Is the Walloon Region's public debt sustainable?

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Contents

2 Glossary

ADR: Adjusted Disposable Revenues

DSA: Debt Sustainability Analysis

DSM: Debt Sustainability Monitor

EC: European Commission

ESA: European System of Accounts

EU: European Union

FPB: Federal Planning Bureau

GDP: Gross Domestic Product

GNP: Gross National Product

IBC: Intertemporal Budget Constraint

IMF: International Monetary Fund

LT: Long-Term

LSF: Loi Spéciale de Financement

NBB: National Bank of Belgium

OLO: Obligations Linéaires

PIT: Personal Income Tax

POWG: Potential Output Working Group

ST: Short-Term

US: United States

3 Introduction

3.1 Global context

"Global public debt is very high. It is expected to exceed $100 trillion (93 percent of global GDP) in 2024 and to keep rising through the end of the decade (approaching 100 percent of GDP by 2030)" (IMF, 2024, p. 10). This is the first sentence of the Executive Summary of the IMF Fiscal Monitor published in October 2024. Public debt has emerged as a significant concern for the future, given the persistent upward trend in debt-to-GDP ratios observed across countries over the past decades. In addition, the report emphasizes that spending pressures are significant for governments due to the multiple challenges our societies face, such as the green transition, ageing, security concerns, and development challenges (IMF, 2024).

Belgium is no exception to this general trend and faces major debt concerns. In its last annual report, the National Bank of Belgium (2025) pointed out that the debt ratio continued to increase in 2024, reaching 104.4% of Belgium's GDP. The report concludes its public finance section by emphasizing the urge to proceed to fiscal consolidation because the debt dynamics are on an unsustainable path (NBB, 2025).

The European Commission (EC) has also pointed out that Belgium is not in a good budgetary position. In its last report on the debt sustainability of member states (European Commission, 2025), Belgium is projected to reach a debt ratio of around 126% of GDP in 2035 at constant policies. Both medium and long-term indicators point to a high risk of non-sustainability of public debt.

These analyses from institutions highlight that public debt is a major issue and at the center of the current public debate.

3.2 Case of the Walloon Region

Therefore, it could be understood that the sustainability of countries' public finance is closely monitored by several institutions. Still, it is not necessarily the case for the entities that compose a country.

Belgium is a federal state where federated entities have been allowed to incur debt without federal guarantees since 1989 (Bourgeois et al., 2019). This institutional arrangement has raised important questions regarding the entities' public debt sustainability. Moreover, the increasing financial autonomy of these entities has led to a redistribution of fiscal responsibility, requiring coordinated budgetary efforts. In this context, the High Council of Finance plays a central role by issuing

recommendations on the budgetary paths each entity must follow to comply with European fiscal rules and ensure the long-term sustainability of public finances (Bourgeois et al., 2019).

The aim of this master's thesis is to carry out a complete analysis of the sustainability of the Walloon Region's public debt by applying and adapting the methodological tools used by institutions such as the European Commission and the IMF. Particular attention will be paid to Belgium's specific institutional features, which make the transposition of international methods complex but essential for a relevant assessment on a regional scale.

4 Literature Review

This section reviews the literature on two concepts related to public debt: optimality and sustainability. Optimality tries to answer the question of the right quantity of public debt required to maximize the economy's welfare. Sustainability addresses issues related to different criteria that would ensure that a government or a state would not default. Finally, a few words about some key elements regarding current discussions on public debt sustainability.

4.1 Optimality of Public Debt

Researchers tried to model and understand the effects that public debt could have on the economy. The objective is to maximize the welfare of the economy as a whole. This first section explores the main papers related to the optimality of public debt through time.

Public debt can be detrimental to welfare by reducing the stock of capital per worker and decreasing their consumption in their lifetime (Diamond,1965). These results come from a highly stylized model but still give useful intuitions for further research. This model, built by Diamond, is a reference for many other articles exploring the effects of public debt on the economy.

From this initial model, with different settings, it can be shown that an increase in government debt does not change the consumption of actual and future generations. If generations are linked by their utility function, households will adjust their consumption and bequest for the next generation. However, in the case of imperfect capital markets, this issuance of bonds for individuals with infinite life can induce a net wealth increase (Barro, 1974). The main intuition behind this result is that agents can perfectly forecast and anticipate the future and adapt their behavior in

the present. Therefore, the emission of public debt has no real effect due to the forecast increase in taxes; this is the Ricardian equivalence. From that perspective, Barro (1979) developed relationships with real variables and showed that it could be optimal for the government to issue debt to reduce the distortion created by taxation (Barro, 1979).

One of the problems linked to the issuance of public debt is the crowding-out effect, which is often responsible for welfare losses. It leads to an increase in interest rates, decreases the assets per capita, and decreases the capital stock per capita (Aiyagari & McGrattan, 1998). The theoretical optimal level is low because public debt crowds out capital, decreasing welfare. The government should even own assets, i.e., a negative public debt, to increase capital stock and decrease the return on capital. It would, therefore, increase wages and benefit the majority (Vogel & European Central Bank, 2014).

However, if public debt is considered as private liquidity, it can also be shown that a higher level of debt can increase the stock of capital per worker and be optimal (Woodford, 1990). In some circumstances, a high level of debt can be effective if it allows economic agents to smooth out their consumption throughout their lifetime.

The optimal level of debt heavily depends on the underlying assumptions of models. Empirically, the crowding effect seems to hold in a small proportion (Aschauer, 1989). However, transfers seem to be better economic tools than public debt to insure against uninsurable risk (Flodén, 2001).

With imperfect competition, it can be shown that there exists an inverse long-term causal relationship between GDP growth rate and the long-run debt ratio. Skott & Ryoo (2014) also show that a reduction in government consumption and entitlement programs for the old increases public debt in the long run. Here again, there is a trade-off between raising interest rates and the easing of friction. The optimal quantity of debt depends on the crisis conditions and the economy's needs because of this trade-off (Angeletos et al., 2023).

More recently, Blanchard (2019) shows that debt rollovers without rising future taxes may be possible and that, given low interest rates, the welfare cost of high debt levels could be lower than usually assumed. Here again, results highly depend on the model specifications and values that the model takes. However, Blanchard (2019) gives interesting insights regarding the interaction between safe and risky interest rates, the nominal GDP growth rate, and the consequences of debt on welfare.

The level of debt on GDP growth rate has also been examined empirically. Woo & Kumar (2015) showed that a high level of public debt has a negative impact on long-term GDP growth. Indeed, it can be shown that a high initial level of public debt has a negative impact on subsequent GDP growth through the channels of lower labor productivity and slower growth of capital stock per worker. Empirical proof from the European Union also supports the hypothesis of a non-linear relationship between the GDP growth rate and the debt-to-GDP ratio and a threshold above which the debt level negatively impacts GDP growth (Mencinger et al., 2014). The common threshold of 90% of the debt-GDP ratio above which the growth rate falls (Reinhart & Rogoff, 2010) is still considered as a reference. However, these results are controversial because of methodological issues (Herndon et al., 2014).

More recently, a meta-analysis has shown that there is a negative relationship between the public debt-to-GDP ratio and the real GDP growth rate: an increase of 10 percentage points in the debt ratio is related to a decrease of the real GDP growth rate by 0.14 percentage points per year (Heimberger, 2023). However, once the publication bias is removed, a zero average effect cannot be rejected (Heimberger, 2023). The existence of a threshold above which the GDP growth rate is reduced is also discussed, and the author points to a lack of robust evidence for a universal threshold (Heimberger, 2023).

The concept of public debt's optimality has been studied for a long time and highly depends on the model chosen by the modeler. However, opposite phenomena are shared across research. On the one hand, the crowding-out effect is detrimental to welfare because less money is available to finance private projects and stimulate the economy. On the other hand, public debt is essential as a risk-free asset. It helps to stabilize and ensure agents against uninsurable risk and reduce financial friction in the market. The importance of those opposite forces determines the final welfare effect of public debt.

4.2 Sustainability of Public Debt

Debt sustainability is not obtained through an optimization process like optimality, and the concept is more difficult to define.

A wide range of approaches has been proposed in the literature to assess the sustainability of public debt. Indeed, compared to other economic agents, there are several difficulties in assessing the sustainability of a country's public debt. This question is more challenging to answer because of the nature of the government. Indeed, a sovereign state has an infinite life; it cannot be liquidated like a company, and the default has multidimensional consequences compared to the default of

other agents (Debrun et al., 2019).

Bohn (1998) tried to estimate the reaction of the US primary balance[1]to an increase in the government debt-to-GDP ratio. The idea is that a positive response of the primary surplus to the debt-income ratio means that the debt is sustainable in the sense that it satisfies the intertemporal budget constraint[2](Bohn, 1998). The results show that the US primary balance reacted positively to an increase in government debt for the 1916-1995 period, implying that the government debt is sustainable (Bohn, 1998). Some nonlinearities are observed in the data, indicating that the reaction of the primary balance is greater when the debt-to-GDP ratio is high (Bohn, 1998).

Blanchard proposed another simple definition of a sustainable fiscal policy. A sustainable fiscal policy is defined as one that leads the debt-to-GNP ratio to converge at its initial level. In other words, the discounted value of the future primary deficit must equal the targeted level of debt. Blanchard proposed forecasting the primary deficits and determining the tax rate necessary to converge to a predefined level of debt. If this tax rate is higher, the current fiscal policy is not sustainable (Blanchard et al., 1991). Contrary to Bohn 1998, this approach is purely forward-looking, which makes the analysis sensitive to assumptions.

To tackle the different issues linked to sustainability analysis, the definition mainly used is the one proposed by the IMF in 2013, which includes more dimensions than previously. Other institutions, such as the European Commission or the European Central Bank, use this definition to assess the sustainability of debt:

In general terms, public debt can be regarded as sustainable when the primary balance needed to at least stabilize debt under both the baseline and realistic shock scenarios is economically and politically feasible, such that the level of debt is consistent with an acceptably low rollover risk and with preserving potential growth at a satisfactory level. Conversely, if no realistic adjustment in the primary balance—i.e., one that is both economically and politically feasible—can bring debt to below such a level, public debt would be considered unsustainable. The higher the level of public debt, the more likely it is that fiscal policy and public debt are unsustainable. This is because—other things equal—a higher debt requires a higher primary surplus to sustain it. Moreover, higher debt is usually associated with lower growth and higher interest rates, thus requiring an even higher primary

[1]The primary balance is the difference between the government's total revenues and primary expenditures, i.e., expenditures net of interest payments. See Section 5.1.2

[2]More details are given on this theoretical concept in section 5.4

balance to service it.
(International Monetary Fund, 2013, p.4).

From this definition, three temporal dimensions of debt sustainability can be drawn. A short-term dimension with the idea of an acceptably low rollover risk. A medium-term dimension, with the requirement of a primary balance stabilizing at least the debt ratio, which must be politically and economically feasible. A long-term dimension with a debt level that must ensure potential GDP growth at a satisfactory level. This definition is the one used in the context of this work.

Ghosh and his co-authors (2013) also try to find a clear debt limit above which the country's solvency can be in doubt. The idea was to study the reaction function of developed countries, i.e., how they adjust their budget after a primary deficit, and deduce from that the limit beyond which the solvency could be questioned, beyond which the dynamic leads the government to default (Ghosh et al., 2013). It represents an interesting approach, even if, here again, it relies on several assumptions and does not consider some short-term risks or dimensions.

Due to the purely forward-looking dimension of this exercise, these analyses appear to perform poorly in crisis economies (Guzman & Heymann, 2015). These bad estimates lead to inappropriate decisions, which can be even worse for economies. The authors also show that the IMF tends to be over-optimistic regarding the consequences of the fiscal adjustments that have to be made to return to a sustainable path of debt. Obviously, the IMF is aware of these issues and constantly improves its methodology. The last update of the whole framework is from 2021 (International Monetary Fund. Strategy & Department, 2021), which adds new tools and requirements to reduce forecast errors. However, it does not radically change the baseline of the approach.

In a few words, if the sustainability of public debt is not ensured, the consequences related to a default are numerous. On average, Borensztein & Panizza (2009) found that default on external debt is associated with a decrease of GDP of 1.2 percentage points per year. After a default, the government's reputation can be damaged, leading to lower credit ratings (Borensztein & Panizza, 2009). The impact on borrowing costs seems to be positive in the short and medium term (Borensztein & Panizza, 2009).

4.3 Current debate

From the two concepts developed above, it is clear that public debt optimality and sustainability are different but related concepts. Optimality refers to the quantity

that maximizes the economy's welfare, for example, by smoothing consumption (Barro, 1979) or providing safe assets for economic agents (Woodford, 1990). On the other hand, sustainability focuses on conditions that ensure the debt level does not increase to excessive debt accumulation (Blanchard et al., 1991) and that fiscal policies are economically and politically feasible to keep the debt dynamic under control (IMF, 2013).

It is clear that sustainability does not necessarily imply that the debt is optimal, but the reverse question is more challenging to answer. From a theoretical perspective, if the government budget constraint is included in the model specifications, the solvency part of the sustainability is ensured. However, the definition of sustainability is broader and considers more dimensions. Sustainability of public debt also depends on macroeconomic parameters and fiscal policy, while optimality results from an optimization process.

Directly related to the quantity of debt and its sustainability, according to Ragot (2022), the demand for safe assets has increased more than the supply due to the decrease in real interest rates and, consequently, the increase in the price of public debt. From these observations comes a new "Triffin dilemma" because there is a short-term need for developed countries' public debt, making them more sensitive and riskier in the future in case of adverse macroeconomic shocks (Ragot, 2022). The original concept was born with the dollar standard, in which Triffin saw a problem. He believed that there was a mismatch between the accumulation of dollars abroad and the real reserve of gold held by the Federal Reserve. The backing of the dollar was not sustainable (Maes, 2012). Nowadays, the same analysis can be made with the accumulation of US external debt and the real backing capacity of the US government and economy. There are also direct links with global financial stability. This problem goes beyond the point of this thesis but remains interesting from the sustainability point of view.

Here comes the theoretical balance between optimality and sustainability. A high level of public debt can be required to provide enough safe assets to agents to insure uninsurable risk. On the other hand, high public debt is usually associated with higher risk and, therefore, higher interest rates, which can make the debt unsustainable.

5 Methodologies

This section is devoted to all theoretical aspects and concepts used in this master's thesis. First of all, the main concepts and equations of public finance are reviewed in Subsection 5.1.

Secondly, the methodologies developed in the literature and used by international institutions are described in Section 5.2 for the short-term indicator, Section 5.3 for the medium-term indicator, and Section 5.4 for the long-term indicator.

5.1 Basic equations [3]

5.1.1 Budget balance

First of all, the budget balance, for the year t, is defined as follows:

$$S_t = T_t - G_t \tag{1}$$

where S is the budget balance, T is the total amount of government revenues, and G is the total amount of government spending.

It is assumed that government revenues are solely derived from taxes on labor, on capital income, and on consumption. These taxes are purely endogenous to economic activity. Indeed, as GDP increases, the tax base also expands. The nominal GDP represents the total amount of resources available for the government to raise taxes. Consequently, T_t can be expressed as the product of the implicit tax rate (τ_t) and the nominal GDP (YN_t).

$$T_t = \tau_t \times YN_t \tag{2}$$

Regarding the government's expenses, G_t can be decomposed into two parts:

$$G_t = PE_t + r_t \times B_{t-1} \tag{3}$$

Where PE_t are the primary expenditures, r_t is the nominal implicit interest rate [4], and B_{t-1} is the accumulated total amount of debt in $t-1$. It will be considered that

[3] The equations in this section are mainly taken from the university course "Politique économique et finances publiques, by Frogneux V. & Kegels C., Master in Economic Sciences, UNamur, 2023." and from Debrun et al., 2019

[4] The implicit interest rate represents the weighted average of financing cost of an economic agent. In the case of public debt, it is calculated through the ratio between the value of the current year's interest and the value of the debt stock of the previous year. (*Implicit Interest Rate* | Glossary | CFP, s.d.)

PE is exogenous to economic activity; primary expenditures are only subject to political decisions, except for automatic stabilizers such as unemployment benefits.

The equation (1) can now be developed as follows:

$$S_t = T_t - (PE_t + r_t \times B_{t-1}) \tag{4}$$

From this equation, if the government's revenues are greater than the government's expenditures, the budget balance will be said to be in *surplus*. If government revenues are smaller than government expenditures, the budget balance will be said to be in *deficit*.

5.1.2 Primary balance

Secondly, the primary balance is defined as the difference between the total revenues and the primary expenditures.

$$PB_t = T_t - PE_t \tag{5}$$

It is the budget balance increased by the interest payments:

$$PB_t = S_t + r_t \times B_{t-1} \tag{6}$$

Like the budget balance, the primary balance will be said to be in *deficit* if negative and will be said to be in *surplus* if positive.

5.1.3 Structural component

Output Gap
One of the key concepts of this section is the *Output-gap*. The output gap is defined as follows:

The output gap is an economic measure of the difference between the actual output of an economy and its potential output. Potential output is the maximum amount of goods and services an economy can turn out when it is most efficient—that is, at full capacity. Often, potential output is referred to as the production capacity of the economy.
(What Is The Output Gap ? - Back To Basics - Finance & Development, September 2013, 2013)

$$OG_t = \frac{YN_t - YN_t^*}{YN_t^*} \tag{7}$$

In this master's thesis, the output gap is expressed in relative terms.

Cyclically Adjusted Budget Balance
It is possible to adjust the primary and budget balances to consider macroeconomic conditions.

The Cyclically Adjusted Budget balance is the one that would be obtained if the economy were at its full potential:

$$CAB_t = \tau_t \times YN_t^* - G_t \tag{8}$$

The cyclical component is obtained by making the difference between the actual budget balance and the cyclically adjusted budget balance, expressed as a percentage of GDP.

$$\frac{S_t}{YN_t} - \frac{CAB_t}{YN_t^*} = (\tau_t - \frac{G_t}{YN_t}) - (\tau_t - \frac{G_t}{YN_t^*}) = -\frac{G_t}{YN_t} + \frac{G_t}{YN_t^*} \tag{9}$$

By solving the equation, the final result, namely the cyclical component, is the following:

$$\frac{-G_t \times YN_t^* + G_t \times YN_t}{YN_t \times YN_t^*} = \frac{G_t}{YN_t} \times \frac{YN_t - YN_t^*}{YN_t^*} = \frac{G_t}{YN_t} \times OG_t \tag{10}$$

Structural Balance
Finally, the CAB_t can be adjusted by taking the one-off (OO_t) into account. These are purely temporary or exceptional measures that affect the budget balance.

$$\frac{S_t^*}{YN_t^*} = \frac{CAB_t}{YN_t^*} - \frac{OO_t}{YN_t} \tag{11}$$

Finally, the budget balance can be divided into three components: the structural balance, the cyclical component, and the one-off.

$$\frac{S_t}{YN_t} = \frac{CAB_t}{YN_t^*} + \frac{G_t}{YN_t} \times OG_t = \frac{S_t^*}{YN_t} + \frac{G_t}{YN_t} \times OG_t + \frac{OO_t}{YN_t} \tag{12}$$

5.1.4 Public debt

Public debt accumulation
Unlike a budget deficit ($S_t < 0$) or a primary deficit ($PB_t < 0$), debt (B_t) is a stock accumulated over time and not an annual flow. The stock of debt in year t (B_t) is equal to the amount of debt accumulated until $t-1$ (B_{t-1}) minus the primary balance of the year (PB_t) plus the interest payments on the existing debt

($r_t B_{t-1}$) plus the stock flow adjustments (F_t). These are financial operations that are affecting the amount of debt without influencing the budget balance. For example, if the government decides to sell some assets, it will be counted in F. The final equation is the following:

$$B_t = B_{t-1} - S_t + F_t = B_{t-1} - PB_t + r_t B_{t-1} + F_t \tag{13}$$

Through this equation, it can be understood that the amount of debt can be reduced either by a positive primary balance or by negative stock-flow adjustments.

In the analysis that will be conducted, the debt is fully issued in national currency; therefore, the exchange rates and interest rates of foreign currencies are not considered.

To compare countries' debt levels, equation (13) is divided by the nominal GDP, which represents the size of the economy and the total amount of resources available for the government to raise taxes.

$$\frac{B_t}{YN_t} = \frac{B_{t-1}}{YN_t} - \frac{PB_t}{YN_t} + \frac{r_t B_{t-1}}{YN_t} + \frac{F_t}{YN_t} \tag{14}$$

The equation expressed in percentage of nominal GDP can now be expressed as:

$$b_t = b_{t-1} - pb_t + r_t b_{t-1} + f_t \tag{15}$$

Public debt dynamics

Debt dynamics are more interesting to analyze and show the causes of the behavior of the debt ratio.

Let's take equation (14) and subtract $\frac{B_{t-1}}{YN_{t-1}}$ from both sides of the equation.

$$\frac{B_t}{YN_t} - \frac{B_{t-1}}{YN_{t-1}} = \frac{B_{t-1}}{YN_t} - \frac{PB_t}{YN_t} + \frac{r_t B_{t-1}}{YN_t} + \frac{F_t}{YN_t} - \frac{B_{t-1}}{YN_{t-1}} \tag{16}$$

Let's denote g_t as the nominal growth rate of YN.

$$\frac{B_t}{YN_t} - \frac{B_{t-1}}{YN_{t-1}} = \frac{B_{t-1}}{(1+g_t)YN_{t-1}} - \frac{PB_t}{YN_t} + \frac{r_t B_{t-1}}{(1+g_t)YN_{t-1}} + \frac{F_t}{YN_t} - \frac{B_{t-1}}{YN_{t-1}} \tag{17}$$

Let's highlight $\frac{B_{t-1}}{YN_{t-1}}$ in the right-hand side:

$$\frac{B_t}{YN_t} - \frac{B_{t-1}}{YN_{t-1}} = \frac{B_{t-1}}{YN_{t-1}} \times \frac{r_t - g_t}{(1+g_t)} - \frac{PB_t}{YN_t} + \frac{F_t}{YN_t} \tag{18}$$

Finally, this decomposition of the variation of public debt expressed in percentage of nominal GDP:

$$\Delta b_t = b_{t-1} \times \frac{r_t - g_t}{1 + g_t} - pb_t + f_t \tag{19}$$

This equation summarizes the different components of debt dynamics:

- $b_{t-1} \times \frac{r_t - g_t}{1+g_t}$: the differential between the implicit interest rate (r_t) and the nominal growth rate of GDP (g_t).
- pb_t: the primary balance. If pb_t is negative (there is a primary deficit), it will increase the debt ratio. The other way around is when the primary balance is positive (there is a primary surplus).
- f_t: the stock flow adjustment.

Snowball effect
From equation (19), let's be more focused on the differential between the implicit interest rate (r_t) and the nominal growth rate of GDP (g_t).

The first case considered is when the implicit interest rate r_t exceeds the nominal GDP growth rate g_t. All else being equal, this leads to an increasing debt-to-GDP ratio. In this context, a positive primary balance is required merely to stabilize the debt ratio. If this primary balance is not achieved, a snowball effect occurs: rising debt leads to higher interest payments, which, in turn, deteriorate the overall budget balance.

The second case corresponds to a situation where the implicit interest rate r_t is lower than the nominal GDP growth rate g_t. This is the more favorable scenario, as nominal GDP growth more than offsets interest payments, thereby reducing the debt ratio over time. In this context, the primary balance required to stabilize the debt ratio is negative. Under such conditions, public debt is less of a concern, as the risks associated with a snowball effect are significantly diminished.

5.2 Short term indicator

This section addresses methodologies developed to predict the short-term risk of non-sustainability of public debt.

Two main types of methodologies are proposed in the literature on early warning systems. The first one is multivariate regression using probit or logit models. The main advantages of these models are that they take the correlation between variables into account. The second type of model is a signaling model, which consists

of observing the past behavior of relevant variables when crises occurred. When a crisis happened, the series of variables takes the value of one; if not, they take the value of zero. The idea is to find some threshold for variables above which there is a signal that could be sent in order to spot vulnerabilities. Indeed, economies tend to behave differently in periods preceding a crisis, and the idea is to find variables that signal that a crisis will occur (Baldacci et al., 2011); (Berti et al, 2012).

In the EC framework, short-term risk is evaluated through the $S0$ indicator, which uses a signaling approach. The methodology of the EC is the one described by Berti et al. (2012). However, regarding data availability, it will be easier to transpose the IMF methodology, which relies on fewer variables.

This section is therefore devoted to describing and explaining the methodology developed by Baldacci et al. (2011), one of the main references of the $S0$ methodology.

The methodology is categorized as a signaling approach, consisting of defining cut-off values for each variable that will discriminate between the predictions. In this context, the definition of a crisis includes four types of events:

- Debt default or restructuring: identified as failure to service debt as payments come due, and also distressed debt exchange.
- Implicit default: identified as high inflation level, more than 35% for advanced economies.
- Recourse to exceptional official financing: identified as a large IMF-supported program.
- Sharp deterioration in the market access: when yields of a specific country go above two standard deviations of the country's mean.

This broader definition of what a fiscal crisis is allows the author to find more data and capture more events that could occur.

The main issue for this signaling approach is to find the right cut-off that balances the two types of statistical error. If the cut-off to detect a crisis is low, many signals will be sent, meaning that type II error decreases (do not reject the null hypothesis while it is false), but type I error will increase (reject the null hypothesis while it is true).

Given that a small number of crises have been observed, the minimization of the total misclassified errors will lead to an over-weighting of the false negatives,

misclassifying crises, which leads to a conservative threshold compared to other methods. The minimization problem is therefore the following:

$$SNR(C) = \frac{TP_C/N_C}{FP_C/N_{NC}} \tag{20}$$

The optimal C^* is the one that minimizes the Signal to Noise Ratio (SNR), which is the ratio between the rate of True Positives and the rate of False Positives (declared as crises while they were not).

Baldacci et al. (2011) use 11 variables classified into three main categories. The basic fiscal variables, namely the 5-year average of interest-growth rate differential, the general government debt to GDP ratio, and the cyclically adjusted primary balance expressed in % of potential GDP. The indicators for asset and liability management are four: the gross financing need in % of GDP, the share of short-term debt in the total debt, the debt held by non-residents in % of total debt, and the weighted average maturity of general government debt expressed in years. The last category evaluates long-term fiscal trends through the fertility rate expressed as the deviation from 2.1, the long-term projections of public health expenditures (% of GDP), the long-term projections of public pension expenditures (% of GDP), and the dependency ratio. All variable values and thresholds for developed economies are summarized in the following figure:

INDICATOR	Direction to be safe	Crisis obs.	Non-crisis obs.	Threshold	Type 1 error	Type 2 error	Signaling Power	Index Weight
Basic Fiscal Variables								**31.59**
r-g (5 year average)	<	21	670	3.6	0.48	0.29		14.9
General government gross debt (percent of GDP)	<	15	561	72.2	0.22	0.67		7.3
Cyclically adjusted primary balance (percent of potential GDP)	>	5	137	-4.2	0.45	0.40		9.4
Asset and Liability Management								**42.56**
Gross financing needs (percent of GDP)	<	6	219	17.2	0.45	0.17		24.6
Share of short term debt as a ratio of total debt	<	15	506	9.1	0.96	0.00		2.8
Debt held by nonresidents (percent of total debt)	<	7	163	83.6	0.06	0.79		10.1
Weighted average maturity of general government debt (years)	>	18	535	3.9	0.09	0.83		5.0
Long-Term Fiscal Trend								**25.85**
Fertility rate (deviation from 2.1)	<	31	950	0.64	0.25	0.71		2.4
Long term projections of public health expenditure (percent of GDP)	<	28	749	4.5	0.53	0.32		9.4
Long term projections of public pension expenditure (percent of GDP)	<	21	600	6.2	0.09	0.76		9.6
Dependency ratio	<	37	1006	36.0	0.06	0.86		4.5

Figure 1: Variables and threshold of the short-term indicator
Source : Baldacci et al, (2011)

Finally, the final fiscal stress index can be computed and compared with other countries. In our case, it will be relevant to compare the results of the Walloon

Region with the Flemish Community to see who is, regarding this indicator, the most at risk of fiscal stress.

5.3 Medium term indicator: Debt Sustainability Analysis (DSA)

This section is devoted to the methodological development of the medium-term public debt sustainability assessment performed with the DSA. The methodology explanations mainly come from the *Debt Sustainability Monitor* (European Commission, 2025) and Bouabdallah et al. (2017).

The DSA approach can be divided into two main parts: the deterministic analysis and the stochastic projections. Section 5.3.1 describes how the reference scenario is built and the main variables and assumptions required. Sections 5.3.2 and 5.3.3 explain the deterministic and stochastic approaches. Finally, Section 5.3.4 describes the risk assessment process.

5.3.1 Reference scenario

The first step of the DSA is to create a reference scenario. It evaluates the evolution of a state's debt ratio under a no-policy-change assumption over a 10-year horizon. This scenario will serve as the point of comparison for measuring the effects of the various shocks applied later in the analysis. The main variables required, as well as their assumptions about their evolution, are as follows (European Commission, 2025, pp. 47-49):

- Real GDP growth rate: short-term rates are taken from the EC's 2024 Autumn Forecast for T+1. For T+2, it is based on the actual real GDP growth of the forecasts but adjusted with the budgetary multiplier of 0.75 * the change in the structural primary balance of the forecasts. Beyond T+2, growth rates are deduced from the POWG "T+10 methodology" report, considering a closed output gap from T+5 (European Commission, 2025, p. 47).

- Inflation: the current rate of each country converges linearly to its level anticipated by the market in T+10 (European Commission, 2025, p. 47).

- Primary balance: structural primary balances before aging-related costs are assumed to remain constant at the value projected by the EC as a % of GDP in T+1 and for the entire forecast period. Aging-related costs are added according to the projections made in a dedicated report (Ageing Report from the Ageing Working Group). Property income is also added. A cyclical

component is also computed according to EC standards. The latter is equal to the product of the output gap and the budget balance semi-elasticity of the country concerned. As the output gap closes in T+5, the cyclical component is zero from T+5. One-offs and temporary measures are assumed to be equal to 0 after T+2 (European Commission, 2025, p. 47).

- Interest rates: long-term interest rates converge from their current value to the value of their forward on the market until T+10. The same reasoning is applied to short-term rates. The implicit debt rate is deduced after applying the procedure for long and short-term rates (European Commission, 2025, p. 48).
- Stock-flow adjustments: these are equal to 0 after T+2 except for certain countries for which information allows deviating from the assumption that they are equal to zero (Greece) (European Commission, 2025, p. 48).

5.3.2 Deterministic approach

The deterministic approach evaluates the impact of various shocks around the baseline scenario. It quantifies the impact of changes in macro-budgetary assumptions on the level of debt as well as on its dynamics compared to the reference scenario. There are five alternative scenarios (European Commission, 2025, p. 48).

Scenario of convergence of the primary balance towards its historical average. In this alternative scenario, the structural primary balance converges in 4 years to its average of the period 2009-2023. The primary balance in T+1 is obtained using the EC's forecasts, and convergence occurs from T+2. This scenario makes it possible, in particular, to see whether the baseline scenario is plausible given past performances (European Commission, 2025, p. 48).

Lower structural primary balance scenario. When a fiscal consolidation is planned between T and T+1, efforts are reduced by 50%. If a degradation is planned in the baseline scenario, the latter is then increased by 50%. This scenario is unfavorable by construction and is not symmetrical between all the Member States. Indeed, it depends on the initial budget trajectory of each country (European Commission, 2025, p. 49).

Unfavorable interest rate-growth differential. This scenario assesses the impact of a deterioration in the interest rate-growth differential. Therefore, the difference $r - g$ is increased by one percentage point by increasing r and decreasing g during the whole projection horizon. The scenario deteriorates the results for all Member States by construction, but not uniformly (European Commission,

2025, p. 49). Indeed, states that are already heavily indebted suffer more from a deterioration in the differential than less indebted states.

Stress on financial markets. An increase of 1 percentage point in the interest rate, only in T+1, is applied to all Member States. The rate increase is greater for countries with a debt level greater than 90% of GDP because a risk premium is also added (European Commission, 2025, p. 49). States with high debt and lower maturities are impacted more heavily by this scenario.

Deterioration of the exchange rate (European Commission, 2025, p. 49). This scenario is not relevant for countries with their debt denominated in national currency, which is the case for Belgium and its federated entities (NBB).

Note that the first two alternative scenarios are budgetary policy shocks, while the next three scenarios are called "stress tests", which simulate unexpected market changes.

Once the reference scenario and the alternative scenarios have been carried out, an analysis of the results based on the following three criteria and thresholds is performed to assign an overall risk level of non-sustainability.

	Criterion	Threshold
Deterministic projections	Debt level in 2035	High: above 90% of GDP Medium: between 60% and 90% of GDP Low: below 60% of GDP
	Debt trajectory (debt peak year)	High: peak year between T+7 (2031) and end of projections (2035), or still increasing by end of projections Medium: peak year between T+3 (2027) and T+6 (2030) Low: peak year within the T+2 forecast horizon (2024-2026)
	Fiscal consolidation space (1 - percentile rank of average SPB in 2025-2035)	High: up to 25% Medium: between 25% and 50% Low: above 50%

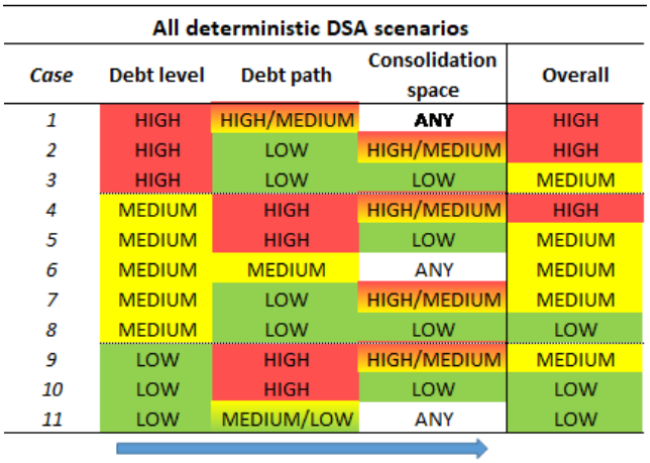
Figure 2: Criteria and thresholds of the deterministic approach
Source: European Commission, 2025, p.235

The first criterion is the level of debt as a percentage of GDP at the end of the projection. The second analyzes the dynamics of the debt ratio through the year in which the maximum level is reached during the projection period. Finally, the third criterion is the space for consolidation, which needs more explanation.

The fiscal consolidation space tells us how plausible or implausible the proposed scenario is, given past structural primary balances. If the projected average structural primary balance is in the 1st quartile of past structural primary balances, this means that the projected path is implausible. Consequently, a country that has

recorded numerous budget surpluses in the past and is recording deficits for the projection will not see its situation deteriorate, as it benefits from consolidation margins based on past experience. Conversely, a country with a large projected fiscal consolidation but a history of deficits will see its projection as unlikely, leading to confirmation of high risk (European Commission, 2025, p.235).

Once the three criteria have been assessed, a decision tree (Figure 3) is used to assign an overall risk to each scenario. It can be seen that the third criterion comes after the two previous ones, when the latter indicate two different levels of risk. It is therefore used to conclude on the final risk of the deterministic projections (European Commission, 2025, p.235).



All deterministic DSA scenarios				
Case	Debt level	Debt path	Consolidation space	Overall
1	HIGH	HIGH/MEDIUM	**ANY**	HIGH
2	HIGH	LOW	HIGH/MEDIUM	HIGH
3	HIGH	LOW	LOW	MEDIUM
4	MEDIUM	HIGH	HIGH/MEDIUM	HIGH
5	MEDIUM	HIGH	LOW	MEDIUM
6	MEDIUM	MEDIUM	ANY	MEDIUM
7	MEDIUM	LOW	HIGH/MEDIUM	MEDIUM
8	MEDIUM	LOW	LOW	LOW
9	LOW	HIGH	HIGH/MEDIUM	MEDIUM
10	LOW	HIGH	LOW	LOW
11	LOW	MEDIUM/LOW	ANY	LOW

Figure 3: Decision Tree for the deterministic approach
Source: European Commission, 2025, p.235

5.3.3 Stochastic Approach

Unlike the deterministic approach, the stochastic approach simulates many random shocks based on historical volatility. 10,000 random shocks are simulated, accounting for country-specific characteristics and variable correlations. Thousands of debt trajectories are plotted in a fan chart, displaying only the values between the 1st and 9th deciles to exclude extreme scenarios (European Commission, 2025, p.39-40).

The shocks are applied to the variables of the debt dynamics equation (19) namely: short- and long-term interest rates that constitute the implicit interest rate (r_t), nominal GDP growth rate (g_t) and the primary balance (pb_t) (European Commission, 2025, p. 255).

Quarterly data from Q1 2000 to Q3 2024 are used. Extreme values below the 5th percentile or above the 95th percentile are replaced with the nearest bound. The shocks series are obtained by taking the first differences of the quarterly data (European Commission, 2025, p. 256):

$$\epsilon_t^z = z_t - z_{t-1}$$

Once the series of shocks are built, the variance-covariance matrix is calculated. A Monte Carlo simulation is used to generate 10,000 random vectors with a joint normal distribution with mean equal to 0 and the same variance-covariance matrix as the historical one (European Commission, 2025, p. 256).

Once the quarterly shocks are generated, they are aggregated into annual shocks.

For non-persistent variables (nominal GDP growth rate, primary balance, short-term interest rate):

$$\epsilon_t^z = \sum_{q=1}^{4} \epsilon_q^z$$

For persistent long-term interest rate shocks in year $t = 1$:

$$\epsilon_1^{iLT} = \frac{1}{T} \sum_{q=1}^{4} \epsilon_q^{iLT}$$

For subsequent years $t > 1$:

$$\epsilon_t^{iLT} = \frac{t}{T} \sum_{q=-(t-1)\times 4}^{4} \epsilon_q^{iLT}$$

The annual shocks are then applied to the reference scenario's values.

Like in the deterministic approach, criteria and thresholds are used to assess the final risk of the stochastic approach. The first criterion is the probability that the level of debt will increase over the projection period. The second criterion assesses the uncertainty associated with the projection by measuring the size of the cones, i.e., the gap between the 1st and 9th decile of the stochastic projections. The size of the cone is then compared with those of the other States. The thresholds to apply to the criteria are summarized in Figure 4.

Stochastic projections			
	Probability of debt not stabilising over the next 5 years, i.e. of debt ratio in 2029 exceeding the initial debt ratio	Initial debt ratio ≥ 90%	**High:** if probability > 30% **Medium:** if 0 < probability ≤ 30% **Low:** if probability = 0
		60 % ≤ initial debt ratio < 90%	**High:** if probability > 60% **Medium:** if 30% < probability ≤ 60% **Low:** if probability ≤ 30%
		Initial debt ratio < 60%	**Medium:** if probability > 70% **Low:** if probability ≤ 70%
	Size of macroeconomic uncertainty (diff. btw 10th and 90th percentiles of the distribution of debt paths)	**High:** the third of the countries with highest dispersion **Medium:** the third of the countries with intermediate dispersion **Low:** the third of the countries with lowest dispersion	

Figure 4: Criteria and thresholds of the stochastic approach
Source: European Commission, 2025, p.235

Again, a decision tree assigns a global sustainability risk level based on these two indicators (European Commission, 2025, p.235).

Probability of debt not stabilising	Size of uncertainty	Overall
HIGH	ANY	HIGH
MEDIUM	HIGH	MEDIUM
MEDIUM	MEDIUM	MEDIUM
MEDIUM	LOW	LOW
LOW	HIGH	MEDIUM
LOW	MEDIUM	LOW
LOW	LOW	LOW

Figure 5: Decision Tree for the deterministic approach
Source: European Commission, 2025, p.235

5.3.4 Final Classification

Once all scenarios (baseline, deterministic, and stochastic) have been assigned a risk level, the overall risk of the DSA is determined through a final decision tree.

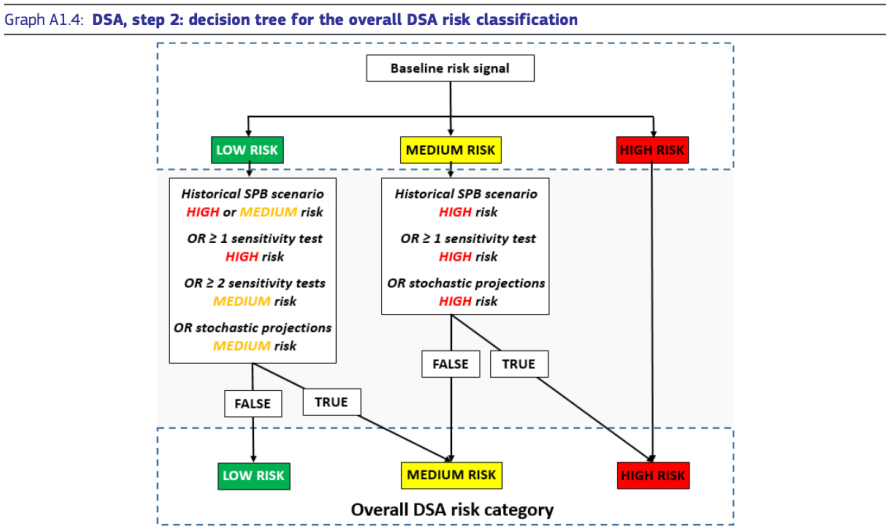


Figure 6: Decision Tree for overall risk of the DSA
Source: European Commission, 2025, p.236

If the baseline scenario already shows a high risk, the final DSA risk is directly classified as high. If the baseline indicates a low or moderate risk, the results of the deterministic and stochastic scenarios are used to refine the classification.

5.4 Long term indicators[5]

The long-term debt sustainability analysis focuses on the *solvency* part of the definition of public debt sustainability. Behind solvency lies the concept of the *intertemporal budget constraint*.

This concept is the application of the budget constraint to the government. However, the government cannot be treated like other economic agents due to its intrinsic characteristics already mentioned in section 4.2.

Let's consider the simplest setup, where the debt accumulated in the past must be reimbursed in the next period:

$$PE_t + B_{t-1} + r_t B_{t-1} = T_t + B_t \tag{21}$$

[5]The equations in this section are mainly taken from (Debrun et al., s.d.)

Where

$$B_t = B_{t-1} + r_t B_{t-1} - PB_t \tag{22}$$

Each year, government primary spending PE_t plus the reimbursement of the debt contracted last year increased by the interest payment $(1 + r_t) \times B_{t-1}$ must be equal to the amount of tax raised T_t plus the new debt B_t.

When equation (22) is expressed as a percentage of GDP, the basic form of debt accumulation is obtained.

$$b_t = \frac{1 + r_t}{1 + g_t} \times b_{t-1} - pb_t \tag{23}$$

Let's assume that

$$R_t = \frac{1 + r_t}{1 + g_t} \tag{24}$$

Therefore, equation (24) can be plugged into the debt accumulation equation and extended for the next period:

$$b_{t+1} = R_{t+1} \times b_t - pb_{t+1} \tag{25}$$

Let's isolate b_t from equation (25)

$$b_t = \frac{1}{R_{t+1}} b_{t+1} + \frac{1}{R_{t+1}} pb_{t+1} \tag{26}$$

Thanks to equation (26), b_t is now expressed as the discounted value of b_{t+1} and pb_{t+1}.

Therefore, equation (26) can be extended for the next period:

$$b_{t+1} = \frac{1}{R_{t+2}} b_{t+2} + \frac{1}{R_{t+2}} pb_{t+2} \tag{27}$$

Equation (27) is plugged into equation (26) such that:

$$b_t = \frac{1}{R_{t+1}} \frac{1}{R_{t+2}} b_{t+2} + \frac{1}{R_{t+1}} pb_{t+1} + \frac{1}{R_{t+1}} \frac{1}{R_{t+2}} pb_{t+2} \tag{28}$$

Let's extend it until period T:

$$b_t = \prod_{j=1}^{T} \frac{1}{R_{t+j}} b_{t+T} + \sum_{j=1}^{T} \prod_{k=1}^{j} \frac{1}{R_{t+k}} pb_{t+j} \tag{29}$$

Then, when T tends to an infinite horizon of time:

$$b_t = \lim_{T\to\infty} \prod_{j=1}^{T} \frac{1}{R_{t+j}} b_{t+T} + \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}} pb_{t+j} \tag{30}$$

The main idea behind this concept is that the present value of future primary balance must cover today's debt. Therefore, the solvency condition lies in this condition (No Ponzi Game condition or Transversality condition):

$$\lim_{T\to\infty} \prod_{j=1}^{T} \frac{1}{R_{t+j}} b_{t+T} = 0 \tag{31}$$

And thus that :

$$b_t = \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}} pb_{t+j} \tag{32}$$

The European Commission has therefore developed 2 indicators for the long-term sustainability of public debt.

5.4.1 S1 Indicator

The $S1$ indicator is defined as follows: "the immediate and permanent one-off improvement in the structural primary balance that is required to bring the debt ratio to 60% of GDP by year t_1 (2070)" (European Commission, 2025, p. 262). $S1$ can therefore be included in the primary balance equation:

$$PB_t = SPB_t + S1 - \Delta A_t + \Delta PI_t + CC_t \tag{33}$$

Where:

- PB_t is the primary balance
- SPB_t is the cyclically adjusted primary balance
- ΔA_t is the change in age-related costs relative to the base year.
- ΔPI_t is the change in property income relative to the base year.
- CC_t is the cyclical component of the general government balance

To find the value of $S1$, this equation is plugged into the intertemporal budget constraint with 2070 as the horizon:

$$b_t = \prod_{j=1}^{T} \frac{1}{R_{t+j}} b_{t+T} + \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(SPB + S1 - \Delta A + \Delta PI + CC \right)_{t+j} \right)$$

$$= \prod_{j=1}^{T} \frac{1}{R_{t+j}} b_{t+T} + \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(SPB_{t+j} + S1 \right) \right) - \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} (\Delta A_{t+j}) \right)$$
$$+ \sum_{j=1}^{T} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} (\Delta PI + CC)_{t+j} \right) \tag{34}$$

For mathematical ease, let's define the discount factor as:

$$\prod_{j=1}^{T} \frac{1}{R_{t+j}} = \beta_T \tag{35}$$

Rewriting equation (34), including the simplification posed in equation (35):

$$b_t = \beta_T b_{t+T} + \sum_{j=1}^{T} \beta_j \left(SPB_{t+j} + S1 \right) - \sum_{j=1}^{T} \beta_j (\Delta A_{t+j}) + \sum_{j=1}^{T} \beta_j (\Delta PI + CC)_{t+j} \tag{36}$$

Where $\beta_T d_{t+T}$ is the present value of future debt:

$$\beta_T b_{t+T} = b_{t+T}^{PV}$$

$S1$ is a constant; therefore, it can be removed from the sum:

$$b_t = b_{t+T}^{PV} + \sum_{j=1}^{T} \beta_j SPB_{t+j} + S1 \sum_{j=1}^{T} \beta_j - \sum_{j=1}^{T} \beta_j (\Delta A_{t+j}) + \sum_{j=1}^{T} \beta_j (\Delta PI + CC)_{t+j} \tag{37}$$

Finally, $S1$ can be isolated:

$$S1 = \frac{b_t - b_{t+T}^{PV}}{\sum_{j=1}^{T} \beta_j} - \frac{\sum_{j=1}^{T} \beta_j SPB_{t+j}}{\sum_{j=1}^{T} \beta_j} + \frac{\sum_{j=1}^{T} \beta_j (\Delta A_{t+j})}{\sum_{j=1}^{T} \beta_j} - \frac{\sum_{j=1}^{T} \beta_j (\Delta PI + CC)_{t+j}}{\sum_{j=1}^{T} \beta_j} \tag{38}$$

5.4.2 S2 indicator

The second indicator $S2$ is defined as: *"the immediate and permanent one-off fiscal adjustment that would ensure that the IBC is met. This indicator is appropriate for assessing long-term fiscal sustainability in the face of ageing costs"* (European Commission, 2025, p.262-263).

Let's take back the equation of the intertemporal budget constraint and conditions expressed above. The amount of debt must be equal to the present discounted value of future primary balances:

$$b_t = \sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} pb_{t+j} \right) \tag{39}$$

Given that $S2$ is a primary balance adjustment, it can be included in the equation similarly to what was done with $S1$:

$$PB_t = SPB_t + S2 - \Delta A_t + \Delta PI_t + CC_t \tag{40}$$

This expression of the primary balance is plugged into the intertemporal budget constraint:

$$b_t = \sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(SPB_{t+j} + S2 - \Delta A_{t+j} + \Delta PI_{t+j} + CC_{t+j} \right) \right) \tag{41}$$

The discount factor is defined as:

$$\prod_{k=1}^{j} \frac{1}{R_{t+k}} = \beta_j$$

Rewriting the equation:

$$b_t = S2 \sum_{j=1}^{\infty} \beta_j + \sum_{j=1}^{\infty} \beta_j \left(SPB_{t+j} - \Delta A_{t+j} + \Delta PI_{t+j} + CC_{t+j} \right) \tag{42}$$

Finally, solving for $S2$:

$$S2 = \frac{b_t}{\sum_{j=1}^{\infty} \beta_j} - \frac{\sum_{j=1}^{\infty} \beta_j \left(SPB_{t+j} - \Delta A_{t+j} + \Delta PI_{t+j} + CC_{t+j} \right)}{\sum_{j=1}^{\infty} \beta_j} \tag{43}$$

6 Applications to the Walloon Region

6.1 Institutional Context

The Walloon Region is part of federal Belgium, which was officially born in 1993 after numerous institutional reforms. The following paragraphs summarize the creation of the federal state of Belgium. This gives the reader a global picture of the institutional context in which the sustainability analysis takes place.

The first revision of the constitution took place in 1970 with the creation of three cultural communities which were in charge of cultural affairs *(The First And Second State Reforms | Belgium.be,* s. d.).

The second reform took place in 1980 and continued what had started in 1970. The cultural communities became "Communities", still in charge of cultural affairs but also of things related to individuals (health and social services). The Flemish Community, French Community, and German-speaking Community were created. Two Regions were also created: the Flemish Region and the Walloon Region, mainly for more economic autonomy. The Flemish Community and the Flemish Region directly merged to become only one entity *(The First And Second State Reforms | Belgium.be,* s. d.).

A third reform occurred in 1988-1989, creating the Brussels-Capital Region. Even more power has been given to the different entities. Belgium became a true federal state in 1993 with the fourth reform with the change into the Belgian Constitution *(The Third And Fourth State Reforms | Belgium.be,* s.d.).

In 2001, the fifth reform transfers even more autonomy to Communities and Regions *(The Fifth State Reform | Belgium.be,*, s.d.) but does not radically change the Belgian state structure.

The sixth reform, the last to date, is very important for Belgium. The part of interest in the context of this paper took place in 2014. Many new powers have been transferred to Communities and Regions; therefore, the financing law has been reviewed to ensure the correct execution of the transferred powers. More than 20 billion euros went from the federal level to the entities, and the fiscal autonomy of them is consolidated by 12 billion euros *(La Sixième Réforme de L'Etat | Belgium.be,* s.d.). In addition to financing the newly transferred powers, the 2014 reform pursued two main objectives:

- Strengthening the fiscal autonomy of the regions so that their own revenues

increase relatively compared to the volume of grants collected via additional personal income tax (PIT) contributions (Clerbois et al., 2014).

- Participation of federated entities in fiscal consolidation and in expenses related to the cost of aging. In practice, the federal government has reduced its burden of grants by combining the reduction of the link coefficient to the real growth rate of some of them and introducing a threshold of +2.25% real GDP growth (Clerbois et al., 2014).

6.2 Evolution of Belgium and Walloon Region Debt

6.2.1 Belgium

The debt was around 300 billion euros between 2000 and 2007, but has increased significantly since the global financial crisis. The impact of COVID-19 is also clear, with an important increase in 2020. In 2024, the aggregate Belgian debt is around 643 billion euros (NBB).

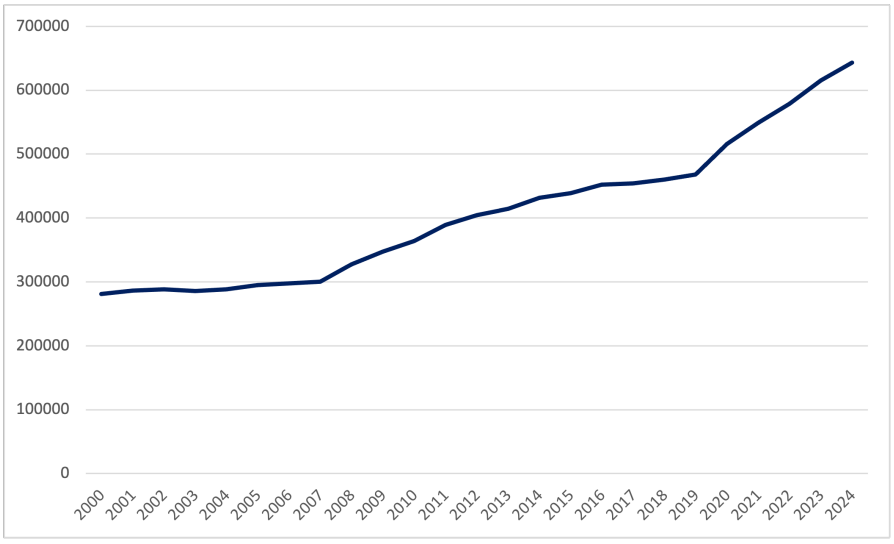


Figure 7: Nominal Belgium's public debt in millions euros
Source: NBB

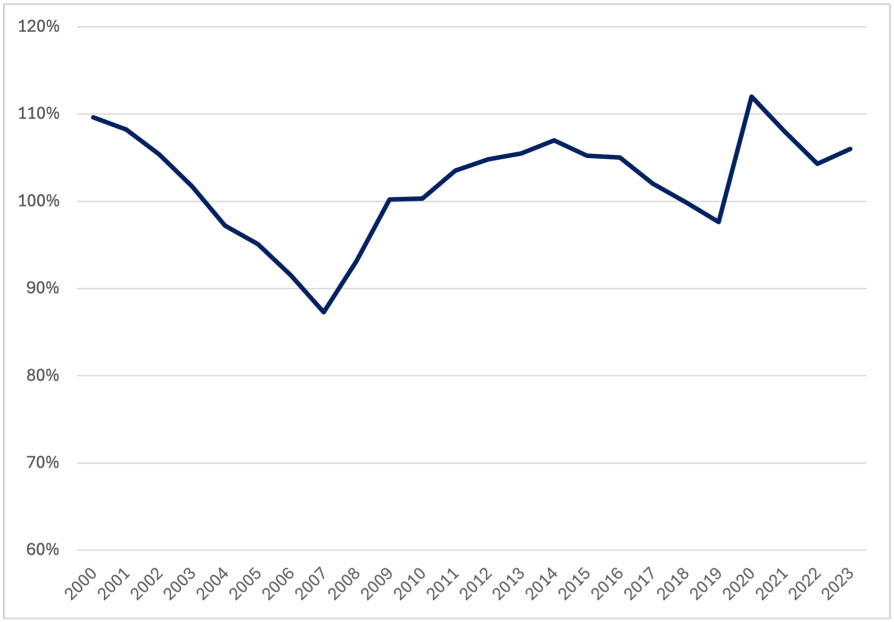


Figure 8: Belgium's public debt to GDP ratio
Source: NBB

However, as mentioned before, a country's public debt is usually expressed as a percentage of GDP because it constitutes the resources on which the government can levy taxes to finance its debt.

The debt-to-GDP ratio was clearly decreasing between 2000 and 2007 from around 110% to less than 90% in 2007. Here again, the global financial crisis is the cause of the increasing debt ratio. The decreasing path that started in 2015 stopped again because of a new crisis: COVID-19. The ratio reached 112% of GDP in 2020. The last data point available is for 2023 and indicates a debt ratio of 106% of GDP (NBB).

6.2.2 The Walloon Region

Given the federal dimension of Belgium, the aggregated debt can be decomposed between power levels. The share of the national debt attributed to communities and regions is around 115 billion euros, around 18% of the total amount. More precisely, the most indebted federated entity in nominal terms is the Flemish Community, with a debt of 47 billion euros, followed by the Walloon Region with 39 billion euros (NBB).

One key element that makes this master thesis relevant is that the share of the aggregate Belgian debt contracted by Regions and Communities is increasing,

mainly since 2008. It emphasizes the increasing importance of the public finance of entities to ensure the sustainability of Belgium's public debt as a whole.

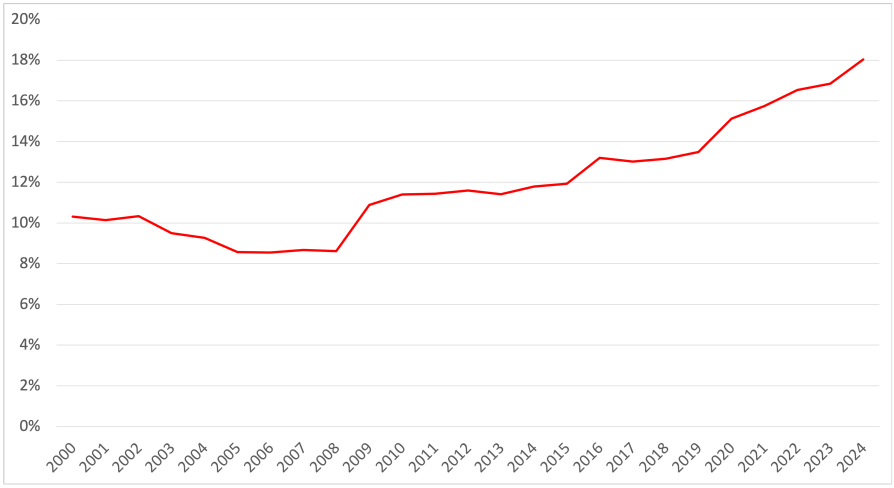


Figure 9: Share of Belgian debt contracted by Regions and Communities
Source: Own calculations, NBB

Regarding the Walloon Region, the debt amount also follows an increasing trend. Indeed, the nominal amount has more than doubled between 2010, when the debt was around 15 billion euros, and 2024, when the debt is around 39 billion euros. This important increase raises the question of the sustainability of the debt of Regions and Communities, with a special focus on the Walloon Region in this master's thesis.

The Walloon Region's debt can be divided into two main categories: direct and indirect. The direct debt is the one that is directly contracted by public authorities, while the indirect one is contracted for the benefit of certain public institutions of public services (CRISP, 2018). In 2023, the direct debt represented 25.3 billion euros; the indirect debt was equal to 11.195 billion euros and is projected to remain stable at this level (Cour des comptes, 2024). The consolidated gross debt is considered in this paper, and all variations are due to changes in direct debt.

Moreover, the risk of Belgium's public debt's non-sustainability is high (European Commission, 2025); therefore, the sustainability of each entity that comprises Belgium must be ensured in order to achieve the national objective of sustainability.

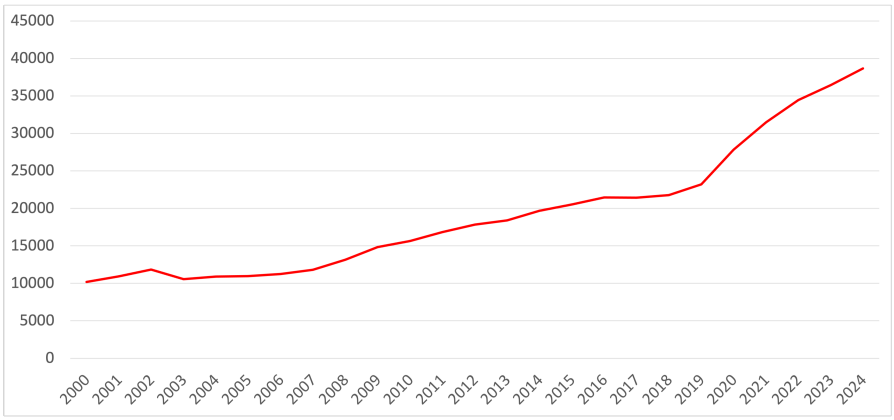


Figure 10: Nominal Walloon's debt in millions euros
Source: NBB

6.3 Main adaptations needed

These sections explain the main adaptations needed to perform the analysis. It mainly comes from the institutional context of Belgium, which implies that an entity cannot be considered as a country. The denominator problem is explained in Section 6.3.1. The financing law of entities (LSF) is addressed in Section 6.3.2. Section 6.3.3 contains a few words about the expenses that entities have to incur to discuss the adaptations required for some built indicators.

6.3.1 Denominator

As mentioned, the country's debt is usually expressed as a percentage of GDP. Indeed, it allows for the comparison between countries, and GDP is a good measure of the tax base on which the government can take its revenue and ensure the financing of its debt (Cornille et al., 2022).

However, regional GDP does not represent a good indicator of the tax base on which entities can raise revenue. Indeed, the revenue that entities have can be decomposed into different components. The revenue mix of each entity is exposed in Figure 11.

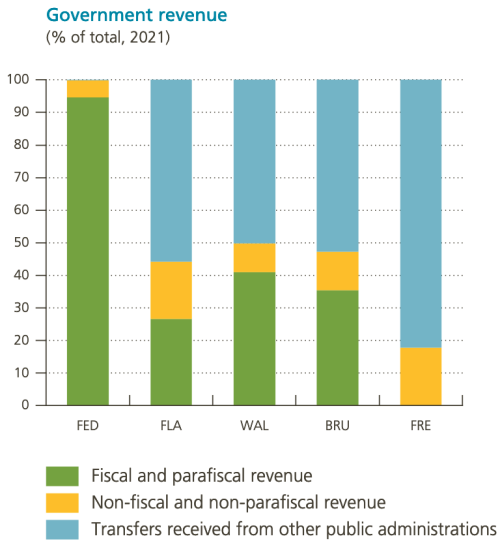


Figure 11: Revenue mix of entities
Source: Cornille et al., 2022

For the Walloon Region (WAL), the share of transfers received by other public administrations is around 40% of the total revenue; it is now clear that expressing the debt as a percentage of regional GDP does not make sense. A region's economic activity is a small determinant of the entity's revenue, given the limited fiscal resources that it can raise. This is even more true for the French Community, which collects no revenue through fiscal and para-fiscal means.

The debt is therefore expressed as the percentage of the adjusted disposable revenue (ADR). Basically, the transfers made by the entity to another are removed from public revenue because the entity cannot use this money to finance policies or debt (Cornille et al., 2022). Additional adjustments are also made to remove some amounts purely due to the rules of ESA 2010.

The following graph shows the Walloon Region's debt ratios expressed as a percentage of ADR and public revenue computed following the ESA 2010.

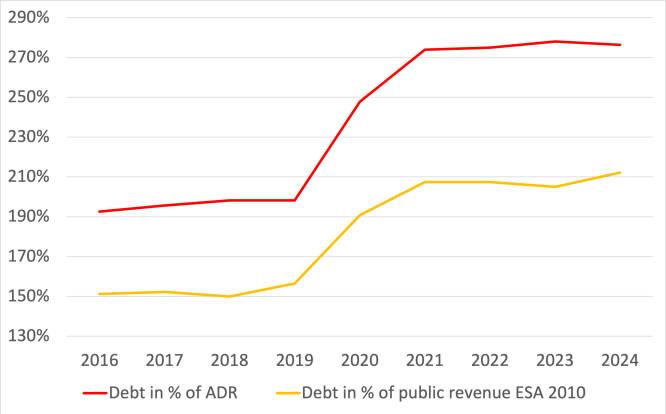


Figure 12: Walloon Region's debt ratio
Source: Own calculations

The two debt ratios mainly follow the same upward trend. The important ratio increase between 2019 and 2021 due to the COVID-19 crisis and floods is shared across these graphs. However, the difference between the two ratios is significant. In 2024, the debt expressed as a percentage of ADR is 276% while it is only 212% if expressed as a percentage of public revenue ESA 2010. Therefore, it seems relevant to use ADR to better reflect the real financing capacity of the Walloon Region.

Given that, at the national level, aggregated adjusted disposable revenue is equal to the national public revenue, and that corresponds to half of GDP (Cornille et al., 2022), all variables and thresholds expressed in percentage of GDP are doubled. All debt equations are also now expressed as a percentage of ADR.

$$\Delta b_t^{ADR} = b_{t-1}^{ADR} \times \frac{r_t - g_t^{ADR}}{1 + g_t^{ADR}} - pb_t^{ADR} + f_t^{ADR} \tag{44}$$

One of the most important elements is that g_t^{ADR} stands for the nominal growth rate of the denominator, which is now the adjusted disposable income.

6.3.2 Loi Spéciale de Financement[6]

As expressed in the previous section, an important part of the revenue that the Walloon Region receives comes from transfers from other entities. The amount received by the entity is determined by the "Loi Spéciale de Financement" (LSF). Therefore, the link between economic activity and the revenue perceived by the entity is not direct. This specificity needs to be considered because it will have an impact when shocks to Belgian GDP are implemented. Indeed, if the nominal GDP growth rate increases, it will not have the same impact if it is due to an inflationary or real GDP shock because of these linking coefficients.

The amount received has been determined by the LSF and mainly evolves according to demographic and economic variables, which are linked by a coefficient. It means that when the Belgian nominal GDP increases by one percentage point, the transfer will increase by less than one percentage point; the elasticity of transfer to nominal GDP is lower than one. An additional threshold mechanism of indexation complicates the link between Belgian nominal GDP growth and the evolution of the transfers. The different coefficients and the mechanism are detailed in Appendix 9.1.

Other components of the adjusted disposable revenue are assumed to have an elasticity equal to one. Elasticities are computed from a baseline where the real GDP growth rate is 2.25% to take the threshold mechanism into account. The average elasticities are summarized in Table 1. The ones that are used in the computation are detailed in Appendix 9.2.

Table 1: Average elasticities of transfers to the Walloon Region

	2025	2026	2027	2028	2029
Real GDP growth +1%	0.63	0.64	0.65	0.66	0.67
Real GDP growth -1%	-0.47	-0.48	-0.49	-0.50	-0.50
Inflation +1%	0.91	0.92	0.93	0.94	0.95
Inflation -1%	-0.91	-0.92	-0.93	-0.94	-0.95

Source: Private computation FPB

[6]Loi spéciale du 6 janvier 2014 portant réforme du financement des communautés et des régions, élargissement de l'autonomie fiscale des régions et financement des nouvelles compétences, *M.B.*, 31 janvier 2014

6.3.3 Social expenditures

For the short-term and long-term indicators, costs related to ageing are needed. Indeed, those indicators require projections on the costs of pensions and health expenditures. However, given Belgium's federal dimension, it is important to have more details on the weight of these expenses in the total expenses of the Walloon Region.

In 2024, social expenditures represent around 42% of the entity ADR's. The share of this type of expense is growing; it represented 37% of the ADR in 2016, five percentage points lower than in 2024. These expenditures can be split into different categories that are summarized in the following figure:

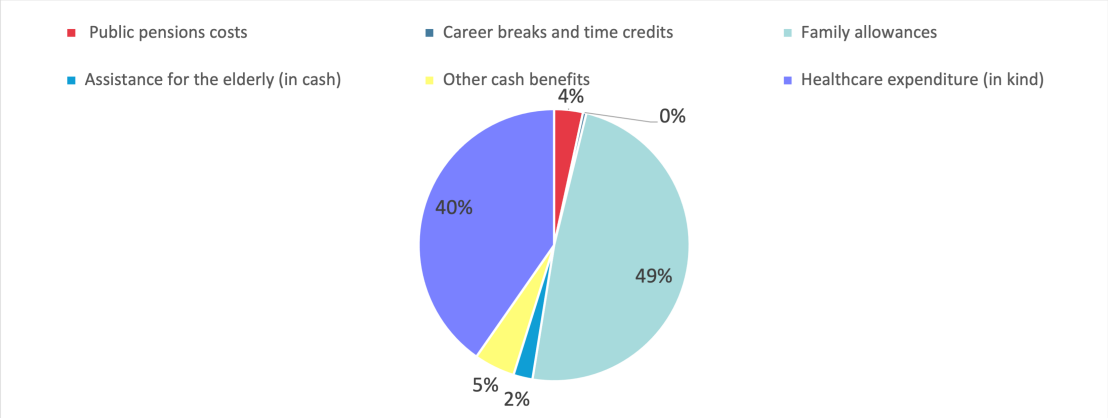


Figure 13: Breakdown of social expenditures of the Walloon Region
Source: FPB

Public pensions costs represent 4% of social expenditures of the entity; then they represent only 1.44% of the ADR. Therefore, in the next sections, the pension cost is considered negligible. Indeed, the pension system is mainly supported by the federal entity (Conseil Supérieur des Finances, 2024).

If only health expenditures are considered, only the two categories are aggregated (Assistance for the elderly and Healthcare expenditures). These expenditures represent 43% of the social expenditures and 18% of the ADR. When the costs of ageing are considered as a whole, only public pensions and other cash benefits are excluded from our projections. Therefore, these expenses represent 92% of the social expenditures; they represent 38% of the ADR. The details about how these data are used in order to build indicators are given in the dedicated sections.

7 Results

This section presents the different sustainability indicators. Section 7.1 is devoted to sustainability assessment in the short term. The methodology is adapted but mainly inspired by the indicator developed in Section 5.2. Section 7.2 develops the medium-term results obtained with the DSA tools explained in Section 5.3. Finally, Section 7.3 is dedicated to the long-term assessment.

7.1 Short-term indicator

First, the variables used by Baldacci et al. (2011) have been adapted or approximated for the Walloon Region. The variables are the following:

- $r - g$ five years average: the five years forward average of the interest-growth rate of the ADR differential. The main adaptation is using the growth rate of ADR instead of the GDP growth rate. Indeed, the debt dynamic directly depends on ADR's growth rate.
- General government gross debt (in percent of ADR).
- Primary balance (in percent of ADR): the cyclical component is unavailable at the entity level. Therefore, the primary balance is used by default.
- Gross financial needs (in percent of ADR).
- Share of short-term debt as a ratio of total debt.
- Debt held by nonresident: the national data is used because no regional data is available.
- Weighted average maturity of general government debt in years.
- Fertility rate for the Walloon Region (deviation from 2,1) of 2023 (last data available).
- Change in long-term public health expenditures (percent of ADR): given the limited data availability, the time horizon has been reduced to 5 years.
- Change in long-term public pension expenditures (percent of ADR): this variable has been ignored for the Walloon Region, and the weight of this variable has been split between the other long-term fiscal trend variables.
- Dependency ratio: number of people older than 65 years old over the population between 15 and 64 years old.

As explained in section 6.2.1, the thresholds expressed in % have been doubled, given that ARD represents around 50% of national GDP.

The threshold of "Change in public health expenditures" is supposed to be used for 30 years ahead projections. To adapt the threshold, it will be assumed that the 30-year change ahead is linear; therefore, since our horizon is 5 years, the threshold expressed as a percentage of GDP is divided by 6 and then doubled to be expressed as a percentage of ADR.

7.1.1 The Walloon Region

The results for the Walloon Region are summarized in the following table:

Variables	Safe Direction	Value	Signal	Thresholds	Weight
Basic Fiscal Variables					
r-g (avg. 5y)	<	-0.23%	0	3.60%	14.90%
Debt ratio	<	276.34%	1	144.40%	7.30%
Primary balance	>	-9.81%	1	-8.40%	9.40%
Asset and Liability Management					
Gross financing needs	<	23.55%	0	34.40%	24.60%
Share of ST debt	<	1.91%	0	9.10%	2.80%
Debt held by non-residents	<	60.79%	0	83.60%	10.10%
Weighted avg maturity	>	15.39	0	3.9	5.00%
Long Term Fiscal Trend					
Fertility rate (deviation from 2.1)	<	0.6168	0	0.64	5.60%
Change in public health expenditures (2029)	<	1.07%	0	1.50%	12.60%
Dependency ratio (2029)	<	34.48%	0	36.00%	7.70%
				INDEX	0.1670

All variables are under the threshold except two basic fiscal variables: the debt ratio and the primary balance. However, the primary balance's threshold has not been adapted to remove the structural component.

The asset and liability management variables are under their respective thresholds, indicating that the Walloon Region's debt is well managed from a financial point

of view.

The long-term fiscal trend variables are neither associated with a signal of short-term risk. The dependency ratio is lower than 36% but is projected to exceed that threshold in 2032.

The final index is 0.1670, which, taken alone, is not very informative. Therefore, the same analysis has been performed on the Flemish Community.

7.1.2 The Flemish Community

Variables	Safe direction	Value	Signal	Threshold	Weight
Basic Fiscal Variables					
r-g (avg. 5y)	<	-0.53%	0	3.60%	14.90%
Debt ratio	<	87,11%	0	144.40%	7.30%
Primary balance	>	-5.85%	0	-8.40%	9.40%
Asset and Liability Management					
Gross financing needs	<	20.80%	0	34.40%	24.60%
Share of ST debt	<	3.94%	0	9.10%	2.80%
Debt held by non residents	<	60.79%	0	83.60%	10.10%
Weighted avg maturity	>	13.16	0	3.9	5.00%
Long Term Fiscal Trend					
Fertility rate (deviation from 2.1)	<	0.6038	0	0.64	5.60%
Change in public health expenditures (2029)	<	0.65%	0	1.50%	12.60%
Dependency ratio (2029)	<	37.60%	1	36.00%	7.70%
				INDEX	0.077

All variables except the dependency ratio are under their threshold. The final index value is 0.077, which indicates that the Flemish Community has a lower short-term risk than the Walloon Region.

7.1.3 Conclusion for short-term indicator

Finally, the Walloon Region does not seem to face a high short-term risk of non-sustainability, given the low number of variables above their threshold. The debt ratio is well beyond the determined threshold; such an excess could open the

discussion concerning the weight given to this criterion, which is only 7.30%. The other variables tend to indicate a low short-term risk.

However, the final index of the Flemish Community is much lower than the one obtained for the Walloon Region, 0.077 against 0.1670, indicating that the Walloon Region faces a higher short-term risk than the Flemish Community.

One problem with that analysis is that entities do not always have the same powers, which makes the comparison between them harder. However, it seems clear that the Walloon Region has a less favorable situation than the Flemish in the short term. The evaluation through the different indicators will allow us to have a global picture regarding the sustainability of the Walloon Region.

7.2 Medium term indicator: DSA

This section is devoted to the medium-term risk assessment. The methodology is the one explained in section 5.3.

The data used come from Bureau Fédéral du Plan et al., 2024. The main macroeconomic assumptions are summarized in the following table.

Table 2: Main macroeconomic assumptions in %

	2025	2026	2027	2028	2029
Real GDP growth rate	1.4	1.4	1.3	1.4	1.3
Inflation rate (CPI)	2.0	1.8	1.8	1.8	1.8
LT interest rate (OLO 10y)	2.8	2.9	2.9	3.0	3.1
ST interest rate (Euribor 3m)	2.9	2.7	2.5	2.3	2.1
Spread direct debt Walloon Region	0.4	0.4	0.4	0.4	0.4
Spread indirect debt Walloon Region	1.4	1.4	1.4	1.4	1.4

Source: BFP et al., 2024., NBB

The time horizon is then 5 years instead of 10 years. No cyclical component is used because none is available for Belgian entities. The primary balance is used by default. It is also assumed that the primary balance, in level, is exogenous. It means that variations in the primary balance in percent of ADR are due to a denominator effect, i.e., a change in the level of ADR.

7.2.1 Reference scenario

The debt ratio was initially high in 2016, around 190% of the ADR. The impacts of COVID-19 and the flood are clear, with a very steep increase in the debt ratio reaching 274% at the end of 2021. There is stabilization between 2022 and 2024. However, the debt ratio is projected to increase in the next five years. The peak is reached in 2027 at 307% of the ADR. The ratio slightly decreases after 2027, thanks to the end of the recovery plan (Cour des comptes, 2024).

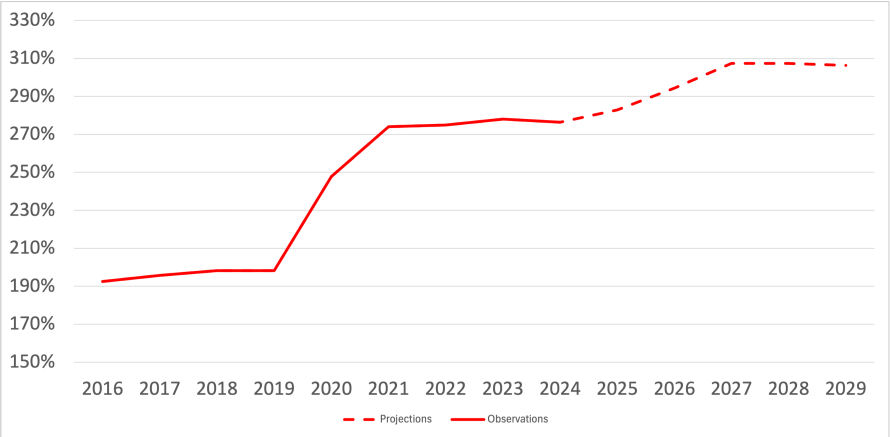


Figure 14: Reference scenario
Source: BFP et al., 2024

The variation of the debt ratio can be decomposed into the different components of the equation, which are represented in Figure 15. The primary balance played a relatively limited role between 2017 and 2019, but it became significantly negative under the impact of the COVID crisis. It therefore significantly increased the debt ratio in 2020 and 2021. The increase was more than 49 percentage points in 2020 and more than 27 percentage points in 2021. It is also clear that the exogenous factor played an important role during these years of high uncertainty.

Five-year projections indicate a gradual improvement in the primary balance, leading to it becoming positive from 2028 onward, which will be favorable for a decrease in the debt ratio. The differential between the interest rates has a fairly limited but favorable effect on the Walloon Region for 2025, 2028, and 2029.

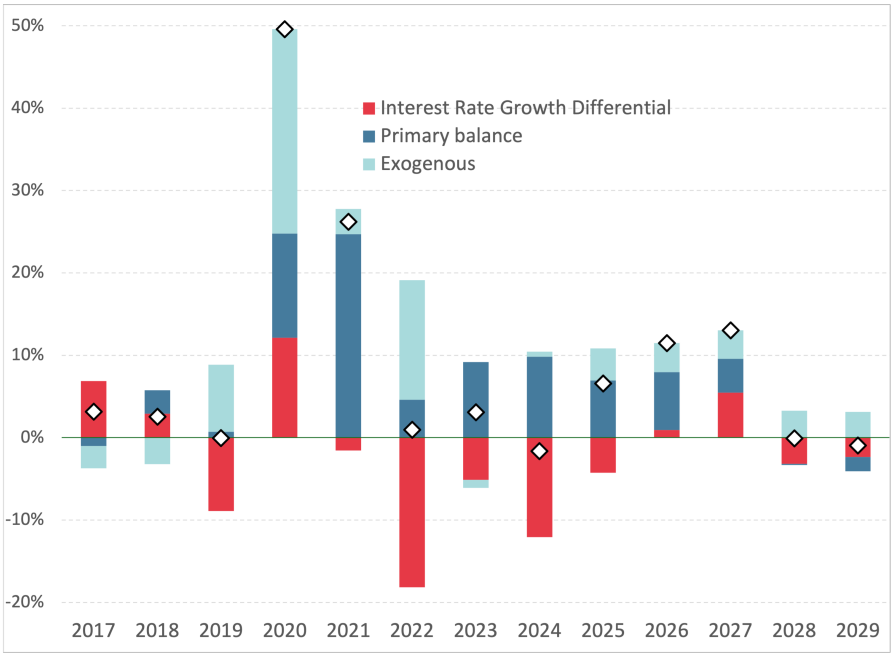


Figure 15: Debt dynamic decomposition
Source: Bureau Fédéral du Plan et al., 2024

7.2.2 Deterministic approach

The deterministic shocks have been implemented on the Walloon Region's reference scenario and make it possible to identify the risks to which the entity is exposed.

Financial Stress
The interest rates on direct and indirect debt are increased by 1 percentage point for 2025. The risk premium has not been applied.

The impact of this stress test is small, given the limited amount of debt that needs to be refinanced. At the end of the period, the debt ratio is 1.42 percentage points higher than the baseline scenario. The implicit interest rate is also slightly higher, given that the debt contracted in 2025 has been refinanced at a higher rate.

Unfavorable shock on $r - g$
In this stress test, interest rates are permanently higher by 0.5 percentage points, and the nominal growth rate of Belgian GDP is decreased by 0.5 percentage points. To take into account that ADR are differently affected by the source of the variation of nominal GDP, it is assumed that the reduction is due to a negative shock of 0.25 percentage points on real GDP growth and a negative shock of 0.25 percentage

points on inflation.

The elasticities provided in Appendix 9.2 are used and applied to the initial growth rate of the revenue from transfers. Therefore, for 2025, there is a reduction of 0.25 percentage points of inflation and real GDP growth, the "after shock" growth rate of LSF revenue is:

$$2.0161\% - (0.25 \times 0.57) - (0.25 \times 0.82) = 1.6686\%$$

For the revenues that are not from transfers, an elasticity equal to 1 is assumed.

This combined shock significantly worsens the debt ratio, bringing it to 315% at the end of the projection, 8.75 points higher than in the baseline. Regarding the debt ratio dynamics, the results show that the increase in the debt ratio is mainly explained by the denominator. Revenues increase less quickly than in the baseline scenario, which slightly weakens the effect of the interest rate-growth differential. This scenario is particularly unfavorable to the Walloon Region due to its high initial debt level.

Lower primary balance
For this scenario, a deterioration in the primary balance equivalent to 1% of the ADR is applied, corresponding to a decrease in the primary balance of €146 million in 2025. It is assumed that the GDP is not affected by this shock.

Following this shock, the primary balance only becomes positive in 2029, resulting in a higher debt ratio than in the baseline scenario. The maximum level is reached in 2028 at 311% of the ADR. The debt ratio is 5.04 percentage points higher in 2029 than in the baseline scenario. This gap equals the sum of the reduction of the primary balance, increased by a few hundredths of a percent from the increase in interest charges.

Historical primary balance: 2016-2023 excluding 2020-2021
Unsurprisingly, 2020, the year of the COVID pandemic, and 2021, marked by the July floods, are extreme values for the primary balances: -12.64% and -24.74% of the ADR. Excluding these two years from the average calculation gives us a historical average of -1.99% of the ADR. An annual improvement of 1.24% of the ADR is then applied to the balance from 2025 to reach the target primary balance of -1.99% in 2028.

The debt ratio is lower from 2025 to 2028 compared to the baseline scenario. The

effort on the primary balance for this scenario is higher than in the baseline projections. However, the situation is less favorable in 2029, as the baseline scenario projects a positive balance for 2029. The projected debt ratio in 2029 is 307%, 1.09 percentage points higher than the baseline.

The momentum is, initially, quite positive; the sooner the adjustment is made, the more beneficial it is. However, the balance would have to continue improving for a truly positive result.

Summary deterministic projections
All deterministic scenarios lead to a worse situation than the baseline. No scenario has a stabilization of the debt ratio; the peak is systematically reached in 2029. The impact of the "Unfavorable $r - g$" is particularly significant because of the high initial level of debt.

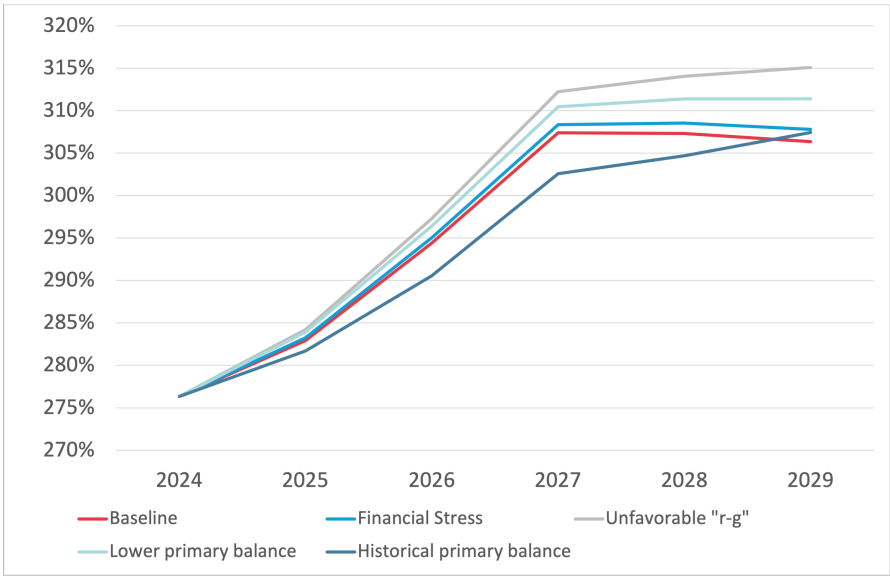


Figure 16: Walloon Region's deterministic projections
Source: Own calculations

Regarding the risk assignment, the results show that the medium-term risk of non-sustainability of public debt is high. Even if there is fiscal space according to the third criterion, the level of debt and the increasing path lead to a high risk.

Therefore, according to the final decision tree, it can be said that the global medium-term risk is high; the stochastic results will not improve or worsen the global risk assigned to the entity.

Walloon Region	Medium term - Debt Sustainability Analysis (DSA)				
	Deterministic scenarios				
	Baseline	Financial Stress	Unfavorable "r-g"	Lower primary balance	Historical primary balance
Overall Risk	High	High	High	High	High
Debt level (in % of ADR in 2029)	306,36%	307,78%	315,11%	311,39%	307,45%
Debt peak	2027	2028	2029	2029	2029
Fiscal consolidation space	64,50%	64,50%	64,70%	75,90%	67,10%

Figure 17: Deterministic decision tree
Source: Own calculation

7.2.3 Stochastic approach

Let's take the debt dynamic equation:

$$\Delta b_t^{ADR} = b_{t-1}^{ADR} \times \frac{r_t - g_t^{ADR}}{1 + g_t^{ADR}} - pb_t^{ADR} + f_t^{ADR} \tag{45}$$

Due to the institutional complexity and the lack of quarterly data at the entity level, variables have been taken at the national level. The sample period is 1991Q2-2023Q4.

The growth rate of ADR (g_t^{ADR}) is shocked through the channel of national real GDP growth (g_t) and inflation rate (π_t). Thanks to this approach, the financing characteristics of the different entities are considered. The detailed computations are in Appendix 9.4.

The implicit interest rate is shocked through short-term interest rate i_{ST} (Euribor 3m) and long-term interest rate i_{LT}(OLO 10y).

Once the shocks have been identified, as the first difference of the series, the variance-covariance matrix is computed.

The point of this historical variance-covariance matrix is to capture the country characteristics and interactions between variables.

Therefore, 10,000 vectors of shocks are generated for each variable for the 20 quarters ahead. A normal distribution with a mean equal to 0 and the same variance-covariance matrix is assumed to generate the shocks. Ultimately, the simulated quarterly shocks have the same variance-covariance matrix as the historical one. The outliers have not been treated as the EC is doing because the objective is

Table 3: Historical variance-covariance matrix

	Real GDP	Inflation	ST interest	LT interest
Real GDP	6.36e-04	8.05-e06	-3.15e-06	-4.33e-06
Inflation	8.5-e06	4.70e-05	1.29e-06	5.98e-07
ST interest	-3.15e-06	1.29e-06	1.09e-05	4.78e-06
LT interest	-4.33e-06	5.98e-07	4.78e-06	1.82e-05

to replicate the volatility of past crises in future projections.

Due to the absence of quarterly data for the primary balance, a relationship with the variation of the real GDP growth rate has been built. The detailed computations are in Appendix 9.5. The exercise replicates the variability of Belgian variables and quantifies the impact on the Walloon Region's debt ratio without fiscal policy, given the absence of the primary balance in the variance-covariance matrix.

The results are exposed on the following fan chart, where the values between the 1st and 9th deciles are represented:

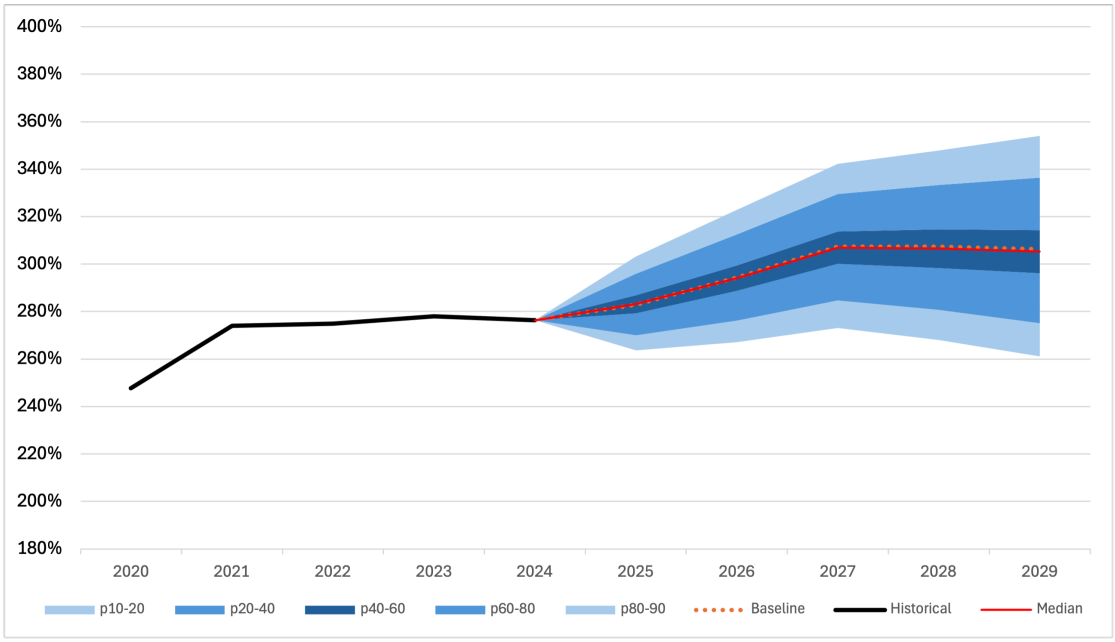


Figure 18: Walloon Region's stochastic projections
Source: Own calculation

The first criterion of the stochastic analysis indicates that the probability that the

debt in 2029 exceeds its level of 2024 is 79.30%.

The size of the uncertainty, i.e., the difference between the 9th and 1st percentile value, is 93 percentage points. For Belgium, the uncertainty obtained in the European Commission (2025) is around 30% of GDP, which would represent 60% of ADR according to our assumption. Therefore, the uncertainty around the baseline seems to be high. Such a large amount of uncertainty means that the level of the debt ratio can deviate (almost) symmetrically by up to 45 percentage points from its 2029 baseline level.

This important uncertainty is mainly explained by the initial high debt level, which creates uncertainty because small shocks on the differential interest rate-growth rate significantly impact the debt ratio. This phenomenon is illustrated by the fact that the shock on $r-g$ is the most unfavorable in the deterministic tests.

The risk associated with the stochastic projections, only analyzed through the first criterion, is high.

7.2.4 Conclusion for medium-term indicator

Finally, the global risk can now be assessed for the Walloon Region. All the results from the medium-term analysis are summarized in Figure 19:

Global Risk	Walloon Region	Medium term - Debt Sustainability Analysis (DSA)					
		Deterministic scenarios					Stochastic projections
		Baseline	Financial Stress	Unfavorable "r-g"	Lower primary balance	Historical primary balance	
HIGH	Overall Risk	High	High	High	High	High	High
	Debt level (in % of ADR in 2029)	306,36%	307,78%	315,11%	311,39%	307,45%	
	Debt peak	2027	2028	2029	2029	2029	
	Fiscal consolidation space	64,50%	64,50%	64,70%	75,90%	67,10%	
	Probability that the debt ratio > in 2029 than in 2024						79,30%
	Difference between the 9th and the 1st decile						92,91%

Figure 19: Final classification tree of the DSA
Source: Own calculation

The medium-term risk of non-sustainability of the Walloon Region's public debt is high according to the DSA final classification. The result was, however, known

since the beginning, given the high risk associated with the baseline.

The deterministic projections emphasized that the entity is particularly vulnerable to an adverse shock on $r-g$ due to its initial high level of debt. The main conclusion is that the Walloon region is facing many risks regarding its debt level and dynamics.

The stochastic projections do not provide additional information regarding the risk assessment but emphasize the uncertainty around the baseline. Here again, the high risk classification is mainly explained by the high initial level that makes small shocks on the interest rate and/or growth rate even more important.

This analysis is alarming regarding the medium-term sustainability of the Walloon Region's public debt. It must push decision-makers to action to, at least, stabilize the debt ratio in the medium term.

7.3 Long-term indicators

The section is dedicated to the two long-term indicators that are, here again, adapted given the data availability and the relevance of the exercise.

In order to carry out this exercise, it is necessary to extend various data series beyond 2029. However, no ADR projections, the denominator of the debt ratio, are available. However, projections of LSF revenues are available, making it possible to extend the reference scenario to 2034 with relatively reasonable assumptions.

Regarding other variables required, additional assumptions are necessary to perform the analysis, as the FPB does not provide detailed forecasts for the Walloon Region's public finance components beyond 2029. Therefore, the additional assumptions are the following:

- Inflation rate is maintained at 2% after 2029.
- Real GDP growth rate is maintained at its 2029 level for the period 2030-2034.
- Interest rates remain at their 2029 level.
- The primary balance is maintained at its 2029 level as a percentage of ADR.
- Revenues coming from transfers are provided thanks to the forecasting model of the FPB.
- Other revenues are following inflation and the real GDP growth rate.

- The stock flow adjustments are set to 0 due to the lack of information after 2029.

With those assumptions, the new reference scenario is the following:

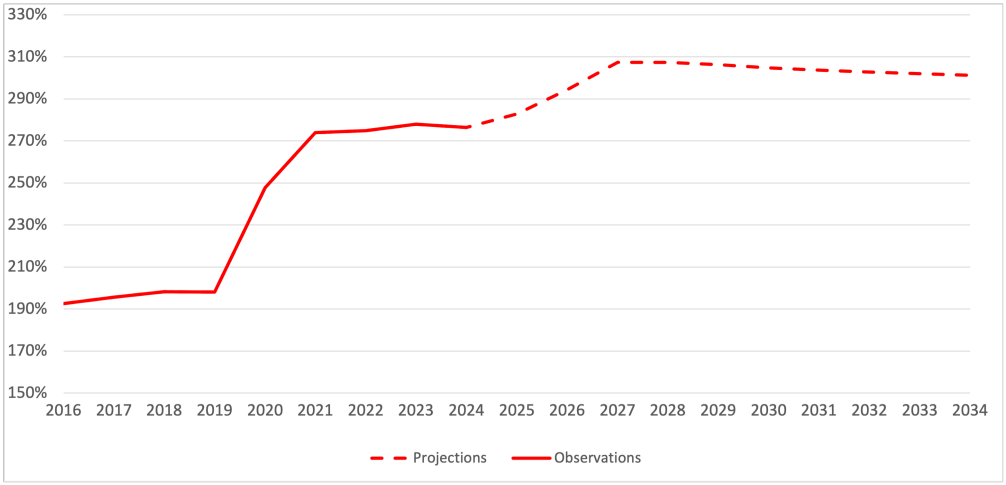


Figure 20: Reference Scenario, 2016-2034
Source: BFP et al., 2024; own calculations

The debt ratio slightly declines during the projection horizon. Indeed, the primary balance is kept at 1.73% which is responsible for the reduction of the debt ratio. In 2034, the debt ratio is projected to be equal to 301% of ADR.

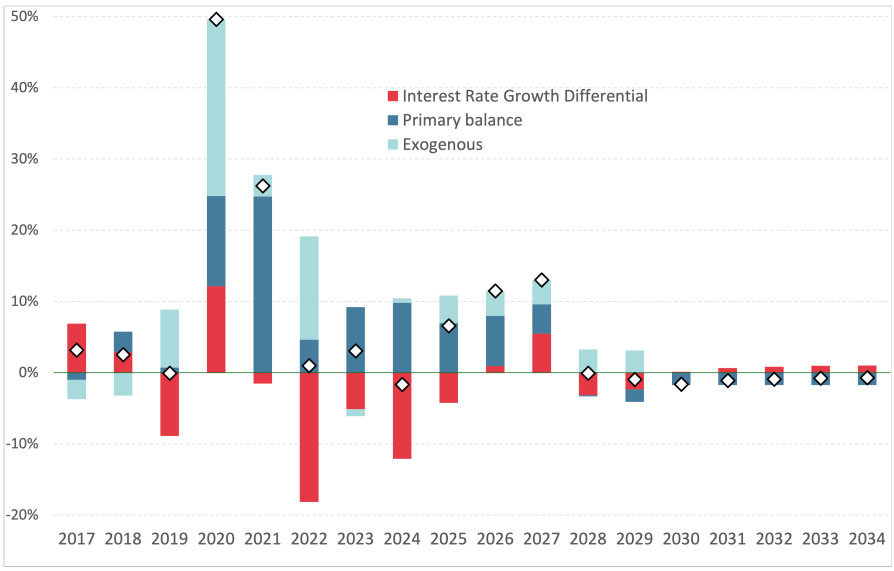


Figure 21: Debt dynamic decomposition
Source: BFP et al., 2024, own calculation

Thanks to the debt decomposition, it can be shown that the interest rate-growth differential is projected to be unfavorable from 2031 to 2034. However, the primary balance is sufficient to decrease the debt ratio.

7.3.1 S1

As mentioned in section 5.4.1, the objective is to find the permanent improvement in the structural primary balance required to bring the debt ratio to 60% of GDP by the year 2070 (European Commission, 2025, p. 262). Let's reconsider equation (38).

$$S1 = \frac{b_t - b_{t+T}^{PV}}{\sum_{j=1}^{T} \beta_j} - \frac{\sum_{j=1}^{T} \beta_j SPB_{t+j}}{\sum_{j=1}^{T} \beta_j} + \frac{\sum_{j=1}^{T} \beta_j (\Delta A_{t+j})}{\sum_{j=1}^{T} \beta_j} - \frac{\sum_{j=1}^{T} \beta_j (\Delta PI + CC)_{t+j}}{\sum_{j=1}^{T} \beta_j}$$

As mentioned earlier, no cyclical component is available for the entity level; therefore, the primary balance of the reference scenario is used by default. It means that $SPB_t = \bar{PB}_t$.

Potential property income is not precisely detailed (Cour des Comptes, 2024); this component is therefore set to zero ($DeltaPI_t = 0$).

Regarding ageing costs, they are included in the primary balance since the beginning of the exercise, contrary to the European Commission's methodology. Therefore, they are included in the primary balance and do not need an additional term. The major part of ageing costs is supported by the Federal level, called "Entity I" (Conseil Supérieur des Finances, 2024). In addition, almost two-thirds of the aging-related costs of "Entity II" (federated entities) are compensated by the decrease in family benefits (Conseil Supérieur des Finances, 2024). Therefore, the projected increase in those costs is very limited for entities. Adding this component would not provide additional relevant information.

Information about stock flow adjustments for years 2025 to 2029 is available. Therefore, they are subtracted from the primary balance. Indeed, they increase the debt if positive and decrease the debt if negative. For the period considered, they are always positive and deteriorate the debt situation; they increase the effort that has to be made. Beyond 2029, it is assumed that they are equal to 0. The final equation used is the following:

$$S1 = \frac{b_t - b_{t+T}^{PV}}{\sum_{j=1}^{T} \beta_j} - \frac{\sum_{j=1}^{T} \beta_j (\bar{PB}_{t+j} - f_{t+j})}{\sum_{j=1}^{T} \beta_j} \tag{46}$$

Due to the shorter time adjustment, 120% of ADR, equivalent to 60% of GDP according to the adapted thresholds, seems implausible. Therefore, the debt objective is set to 200% of ADR, corresponding to the pre-crisis debt level.

$S1$ is equal to 10.06% of the ADR. $S1$ is equal to 10.06% of ADR. This means that the Walloon Region must increase its primary balances over the next 10 years by 10.06 percentage points, bringing the debt ratio to 200% of ADR in 2034. The debt path obtained is the following:

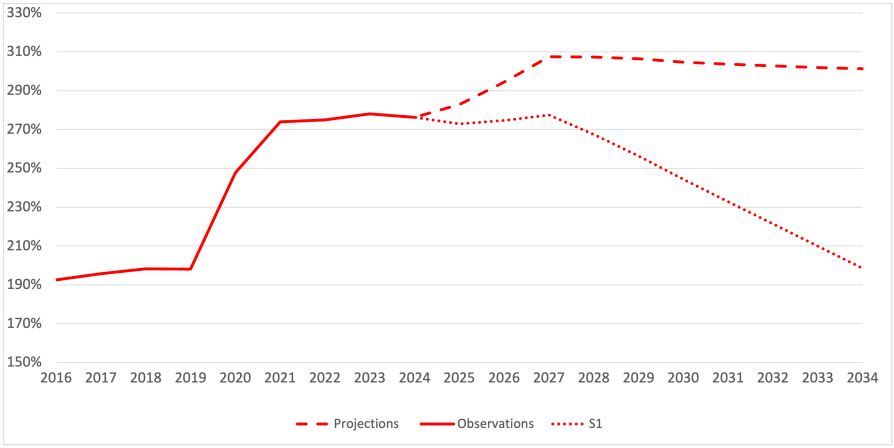


Figure 22: Debt projections
Source: BFP et al., 2024; own calculations

Given the magnitude of the adjustment needed to reach a debt ratio of 200%, this 10-year target seems impossible and unrealistic. The result further highlights the scale of the challenge facing the Walloon Region.

If the objective of 250% of debt ratio is targeted for 2034, $S1$ equals 5.09%. This adjustment is still very significant. Given the results achieved, an ambitious reduction in the debt ratio seems an almost impossible challenge in this macroeconomic context. The necessary fiscal consolidation, whether to reach 200% or 250%, has never been observed in the past.

Moreover, it can be understood that bringing the debt level below the high-risk threshold is a challenge that will require more than 10 years and will require a massive fiscal consolidation. Therefore, fiscal consolidation will be long, an adjustment lasting beyond 10 years, and challenging to achieve.

7.3.2 S2

Given the longer-term dimension of the $S2$ indicator, it needs even more assumptions. Without sufficient data on the evolution of ADR and the many assumptions required, this exercise is more of a theoretical exploration than a practical application.

The spirit of this exercise is also in line with that carried out by the European Commission. It is still inspired by the methodology used in the report: Commission externe de la dette et des finances publiques. (2021). *La soutenabilité de la dette de la Région Wallonne.*

Let's take back equation (41) and, like in $S1$, the SPB is replaced by the primary balance $\bar{PB}$; ΔPI, CC, ageing costs are assumed to be equal to 0, and stock-flow adjustments are subtracted from the primary balance but set to 0 beyond 2029:

$$b_t = \sum_{j=1}^{\infty} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(\bar{PB}_{t+j} + S2 - f_{t+j} \right) \right)$$

The projection horizon can be divided into two parts: the first one going until 2034, for which data are more detailed, and after 2034, for which new assumptions are required:

- The primary balance is constant at its 2034 level.
- The growth rate of the ADR is constant and equal to the 2025-2034 average.
- The implicit interest rate is equal to the average of the Belgian projected implicit interest rate until 2070 (Conseil Supérieur des Finances, 2024).
- Stock-flow adjustments are equal to zero beyond 2029.

It means that all variables after 2034 are constant, which will be useful for mathematical computation.

$$b_t = \sum_{j=1}^{N} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(PB_{t+j} + S2 - f_{t+j} \right) \right) + \sum_{i=N+1}^{\infty} \left(\prod_{k=1}^{i} \frac{1}{R_{t+k}} \left(PB + S2 \right) \right) \quad (47)$$

The product $\prod_{k=1}^{j} \frac{1}{R_{t+k}}$ can be decomposed into the period from 2025 to 2034 and the period going to infinity.

$$\prod_{k=1}^{i} \frac{1}{R_{t+k}} = \prod_{k=1}^{N} \frac{1}{R_{t+k}} \times \prod_{k=N+1}^{i} \frac{1}{R_{t+k}} = \prod_{k=1}^{N} \frac{1}{R_{t+k}} \times \left(\frac{1}{R} \right)^{i-N} \quad (48)$$

Let's assume that $l = i - N$ and plug equation (49) into equation (48):

$$b_t = \sum_{j=1}^{N} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(PB_{t+j} + S2 - f_{t+j} \right) \right) + \sum_{l=1}^{\infty} \left(\prod_{k=1}^{N} \frac{1}{R_{t+k}} \left(PB + S2 \right) \right) \left(\frac{1}{R} \right)^{l} \quad (49)$$

The infinite sum only concerns the last term :

$$b_t = \sum_{j=1}^{N} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(PB_{t+j} + S2 - f_{t+j} \right) \right) + \left(\prod_{k=1}^{N} \frac{1}{R_{t+k}} \left(PB + S2 \right) \right) \sum_{l=1}^{\infty} \left(\frac{1}{R} \right)^{l} \quad (50)$$

Using the geometrical convergence of series :

$$\sum_{l=1}^{\infty} \left(\frac{1}{R} \right)^{l} = \left(\frac{1/R}{1 - (1/R)} \right) = \frac{1}{R-1} \quad (51)$$

The equation from which $S2$ can be isolated is:

$$b_t = \sum_{j=1}^{N} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(PB_{t+j} + S2 - f_{t+j} \right) \right) + \left(\prod_{k=1}^{N} \frac{1}{R_{t+k}} \left(PB + S2 \right) \right) \frac{1}{R-1} \quad (52)$$

$S2$ can be highlighted from equation (53):

$$\begin{aligned} b_t = & \sum_{j=1}^{N} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(PB_{t+j} - f_{t+j} \right) \right) + \left(\prod_{k=1}^{N} \frac{1}{R_{t+k}} PB \right) \frac{1}{R-1} \\ & + S2 \left(\sum_{j=1}^{N} \prod_{k=1}^{j} \frac{1}{R_{t+k}} + \prod_{k=1}^{N} \frac{1}{R_{t+k}} \frac{1}{R-1} \right) \end{aligned} \quad (53)$$

Finally, $S2$ is equal to the following expression:

$$S2 = \frac{b_t - \sum_{j=1}^{N} \left(\prod_{k=1}^{j} \frac{1}{R_{t+k}} \left(PB_{t+j} - f_{t+j} \right) \right) - \left(\prod_{k=1}^{N} \frac{1}{R_{t+k}} PB \right) \frac{1}{R-1}}{\left(\sum_{j=1}^{N} \prod_{k=1}^{j} \frac{1}{R_{t+k}} + \prod_{k=1}^{N} \frac{1}{R_{t+k}} \frac{1}{R-1} \right)} \quad (54)$$

The results indicate that $S2$ is equal to 0.707% of ADR. This means that the primary balance must be higher by 0.707 percentage points from 2025 and forever. The primary balance from 2029 to infinity equals 2.44%. Note that this primary balance, which does not seem impossible, has never been reached since 2016. This improvement of the primary balance allows for a decreasing debt path and ensures the long-term solvency of the Walloon Region's public debt.

7.3.3 Conclusions for long-term indicators

Long-term indicators seem to indicate that the solvency of the Walloon Region is not guaranteed.

The adapted $S1$ indicates that the fiscal consolidation required to bring back the debt level to 200% of ADR, which would still be considered as a high level of debt for the European Commission threshold, within 10 years is enormous and probably impossible to achieve. Even with a less ambitious objective of 250%, the fiscal consolidation remains significant and has never been observed in the past.

This result is mainly due to the initial debt level, which is very high, and the increasing proportion of interest payments. Indeed, the implicit interest rate is increasing during the projection horizon, and the amount of debt is also increasing.

Regarding $S2$, which is the immediate and permanent improvement of the primary balance required to respect the IBC, the result indicates an increase of 0.707% of ADR of the primary balance. This result seems achievable and could represent a good signal for the debt sustainability of the Walloon Region's public debt. However, as mentioned, this indicator relies on numerous assumptions and should be regarded more as a theoretical construct than a practical tool for real-world application.

8 Conclusions

8.1 Sustainability of the Walloon Region's public debt

After analyzing the three time dimensions of public debt sustainability, it appears that the Walloon Region's public debt is non-sustainable, at least over the medium and long term.

The first indicator does not signal a risk of default, in the broad sense, in the short term. Indeed, most variables are under their adapted threshold. However, the Walloon Region index is worse than the Flemish Community index. Two of three fiscal variables are above the threshold, and the debt ratio is almost twice the risk threshold. Other variables do not exceed their risk threshold, which can be seen as a rather positive sign.

The medium-term analysis, which is performed through the DSA toolkit developed by the European Commission, indicates that the risk of non-sustainability of the Walloon Region's public debt in the medium term is high. From the reference

scenario, it was already known that the debt ratio is too high and the debt peak is reached too late, which led to a high global risk. The deterministic analysis has highlighted the risks that the entity is facing. Indeed, even a small shock can significantly impact the debt level and debt path, mainly due to the high initial debt level of the Walloon Region. The stochastic projections confirm that the probability that the debt ratio in 2029 will be higher than that of 2024 is very high. Moreover, the uncertainty around the baseline is around 90% of ADR, which means the debt ratio may deviate by 45 percentage points from the reference scenario, both upwards and downwards in 2029. All these results put together, the DSA indicates that the risk of non-sustainability, in the medium term, of the Walloon Region's public debt is high.

Long-term indicators show that the fiscal consolidation the Walloon Region needs to do is significant. $S1$, which is the primary balance improvement required to reach a debt ratio equal to 200% of ADR within 10 years, is equal to 10.06 percentage points. If the debt ratio target is 250% of ADR, the primary balance adjustment is equal to 5.09% of ADR, which is still very significant. The main information that could be extracted from that exercise is that the required fiscal consolidation is important and will last a long time, more than 10 years. Regarding $S2$, the improvement of the primary balance required to respect the intertemporal budget constraint is equal to 0.707 percentage points, which is not unachievable and could, theoretically, ensure the solvency of the Walloon Region's public debt.

Finally, the Walloon Region is facing a real problem regarding the sustainability of its public debt, at least in the medium and long term. The indicators converge to say that the sustainability of the Walloon Region's public debt is not ensured.

8.2 Policy recommendations

Given the results exposed in this paper, some policy recommendations could be relevant to improve the fiscal situation and the sustainability of the Walloon Region's public debt.

The first thing that seems most logical is to increase the primary balance of the entity. However, according to Ando et al. (2024), on average, fiscal consolidation does not lead to a meaningful reduction in the debt ratio. Therefore, the authors try to identify factors that make fiscal consolidation successful, meaning a reduction of the debt ratio. To succeed, fiscal consolidation must occur in good times, i.e., during economic expansions, and should be well-designed, i.e., it should not kill GDP growth and be expenditure-based (Ando et al., 2024).

The full transposition of those results is not obvious given the institutional context

in which this analysis takes place. However, a few intuitions and general recommendations can be made. More precise choices are a matter of political choices that are somewhat beyond the scope of this work.

Even if the economic context is not particularly favorable, a fiscal consolidation is essential to ensure the debt sustainability of the Walloon Region's public debt.

Fiscal consolidation must focus primarily on reducing expenditures. The choice of cuts, on the other hand, is more political, but must not have the effect of reducing the Region's economic development.

On the revenue side, it would be more relevant to examine the various forms of transfers from other levels of government to increase revenue without raising taxes. This option would, however, require negotiations with other levels of government, or even a new state reform. Once again, such considerations are beyond the scope of this work. An increase in revenue from the entity's fiscal capacity is also conceivable, but it requires political choices and trade-offs that are beyond an economist's role. A more in-depth analysis of the various revenue streams is required to assess the impact of such measures.

Finally, in the short term, acting on the expenditures side seems more relevant to improve the primary balance as soon as possible. The sooner the adjustment is engaged, the sooner the increase in the debt ratio will be limited. In the medium term, it is crucial to make efforts to put the region on a downward debt trajectory. This will enable the Walloon Region to redeploy fiscal space over the long term.

8.3 Limitations

This section is dedicated, first, to explaining the methodological limitations and potential improvements that could be made. Second, it addresses the broader issue of debt sustainability.

8.3.1 Methodologies

One of the main problems when analyzing the sustainability of public debt of federated entities is the possibility of having points of comparison. This is particularly true within Belgium, where responsibilities are sometimes not completely identical between regions and communities. The characteristics of each entity's financing also complicate the possibilities of comparison.

Regional data are also not as rich as those available nationally. This lack of data also limits the overall analysis.

Directly related to the previous points, the informative power of the short-term indicator seems limited due to the lack of data, but also because of the lack of comparison points. The European Commission's indicator $S0$ requires many variables that are unavailable at the entity level and could have given more insights about the short-term risk. The idea of this paper was to use similar methodologies to those used by the EC, but exploring probit or logit models to assess the short-term risk could also be relevant.

Regarding the DSA, which includes deterministic and stochastic projections, it requires an important set of assumptions that could always be discussed. However, when used in an informative framework and to assign an overall sustainability risk, most assumptions appear reasonable and will not radically change the results. The main improvements that could be made would be integrating a cyclical component to the primary balance and a fiscal multiplier at the entity level. It would probably increase the reliability of the results and allow relaxation of the assumptions of a pure exogenous primary balance, which is restrictive. The major improvement for the stochastic projections would be to have a quarterly series of the primary balance to capture the interaction with the other variables. As mentioned, only the variability of Belgium's macroeconomy influences the debt path; no budgetary policy is captured in this exercise. The normality hypothesis of the generated shocks is also imperfect and could be relaxed by using other statistical methods, like the bootstrapping method.

The construction of $S1$ and $S2$ required even more assumptions on the projected variables. Therefore, the results must also be interpreted by keeping that in mind. Here again, longer projection series would allow for more reliable indicators.

Such an exercise aims to find the right trade-off between complexity and informative power. Sometimes, adding complexity to the model is not necessary because the cost is not offset by the additional informative power provided. The potential improvements proposed above are not too costly and could make results more robust.

8.3.2 Discussion

The analysis carried out in this article is valid in the current institutional context. Any reform on the financing or on the powers of the federated entities would make this analysis obsolete.

From a more general perspective, the risk thresholds can be subject to discussion. While the initial thresholds used in the short-term indicator are obtained by minimizing the SNR, the risk thresholds of the DSA are not clearly justified. These are linked to the Stability and Growth Pact; therefore, these are arbitrary rules that can always be subject to discussion. Blanchard et al (2021) discuss European fiscal rules that have been unsuccessful and propose to add fiscal standards to allow for more judgment and flexibility.

The European fiscal rules and methodology do not take several dimensions into account. In his report for the European Commission, Draghi (2024) requires an additional investment in the EU corresponding to 4.4 to 4.7% of EU GDP. Even if these additional investments are also pushed by the private sector, the public sector has to take its part. The report mentions policies such as subsidies, corporate tax reduction, or an increase in direct public investment. This will obviously have fiscal costs (Draghi, 2024). Even if required, the actual methodology and framework appear very rigid, which could represent an obstacle for investment policies. The methodologies used are purely for accounting purposes and do not give a full picture of the sustainability of public debt (Timbeau, 2022).

Public debt sustainability is also at the heart of public investment policies and represents another dimension that would be interesting to explore in further research.

9 Appendix

9.1 Linking coefficients of the LSF

Table 4: Linking coefficients of LSF transfers to the Walloon Region

	Real GDP growth rate g_t^Y	Inflation rate π_t	Distribution key
"Dotation emploi"	0.55 if $g_t^Y < 2.25\%$	1	Resident PIT
"Dépenses fiscales"	0.55 if $g_t^Y < 2.25\%$	1	Resident PIT
"Solidarité nationale"	1	1	None
"Compétences diverses"	0.55 if $g_t^Y < 2.25\%$	1	Fixed

Source: Loi Spéciale du 6 janvier 2014

The nominal amount of the transfer is therefore set by the LSF and is then indexed based on coefficients linked to economic variables. When $g_t^Y > 2.25\%$, the law stipulates that the linkage coefficient increases to 1 for the real GDP growth share exceeding 2.25%. This mechanism allows the federal government to limit the increase in transfers during periods of low real GDP growth, while maintaining adequate funding during periods of strong real GDP growth.

To illustrate how the link coefficients work, let us take the WAL employment allocation assuming an initial amount of 100, real growth of 2% and inflation of 2%. Assuming the distribution key remains unchanged, the final amount will be:

$$Dot_{WAL,t}^{Emploi} = 100 \times (1 + 0.55 \times 0.02) \times (1 + 1 \times 0.02) = 103.122$$

When the real growth rate is greater than 2.25%, assuming 3%, therefore an increase of 0.75%, the threshold mechanism is applied, which gives us the following result:

$$Dot_{WAL,t}^{Emploi} = 100 \times ((1 + 0.55 \times 0.0225) + (1 \times 0.0075))(1 + 1 \times 0.02)) = 104.03$$

Another subtlety is that this threshold mechanism does not apply when real GDP growth rate falls into negative territory. If, in one year, growth is negative beyond -2.25%, and a catch-up effect occurs the following year, the entity will be 'refinanced'.

Let's take the example of negative real GDP growth of -5% in one year and zero inflation to simplify the calculation. The allocation therefore becomes:

$$Dot_{WAL,t}^{Emploi} = 100 \times (1 + (0.55 \times (-0.05)) = 97.25$$

The threshold is not applied for anything exceeding -2.25%. The following year, real GDP growth rebounds to +5.26%, returning to pre-crisis GDP. Inflation is still set to zero.

$$Dot_{WAL,t}^{Emploi} = 97.25 \times [1 + (0.55 \times 0.0225) + 1 \times (0.0526 - 0.0225)] = 101.38$$

The "Sainte-Émilie" transfers follow the same logic but with even more complex mechanisms.

These mechanisms pose a problem when calculating the elasticities of revenues to GDP and inflation because many parameters are considered to determine their amounts. Therefore, the elasticities of revenues to GDP and inflation had to be calculated to implement the different shocks while taking into account institutional complexities.

9.2 Detailed elasticities

Table 5: Elasticities of the growth rate of LSF Revenues

	2025	2026	2027	2028	2029
Real GDP growth +1%	0.82	0.85	0.87	0.89	0.91
Real GDP growth -1%	-0.57	-0.59	-0.60	-0.62	-0.63
Inflation +1%	0.82	0.84	0.86	0.88	0.90
Inflation -1%	-0.82	-0.84	-0.86	-0.88	-0.90

Source: Private computation FPB

Table 6: Elasticities of the growth rate of "Sainte-Émilie"

	2025	2026	2027	2028	2029
Real GDP growth +1%	0.49	0.50	0.50	0.51	0.52
Real GDP growth -1%	-0.41	-0.41	-0.41	-0.42	-0.42
Inflation +1%	0.98	0.98	0.98	0.98	0.98
Inflation -1%	-0.98	-0.98	-0.98	-0.98	-0.98

Source: Private computation FPB

9.3 Interest expenses computation

This annex is dedicated to details concerning the calculation of interest charges on the debt, which causes a technical problem that deserves to be detailed.

ESA 2010 calculates interest charges according to an "interest run on December 31". It follows that the paired interest charges are a function of the amount of debt accumulated during the year. This debt is itself a function of the annual budget deficit. However, the extent of the budget deficit itself depends on interest charges. There is, therefore, a circular relationship between our two variables.

This relationship, therefore, breaks down a technical problem during the exercise, which is why working with "expired interest" has been chosen. The differences observed between our calculation method and the projections have been reclassified in the stock-flow adjustment.

The projections of debts used come from the FPB. Let's place ourselves in 2024 to illustrate the reasoning. A new debt had to be issued during the year 2023 to fill the need for funding. This is the difference between the total debt amount of 2023 B_{t-1} and that of the year 2022 B_{t-2}.

$$NewDebt_{t-1} = \Delta B_{t-1} = B_{t-1} - B_{t-2}$$

Part of the 2022 debt securities reached maturity and had to be refinanced during 2023. It is assumed that the proportion of debt to be refinanced corresponds to the total debt divided by its average maturity. For example, if the average maturity of the assets constituting the debt is 10 years, the annual refinancing share is 10%.

$$\alpha = \frac{1}{\text{Average Maturity}}$$

The debt to be refinanced in 2023 is therefore the roll-over rate α multiplied by the 2022 debt stock:

$$\text{Refinanced Debt}_t = \alpha \times B_{t-1}$$

The total borrowing for 2023 is obtained, which is the sum of the refinanced debt and the new debt issuance.

The interest charges for 2024 are calculated as the 2023 interest charges multiplied by the proportion of non-refinanced debt $(1 - \alpha)$. The total 2023 borrowing refinanced at the 2024 interest rate is added. The complete formula is thus:

$$\text{Interest Expenses}_t = (1-\alpha)\times\text{Interest Expenses}_{t-1}+(r_t+\text{spread})\times\text{Total Borrowing}_{t-1}$$

It is worth noting that a spread is applied to the reference interest rate, which is the 10-year yield on the linear bonds issued by the Belgian State. This spread is assumed constant throughout the projection period and varies depending on the type of debt (direct or indirect).

The choice to apply the year-t interest rate to the total borrowing of year $t-1$ may be subject to debate. Still, it is justified as it allows us to immediately transmit an interest rate shock to the implicit interest rate in year t. Without this mechanism, an increase in the interest rate in year t would only affect the interest charges in $t+1$.

Thanks to this calculation method and the projections from the Federal Planning Bureau, the results are close to those obtained via iterative calculation. However, when adjusting interest charges, the initial consistency between the budget balance and the primary balance, which was based on the initial interest charges, is lost. Indeed, the difference between the initial primary balance and the corrected interest charges no longer equals the initial budget balance.

9.4 Stochastic shock on ADR

As explained previously, the denominator is the ADR; its growth rate is required for the debt accumulation equation. However, the evolution of the ADR depends on the origin of the nominal GDP shocks. This is due to the link coefficients set in the LSF. It is therefore important to distinguish between shocks of real origin and shocks of inflationary origin.

Real shock on LSF revenue growth ($e_t^{g^{\mathbf{LSF},Y}}$)
This must take into account both high and low elasticities. Define the shocked real GDP growth rate as:

$$\tilde{g}_t^Y = \bar{g}_t^Y + e_t^Y$$

where $\tilde{g}_t^Y$ is the shocked real GDP growth rate, $\bar{g}_t^Y$ is the baseline real GDP growth rate, and e_t^Y is the real GDP growth rate shock.

When $\tilde{g}_t^Y > 2.25\%$, the real shock on LSF revenue growth is:

$$e_t^{g^{\text{LSF},Y}} = (\tilde{g}_t^Y - 0.0225) \times \varepsilon_t^{\text{LSF},Y+} + (0.0225 - \bar{g}_t^Y) \times \varepsilon_t^{\text{LSF},Y-}$$

Where $\varepsilon_t^{\text{LSF},Y+}$ is the elasticity of the transfer growth when real GDP growth exceeds 2.25% and $\varepsilon_t^{\text{LSF},Y-}$ is the elasticity of the transfer growth when real GDP growth is under 2.25%.

If the threshold is not exceeded, then:

$$e_t^{g^{\text{LSF},Y}} = e_t^Y \times \varepsilon_t^{\text{LSF},Y-}$$

Inflation shock on LSF revenue growth ($e_t^{g^{\mathbf{LSF},P}}$)
This is the product of the inflation shock (e_t^P) and the price elasticity:

$$e_t^{g^{\text{LSF},P}} = e_t^P \times \varepsilon_t^{\text{LSF},P}$$

where $\varepsilon_t^{\text{LSF},P}$ is the elasticity of the transfer growth to inflation.

Total post-shock growth rate of LSF revenues:

$$\tilde{g}_t^{\text{LSF}} = \bar{g}_t^{\text{LSF}} + e_t^{g^{\text{LSF},Y}} + e_t^{g^{\text{LSF},P}}$$

Post-shock level of LSF revenues:

$$\widetilde{LSF}_t = \widetilde{LSF}_{t-1} \times (1 + \tilde{g}_t^{\text{LSF}})$$

Non-LSF revenues: Unit elasticity is assumed for both real GDP and inflation. Their growth rate is:

$$\tilde{g}_t^{\text{Non-LSF}} = \bar{g}_t^{\text{Non-LSF}} + e_t^Y + e_t^P$$

Post-shock level of non-LSF revenues:

$$\widetilde{\text{Non-LSF}}_t = \widetilde{\text{Non-LSF}}_{t-1} \times (1 + \tilde{g}_t^{\text{Non-LSF}})$$

Reconstruction of the ADR: The shocked ARD is the addition of the two other components:

$$\widetilde{\text{ADR}}_t = \widetilde{\text{LSF}}_t + \widetilde{\text{Non-LSF}}_t$$

The shocked growth rate of ADR is then:

$$\tilde{g}_t^{\text{ADR}} = \frac{\widetilde{\text{ADR}}_t - \widetilde{\text{ADR}}_{t-1}}{\widetilde{\text{ADR}}_{t-1}}$$

9.5 Stochastic shock on the primary balance

As mentioned, there is no quarterly data on the primary balance of Belgian entities. Therefore, it has not been possible to build the variance-covariance matrix with the primary balance, like in the EC methodology.

The idea is therefore to use the relationship between the ADR and the primary balance. The idea is that the primary balance equals the ADR minus a residual part, μ, which is assumed constant.

$$\frac{PB_t}{ADR_t} = \frac{ADR_t - \mu_t}{ADR_t} = 1 - \frac{\mu_t}{ADR_t}$$

It is assumed that only real GDP growth shocks affect the primary balance. The

shock is then identified as follows:

$$
\begin{aligned}
e_t^{\mathrm{pb}^{\mathrm{ADR}}} &= \frac{\widetilde{\mathrm{PB}}_t}{\widetilde{\mathrm{ADR}}_t} - \frac{\bar{\mathrm{PB}}_t}{\bar{\mathrm{ADR}}_t} \\
&= \frac{\widetilde{\mathrm{ADR}}_t - \mu_t}{\widetilde{\mathrm{ADR}}_t} - \frac{\mathrm{ADR}_t - \mu_t}{\mathrm{ADR}_t} \\
&= \left(1 - \frac{\mu_t}{\widetilde{\mathrm{ADR}}_t}\right) - \left(1 - \frac{\mu_t}{\mathrm{ADR}_t}\right) \\
&= \frac{\mu_t}{\mathrm{ADR}_t} - \frac{\mu_t}{\widetilde{\mathrm{ADR}}_t} \\
&= \frac{\mu_t}{\mathrm{ADR}_t} \times \left(1 - \frac{\mathrm{ADR}_t}{\widetilde{\mathrm{ADR}}_t}\right) \\
&= \frac{\mu_t}{\mathrm{ADR}_t} \times \left(\frac{\widetilde{\mathrm{ADR}}_t - \mathrm{ADR}_t}{\widetilde{\mathrm{ADR}}_t}\right) \\
&= \frac{\mu_t}{\mathrm{ADR}_t} \times \left(\frac{\frac{\widetilde{\mathrm{ADR}}_t}{\mathrm{ADR}_{t-1}} - \frac{\mathrm{RDA}_t}{\mathrm{ADR}_{t-1}}}{\frac{\widetilde{\mathrm{ADR}}_t}{\mathrm{ADR}_{t-1}}}\right) \\
&= \frac{\mu_t}{\mathrm{ADR}_t} \times \left(\frac{(1 + \tilde{g}_t^{\mathrm{ADR}}) - (1 + \bar{g}_t^{\mathrm{ADR}})}{1 + \tilde{g}_t^{\mathrm{ADR}}}\right) \\
&= \frac{\mu_t}{\mathrm{ADR}_t} \times \left(\frac{\tilde{g}_t^{\mathrm{ADR}} - \bar{g}_t^{\mathrm{ADR}}}{1 + \tilde{g}_t^{\mathrm{ADR}}}\right) \\
&= \frac{\mu_t}{\mathrm{ADR}_t} \times \left(\frac{e_t^{g^{\mathrm{ADR},Y}}}{1 + \tilde{g}_t^{\mathrm{ADR}}}\right)
\end{aligned}
$$

Therefore, the shock applied to the primary balance is:

$$
\tilde{pb}_t = \bar{pb}_t + e_t^{pb^{ADR}} = \bar{pb}_t + \frac{\mu_t}{\mathrm{ADR}_t} \times \left(\frac{e_t^{g^{\mathrm{ADR},Y}}}{1 + \tilde{g}_t^{\mathrm{ADR}}}\right)
$$

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EXECUTIVE SUMMARY

This master's thesis investigates the sustainability of the Walloon Region's public debt. This research topic was motivated by the current challenges our societies face and the related financing issues that are at the center of many public debates. The first part was dedicated to exploring the literature on the concepts of optimality and sustainability of public debt, with the aim of gaining a broad understanding of the topic. The second part defines all the concepts necessary to understand the methodologies used to answer the sustainability question. A contextualization of Belgian debt figures and the growing share of federated entity debt in the national total shows the relevance of the research topic. Therefore, the main obstacles, due to the institutional context in which the assessment takes place, to replicating methodologies are explained, and the solutions found are also provided. The short-term analysis revealed that the situation was not particularly alarming, although it was worse than in the Flemish Community. However, the results from the medium and long-term analyses indicate that the sustainability of the Walloon Region's public debt is not guaranteed. The debt level is too high, and the debt stabilization is too sensitive to any small adverse shock. The fiscal consolidation required is significant and will need to last for more than ten years. Finally, some policy recommendations are developed in the last section. A broader discussion on public debt follows the main limitations and potential improvements of the methodologies.

KEYWORDS: Public debt, Sustainability, Walloon Region, DSA

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