The BL and RP strategies maintain relatively steady performances, finishing almost equal, but neither matches the peak returns achieved by the SR portfolio.

IP-3-MAX SR vs. RP, M, BL Strategies:
In Figure 8, the comparison of IP-3-MAX SR with RP, M-3, and BL-3 portfolios shows a more complex pattern. Up until the first quarter of 2020, the SR portfolio underperforms compared to both RP and BL strategies. However, from the first quarter of 2020 onwards, the SR portfolio begins to peak, quickly surpassing the individual strategies. This shift in performance highlights the SR portfolio's resilience and capacity to capitalize on market recovery post-crisis.

- **Performance Insights:** The SR portfolio's sharp increase in cumulative returns post-2020 underscores its effectiveness in managing risk while achieving higher returns. Although both the RP and BL strategies perform well during the earlier part of the study, the SR portfolio overtakes them, particularly in the context of the market's response to the COVID-19 pandemic. It's important to note that BL slightly outperforms RP during this period, making the SR's eventual dominance even more notable.

IP-5-MAX SR vs. RP, M, BL Strategies:
In Figure 9, contrary to what might be expected, the SR portfolio does not show dominance from the beginning. Instead, it lags behind both the BL and RP strategies until the end of 2020 (same scenario as other risk aversion levels). It is only in the latter stages of the study, particularly from late 2020 onwards, that the SR portfolio gains momentum and eventually surpasses the other strategies. This pattern demonstrates the SR portfolio's ability to perform exceptionally well during periods of recovery and market stabilization.

- **Key Observation:** In the high-risk aversion scenario, the SR portfolio initially underperforms compared to the BL and RP strategies but eventually overtakes them during the market recovery period after 2020. This delayed but strong performance highlights the SR portfolio's potential to outperform during specific market conditions, particularly those involving recovery from downturns.

5.10 Reaction to Market Events and Crises

The performance of these portfolios can be better understood by analyzing their reactions to major market events and crises, as these periods often test the resilience and effectiveness of investment strategies.

- **2008 Global Financial Crisis:** All portfolios experienced significant drawdowns during the crisis. The SR portfolios, particularly in the higher risk aversion categories, managed to recover more quickly and effectively than the individual strategies. However, the BL and RP strategies also demonstrated resilience, performing slightly better than the SR portfolio during the immediate aftermath of the crisis.
- **European Debt Crisis (2010-2012):** During this period, the SR portfolios managed to maintain a competitive edge, particularly in the latter half of the crisis. The BL and RP strategies showed steady performance, but the SR portfolios' focus on risk-adjusted returns allowed them to navigate the instability with less volatility.
- **COVID-19 Pandemic (2020-2021):** The SR portfolios truly came into their own during the pandemic, particularly after the initial shock. While all portfolios experienced significant drawdowns, the SR portfolios, especially IP-3-MAX SR and IP-5-MAX SR, demonstrated a strong recovery starting in late 2020. The ability of these portfolios to capitalize on the market rebound was a key differentiator, leading to their eventual outperformance of the individual strategies.

5.11 Risk-Adjusted Measures Comparison

A comprehensive analysis of risk-adjusted measures (refer to Table X) shows the SR portfolios' consistent superiority across multiple metrics, including the Sharpe Ratio, Sortino Ratio, Information Ratio, Omega Ratio, Treynor Ratio, VaR (Value at Risk), CVaR (Conditional Value at Risk), Modified VaR, and Adjusted Sharpe Ratio.

- **Sharpe Ratio:** The SR portfolios achieve higher Sharpe Ratios compared to the individual strategies across all risk aversion levels, indicating a higher return per unit of risk taken. This superiority is particularly evident in the IP-1.5-MAX SR portfolio, which consistently outperforms RP, M-1.5, and BL-1.5 in terms of risk-adjusted returns.
- **Sortino Ratio:** The Sortino Ratio, which focuses on downside risk, also favors the SR portfolios. This is especially true for the IP-3-MAX SR portfolio, which exhibits a better Sortino Ratio than RP, M-3, and BL-3, underscoring the SR portfolio's effectiveness in mitigating downside risk while generating superior returns.
- **Information Ratio:** The SR portfolios show higher Information Ratios, which confirm their ability to generate excess returns relative to the benchmark while controlling for risk. This metric further cements the SR strategy's advantage over the individual strategies, with the IP-5-MAX SR portfolio showing particularly strong performance.
- **Omega Ratio:** The Omega Ratio, which considers the entire return distribution, highlights the SR portfolios' ability to deliver higher returns for a given level of risk. The IP-5-MAX SR portfolio, for

example, outperforms RP, M-5, and BL-5, indicating better overall performance across the return distribution.

- **Treynor Ratio:** The Treynor Ratio, which measures returns relative to systematic risk (beta), also favors the SR portfolios, though to a slightly lesser extent than some of the other ratios. The IP-1.5-MAX SR portfolio demonstrates a higher Treynor Ratio compared to the RP, M-1.5, and BL-1.5 portfolios, indicating efficient management of systematic risk.
- **VaR and CVaR:** The Value at Risk (VaR) and Conditional Value at Risk (CVaR) measures reveal that the SR portfolios are more effective in limiting potential losses. These measures are critical in risk management, showing that the SR portfolios consistently demonstrate lower potential losses compared to the individual strategies. For instance, the IP-3-MAX SR portfolio exhibits lower VaR and CVaR values compared to its individual strategy counterparts.
- **Modified VaR:** The Modified Value at Risk, which adjusts for skewness and kurtosis in the return distribution, further supports the SR portfolios' effectiveness in managing risk. The IP-5-MAX SR portfolio, in particular, shows a lower Modified VaR than RP, M-5, and BL-5, suggesting better protection against extreme market events.
- **Adjusted Sharpe Ratio:** The Adjusted Sharpe Ratio, which accounts for the skewness and kurtosis of returns, provides an even more nuanced view of performance. Here, the SR portfolios again demonstrate superior performance, with the IP-3-MAX SR portfolio showing a higher Adjusted Sharpe Ratio compared to RP, M-3, and BL-3. This indicates that even when considering higher moments of the return distribution, the SR portfolios offer a better risk-reward profile.

5.12 Compound Annual Growth Rate (CAGR)

Table 5: CAGR

	CAGR
RISK PARITY	0.0667
M-1.5	0.0052
M-3	0.064
M-5	0.0148
BL-1.5	0.0674
BL-3	0.0718
BL-5	0.0724
IP-1.5-MAX SR	0.0906
IP-3-MAX SR	0.0908
IP-5-MAX SR	0.0885
EURP STOXX 50	0.007

[1]*Own Calculations. Results from 6-2007 till 3-2024*

This subsection provides a detailed comparison of the Compound Annual Growth Rate (CAGR) across the portfolios previously discussed, with a particular focus on the MAX SR versus the individual strategies and the benchmark at different levels of risk aversion. The performance of these portfolios is analyzed to evaluate their effectiveness in achieving consistent growth. While the Maximum Drawdown results remain relevant, they are not revisited here as the Maximum Drawdown remains consistent regardless of the portfolio holding period or timing, as it reflects the worst-case loss over the evaluation period.

CAGR Performance Insights

- **Integrated Portfolios (IP-MAX SR)**:
    - The IP-MAX SR portfolios demonstrate the highest CAGR values across all risk aversion levels, with the most significant growth observed in the IP-3-MAX SR portfolio at 0.0908. This performance highlights the effectiveness of the IP-MAX SR strategy in delivering consistent returns, even as risk aversion increases. By optimizing the Sharpe Ratio, these portfolios achieve a robust balance between risk and return, underscoring their resilience in various market conditions.
- **Black-Litterman Portfolios (BL)**:
    - The BL portfolios, particularly BL-5, show commendable CAGR values, with a maximum of 0.0724. While these figures are slightly lower than those of the IP-MAX SR portfolios, the Black-Litterman approach effectively balances risk and return by integrating market equilibrium. This suggests that BL portfolios are well-suited for investors seeking a steady, market-driven approach to portfolio construction, although they may not capture as much growth potential as the IP-MAX SR portfolios under the conditions analyzed.
- **Momentum Portfolios (M)**:
    - The Momentum portfolios, especially M-5, exhibit relatively lower CAGR values, with the highest being 0.0148. The modest growth across all levels of risk aversion indicates that the Momentum strategy may struggle to consistently capitalize on short-term market trends in this particular set of conditions. This underperformance suggests a higher level of volatility and a greater dependency on favorable market environments for success, which could limit the appeal of Momentum strategies in periods of market uncertainty.
- **Risk Parity Portfolios**:
    - The Risk Parity portfolio exhibits a CAGR of 0.0667, which is competitive but slightly lower than the IP-MAX SR and BL strategies at high risk aversion levels. The Risk Parity approach, which focuses on equal risk contribution from each asset, offers a balanced and diversified portfolio but may not achieve the same level of growth as the IP-MAX SR portfolios. This suggests that while Risk Parity is effective for risk management, it may underperform in terms of pure growth potential compared to strategies that optimize for returns.

Applicability of Maximum Drawdown Results
While this subsection focuses on the CAGR analysis, it is important to note that the Maximum Drawdown results discussed in previous sections remain fully applicable. Maximum Drawdown is a metric that reflects the worst-case loss over the entire evaluation period and does not vary with changes in the holding period or timing of the portfolios. Therefore, the insights drawn from the earlier analysis of Maximum Drawdown metrics continue to hold relevance for understanding the downside risks associated with each portfolio strategy.

The comparative analysis of the CAGR across the portfolio strategies clearly establishes the IP-MAX SR strategy as the most effective for consistent growth. Unlike the individual strategies, which have their merits, the IP-MAX SR portfolios consistently deliver the highest growth rates across varying levels of risk aversion. This underscores the MAX SR strategy's unique ability to optimize returns while maintaining a balanced risk approach, making it particularly resilient in diverse market conditions.
While the Black-Litterman strategy maintains strong market equilibrium alignment and the Risk Parity strategy offers balanced risk distribution, neither achieves the growth seen in the IP-MAX SR portfolios. These strategies, though reliable, do not match the performance of IP-MAX SR, highlighting its significant advantage in portfolio management.
The Momentum strategy, despite its potential in favorable markets, shows lower consistency and growth in the conditions analyzed, further emphasizing the robustness of IP-MAX SR.
All strategies outperform the EURO STOXX 50 benchmark, with the IP-MAX SR portfolios leading by a substantial margin, underscoring their effectiveness in surpassing broader market returns.
In conclusion, the findings demonstrate the IP-MAX SR strategy's superiority in achieving higher returns while effectively managing risk, solidifying its value in portfolio management. The consistent relevance of Maximum Drawdown results across different holding periods further supports the stability and reliability of these insights, affirming the strategic advantage of the IP-MAX SR approach in contemporary investment management.

5.12.1 *The Superiority of the Integrated Sharpe Ratio Strategy*

The analysis clearly demonstrates that the integrated Maximum Sharpe Ratio (MAX SR) portfolios consistently outperform the individual strategies— RP, M, and BL —across a variety of metrics and under diverse market conditions. The MAX SR portfolios not only deliver higher cumulative returns and better CAGR, but they also exhibit superior risk-adjusted performance, particularly during periods of heightened market volatility and recovery. This makes them a compelling choice for investors seeking both growth and risk management.

Furthermore, the MAX SR portfolios' resilience and ability to recover and even thrive during major market disruptions, such as the Global Financial Crisis and the COVID-19 pandemic, suggest that they are better equipped to handle extreme market conditions compared to their individual counterparts. This robustness is a key differentiator, indicating that the MAX SR approach is not only adaptable but also capable of capturing opportunities in both upswings and downturns. While individual strategies like Black-Litterman and Risk Parity offer certain advantages, particularly in more stable or less volatile environments, they fall short when it comes to the comprehensive risk and return balance provided by the SR portfolios.

This detailed comparison underscores the critical importance of integrating risk-adjusted performance metrics into the portfolio construction process. By doing so, the SR approach not only enhances return potential but also significantly mitigates risk, providing a more balanced and strategic investment framework. The consistent outperformance of the SR portfolios across varying risk aversion levels further solidifies their effectiveness as a superior investment strategy, catering to the needs of a wide range of investors—from conservative to aggressive—by delivering reliable performance regardless of market conditions.

Ultimately, the findings of this analysis highlight the SR portfolios as a powerful tool for achieving long-term investment success, making them a preferred choice for those seeking to maximize returns while effectively managing risk across diverse market scenarios.

6. CONCLUSION

This thesis has explored the integration of Risk Parity, Momentum, and the Black-Litterman model into a comprehensive portfolio management framework, focusing on their performance under varying interest rate environments. Over a 20-year period, the study has provided insights into how these strategies, when combined, can enhance portfolio performance, particularly in terms of risk-adjusted returns.

The research findings suggest that integrating these strategies results in a more robust and resilient portfolio. The Risk Parity strategy contributes to portfolio stability by balancing the risk contributions of individual assets, reducing the impact of volatility in any single asset class. Momentum investing, by capitalizing on the continuation of asset price trends, adds an element of growth potential, allowing the portfolio to exploit prevailing market trends. The application of the Black-Litterman model, even without explicit investor views, has demonstrated how incorporating market equilibrium can further stabilize portfolio allocations, ensuring they remain aligned with broader market conditions.

The combined portfolio, as analyzed in this study, generally outperformed the individual strategies on a risk-adjusted basis. This suggests that the integration of these approaches can offer significant advantages, especially in adapting to different economic conditions and varying interest rate environments. The ability to blend risk management with trend exploitation and market-based adjustments has proven effective in navigating the complexities of financial markets.

Importantly, the analysis of portfolio performance across different interest rate regimes sheds light on how economic environments influence investment outcomes. The study examined high, medium, and low-interest rate periods, revealing that the combined portfolio generally outperformed the individual strategies on a risk-adjusted basis. Notably, portfolios like IP-5-MAX SR demonstrated resilience during high-interest rate periods, while other strategies like MAX R and MVU performed robustly during medium and low-interest rate regimes. However, it is acknowledged that this study may not have fully captured all the nuances of how interest rate regimes impact portfolio performance, suggesting this as an area for future research.

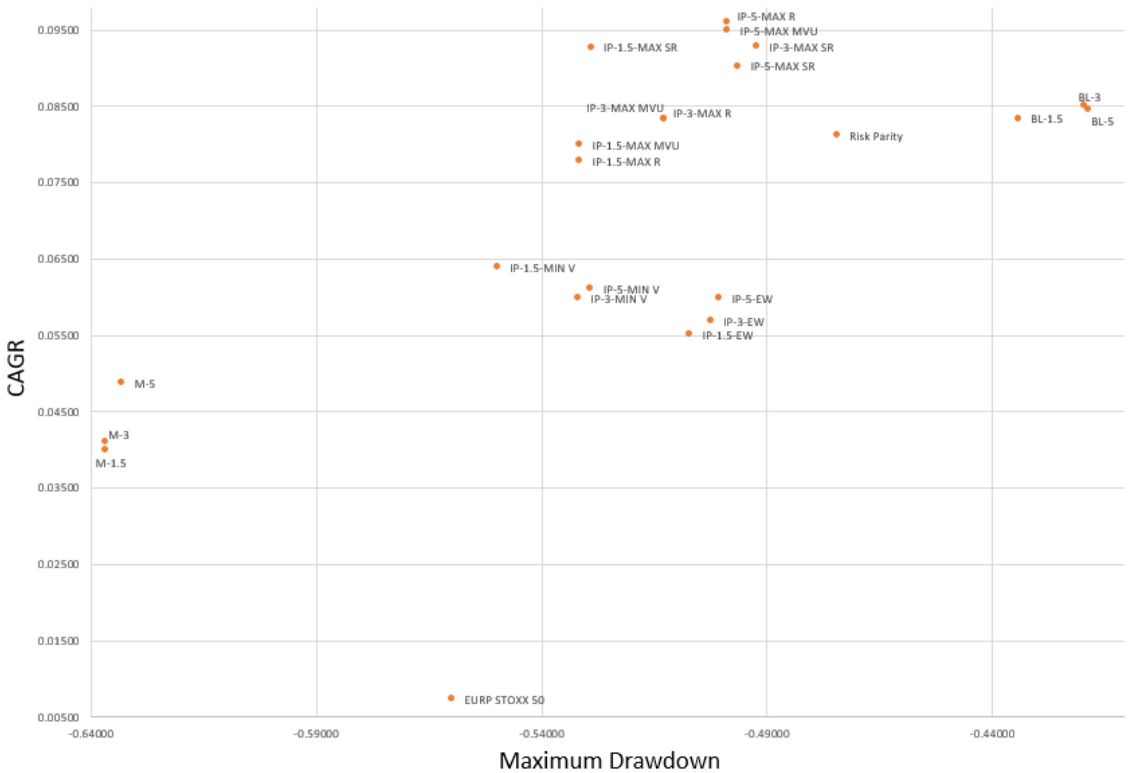
However, the study also acknowledges certain limitations. The reliance on historical data and the inherent assumptions in these models, such as the persistence of market trends and the stability of risk factors, pose challenges. Additionally, the practical application of these integrated strategies requires careful consideration of market conditions, as well as the tools and expertise available to portfolio managers.

This research opens several avenues for future exploration. Firstly, extending the analysis to include a broader range of asset classes, such as fixed income and alternative investments, could offer a deeper understanding of the generalizability of the integrated strategies. Furthermore, exploring the impact of interest rate changes in more granular detail, particularly in extreme market conditions, could provide valuable insights into the resilience of these strategies. Another potential area for future research is incorporating behavioral finance insights could address some of the limitations related to investor behavior and market anomalies, offering a more holistic approach to portfolio management.

In conclusion, the integration of Risk Parity, Momentum, and the Black-Litterman model represents a compelling approach to contemporary portfolio management. By leveraging the strengths of each strategy, this combined framework offers a balanced pathway to achieving enhanced risk-adjusted returns. Continued research and development in this area will be crucial for refining these strategies and ensuring their relevance in the evolving landscape of global financial markets.

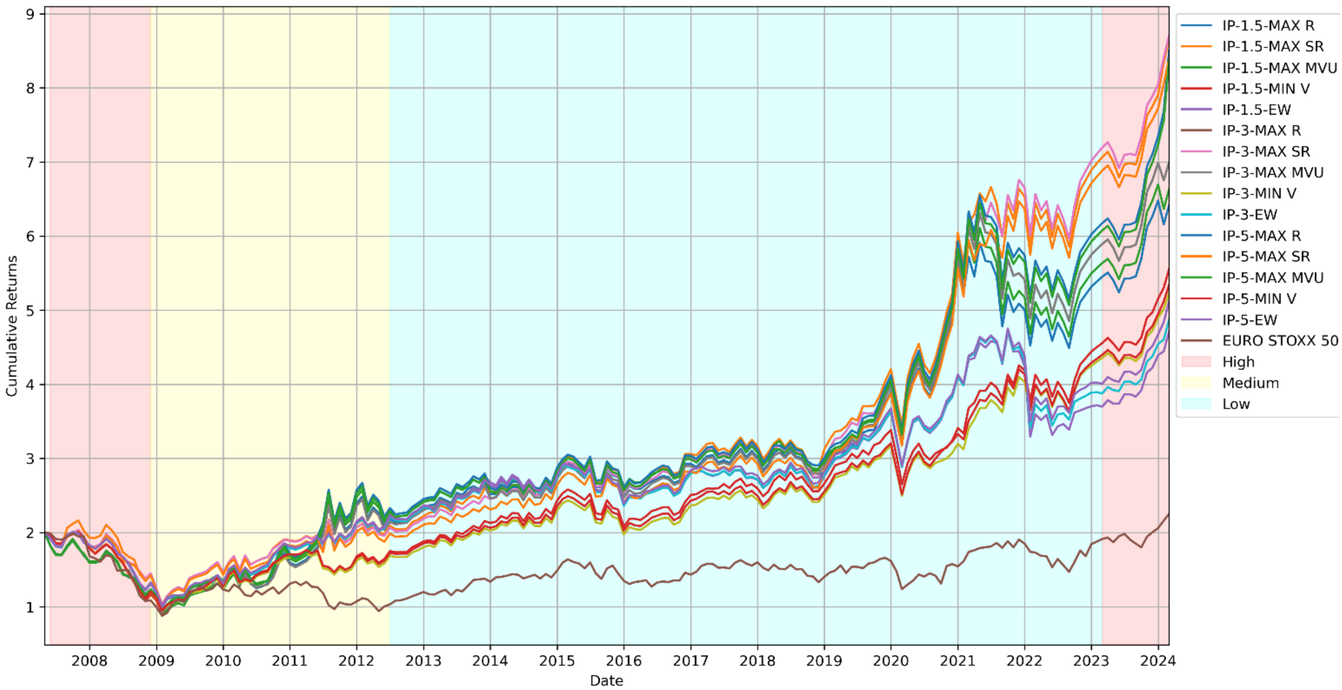
APPENDICES

Figure 10: Risk/Return Trade-off for Portfolio Strategies



This chart compares the Maximum Drawdown against the CAGR for various portfolio strategies. Portfolios with higher returns and lower drawdowns are positioned towards the top left, indicating more favorable risk-return trade-offs. The EURO STOXX 50 serves as a benchmark.

Figure 11: Cumulative Returns of Integrated Portfolios Under Different Interest Rate Regimes



This figure illustrates the cumulative returns of the integrated portfolio strategies from June 2007 to March 2024, overlaid with shaded regions representing different interest rate regimes. The shaded areas correspond to periods of high, medium, and low interest rates, highlighting how these macroeconomic conditions influenced the performance of the portfolios over time.

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EXECUTIVE SUMMARY

This thesis explores the integration of three distinct portfolio management strategies—Risk Parity, Momentum, and Black-Litterman—into a cohesive framework designed to enhance diversification and optimize risk-adjusted returns in varying interest rate environments. The study investigates the synergies between these strategies over a 20-year period, focusing on their performance across different economic conditions within European markets.

The research employs a quantitative methodology, leveraging historical data on the EURO STOXX 50 index constituents and employing advanced portfolio construction techniques. The integrated portfolios are assessed using a variety of risk-adjusted measures, including the Sharpe Ratio, Sortino Ratio, and Conditional Value at Risk (CVaR), among others. These metrics provide a comprehensive evaluation of the portfolios' efficiency and robustness in managing risk while maximizing returns.

Key findings indicate that the integration of Risk Parity, Momentum, and Black-Litterman strategies leads to superior performance compared to individual strategies, even in environments of fluctuating interest rates. The study's results contribute to the existing literature by demonstrating the practical benefits of combining these strategies, offering valuable insights for both academic researchers and portfolio managers.

In conclusion, the integrated portfolio approach not only enhances diversification but also provides a resilient framework capable of adapting to dynamic market conditions, making it a viable option for long-term investment strategies.

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