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Towards redefining the second using optical clocks

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Introduction

The measurement of time is one of the oldest human concerns and is also a prominent topic in physics and engineering, particularly in physics and engineering.

From the observation of the Sun's movement on the celestial sphere in Antiquity to the development of atomic clocks in the 20th century, the history of time measurement is essentially linked to the evolution of scientific knowledge, particularly in astronomy, celestial mechanics, and, more recently, quantum mechanics and atomic physics.

Initially, the definition of time was based on the observation of the timekeeping relied on observations of celestial motions, especially the Sun and fixes stars, in order to define solar time and sidereal time. These concepts, based on the Earth's rotation, led to the establishment of globally adopted time scales, notably Universal Time (UT) [1].

It is with this in mind that the scientific community sought to precisely define the units of its International System of units, and in particular, the second, initially defined as a fraction of the mean solar day, giving rise to UT [2]. However, the Earth's rotation exhibited irregularities, and in response to these instabilities, in 1956 the Ephemeris Second was defined through the Ephemeris Time (ET), before the atomic definition allowed, in 1967 [1, 2, 3]. This definition, still used today, is based on the frequency of the hyperfine transition of the cesium-133 atom [4, 5].

Since then, time measurement relies on atomic standards, particularly cesium fountain clocks. Thanks to their high stability and accuracy, these devices enabled the computation of a new time scale : International Atomic Time (TAI), from which Coordinated Universal Time (UTC), the time scale used today, is derived. Although based on the atomic definition of time, the latter has the interesting property of being kept in phase with the Earth's rotation by inserting leap seconds [2, 6].

However, advances in quantum and atomic physics, and in optical technologies (particularly lasers), have led to the development of so-called optical clocks, which use much-higher frequency, intrinsically narrower optical transitions than the microwave transition used in cesium primary standards. These clocks possess stability and accuracy that surpass the best microwave clocks [7, 8].

In this new technological context, the General Conference on Weights and Measures (CGPM) invited steps towards adopting a new definition at the CGPM 2030. This turning point raises several questions, including : which transitions will be adopted? What types of clocks? What criteria should guide the thinking and initiate this choice [7, 8] ?

This master's thesis is in line with this reflection, through three chapters, each playing a role in understanding the past, present and future concepts.

The first chapter deals with the notion of UT. It goes back to the origins of timekeeping through the observation of the stars, presenting the concepts of sidereal time and solar time. It also highlights the limitations of this time scale, notably the irregularities of the Earth's rotation, which led to the redefinition of the second thanks to atomic physics.

The second chapter introduces the concept of the second using the hyperfine transition of cesium-133 and briefly describes the operating principle of cesium atomic clocks. It then explains how TAI is computed using the Bureau International des Poids et Mesures (BIPM) time-scale algorithms, ALGOS . It also describes the link between TAI and UTC by introducing the concept of leap seconds and expands on the notions of stability and accuracy of these clocks using the Allan variance, which also allows for a comparison between cesium clocks and those based on hydrogen and rubidium.

Finally, the third and final chapter constitutes a personal contribution to the current issue of redefining the second. It provides an analysis of the limitations of the current definition based on cesium, before presenting in detail optical lattice clocks and trapped-ion clocks, their physical principles, and candidate transitions. It also draws on a number of criteria established by the CGPM to compare the different redefinition options and assesses the performance, robustness, reproducibility, and challenges associated with this definition.

Chapter 1

The concept of Universal Time

This chapter aims to provide a comprehensive overview of the concept of Universal Time (UT), from both historical and scientific perspectives. It traces the evolution of timekeeping approaches, from ancient observational practices to modern standards, emphasizing the transition from astronomical to atomic definitions of time. The chapter is structured to introduce the reader progressively to mathematical and astronomical concepts underlying the definition and realization of Universal Time, including spherical geometry, sidereal and solar time. It also highlights the limitations of UT due to irregularities in Earth's rotation, which ultimately led to the introduction of ephemeris and, later, atomic time scales. This foundational material is essential for understanding the motivations behind current definitions of time and the standards used in modern timekeeping.

1.1 Introduction

1.1.1 Origin of time measurement

Since antiquity, civilizations have sought to structure timekeeping around natural cycles, particularly the alternation of day and night. Initially, daytime was the period during which the Sun illuminated a specific location, and day and night were each divided into twelve hours [5]. Many early cosmologies assumed a stationnary, geomcentric cosmos, where the Earth was flat. Consequently, it was assumed that the entire world experienced daylight or darkness simultaneously. The length of temporal hours varied with the seasons : daytime and nighttime hours had equal length only near the equinoxes.

Early systematic astronomical reckoning methods used the sexagesimal (base-60) system developed by Mesopotamian astronomers and later adopted by the Greeks. In this tradition, equal (also called equinoctial) hours and their sexagesimal subdivisions, minutes and seconds, were adopted in astronomical practice. Civil use of these became widespread much later [1, 9].

1.1.2 Time measurement in the Geocentric model

In the 2nd century AD, Claudius Ptolemy proposed a geocentric model in his *Almagest*, placing Earth at the center of the universe. In this model, the Sun and the Moon, as well as Mercury, Venus, Mars, Jupiter and Saturn, were taken to orbit Earth on deferents and epicycles. Ptolemy also introduced the concept of the celestial sphere, an idealized geometric construct representing the night sky, onto which the stars appear to be projected, as illustrated in Figure 1.1. [1, 10].

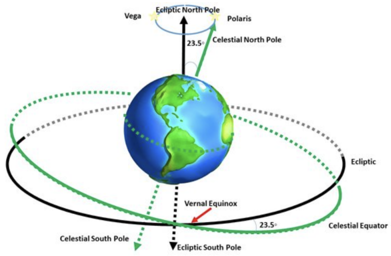

Figure 1.1: Celestial sphere (schematic) [11]

This framework played a key role in structuring time measurement : the apparent motion of the stars across the celestial sphere provided reliable reference points, allowing astronomers to associate the daily cycle, later understood as Earth's rotation, with observable celestial patterns. These observations laid the groundwork for time scales tied to Earth's rotation, such as Universal Time (UT)

1.1.3 Historical emergence of Universal Time

The growing need for a standardized reference time became critical with the development of railways and telecommunications in the 19th century. Until then, each locality used its own local mean solar time, which made it difficult to coordinate train schedules and long-distance communication.

To address this issue, Greenwich Mean Time (GMT), the mean solar time at Greenwich, became the de facto international reference. The term UT was introduced later by astronomers to designate timekeeping based on Earth's rotation [2].

The 1884 International Meridian Conference designated the Greenwich meridian as the prime meridian and recommended a universal day beggining at midnight [12, 13]. Time zones and the civil adoption of standard time were implemented separately by railways and governements [14].

1.2 Spherical Geometry and Time Measurement

1.2.1 Introduction to spherical geometry

This section [9] introduces spherical geometry and geodesy, as the quantities defining the time scale are affected by Earth's non-flat, oblate spheroidal shape and by the curvature of spacetime as described by relativity. Spherical geometry, a branch of non-Euclidean geometry, helps to understand these spherical triangles. This part of geometry allows understanding the relationship between points on Earth's curved surface and their directions on the celestial sphee.

In this framework, the first four of Euclid's postulates remain valid:

1. A straight line can always be drawn through any two points.
2. Any line segment can be extended indefinitely in its direction.
3. A circle can be drawn with any center and radius.
4. All right angles are equal.

In spherical geometry, there are no parallel geodesics on the sphere, all great circles intersect (twice). Instead, geodesics, which are the shortest paths between two points on a curved surface, are used. These geodesics replace the straight lines of Euclidean geometry and underpin geodesy and celestial navigation, which support Earth-rotation-based timekeeping (UT) [10].

This concept of geodesics is not only fundamental in spherical geometry but also plays a crucial role in Hamiltonian mechanics and general relativity, where spacetime itself is curved. In these fields, geodesics help explain how objects move through space-time and, in the case of timekeeping, how time is measured on a rotating Earth.

1.2.2 The spherical triangle

A spherical triangle is a figure formed by three arcs of great circles on the surface of a sphere (see Figure 1.2.). Unlike a Euclidean triangle, its angles sum to more than 180°[9, 15]. Consider a unit sphere centered at O and a spherical triangle ABC with sides a, b, c opposite to vertices A, B, C respectively.

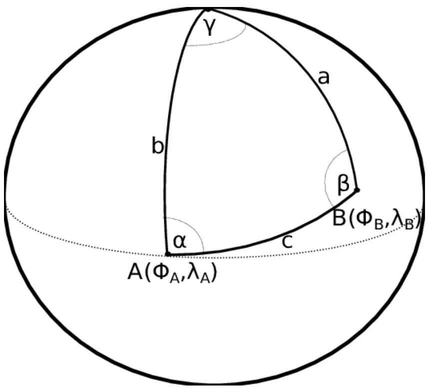

Figure 1.2: Spherical triangle ABC on the unit sphere [16]

It is worth recalling two fundamental identities in spherical trigonometry.

The first is the spherical law of cosines (for sides), relating sides and opposite angles [9, 15].

$$\begin{aligned}
\cos a &= \cos b \cos c + \sin b \sin c \cos A && (1.1)\\
\cos b &= \cos c \cos a + \sin c \sin a \cos B && (1.2)\\
\cos c &= \cos a \cos b + \sin a \sin b \cos C && (1.3)
\end{aligned}$$

The second is the spherical sine rule:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \quad (1.4)$$

1.3 Universal Time

The concept of timekeeping emerged from the observation of natural cycles, particularly the alternation of day and night and the changing seasons. These observations became crucial with the development of agriculture, as farmers needed reliable markers to determine optimal periods for planting and harvesting crops.

Early civilizations devised rudimentary timekeeping instruments, such as sundials and gnomons, to infer the Sun's apparent position and indicate the time of day. Because the day and the night were traditionally each divided into twelve temporal hours, whose lengths vary with the seasons, the hour was not of fixed duration. Sundials indicate local apparent solar time through the shadow's direction (hour angle) and length. However, their indication differs from mean time [1, 9].

1.3.1 Sidereal time

Sidereal time is a time scale used primarily in astronomy. Unlike solar time, which is based on the Sun's apparent motion, sidereal time measures Earth's rotation relative to the fixed stars. Although expressed in hours, minutes, and seconds, it represents an angle (with 24h = 360°) rather than a physical duration : it quantifies Earth's orientation in its daily rotation with respect to the celestial sphere. Formally, the local sidereal time (LST) [2] at a given location and instant is defined as the hour angle of the vernal equinox γ. Geometrically, this is the angle in the celestial equatorial plane between the local meridian and the direction of γ, the origin of right ascension (α) in the equatorial coordinate system.

Let H denote the hour angle of a celestial object. The fundamental relation is

$$S = H + \alpha, \quad (1.5)$$

where S is the local sidereal time, H the object's hour angle, and α its right ascension and with H increasing through west. Thus, when an object transits the local meridian ($H = 0$), the sidereal time equals its right ascension. Conversely, if the sidereal time is known at a given moment, one can immediately deduce which celestial objects are culminating. This property makes sidereal time a crucial tool for astronomers, particularly for telescope pointing and observation planning. Indeed, knowing the sidereal time provides direct access to the orientation of the celestial sphere relative to the observer's meridian. Sidereal time advances uniformly due to the Earth's nearly constant angular velocity of rotation. Since the Earth completes one full turn with respect to the fixed stars slightly faster than with respect to the Sun, a sidereal day, defined as the interval required for the Earth to complete one full rotation relative to the fixed stars, is shorter than a solar day, defined as the interval between two successive transits of the Sun across the local meridian [17, 18].

Because Earth advances along its orbit, it must rotate slightly more than 360°each mean solar day to bring the Sun back to the meridian. The additional daily angle is approximately

$$\Delta\varphi = \frac{360^\circ}{365.25} \approx 0.9856^\circ \tag{1.6}$$

Consequently, the mean solar day (approximately 86 400 s) is about 3.94 minutes (3min and 56 s) longer than the sidereal day (≈ 86,164 s, or 23h56min4s). There are thus about 366.242 sidereal days in one tropical year (compared to 365.242 mean solar days), so sidereal time gains roughly 4 minutes per solar day. This daily shift accumulates so that after one year, sidereal and solar indications return to near-synchrony.

A distinction is made between local sidereal time and Greenwich Mean Sidereal Time (GMST), the latter referring to sidereal time at the Greenwich meridian. Apparent sidereal time (GAST) also exists, incorporating corrections for nutation and precession [17]. These refined definitions are necessary for high-precision applications, especially in radio astronomy and space geodesy [3]. Sidereal time thus constitutes a practical angular measure of Earth's rotation relative to the stellar background. It is intimately connected to the equatorial coordinate system, where right ascension and declination define the positions of celestial objects. Because it progresses uniformly, sidereal time underlies many astronomical algorithms and ephemerides, and remains indispensable in celestial mechanics.

1.3.2 Solar time

Before discussing solar time [2], we must define the notions of the true Sun and the mean Sun. The true Sun is the actual celestial body moving along the ecliptic, whereas the mean Sun is a fictitious construct introduced to provide a uniform reference for timekeeping. The discrepancy between the angular positions of the true and mean Sun, known as the equation of time, arises from two primary effects: the eccentricity of Earth's orbit and the obliquity of the ecliptic [19, 20]. While sidereal time tracks Earth's rotation relative to the fixed stars, solar time is based on the apparent motion of the Sun in the sky. It provides a more intuitive measure of time for daily life, aligned with the alternation of day and night as experienced on Earth.

Solar time is defined as the hour angle of the Sun:

$$H_\odot = S - \alpha_\odot, \tag{1.7}$$

where S is the local sidereal time and $\alpha_\odot$ the right ascension of the Sun. This formulation is consistent with the general expression $H = S - \alpha$, valid for any celestial object. However, the Sun's apparent motion is not uniform. This irregularity results from two main causes. First, due to the elliptical shape of Earth's orbit, the orbital speed varies:

it is faster at perihelion (when Earth is closest to the Sun) and slower at aphelion (when Earth is farthest) [1]. This variation causes a sinusoidal lag of the true Sun relative to the mean Sun over the course of a year, with an amplitude of about 7.66 minutes.

Secondly, the Earth's axial tilt causes the Sun's projection onto the celestial equator to vary. Near the equinoxes, the Sun moves diagonally with respect to the equator, slowing the rate of change in right ascension. Near the solstices, it moves nearly parallel to the equator, accelerating it [2, 21]. As a consequence, the Sun's right ascension $\alpha_\odot(t)$ does not increase linearly throughout the year. Although the mean Sun's ecliptic longitude $\lambda_\odot(t)$ increases uniformly, whereas the true Sun's ecliptic longitude is non-uniform, due to orbital eccentricity :

$$\tan \alpha_\odot(t) = \tan \lambda_\odot(t) \cos i, \tag{1.8}$$

where i is the obliquity of the ecliptic. From spherical trigonometry, using cosine and sine laws applied to a suitably constructed triangle, we obtain:

$$\begin{aligned} \cos \delta_\odot(t) &= \cos \alpha_\odot(t) \cos \lambda_\odot(t) + \sin \alpha_\odot(t) \sin \lambda_\odot(t) \cos i && (1.9)\\ \cos \lambda_\odot(t) &= \cos \alpha_\odot(t) \cos \delta_\odot(t) && (1.10) \end{aligned}$$

Substituting equation (1.10) into (1.9) and simplifying:

$$\begin{aligned} \frac{\cos \lambda_\odot(t)}{\cos \alpha_\odot(t)} &= \cos \alpha_\odot(t) \cos \lambda_\odot(t) + \sin \alpha_\odot(t) \sin \lambda_\odot(t) \cos i && (1.11)\\ \Rightarrow \cos \lambda_\odot(t) &= \cos^2 \alpha_\odot(t) \cos \lambda_\odot(t) + \cos \alpha_\odot(t) \sin \alpha_\odot(t) \sin \lambda_\odot(t) \cos i && (1.12) \end{aligned}$$

Isolating the term with $\cos i$:

$$\begin{aligned} \cos \lambda_\odot(t) - \cos^2 \alpha_\odot(t) \cos \lambda_\odot(t) &= \cos \alpha_\odot(t) \sin \alpha_\odot(t) \sin \lambda_\odot(t) \cos i && (1.13)\\ \Rightarrow \cos \lambda_\odot(t) \sin^2 \alpha_\odot(t) &= \cos \alpha_\odot(t) \sin \alpha_\odot(t) \sin \lambda_\odot(t) \cos i && (1.14) \end{aligned}$$

Applying the Pythagorean identity $\sin^2 \alpha = 1 - \cos^2 \alpha$:

$$\begin{aligned} \cos i &= \frac{\cos \lambda_\odot(t) \sin^2 \alpha_\odot(t)}{\cos \alpha_\odot(t) \sin \alpha_\odot(t) \sin \lambda_\odot(t)} && (1.15)\\ &= \frac{\cos \lambda_\odot(t)}{\sin \lambda_\odot(t)} \cdot \frac{\sin \alpha_\odot(t)}{\cos \alpha_\odot(t)} && (1.16)\\ &= \cot \lambda_\odot(t) \tan \alpha_\odot(t) && (1.17)\\ &= \frac{\tan \alpha_\odot(t)}{\tan \lambda_\odot(t)} && (1.18) \end{aligned}$$

Finally, this confirms the identity:

$$\tan \alpha_\odot(t) = \tan \lambda_\odot(t) \cos i \tag{1.19}$$

The two effects responsible for the deviation between the true and mean Sun produce a semi-annual sinusoidal pattern known as the equation of time [19, 22] :

$$E(t) = \overline{H_\odot}(t) - H_\odot(t), \tag{1.20}$$

where $\overline{H_\odot}(t)$ is the hour angle of a fictitious mean Sun moving uniformly along the celestial equator.

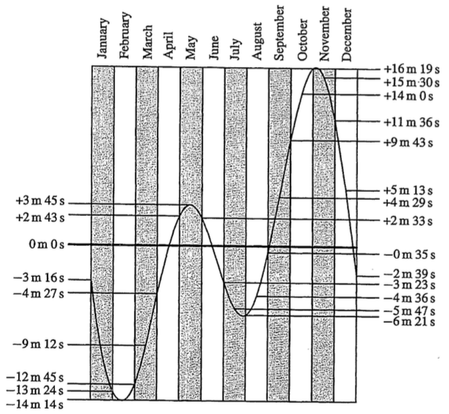

Figure 1.3: Equation of time $E(t)$ [2]

This equation, which can reach up to ±16 minutes, as illustrated in Figure 1.4., explains why sundials and mechanical clocks may disagree. The cumulative effect over a year produces the analemma [2] , a figure-eight-shaped curve traced by plotting the Sun's position in the sky at the same mean time each day, represented in Figure 1.4.

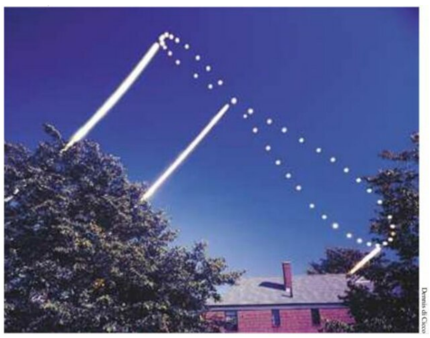

Figure 1.4: Solar analemma (mean-time sampling) [23]

To correct for these irregularities, mean solar time was introduced. It is based on the transit of a fictitious mean Sun moving at constant angular velocity along the celestial equator. Noon in mean solar time corresponds to the instant this mean Sun crosses the local meridian. By construction, the mean Sun is defined to move at a uniform angular velocity. In Newcomb's convention, a fictitious Sun (FS) moves uniformly along the ecliptic at the average angular speed of the true Sun. The actual mean Sun (MS) is then defined as a fictitious point that moves at the same angular speed as FS, but along the celestial equator, starting from the vernal equinox. The mean right ascension is determined such that at each moment, it matches the point where the ecliptic longitude of FS equals the equatorial position of MS [1, 2]. The angular velocity of this fictitious mean Sun defines the length of a mean solar day:

$$\omega = \frac{360°}{24\,\mathrm{h}} = \frac{360°}{86\,400\,\mathrm{s}} = \frac{1}{240}\mathrm{deg/s}. \tag{1.21}$$

Despite being an artificial construct, mean solar time was widely adopted for civil and legal purposes due to its uniformity and its intuitive alignment with the natural day-night cycle. However, long-term variations in Earth's orbit and axial tilt, due to precession, nutation, and planetary perturbations, render even mean solar time imperfect over millennia [3].

These limitations led to the eventual adoption of ephemeris time and, ultimately, atomic time standards, which offer far superior long-term stability. Nevertheless, solar time remains fundamental to the historical and conceptual framework of timekeeping and continues to underlie the civil time used in daily life.

1.3.3 Civil time, Local time and GMT

Solar time, although directly derived from astronomical observation, proved increasingly impractical as societies became more interconnected. Until the 19^{th} century, each locality used its own local time, defined by the apparent daily transit of the Sun across the local meridian. This meant that noon occurred at slightly different moments in neighbouring cities, a discrepancy that had little impact in isolated contexts but became problematic with the development of railways, telecommunications, and national infrastructure [1, 2].

To address these difficulties, civil standard time was introduced. Unlike local time, civil time did not follow the Sun's position at each location but was instead based on mean solar time referred to a chosen reference meridian. This provided a uniform legal and social time within a given region, allowing clocks to be synchronized independently of local solar variations. However, in large countries extending accross several longitudes, discrepancies remained between regions adopting different local mean times. This situation led to the formal introduction of time zones. Earth was ideally divided into 24 zones, each spanning 15°of longitude and corresponding to a one-hour offset. In practice, political, social, and economic factors produced deviations from this geometric ideal [1, 2, 14].

The International Meridian Conference held in Washington, D.C., in 1884 designated the Greenwich meridian (0°longitude) as the prime meridian for global navigation and timekeeping. From then on, Greenwich Mean Time (GMT), the mean solar time at Greenwich, became a widely used reference for naviagation and civil timekeeping, while time zones were adopted via national legislation [17, 24]. GMT enabled the synchronization of schedules across wide areas and replaced the fragmented system of local times. However, because it remained based on Earth's irregular rotation, it eventually proved inadequate for applications requiring uniform time intervals, such as precise astronomical measurements and telecommunications [1, 2].

1.3.4 Difficulties with Universal Time

Until 1960, the definition of the second was based on the Earth's rotation. For a long time, it was assumed that this rotation was uniform, a view supported notably by Leibniz in the 18th century. Consequently, mean solar time was considered constant. However, it became evident that maintaining this definition with high precision required empirical corrections over time. Although widely adopted, the definition of the second as 1/86 400 of the mean solar. While uniformity in Earth's rotation was generally accepted, Kepler had already expressed doubts regarding its constancy. These concerns, however, were largely disregarded by the scientific community at the time [2, 9].

By analyzing ephemeris tables developed by Hansen, Newcomb identified discrepancies between predicted and observed positions of celestial bodies, particularly the Moon. These inconsistencies led him to suspect that time itself was not being measured consistently. If time were properly defined, the Moon's observed positions would align precisely with its ephemerides. Further investigations by de Sitter and Brown on additional celestial bodies confirmed Newcomb's hypothesis. Their findings pointed to fundamental flaws in defining time intervals, primarily resulting from irregularities in Earth's rotation [2, 9].

The length of day (LOD), defined as the actual duration of one Earth rotation, often deviates from the SI-defined 86 400 seconds [25, 26]. These variations result from multiple complex geophysical processes:

- **Lunar tides and secular braking:** Tidal friction transfers angular momentum from Earth to the Moon, lengthening Earth's rotation period by approximately 1.4 to 1.7 ms per century.
- **Atmosphere and oceans:** Momentum exchanges between Earth's solid body and the fluid systems induce seasonal variations. Large-scale climate phenomena, such as El Niño or La Niña, also influence LOD.
- **Core–mantle coupling:** Interactions between Earth's internal layers generate oscillations in LOD on decadal timescales.
- **Mass redistribution:** Processes like glacial melting or hydrological cycles alter Earth's moment of inertia, affecting rotation through conservation of angular momentum.
- **Major earthquakes:** Though small, seismic events like the 2004 Sumatra earthquake have measurably modified LOD.

On average, these processes cause the length of day to increase by about between 1.7 and 2.3 milliseconds per century. Recast in hours, this means the day lengthens by roughly 76 minutes, or 1 hour and 16 minutes, every 200 million years. Despite this being an average trend, the temporal evolution of LOD appears irregular, resembling a noisy yet periodic signal with distinguishable peaks.

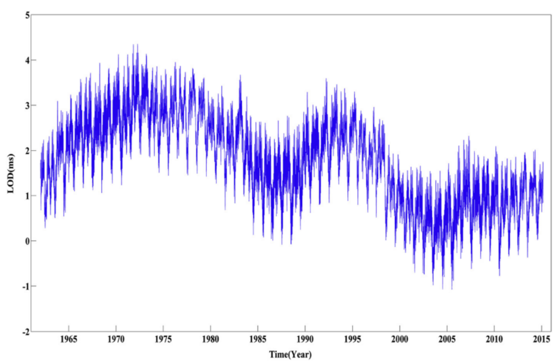

Figure 1.5: Length-of-day (LOD) variations relative to 86 400 s, 1962-2023 [25]

As illustrated in the Figure 1.5., in 1973 the day was 0.004 ms longer than average, whereas in 2022 it was 0.002 ms shorter, indicating a slightly faster Earth rotation in the latter.

1.3.5 Ephemeris Time

Ephemeris Time (ET) has been introduced in 1952 by the CGPM recommended to define the SI second from it in 1956. The definition of ET is derived from Sun's geometric mean longitude

$$L = 279°41'48.08'' + 129\,602\,768.13''T + 1.089T^2, \tag{1.22}$$

where T is measured in Julian centuries (each consisting of 36 525 ephemeris days), starting from 12:00 on January 0, 1900, when the Sun's longitude was 279°41′48.08″ [27].

By 1989, the discrepancy between Ephemeris Time and Universal Time had reached 56 seconds, illustrating the insufficiency of Earth-rotation-based definitions.

The 10$^{\text{th}}$ CGPM (1954) defined the second as:

$$\frac{1}{31\,556\,925.9747}, \tag{1.23}$$

of the tropical year [28] at 12:00 on January 0, 1900, ephemeris time. This definition was formalized by the 11$^{\text{th}}$ CGPM in 1960. Initially, the sidereal year of 1900 was considered, but the tropical year, corresponding to the return of the Sun to the same point on the ecliptic, was ultimately preferred for its greater long-term stability. Nonetheless, this definition was short-lived. In 1967, it was superseded by the atomic definition of the second, offering superior precision and long-term consistency. The tropical year itself is not constant. Its duration is affected by nutation and planetary perturbations, making it difficult to model precisely. Today, it is defined as the interval over which the Sun's mean tropical longitude increases by 360°. Due to the secular change in the precession rate, the tropical year is gradually shortening. As of the year 2000, the tropical year was estimated at 365.24219 days and is decreasing by about 0.52820 seconds per century. This variation can be modeled using Taylor expansions, but it remains inherently complex. The associated motion of the vernal point, used as a reference in defining the tropical year, is also difficult to predict. These limitations ultimately rendered ephemeris time unsuitable and paved the way for the adoption of atomic time standards [3].

1.3.6 Transition to the Atomic time

Among the various concepts considered for defining time scales, one in particular is unsuitable as a basis for UT. Sidereal time, based on the apparent motion of stars relative to the vernal equinox, cannot serve as a stable reference because the vernal point undergoes non-uniform motion, due to precession and nutation, predictable but time-dependant motion. Solar time, by contrast, is grounded in the observation of the Sun's passage over the local meridian. This allows for the establishment of a coherent time scale known as UT. However, due to the variability in Earth's rotational speed, even this time scale exhibits long-term drift. To provide a uniform dynamical time scale independant of Earth's rotation, ET was introduced, based on the Earth's orbital motion, yet it too is affected by long-term changes such as the gradual shortening of the tropical year. To overcome these limitations, the SI second was redefined in 1967. It is based on atomic physics, specifically the hyperfine transition frequency of the cesium-133 atom. This atomic definition offers unmatched precision and stability and is being considered for redefinition un the 2030s, contingent on CGPM criteria [29, 30].

Chapter 2

The Universal Time Coordinated

This chapter provides all the elements necessary to understand the current definition of the second and its realization and international dissemination. It begins with the definition of the second and practical realizations, based since 1967 on the hyperfine transition of cesium-133's ground state, then describes the operation of the atomic clocks used to realize this definition, particularly cesium fountain clocks.

The second part is devoted to the computation of International Atomic Time (TAI) using the BIPM time-scale alogorithm, ALGOS, before introducing the establishment of Coordinated Universal Time (UTC) as the civil time scale derived from TAI, including the issue leap seconds inserted to keep UTC close to Earth's rotation.

Finally, the chapter addresses the stability and accuracy of clocks, introduces the Allan variance, and discusses applications and performances metrics of these clocks.

2.1 International Atomic Time and the caesium second

2.1.1 Definition of the SI second

The second is currently defined based on an atomic transition of cesium-133. Historically, as we saw in the previous chapter, and until the 1960s, the second was defined based on the rotation of the Earth. However, there are irregularities in this rotation, which meant that this definition was not the most stable [5, 31]

In 1967, the CGPM adopted a new definition of the second, based on atomic physics, following the major advances in atomic physics and quantum mechanics throughout the 20th century [31, 32]. This definition is as follows: **The second is the duration of** 9 192 631 770 **periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom**. This means that in one second, there are about exactly 9 192 631 770 oscillations of the electromagnetic wave during the hyperfine transition [31].

This hyperfine transition corresponds to the energy difference between two energy levels of the cesium atom in its ground state: the hyperfine levels $F = 3$ and $F = 4$. This definition is specific to a cesium atom which is at 0K [5, 33]. Thus, the second is defined by setting the numerical value of the hyperfine frequency of cesium, denoted $\Delta\nu_{\mathrm{Cs}} = 9\,192\,631\,770$ Hz. Since this value defines the SI unit of second, it is set as exact, without uncertainty, as illustrated in Figure 2.1.

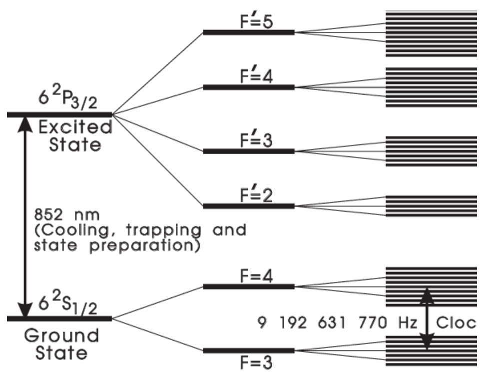

Figure 2.1: Hyperfine structure of the states $6\,^2S_{1/2}$ and $6\,^2P_{3/2}$ of the cesium-133. The transition used in the definition of second is realised between $F = 3$ and $F = 4$ [34]

2.1.2 Cesium Atomic Clocks

A cesium atomic clock uses the hyperfine transition of the cesium atom as a frequency standard for measuring time. In a conventional cesium clock, called a thermal beam clock, cesium atoms are projected in the form of a beam passing through a microwave cavity. Two separate interaction zones are then used to produce a disturbance of the atoms by the electromagnetic wave produced by the quartz, locked to the atomic resonance. This cavity is called a Ramsey cavity [4, 35]. Thus, the atoms, initially prepared in a hyperfine state, will interact with the microwave field whose frequency is close to $\Delta\nu_{\mathrm{Cs}}$. If the frequency is exactly the same as the transition frequency, and is therefore resonant, the atoms will undergo a transition between the two levels, before being able to evolve freely and undergo the electromagnetic interaction again. This has the effect of having atoms in one of the hyperfine states, which is the other state than the one in which the atoms were prepared. A selection system will then make it possible to determine whether the atoms have actually passed into the other state or not. The quartz oscillator that produced the initial wave is then controlled at this resonance frequency so that the oscillator can produce an oscillation identical to the frequency of cesium [4, 5]. The complete representation of a cesium beam clock is illustrated in Figure 2.2.

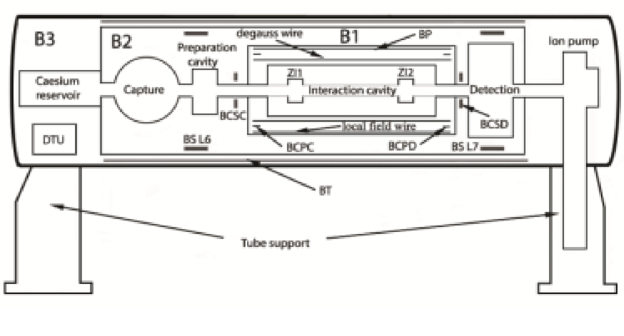

Figure 2.2: Schematic of a cesium beam atomic clock [36]

The oscillator in question will have a certain precision, which depends on the width of the resonance line. The width of this line, obtained through the Heisenberg uncertainty principle for the energy

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{2.1}$$

is determined by the duration of the interaction of the atoms with the microwave field, and more specifically the free zone between the two branches of the Ramsey cavity and given by

$$\Delta \nu \sim \frac{1}{T} \tag{2.2}$$

. In this type of standard, the optimal distance was of the order of a meter, in order to have a narrow line width, increasing precision [35, 37]

The development of laser atom cooling techniques has enabled the evolution of cesium beam clocks into fountain clocks. In this case, the atoms are cooled to temperatures in the microkelvin range, based on Doppler cooling, to reduce their speed to a few cm/s. These atoms move more slowly and allow for a longer interrogation time between the two passages through the electromagnetic wave. This technique led to the creation of fountain clocks themselves. The principle of these clocks is to use gravity to extend the interrogation time of the atoms, once cooled, following a vertical round trip. Thus, to be cooled, the atoms are first trapped in a magneto-optical trap before undergoing Doppler cooling by three pairs of two counter-propagating laser beams, installed in three spatial directions. In addition to the residual displacement due to the quantum nature of these atoms, velocity greatly reduced (few cm/s), not stricly zero. The two vertical lasers will then be slightly detuned in frequency to propel the atoms upward, turning off the other lasers so they can move freely. The atoms then rise to a typical height of 1 meter before descending, producing a trajectory of about one second. During this flight, the atoms will pass twice through a microwave cavity placed at the base of the fountain. This cavity is tuned to the frequency $\Delta\nu_{\mathrm{Cs}}$. The first rise acts as a first impulse in the case of jet clocks; they evolve freely, before passing through the cavity again [35, 38, 39]. The atoms are thus prepared in the state $|F = 3\rangle$. They will interact with the microwave field in the cavity for a time τ, such that their state will be a superposition of the two hyperfine states, the transition being expected to induce a transition, if it is close to resonance:

$$|\psi(\tau)\rangle = \alpha(\tau)|F = 3\rangle + \beta(\tau)|F = 4\rangle \tag{2.3}$$

During the free evolution T, the state will evolve according to the Schrodinger equation:

$$|\psi(\tau + T)\rangle = \alpha(\tau + T)|F = 3\rangle e^{-i\frac{E_3}{\hbar}T} + \beta(\tau + T)|F = 4\rangle^{-i\frac{E_4}{\hbar}T} \tag{2.4}$$

The second interaction will allow the amplitudes to be recombined. Once the process is complete, the atoms will interact with a laser, which will cause fluorescence when the atoms are in the state $|F = 4\rangle$. This illumination makes it possible to measure the rate of photons emitted and thus indicate the proportion of atoms that have changed state under the effect of the wave [4, 35, 40]. We have a schematic representation of this type of clock in Figure 2.3.

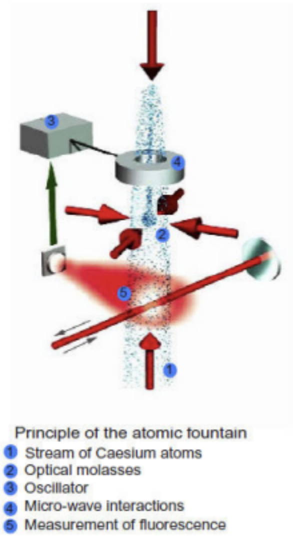

Figure 2.3: Operating principle of a cesium fountain clock [36]

2.1.3 Construction of TAI

International Atomic Time (TAI) is an atomic time scale computed by the BIPM. It is a uniform, continuous time, realized on the basis of the cesium atomic second and achieved by pooling a large number of atomic clocks around the world. TAI is not the time in common use, but it helps define UTC by being a stable and precise frequency reference [32, 41]. Currently, more than 400 clocks are involved in the implementation of TAI. This is achieved by constructing a time scale [32, 42].

To construct a time scale, it is necessary to perform a weighted average of N independent clocks. Let $h_i(t)$ be the reading of clock $H_i(t)$ at time t, $w_i(t)$ the statistical weight of the clock, we define a time scale $TS(t)$ as

$$\mathrm{TS}(t) = \sum_{i=1}^{N} w_i h_i(t) \tag{2.5}$$

assuming the sum of the weights is equal to 1:

$$\sum_{i=1}^{N} w_i(t) = 1 \tag{2.6}$$

This definition of a time scale is ideally given by a weighted sum of readings. However, in practice, clocks have different precision qualities and there may be drift, failures, additions or removals of clocks, it is necessary that this time scale continues to function all the same [4, 43].

To account for these realities, the BIPM proposes an algorithm for implementing TAI, and by extension UTC, based on a time scale called the free atomic scale (EAL). This algorithm, called ALGOS, proposes writing the atomic scale as follows:

$$\mathrm{EAL}(t) = \sum_{i=1}^{N} w_i \left[h_i(t) + h'_i(t)\right] \tag{2.7}$$

where $h'_i(t)$ is a prediction term for the clock $H_i(t)$. This added term prevents time or frequency discontinuities from occurring in EAL. Thus, each clock is applied to ensure that no jumps occur in the time scale [32, 42, 43].

In the ALGOS algorithm, the data involved are the time differences measured between pairs of clocks. We denote by $x_{ij}(t) = h_j(t) - h_i(t)$ the difference in reading between clocks H_j and H_i at time t. ALGOS then requires the introduction of $x_j(t) = EAL(t) - h_j(t)$. We then have, for each clock $H_j(t)$

$$EAL(t) - h_j(t) = \sum_{i=1}^{N} w_i \left[hi_(t) + h'_i(t)\right] - h_j(t) \tag{2.8}$$

$$= \sum_{i=1}^{N} w_i \left[h_i(t) - h_j(t) + h'_i(t)\right] \tag{2.9}$$

$$= \sum_{i=1}^{N} w_i \left[h'_i(t) - x_{ij}(t)\right] \tag{2.10}$$

In practice, we solve this equation iteratively over the monthly computation interval to obtain the values of $EAL(t)$ for the period in question.

There are then other algorithms that come into play in ALGOS. First, we have the prediction algorithm, which calculates the term $h'_i(t)$ for each clock to ensure continuity in time and frequency for each period and avoid jumps, for the reasons already mentioned.

To avoid these discontinuities, the BIPM models the evolution of each clock using a polynomial model. Thus, if clock H_i has, at the beginning of the interval, a deviation a_i from EAL and a relative frequency B_{ip}, then the prediction term is given by

$$h'_i(t) = a_i + B_{ip}(t - t_i) \tag{2.11}$$

where t_i is the beginning of the interval under study. In some models, a third term accounts for frequency drift. This term, C_i, is a quadratic term, and occurs in the case of clocks with significant drift, which is weaker in the case of cesium clocks than in others microwave clocks. In this case, we have

$$h'_i(t) = a_i + B_{ip}(t - t_i) + \frac{1}{2} C_i (t - t_i)^2 \tag{2.12}$$

These terms are chosen precisely: a_i is chosen so that at time t_i, EAL and H_i coincide. This term is called "phase correction" [32]. Then, B_{ip} is chosen to prevent frequency hopping. Thus, EAL will be continuous in time and frequency from one month to the next, even if the practical definition of EAL changes [42, 43].

The second useful algorithm for ALGOS is the choice of weights w_i assigned to each clock. The objective of this choice is to maximize the stability of the timescale by giving more weight to clocks whose stability is proven. In practice, the weight w_i of a clock H_i is determined based on its frequency instability. Thus, the more stable a clock, the greater its weight. The BIPM will thus estimate the stability of each clock using the coefficient B_{ip}. Thus, the BIPM will determine the difference between the measured frequency and the frequency predicted by the prediction algorithm over several past intervals. The algorithm will use these deviations to calculate a variance σ_i^2, using the different deviations over the previous 12 months. A weight is then assigned such that

$$w_i \propto \frac{1}{\sigma_i^2} \tag{2.13}$$

The weights are then calculated monthly, taking into account the clocks' performance. Thus, the weight is modified according to the improvement or degradation of the clocks' performance [41, 43, 44].

Thus, once EAL is calculated, its frequency is adjusted to conform to the SI definition of the second. The BIPM thus compares the frequency of EAL to that of the primary cesium standards [32, 45]. An estimate of the difference

$$\Delta = f(EAL) - f(\mathrm{SI}) \tag{2.14}$$

is made. If $\Delta \neq 0$, beyond a certain degree of uncertainty, then a frequency correction is applied such that

$$f(EAL) \approx f(\mathrm{SI}) \tag{2.15}$$

These corrections are nevertheless very small, of the order of 10^{-15}, and are made after several months so as not to permanently modify TAI, and so that the drift of TAI relative to the second is a maximum of 10^{-13} [32, 45].

2.2 Setting up UTC

TAI thus provides an ideal uniform scale, but it is not linked to any physical phenomenon directly observable by an average human, and has no officially established link with the time scale used at the time, namely UT [31, 32].

Coordinated Universal Time (UTC) was then introduced in 1961 to link TAI and UT. UTC is therefore defined as an atomic scale with the same unit and speed as TAI, except that it differs from TAI by an integer number of seconds, which are leap seconds, added to maintain the condition [31]

$$|\mathrm{UT} - \mathrm{UTC}| < 0.9\,\mathrm{s} \tag{2.16}$$

Concretely, UTC was established on January 1, 1972, with a 10-second offset from TAI, in order to compensate for existing differences. Seconds were then added to UTC at regular intervals (every 18 months) to compensate for the slowing of the Earth's rotation. As of January 1, 2017, the difference between TAI and UTC was 37 seconds, with the last second being added on December 31, 2016 [2, 31]. Currently, no more leap seconds have been added, and it is increasingly common within the scientific community to suggest that leap seconds could be eliminated. Leap seconds could be eliminated in the event of a sustained acceleration of the Earth's rotation, which cannot be ruled out and has already been partially observed for several years [7, 31]. It can be concluded that this shift requiring the addition of leap seconds is due to the change in the length of the day. On average, over a 24-hour day, or 84 600 seconds, the length of the day is exceeded by an average of 2.1 ms, mainly due to tidal effects, over a century. This excess duration, multiplied by the number of days in a year, i.e., 365.25, leads to an accumulated delay of approximately 0.76 s, compared to TAI [31]. The addition of leap seconds thus reduces this difference to 0.9 s. Leap seconds were thus added following Table 2.1. In addition, the difference between TAI and UTC is simulated in Figure 2.4.

Date (UTC)	**Leap second**	**Date (UTC)**	**Leap second**
1972-06-30	+1 s	1985-06-30	+1 s
1972-12-31	+1 s	1987-12-31	+1 s
1973-12-31	+1 s	1989-12-31	+1 s
1974-12-31	+1 s	1990-12-31	+1 s
1975-12-31	+1 s	1992-06-30	+1 s
1976-12-31	+1 s	1993-06-30	+1 s
1977-12-31	+1 s	1994-06-30	+1 s
1978-12-31	+1 s	1995-12-31	+1 s
1979-12-31	+1 s	1997-06-30	+1 s
1981-06-30	+1 s	1998-12-31	+1 s
1982-06-30	+1 s	2005-12-31	+1 s
1983-06-30	+1 s	2008-12-31	+1 s
		2012-06-30	+1 s
		2015-06-30	+1 s
		2016-12-31	+1 s

Table 2.1: List of positive leap seconds added to UTC (1972–2025) [46]

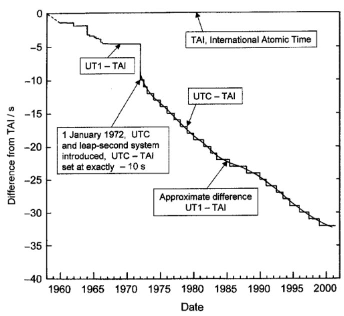

Figure 2.4: Accumulated difference between TAI and UTC since 1972[46]

2.3 Accuracy and stability of atomic clocks

2.3.1 Definition

Atomic clocks are the most accurate and available frequency standards currently used to implement the definition of the second. To characterize them, the concept of stability is used [5, 45]. Stability refers to a clock's ability to provide a constant frequency. In other words, it is the ability to achieve the smallest possible random fluctuations in the instantaneous frequency. It is typically assessed by frequency instability, generally through the Allan variance. In practice, the performance of an atomic clock is measured by its frequency instability, as a function of the integration time, and by its residual systematic uncertainty. For example, cesium fountain clocks achieve a systematic uncertainty of the order of 10^{-16}. This means that atomic clocks with this uncertainty drift by a maximum of one second every 100 million years. We will see in the next chapter that new optical clocks offer better performance: uncertainties reach the order of 10^{-18}, equivalent to a shift of one second over the entire age of the Universe, which is of the order of tens of billions of years [33, 47]. Also, the short-term stability determined by the Allan deviation is 10^{-16} for one second of integration, while for cesium atomic clocks it is 10^{-14} for one second of integration.

2.3.2 Allan Variance and deviation

To quantify the frequency stability of atomic clocks, it is common to use the Allan variance, denoted by $\sigma_y^2(\tau)$. We also introduce the Allan deviation, which is the square root of the Allan variance, denoted by $\sigma_y(\tau)$ [37]. Formally, the Allan variance is defined as follows:

$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{i=1}^{M-1} (y_{i+1} - y_i)^2 \tag{2.17}$$

where M is the number of consecutive measurements of the average frequency over τ. We also define $y(t)$ as the instantaneous fractional frequency of a clock, defined as the ratio of the difference between the clock frequency at time t, $\nu(t)$, and the nominal frequency ν_0 which is the reference atomic frequency, divided by the nominal frequency. Thus, $y(t) = \frac{\nu(t)-\nu_0}{\nu_0}$ [5, 37]. Statistically, the Allan variance measures the magnitude of frequency variations from one interval to another. This is the advantage of the Allan variance over the classical variance: this variance is calculated over two intervals, not just one. Then, to determine the Allan deviation, we simply take the square root of the Allan variance. We therefore have [37]

$$\sigma_y(\tau) = \sqrt{\frac{1}{2(M-1)} \sum_{i=1}^{M-1} (y_{i+1} - y_i)^2} \tag{2.18}$$

2.3.3 Application to atomic clocks

We then apply these concepts to the atomic clocks currently used in the definition of the second. These clocks include hydrogen masers, rubidium clocks, and cesium clocks.

Let's start by briefly describing rubidium atomic clocks. These clocks use the hyperfine transition of rubidium-87 as a reference. This transition corresponds to the energy difference between two hyperfine levels of the ground state of rubidium-87 (typically between the $F = 2$ and $F = 1$ levels), with a frequency of about 6.83 GHz. The physical apparatus of rubidium clocks are vapor cell devices. These cells contain gaseous rubidium in a glass cell maintained at room temperature. A discharge lamp or laser tuned to the rubidium frequency illuminates the cell to prepare the atoms in a hyperfine state by optical pumping. When the atoms are subjected to microwave interaction, they absorb energy in order to move from one level to another of the transition, in the same principle as cesium clocks, as represented in Figure 2.5. The intensity of the light in the cell is then observed, since the rubidium atoms will absorb more light when the light source is close to the resonant transition. This decrease in light intensity is measured using a photodetector that allows a servo system to calibrate the quartz based on the frequency of the rubidium transition. [4, 35, 48]

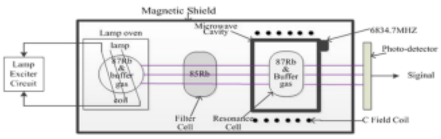

Figure 2.5: Principle of a rubidium vapor cell clock [48]

Let's now look at hydrogen masers. These are a type of clock that uses the principle of stimulated emission to generate a coherent signal from hydrogen atoms. Based on its origin, the word maser is identical to the word laser, except that lasers use amplification of the emitted light through stimulated emission, whereas in the case of the hydrogen maser, it is the microwave that is amplified through stimulated emission. In this case, the atomic transition is the hyperfine transition of the ground state of the hydrogen atom between the states $F = 0$ and $F = 1$, with a frequency of 1.42 GHz, which corresponds to a wavelength of 21 cm. In principle, hydrogen masers consist of a source of dihydrogen with a discharge system that dissociates the molecules into hydrogen atoms, a furnace, and a collimator to form a hydrogen beam. This entire device is associated with a microwave cavity: the gas is injected and dissociated.

A large number of atoms then pass through a region equipped with magnetic fields, produced by Stern-Gerlach states, which deflect and eliminate the atoms in the lowest hyperfine state, retaining only the atoms in the upper hyperfine state. It then passes through a microwave cavity tuned to the transition frequency before being trapped in a storage cell. A principle scheme can be seen at Figure 2.6. [4, 5, 49].

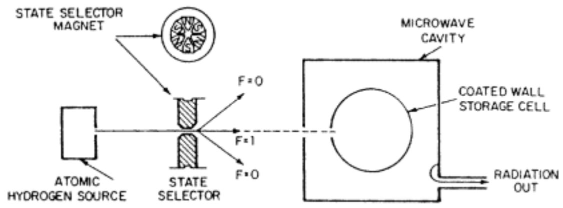

Figure 2.6: Principle of a hydrogen maser [49]

Meanwhile, a certain number of atoms will spontaneously emit a microwave photon upon falling back into the ground state, and these photons will induce stimulated emission of other photons in the proportion of atoms that are still excited. The production of photons will produce an amplification phenomenon in the cavity to amplify the microwave field. When this amplification is sufficient, the cavity begins to oscillate on the hydrogen hyperfine transition, which thus produces a microwave signal at the hyperfine frequency. The signal is then extracted and used in a quartz servo system. Moreover, the generic form of the Allan deviation used to determine the stability is given by [5, 37]

$$\sigma_y(\tau) \approx \frac{\sigma_0}{\sqrt{\tau}} \tag{2.19}$$

where σ_0 is the Allan deviation for one second, and τ is the interrogation time.

The differences in frequency and stability are thus given in the Table 2.2., and a comparison between interesting clocks is given in Figure 2.7. [5, 35, 45]

Clock type	Allan deviation (1s)	Allan deviation at 1 day	Accuracy
Commercial cesium beam	$\sim 10^{-11}$ – 10^{-12}	$\sim 10^{-14}$	$\sim 10^{-13}$
Cesium fountain	5×10^{-13} – 10^{-13}	$\sim 10^{-16}$	$\sim 10^{-16}$
Rubidium vapor cell	$\sim 10^{-12}$	10^{-15} – 10^{-14}	10^{-11} – 10^{-12}
Active hydrogen maser	$\sim 10^{-13}$	$\sim 10^{-15}$	$\sim 10^{-12}$

Table 2.2: Typical performance of selected atomic clocks used in timekeeping applications [5, 35, 45]

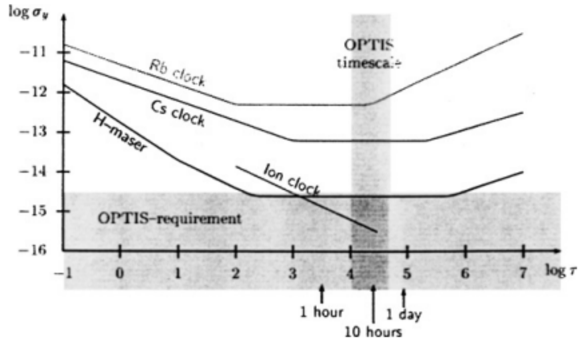

Figure 2.7: Typical log-log plot of Allan deviation versus integration time τ for various types of atomic clocks [46]

We can also see that studying this statistical estimator over time is also important. Indeed, in cases of relatively short time, hydrogen clocks have a certain advantage, while the others have a lower accuracy. On the other hand, when we evolve over time, we notice that the accuracy decreases for these types of clocks, so much so that it will go back above those of cesium. [5, 35, 45]

2.3.4 Example of numerical estimation

To illustrate the calculation of Allan's variance, consider a simulated data set representing the frequency fluctuations on a clock over 10 seconds. Suppose that each second (with $\tau_0 = 1\text{s}$), we measure the deviation $y_i = \frac{\Delta\nu}{\nu}$. Consider a set of 1 points, expressed in 10^{-16}, to correspond to the order of magnitude corresponding to cesium clocks:

$$y = [2.0,\ 1.5,\ 0.9,\ -0.1,\ -0.5,\ -0.3,\ 0.2,\ 0.4,\ 1.1,\ 0.7] \times 10^{-16} \tag{2.20}$$

Let us calculate the Allan variance for $\tau = 1$ from these data. According to the definition, we calculate for each data $y_{i+1} - y_i$, with

$$\begin{aligned} \Delta y &= [1.5 - 2.0,\ 0.9 - 1.5,\ -0.1 - 0.9,\ -0.5 + 0.1, & (2.21)\\ &+ -0.3 + 0.5,\ 0.2 + 0.3,\ 0.4 - 0.2,\ 1.1 - 0.4,\ 0.7 - 1.1] \times 10^{-16} & (2.22)\\ &= [-0.5,\ -0.6,\ -1.0,\ -0.4,\ 0.2,\ 0.5,\ 0.2,\ 0.7,\ -0.4] \times 10^{-16} & (2.23)\end{aligned}$$

The square of these values is given by

$$\Delta y^2 = [0.25,\ 0.36,\ 1.0,\ 0.16,\ 0.04,\ 0.25,\ 0.04,\ 0.49,\ 0.16] \times 10^{-32} \tag{2.24}$$

Thus, the Allan variance is given by

$$\sigma_y^2(1\text{s}) = \frac{1}{2\times 9}\sum_{i=1}^{9}(\Delta y_i)^2 \tag{2.25}$$

$$= 2.75 \times 10^{-32} \tag{2.26}$$

The Allan deviation is then equal to

$$\sigma_y(1\text{s}) = 1.24 \times 10^{-16} \tag{2.27}$$

which is perfectly consistent with the results proposed for a cesium clock.

2.4 Transition towards optical clocks

The standards currently used to define the second are high-performance. However, the scientific community is considering a redefinition of the second under consideration for the 2030s based on optical clocks. These clocks, instead of using a microwave hyperfine transition, will use optical transitions with frequencies higher than the microwave transition, and located in the visible or near-infrared to ultraviolet range [7, 38, 50]. For example, we find optical lattice clocks, as well as ion clocks, which will use various types of transitions that will be explained in the next chapter. Their key advantage of these transitions is a much higher oscillation frequency and ultra-narrow natureal linewidths, yielding a very large quality factor $Q = \nu/\Delta\nu$. They can thus achieve greater precision [38, 39]. In practice, optical clocks achieve fractional systematic uncertainties at the 10^{-18} level, which improves the accuracies of cesium clocks by two orders of magnitude, but also short-term instabilities of order of 10^{-16} - 10^{-17} over average interrogation times $\tau \approx 1$s, depending on the design. We therefore have prospects for further improvements by one to two orders of magnitude [50, 51, 52].

Chapter 3

Towards redefining the second in the 2030s

The upcoming redefinition of the second represents a decisive step in the evolution of the International System of Units. By 2030, a transition from the cesium-133 reference to optical references marks a technological and conceptual turning point in time metrology [7, 50, 53]. After a detailed study of the architecture of atomic clocks, candidate transitions, systematic effects, and the criteria imposed by the CGPM [38, 39], it is appropriate to take a position on the possible paths.

This chapter constitutes a personal and well-argued contribution to this issue. It aims to compare the different redefinition scenarios with the available experimental data, metrological constraints, definition challenges, and operational practicality [38, 50, 54, 55]. Each option is analyzed with regard to the performance of the corresponding clocks, and the feasibility and stability of the clocks and their implementation [38, 50, 56, 57]. Through this reflection, I develop a position based on an informed approach, drawing on relevant indicators: Allan variance, systematic uncertainty, robustness, reproducibility, not forgetting institutional timelines and practical considerations [6, 37, 50, 58, 59]. The goal is to demonstrate why certain options appear more robust, more coherent, and more realistic by 2030.

3.1 The need for redefinition

3.1.1 Limits of the current definition

The second of the International System in its current form has been defined since 1967. It is defined on the basis of the hyperfine transition of the hyperfine levels of the ground state of cesium-133 [7, 38, 50]. As technology continues to evolve, the most advanced cesium-133 clocks available today are the fountain clocks already discussed in the previous chapter [35, 54]. These clocks define the second with a fractional uncertainty of order 10^{-16}s, corresponding to a slight drift over time of around 1 second every 100 million years [38, 50]. Cesium clocks are thus approaching their fundamental limits, despite the many technological improvements made to the components of these clocks, such as the lasers used to cool the atoms [35, 38]. Further improvements in the accuracy of cesium clocks are therefore rather complicated, not least because of the systematic errors present in the device, such as the Doppler effect and the limited free-evolution time in Ramsey interrogation [58].

The aim is therefore to develop a new definition of the second, which would no longer be based on a microwave transition (9.192 631 770 GHz for Cs-133), as is currently the case, but rather on clocks using optical transitions, which have transition frequencies much higher than the hyperfine frequency of cesium [7, 39, 50, 56].

3.1.2 Motivations and challenges

The redefinition of the second is based on several reasons. Technological progress allows us to think about potential and future evolutions of this definition, notably by using new clocks that emerged over the past two decades, allowing a greater stability of the second, and for which systematic uncertainties at the level 10^{-18}, that is, a drift smaller than one second over the entire age of the Universe[38, 50, 60]. Furthermore, the international system of units is constantly evolving in order to be able to integrate the numerous advances in the fields of fundamental physics and in particular by using the fundamental quantities of quantum mechanics and general relativity: the definition of the meter is based on the speed of light c (1983), and that of the kilogram is based on that of the Planck constant h (2018) [53], in order to allow better temporal reliability, or even to tie units to invariant constants and improve long-term stability. Obviously, the stakes are high: firstly, this new definition, although the previous one is sufficiently precise for the uses currently made of it, will allow the development of more reliable processes for verifying fundamental theories: we are thinking in particular of more in-depth tests of general relativity on smaller scales, but also the installation of sensors used to determine more precisely the gravitational shift and thus evaluate more precisely the curvature of space-time [7, 38, 39]. It is also possible to implement more efficient geolocation processes, particularly at the millimeter scale, although this use is not of great interest at present [50].

3.2 Cesium atomic clocks

As already explored in the previous chapter, the most accurate primary standards realizing the SI second are cesium fountain clocks. They are based on the cooling of cesium-133 atoms launched vertically into a microwave cavity where they are interrogated by a microwave field derived froma quartz and locked to the atomic resonance [35, 38, 39]. The cesium atoms will thus interact with the electromagnetic wave for the first time when they are launched into the cavity, before moving freely, and will interact with the wave a second time when they descend back into the cavity, under the effect of gravity [35]. A detector formed by a cycling laser that measures the state in which the cesium atom is placed after the second interaction and makes it possible to verify whether the quartz frequency is thus correct [35, 55]. The best cesium fountains (notably NIST-F2, the American primary standard, or PTB-CSF2, a German standard) achieve fractional uncertainties 10^{-16}s [35, 38, 50].

Optical atomic clocks are clocks that exploit very narrow electronic transitions. There are two types: optical lattice clocks and trapped-ion clocks [38, 39, 50]. Before exploring the two types of optical clocks, it's useful to remember that the operation of a clock, whether microwave or optical, is based on the control of a quartz crystal by a mechanism that regulates its frequency based on the signals given by the clock [4, 38]. Thus, the quartz is placed in a piezoelectric quartz crystal resonator. It will oscillate at its own frequency. The frequency of the quartz will then be extracted and synthesized so that it can be used in the form of an electromagnetic wave. This frequency will be close to the resonant transition of the atom or ion used in the clock and we will be able to determine during the interrogation phase whether the frequency of the quartz is correct. Whether it is correct or not, an electronic system will thus make it possible to control the quartz thanks to the piezoelectric system in which it is placed so that its frequency is adapted, if necessary [4, 5, 35].

3.3 Optical lattice clocks

Optical lattice clocks exploit stable transitions of neutral atoms trapped in optical lattices. Because the atoms are neutral and spin-polarized, there are weak interactions between these atoms and it is possible to use a large number of atoms, improving the stability of the clock [38], represented in Figure 3.1.

3.3.1 Principle of optical trapping in a lattice

Optical trapping of neutral atoms uses the alternating dipole potential induced by a laser electromagnetic field that is frequency-shifted relative to the resonant transition of the atom in question. In this case, the potential energy of an atom in the laser field is given by

$$U(\mathbf{r}) = -\frac{1}{4}\alpha(\omega_L)\left\langle E^2(\mathbf{r})\right\rangle \tag{3.1}$$

where $\alpha(\omega_L)$ is the dynamic polarizability of the state, dependent on the laser frequency used, and $E(\mathbf{r})$ is the amplitude of the electric field [61]. The sign of the polarizability will thus make it possible to determine whether the atom is attracted towards the antinodes of the field ($\alpha > 0$, so-called red trap) or repelled towards the nodes ($\alpha < 0$, so-called blue trap).

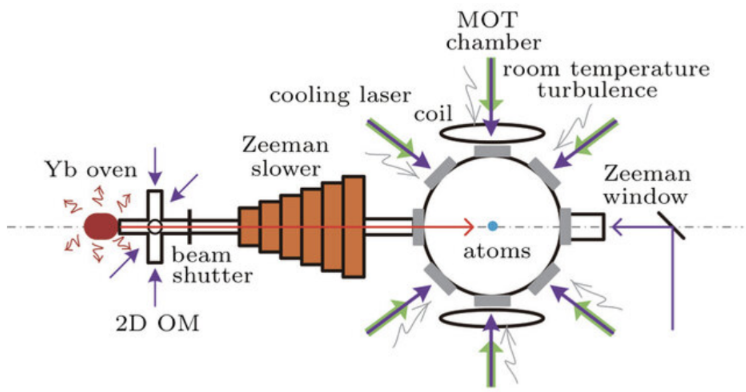

Figure 3.1: Experimental realization of a ytterbium optical lattice using laser coolling in a 1D optical lattice [62]

We can then study the 1D optical lattice [63]. An example of a 3D optical lattice can be seen in Figure 3.2. It is formed by two coaxial lasers facing each other, with the same wavelength λ and linearly polarized. These conditions imply that the lasers create a standing wave. The atom will then experience a periodic potential along the z axis given by

$$U(r,z) = U_0 \exp\left(-\frac{2r^2}{\omega_0^2}\right)\sin^2(kz) \tag{3.2}$$

where U_0 is the definition of the intensity of a laser beam. Indeed, we have

$$I = c\varepsilon_0 \frac{\langle E^2 \rangle}{2} \quad \Leftrightarrow \quad \langle E^2(\mathbf{r})\rangle = -\frac{4U(\mathbf{r})}{\alpha(\omega_L)} \tag{3.3}$$

Given the definition of $U(\mathbf{r})$, we have

$$U = -\frac{1}{2\varepsilon_0 c}\alpha I \tag{3.4}$$

which corresponds to U_0 of the potential, and which is the depth of the well. Furthermore, ω_0 is the beam width, I_0 is the laser intensity, and $k = 2\pi/\lambda$ is the wavenumber. This optical lattice then has a mesh of length $\lambda/2$. For a red trap, $\alpha > 0$ and the atoms are trapped at the intensity peaks $z = \frac{n\lambda}{2}$, forming pancakes separated by $\lambda/2$. For blue traps, the intensity minima are trapping. At the bottom of the well, the potential is similar to a harmonic oscillator of frequency

$$\omega = \sqrt{\frac{2U_0k^2}{2M}} \tag{3.5}$$

where M is the mass of the atom. If we define the recoil energy as $E_R = \frac{\hbar^2 k^2}{2M}$ and we have such an energy much lower than $\hbar\omega$, the system satisfies the Lamb-Dicke condition. In this case, the absorption of a photon by a trapped atom does not change its state: the absorbed frequency then corresponds to the internal atomic frequency (i.e. the frequency of the atomic transition). This can also be seen as a displacement of the atom considered over a distance less than the wavelength of light. This actually implies that the atom is trapped in the lattice [38, 64].

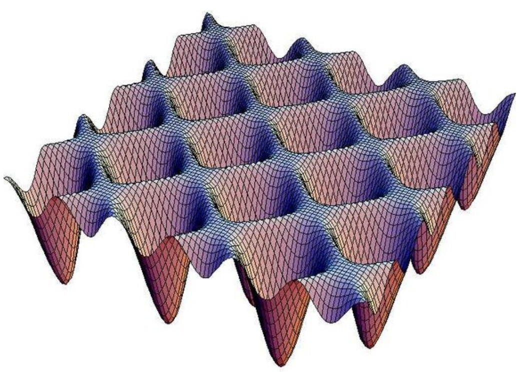

Figure 3.2: Schematical representation of an optical lattice [65]

3.3.2 Magic wavelength and dynamical polarizability

The main problem with lattice clocks is that the laser lattice field induces an Autler-Townes effect [66]: the oscillating electric field produced by the laser will modify the energy levels of an atom, causing a frequency shift of the atomic transition, particularly significant when the transition frequency is close to the laser frequency. This effect is actually the alternating counterpart of the Stark effect. This frequency shift will be different at the two levels of the transition, resulting in a drift in the clock frequency, since the transition is disturbed. This frequency deviation is given by

$$\begin{aligned}
\Delta\nu &= \frac{\Delta E}{h} && (3.6)\\
&= \frac{1}{h}\left[U_e(\omega_L) - U_g(\omega_L)\right] && (3.7)\\
&= -\frac{1}{4h}\left[\alpha_e(\omega_L) - \alpha_g(\omega_L)\right]\langle E^2\rangle && (3.8)\\
&= -\frac{1}{4h}\Delta\alpha_i(\omega_L)\langle E^2\rangle && (3.9)
\end{aligned}$$

where e and g denote excited and ground state respectively. Using the definition of intensity, we have

$$\Delta\nu = -\frac{1}{2\varepsilon_0 ch}\Delta\alpha(\omega_L)I \tag{3.10}$$

However, it is possible to eliminate this effect by tuning the wavelength of the laser creating the optical lattice to the magic wavelength λ_m. This wavelength will thus allow the same shift to be imposed on the transition levels [63, 67]. Let's take ^{87}Sr for example: the studied transition of ^{87}Sr, which will be detailed later, is

$$|g\rangle =^1 S_0 \leftrightarrow |e\rangle =^3 P_0 \tag{3.11}$$

with a transition frequency given by $\nu_0 \approx 429 \times 10^{12}$ Hz, or $\lambda_0 \approx 698$ nm. Furthermore, the magic wavelength of this atom has been experimentally measured to be $\lambda_m \approx 813.43$ nm. Let's also assume a typical laser beam intensity of $I = 10^7$ W/m^2. Under these conditions, it was experimentally determined that

$$\frac{d\Delta\nu}{d\lambda} \approx 2.5\,\text{Hz/mW.nm} \tag{3.12}$$

Thus,

$$\begin{aligned}
\Delta\nu &\approx 2.5\frac{\text{Hz.nm}}{\text{mW}}\Delta\lambda P && (3.13)\\
&\approx \frac{\text{Hz}}{\text{kW/cm}^2\text{.nm}} I\Delta\lambda && (3.14)\\
&\approx 1.25\text{Hz} && (3.15)
\end{aligned}$$

where $\Delta\lambda = 0.05$ nm was experimentally determined based on the conditions imposed on the system [67]. In this case, such a shift will induce an error of

$$\frac{\Delta\nu}{\nu_0} \approx \frac{1.25}{4.292 \times 10^{14}} \approx 2.9 \times 10^{-15} \tag{3.16}$$

Thus, we note that the error for an optical clock in this case is greater than in the best cases of cesium clocks, which are of the order of 10^{-16}. There is therefore little point in developing such a technology as it stands. In addition, calibration based on the magic wavelength allows $\Delta\alpha(\omega_L) = 0$, and therefore avoids any frequency shift that would affect the clock's accuracy. At the magic wavelength, the two levels of the transition experience the same optical potential: their wells are therefore identical [68].

3.3.3 Atoms preparation cycle

A lattice clock involves a series of important steps. First, a beam of atoms is cooled to a temperature in the mK range using a magneto-optical trap on a wide transition. A second cooling process is then performed on a narrower transition to reach the μK range, creating a cloud of ultracold atoms [67, 69]. The cloud of ultracold atoms is then placed in the optical lattice calibrated to the magic wavelength. The atoms are then trapped in the wells of the optical lattice to confine them [8].

3.3.4 Spectroscopy on optical transition

When atoms are trapped in the lattice, a laser is used whose frequency is close to the atomic transition. As in the case of cesium clocks, this will be modeled by a two-level system $|g\rangle$ and $|e\rangle$ coupled by a Rabi oscillation Ω and with a laser detuning given by $\Delta = \omega_L - \omega_0$ [8, 64]. The transition probability is the typical probability of a Rabi oscillation [64], which can be solved using the Schrödinger and Bloch equations, given by

$$P = \frac{\Omega^2}{\Omega^2 + \Delta^2} \sin^2\left(\frac{1}{2}\sqrt{\Omega^2 + \Delta^2}t\right) \tag{3.17}$$

For $\Delta = 0$, we have complete Rabi oscillations. Thus, a pulse $\Omega t = \pi$ transfers the entire atomic population from the ground state to the excited state.

Alternatively, we use Ramsey spectroscopy: two pulses $\pi/2$ separated by a free time T. The first pulse creates a superposition of states. The free time then allows the state to evolve freely and acquire a phase. The second pulse reinterferes with the amplitudes. The principle of Ramsey spectroscopy is identical to that used in cesium beam clocks.

Generally, the protocol used in optical clocks is Ramsey spectroscopy. Indeed, from a spectroscopic point of view, the resonance lines are narrower (and therefore the resolution is better) but this depends on the design of the laser that is used to interrogate the atoms. Thus, if the laser used for interrogation is coherent, we will preferentially use Ramsey spectroscopy to take advantage of the better resolution it allows [70]. Conversely, if laser coherence is not of interest, Rabi spectroscopy is used to remain in a time domain corresponding to the laser's coherence duration. It is therefore preferable to use this process in the short term, while Ramsey spectroscopy is more suitable for the long term [71]. In this case, the final transition probability is given by

$$P = \frac{1}{2}\left[1 + \cos(\Delta T + \phi_0)\right] \tag{3.18}$$

Thus, there is a certain probability that the interrogation laser will allow the atoms to return to their excited state. If the atoms actually reach the excited state, they will de-excite and emit fluorescence photons. A detection method based on counting the emitted photons is then implemented to determine whether the laser is indeed at the correct transition frequency. The frequency error is then analyzed and controlled by an electronic loop system, which locks the laser to the atomic frequency. In practice, this fluorescence photon detection system is implemented either by a camera or by photodiodes [71].

3.3.5 Major systematic effects

The high precision of optical clocks requires control over systematic effects, which can have diverse origins: environmental, related to trapping, or the integrating laser. For example, for environmental effects, we have blackbody radiation. Indeed, at room temperature, the chamber induces a mean electric field, which produces a Stark effect, producing a frequency variation given by

$$\Delta\nu_{BB} = -\frac{1}{2h}\Delta\alpha_s\langle E^2(T)\rangle(1+\eta(T)) \tag{3.19}$$

where $\eta(T)$ is the dynamic correction due to the frequency dependence of the spectrum. This results in an inaccuracy in the frequency value due to the temperature perceived by the trapped atoms. This effect can usually be erased, since the atoms are cooled in order to trap them [72, 73]. Another interesting and important effect is gravitational shift. Indeed, two identical clocks at different altitudes will exhibit a frequency shift given by

$$\frac{\Delta\nu}{\nu} = \frac{g\Delta h}{c^2} \tag{3.20}$$

where g is the gravitational acceleration. For example, if we take an altitude difference of one meter (i.e., one clock on a table and the other on the ground), we obtain a frequency variation of around 10^{-16}m [74]. Finally, optical clocks, like cesium clocks, are sensitive to external magnetic fields since these produce effects within the energy structure of atoms, such as the Zeeman effect, given up to second order by

$$\Delta\nu \approx \frac{m_F g_F \mu_B B}{\hbar} + k_Z B^2 \tag{3.21}$$

where m_F is the magnetic quantum number associated with the total spin F, g_F is the Landé factor of the species considered, μ_B is the Bohr magneton, and k_Z is a quantity characterizing the atom under study [8]. Atoms with $F = 0$ are therefore generally chosen, where $m_F = 0$ if $F \neq 0$ in order to eliminate the frequency shift to first order. The uncertainty is thus limited to the second-order term, thereby limiting the frequency uncertainty [38].

There are also effects related to trapping. For example, and this effect has already been explored previously, we have a dynamic Stark effect that is canceled out by calibrating the lasers based on the magic wavelength λ_m. In the same vein, it is possible that residual electrostatic fields (particularly due to static charges present on the clock surfaces) produce a Stark effect that will lower the frequency of the energy levels. Furthermore, as we have already indicated previously, there are a fairly large number of atoms, between 1,000 and 100,000 atoms, trapped in the lattice. Furthermore, these atoms are not perfectly static either and undergo a residual displacement due to the quantum nature of the particles, these not being at a perfectly zero temperature. These different atoms can thus collide with each other in an elastic manner, and thus produce a frequency shift.

For fermions, s-wave collisions are forbidden due to the Pauli exclusion principle, making the effect quite small, with a frequency uncertainty of the order of 10^{-18}. On the other hand, for bosons, collisions can generate quite large effects, of the order of 10^{-16}, which can make the clock quite inefficient, given the precision achieved by cesium clocks [75, 76]

Finally, there are effects due to laser interrogation, notably a significant shifting effect of the probe light: the interrogation laser will thus cause a shift in the energy levels given by

$$\Delta\nu_{\text{laser}} = \frac{2\delta\Omega_R^2}{4\delta^2 + \Gamma^2} \tag{3.22}$$

where Γ is the linewidth, δ is the frequency detuning, and Ω_R is the laser frequency. For an optical clock, the desired transition must be as narrow as possible, and therefore $\Gamma \to 0$ [69]. Thus,

$$\Delta\nu_{\text{laser}} = \frac{2\delta\Omega_R^2}{4\delta^2} \tag{3.23}$$

$$= \frac{\Omega_R^2}{2\delta} \tag{3.24}$$

In practice, all other lasers other than the one used for interrogation are turned off during the interrogation phase to limit frequency shifts that could be too large to impact clock stability. Frequency uncertainty is generally achieved below 10^{-18}, which is entirely acceptable given the desired accuracy levels.

Furthermore, we saw earlier that the lattice clock operates based on the Lamb-Dicke principle when the residual atomic displacement is less than the wavelength of light. In this case, the Doppler effect is canceled to first order. However, there are relativistic effects caused by the relative motion between the source and the atom that involve changes in the frequency of the light that forms the light potential. The frequency shift is then given by

$$\frac{\Delta\nu}{\nu} = -\frac{\langle v^2 \rangle}{2c^2} \tag{3.25}$$

which is of the order of -10^{-18} for a residual atomic velocity in the range of 10^{-9} m/s [38].

3.3.6 Temporal stability and Allan variance

The stability of lattice clocks is very high due to the very large number of atoms interrogated. In the absence of technical noise, the fundamental quantum limit gives an estimate for the Allan derivation of

$$\sigma_y(\tau) \approx \frac{1}{2\pi\nu_0 T}\sqrt{\frac{T_c}{N\tau}} \tag{3.26}$$

where T is the atom interrogation time by the clock laser (i.e., the free evolution time in Ramsey spectroscopy or the laser pulse duration in Rabi spectroscopy), T_c is the total measurement time, i.e., the time for signal preparation, interrogation, and reading, N

is the number of atoms, and τ is the total integration time, i.e., the time over which successive measurements are made and over which the Allan variance is measured [77]. Thus, for $N \sim 10^4$, $\nu_0 \sim 4 \times 10^{14}$ Hz, and $T_c \sim 1$ s. Thus, in this case, we have

$$\sigma_y(\tau) \approx \frac{10^{-15}}{\sqrt{\tau}} \tag{3.27}$$

meaning that the Allan deviation of this type of clock is at least 10^{-15}, and can be even lower. For example, after a few hours, it is possible to achieve an accuracy of the order of 10^{-17}, which is lower than what was then possible with cesium beam clocks [38].

3.4 Ion trap clocks

3.4.1 Ion traps

As their name suggests, trapped-ion clocks are based on ion traps. Two main types exist: Paul traps, represented in Figure 3.2., and Penning traps [78]. Firstly, Paul traps are ion traps composed of three electrodes: two outer electrodes and one central electrode. The two outer electrodes are held at the same potential, and a potential difference appears between them and the central electrode, given by:

$$\Phi_0(t) = U - V\cos(\Omega t) \tag{3.28}$$

where U and V are voltage parameters chosen according to the desired behavior of the trap, and especially depending on the charge of the particle to be confined. This potential generates a time-dependent quadrupole electric field of the form:

$$\Phi(r, z, t) = \Phi_0(t)\,\frac{r^2 - 2z^2}{2r_0^2} \tag{3.29}$$

where $\Phi_0(t)$ includes both a constant and an oscillating component, r and z denote the radial and axial directions in the trap, respectively, and r_0 is the characteristic distance between the central electrode and the trap center [78]. In a Paul trap, Newton's equation takes a so-called Mathieu form:

$$\frac{d^2u}{d\tau^2} + \left[a_u - 2q_u\cos(2\tau)\right]u = 0 \tag{3.30}$$

where u is the generic quantity defining r or z, $\tau = \Omega t/2$, and $a_u \propto U$ and $q_u \propto V$ [79]. There is thus one equation to solve in the r direction and one in the z direction. The solutions to the Mathieu equations then make it possible to define stability zones, where the ion is trapped in the r direction and in the z direction [80]. This is, however, valid in two dimensions. In practice, we work in three dimensions, with a quadratic potential given by

$$\Psi(x, y, z) = \frac{Q^2}{4m\Omega^2}|\nabla\Phi(x, y, z)|^2 \tag{3.31}$$

but which has the same characteristics as the 2D trap. In this case, Q is the charge avec the particle.

The other type of trap is the Penning trap. This type of trap has a similar structure to a Paul trap but $V = 0$ and an axial magnetic field B is applied. This type of trap therefore sees the application of a quadrupole electric potential and a constant axial magnetic field. The ion will thus undergo a complex motion resulting from three movements: an axial harmonic oscillation (of frequency $\omega_z = \sqrt{\frac{2qU}{mr_0^2}}$) and two radial movements: a fast cyclotron rotation and a slow magnetron rotation [78].

3.4.2 Laser ion cooling

In order to make a clock, it is necessary for the ions to be as immobile as possible. Indeed, the Paul and Penning traps will confine the ions in a region of space but do not make them almost immobile. It is therefore necessary to use Doppler cooling: the ions will be illuminated by a laser beam detuned towards the red (such that $\Delta = \omega_L - \omega_0 < 0$) of the resonant transition of the ion [2]. The electromagnetic field absorbed by the ion during counter-propagation with respect to the laser will thus produce an impulse kick in the opposite direction to the propagation, inducing a braking force, which imposes a zero average speed, within the limits of the Heisenberg uncertainty principle. It is then possible to estimate the Doppler limit, which is the temperature reached during Doppler cooling. This is given by

$$T_D = \frac{\hbar\Gamma}{2k_B} \tag{3.32}$$

where $\Gamma = \tau^{-1}$, which is the desintegration rate, corresponding to the inverse of the excited state lifetime and can be defined as the width of the line associated with the transition [81]. If we take a lifetime of 10^{-8} s, or a line width of 100 MHz, we obtain, with $k_B = 1.38 \times 10^{-23}$ J/K, the Boltzmann constant, we obtain that $T_D \sim 3.8 \times 10^{-4}$ K.

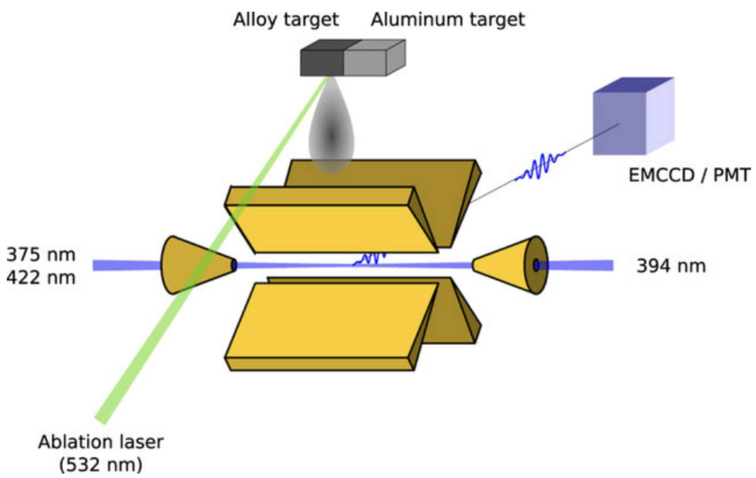

Figure 3.3: Schematic view of a single-ion clock using a linear Paul trap [82]

There is, however, another type of cooling that can be very useful in the case we are studying, as it can occur with one of the ions potentially used in the redefinition of the second. Indeed, some ions, such as the Al^+ ion, do not have an accessible transition [83]. In this case, what is known as sympathetic cooling is used. In this case, another ion is used, which has a transition that allows cooling. However, the two ions in question are linked by a Coulomb interaction and will share the trap vibration modes, automatically leading to the cooling of the coupled ion.

3.4.3 Laser interrogation

In the case of ions, the laser interrogation procedure is identical to that of lattice clocks. Thus, either a single pulse is applied for a time τ (a so-called π pulse) and for which the transition probability between the two levels of the transition is given by

$$P = \frac{\Omega^2}{\Omega^2 + \Delta^2} \sin^2 \left(\frac{1}{2} \sqrt{\Omega^2 + \Delta^2} t \right) \tag{3.33}$$

It is also possible to use Ramsey spectroscopy, where two $\pi/2$ pulses are applied for a time $\tau/2$. These two pulses are separated by a free evolution time T. The transition probability is then given by

$$P = \frac{1}{2} \left[1 + \cos(\Delta T) \right] \tag{3.34}$$

Then, the principle is identical to that of lattice clocks. The ions that are excited by the laser, which is at a frequency close to the resonant transition, and transition to the $|e\rangle$ state, will de-excite by emitting photons. These fluorescence photons can then be counted by a camera or a diode, and thus, by an electronic control system, verify whether the laser is correctly calibrated at the correct frequency, and therefore whether the quartz is correctly locked to the atomic transition [78].

3.4.4 Major systematic effects

As with lattice clocks, there are three types of systematic effects that can impact the stability of optical clocks: environmental effects, trapping effects, and interrogation laser effects.

Thus, environmental effects are similar to those found with lattice clocks. Thus, we have blackbody radiation, whose frequency variation due to it remains [84]

$$\Delta\nu_{\mathrm{BB}} = -\frac{\Delta\alpha_s E^2}{2h} \tag{3.35}$$

For example, for the Al^+ ion, we find that $\Delta\nu_{\mathrm{BB}} \approx -4 \times 10^{-3}$ Hz, or

$$\frac{\Delta\nu_{\mathrm{BB}}}{\nu} \approx -3.8 \times 10^{-18} \tag{3.36}$$

Gravitational shift is also present, for which the inaccuracy is of the same order as for lattice clocks, namely 10^{-17}. Finally, external magnetic fields can also cause unwanted Zeeman effects. As with lattice clocks, ion transitions are chosen to have a total electron spin $J = 0$ in order to suppress the linear Zeeman effect. The quadratic effect is still present. Taking the example of the Yb^+ ion, it is determined that the constant k_Z is given by -0.72 Hz$/(\mu\mathrm{T})^2$, producing an uncertainty of the order 10^{-17} for fields of 50μT. Furthermore, the errors determined on the different ions used are of the order 10^{-17}-10^{-18} [38].

Then, we have effects related to trapping. The most important being the one inherent in the nature of the trap itself. Indeed, we have seen that ion traps can be either Paul traps or Penning traps. We will only study Paul traps, Penning traps being generally separated due to the magnetic field that must be applied to trap the ion, which could cause a Zeeman effect, which must be eliminated, at least to the first order. Thus, in the case of the Paul trap, the ion is subjected to an oscillating potential. This ion is not perfectly immobile and evolves in the first stability zone, carrying out a tiny residual movement. This movement therefore causes two effects that we have already observed in the context of optical clocks: the second-order Doppler shift, which is due to relativistic effects, as well as an Autler-Townes effect. Experimentally, uncertainties of the order of 10^{-18} or less have been achieved, which is perfectly usable in the context of these clocks [80]. We also find collisions between ions, which are extremely weak given that we are trying to trap a single ion and do not impact the frequency.

Finally, there are systematic errors due to laser interrogation, which are identical to those for optical clocks.

It is therefore clear that the predominantly different effect is due to the nature of the trap and the physical phenomenon used to trap the atoms or ions [85].

3.4.5 Stability and Allan variance

The frequency stability of the trapped-ion clock is also given by the Allan variance $\sigma_y(\tau)$. For a clock that only includes a single ion, the Allan variance is given by

$$\sigma_y(\tau) \approx \frac{1}{2\pi\nu_0\sqrt{NT_{\text{probe}}\tau}} \tag{3.37}$$

where N is the number of ions (here, $N = 1$) and T_{probe} is the interrogation time [86]. Thus, for typical characteristics of an ion clock, i.e. $\nu_0 \sim 10^{15}$ Hz and $T_{\text{probe}} \sim 0.1$ s, we arrive at Allan variances of the order of

$$\sigma_y(\tau) \approx \frac{10^{-15}}{\sqrt{\tau}} \tag{3.38}$$

If we take a measurement lasting approximately 3 hours, we will obtain a frequency uncertainty of the order of 10^{-18}, which is perfectly usable and even sought after in the context of implementing optical clocks [87]. A comparison between different types of optical clocks can be studied in Figure 3.3.

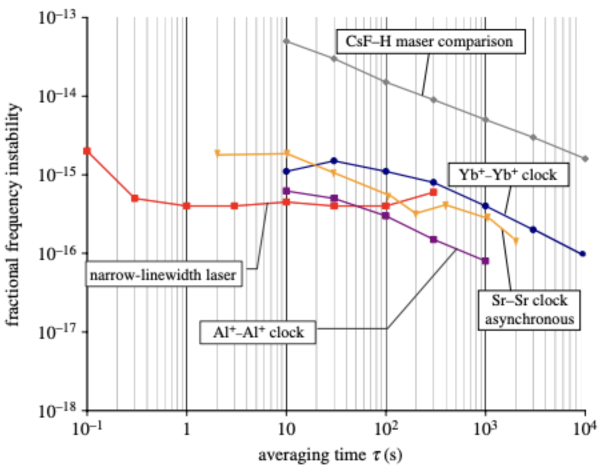

Figure 3.4: Allan deviation $\sigma_y(\tau)$ versus averaging time τ for representative optical clocks : Yb, Sr and Al^+ [7]

3.5 Reference transitions being evaluated for redefinition

As we will see later, the choice of the transition (or transitions) is made based on criteria established by the BIPM [88]. Several requirements must therefore be met.

First, the transition frequency must be stable: the spectral line studied must be very narrow (it must therefore have a very long lifetime, generally a few seconds or minutes) and possess a very high signal-to-noise ratio, meaning that the noise is completely negligible in this case [89]. Furthermore, the transition must be as insensitive as possible to external disturbances. This means that the frequency must be relatively insensitive to any fields that may appear during the measurement process in order to minimize systematic effects. This involves choosing candidates that possess certain spin characteristics, notably a suppression of the magnetic moment to counter certain undesirable effects such as the Zeeman effect, and that are little influenced by electric fields to minimize the Stark effect [38]. It is also essential that the chosen transition be experimentally accessible, meaning that it must be capable of being implemented by stable lasers, whose source must be calibrated to achieve the desired transition. Finally, and in accordance with BIPM requirements, the chosen reference must be reproduced and used by laboratories around the world to obtain the same effects and conclusions. This requires a transition in an atom that exists in significant quantities on Earth, as well as the necessary equipment, including lasers, ion traps, or optical lattices [90].

3.5.1 Transitions used in atomic clocks

Atomic clocks use atomic transitions that are forbidden or unlikely, as they allow for the narrowest possible spectral lines. In laser spectroscopy, the transitions generally used are forbidden E1 electric dipole transitions, which are materialized by selection rules given by $\Delta J = 0, \pm 1$ (except for $J = 0 \leftrightarrow J = 0$, resulting from a spin or parity change) [91]. Clock transitions, on the other hand, are generally transitions of higher order (quadrupolar or octuple, in particular). E1 transitions, generally called forbidden electric dipole transitions, only become accessible using hyperfine coupling or an external magnetic field, the former occurring for nuclei with non-zero spin and the latter for those with zero spin [38]. For example, we can take the transition $^1S_0 \rightarrow ^3 P_0$ in two-electron atoms such as those present in ^{87}Sr, ^{171}Yb or Al^+ [86]. This transition is also called "doubly forbidden". Indeed, we go from a singlet state to a triplet state (i.e. $S = 0 \rightarrow S = 1$), so $\Delta S = 1$. Furthermore, regarding parity, we go from $J = 0$ to $J = 0$, which is forbidden by the selection rule E1. However, the presence of a hyperfine interaction or an external magnetic field will make this transition very weakly allowed, with very long lifetimes (of the order of a minute). The Heisenberg uncertainty principle from an energy perspective

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{3.39}$$

thus allows us to conclude that ΔE is very small, and therefore the transition is very narrow [89].

Next, we can study electric quadrupole (E2) transitions [92]. These transitions are exploited in the case where E1 transitions are effectively forbidden. This type of transition implies a change in angular momentum given by $\Delta J = \pm 2$, which is the selection rule that establishes electric quadrupole transitions. There are a few transitions that respect this selection rule. In particular, we find the 4s($^2S_{1/2}$) $\rightarrow$ 3d($^2D_{5/2}$) transition of Ca^+. This 729 nm transition indeed respects the selection rule since we go from $J_1 = 1/2$ to $J_2 = 5/2$, or $\Delta J = 2$. Experimentally, it has been determined that this transition's lifetime is of the order of a second, i.e., with a lower transition probability than that of an accessible E1 transition, but obviously much higher than a strictly forbidden E1 transition (since its lifetime is extremely short, the excited state not being accessible) [68]. These transitions are narrower than E1 but larger than forbidden E1 transition Furthermore, the most well-known E2 transitions are 729 nm (Ca^+), 674 nm (Sr^+) or 436 nm (Yb^+). Knowing the electromagnetic spectrum, especially in the visible range, we know that these transitions are located in the near infrared, deep red and blue-violet respectively. It is therefore easier to produce lasers capable of exciting this transition [87]. However, they can produce a fairly restrictive systematic effect, since these transitions in the case of ions are very sensitive to the electric field.

Next, the higher-order transitions are the electric octupole (E3) transitions, which follow the selection rule $\Delta J = \pm 3$. Just as E2 transitions have a lower transition probability than E1 transitions, E2 transitions have a lower transition probability than E2 transitions. This also means that the transition's excited state lifetime is very long, sometimes reaching several years [93]. A typical example of a transition, and the best-known of the E3 transitions, is given by 6s($^2S_{1/2}$) $\rightarrow$ 4f^{14}5d($^2F_{7/2}$) at a wavelength of 467nm. [94] Thus, this transition follows the E3 rule, since $\Delta J = 7/2 - 1/2 = 3$. The particularly long lifetimes of these transitions make them attractive candidates for the use of optical clocks. This implies that the transition is very narrow. However, such a long lifetime requires achieving technological feats that are difficult to achieve. In particular, it is necessary to maintain laser stability over a fairly long period of time. It is also necessary to be able to control systematic effects over a period of time as long as the laser's stability.

Thus, optical clocks use either E1 transitions, in the case of lattice clocks or ion clocks when the candidates have two electrons, or multipole transitions in the case of trapped ion clocks.

3.5.2 Main candidate transitions for redefinition

Currently, the CGPM has designated several candidates for inclusion in the process of defining the second [94]. This candidate status indicates that the frequency of the chosen transition has been measured with sufficient precision to contribute to the redefinition of the second [8]. The optical transitions of certain atoms and ions are designated as candidates for redefining the second. In atoms, these include the optical transitions of strontium, ytterbium, and mercury, while in ions, these include the optical transitions of aluminum, calcium, strontium, ytterbium, and mercury, all of which are ionized once. A microwave transition, that of rubidium, has also been selected as a candidate [95]. To make it easier to understand, it is best to draw up a table with the different transitions, their frequency, wavelength, type of transition, half-life time, accuracy and the laboratories that work on these transitions [38, 95], listed in Table 3.1.

Species	**Transition**	ν **(Hz)**	λ **(nm)**	**Type**	τ	**Uncertainty**	**Lab**
$^{27}Al^+$	$^1S_0 \rightarrow {}^3P_0$	1.121×10^{15}	267	E1	∼20 s	$9{,}4 \times 10^{-19}$	1
^{87}Rb	$^1S_0 \rightarrow {}^3P_0$	6.834×10^{9}	4.4 cm	Hyperfine	∼100 ans	$\sim \times 10^{-13}$	3
^{87}Sr	$^1S_0 \rightarrow {}^3P_0$	0.429×10^{15}	698	E1	∼150 s	$2{,}0 \times 10^{-18}$	8
^{171}Yb	$^1S_0 \rightarrow {}^3P_0$	0.518×10^{15}	578	E1	∼20 s	$1{,}4 \times 10^{-18}$	5
^{199}Hg	$^1S_0 \rightarrow {}^3P_0$	1.129×10^{15}	265,6	E1	∼1–2 s	$\sim 1 \times 10^{-15}$	2
$^{40}Ca^+$	$^2S_{1/2} \rightarrow {}^2D_{5/2}$	0.411×10^{15}	729	E2	1,1 s	$3{,}0 \times 10^{-18}$	3
$^{88}Sr^+$	$^2S_{1/2} \rightarrow {}^2D_{5/2}$	0.445×10^{15}	674	E2	0,4 s	2×10^{-17}	2
$^{171}Yb^+$	$^2S_{1/2} \rightarrow {}^2D_{3/2}$	0.688×10^{15}	436	E2	∼0,05 s	$\sim 1 \times 10^{-16}$	2
$^{176}Lu^+$	$^1S_0 \rightarrow {}^3D_2$	0.373×10^{15}	804	E2	$\sim 10^{-3}$ s	1×10^{-17}	1
$^{199}Hg^+$	$^6S_{1/2} \rightarrow {}^2D_{5/2}$	1.065×10^{15}	282	E2	∼0,1 s	2×10^{-16}	2
$^{171}Yb^+$	$^2S_{1/2} \rightarrow {}^2F_{7/2}$	0.642×10^{15}	467	E3	$\gg 10^3$ s	$2{,}7 \times 10^{-18}$	2

Table 3.1: Optical transitions classified by type (E1, E2, E3), then by increasing atomic number.

3.6 Criteria for a new definition

In order to achieve a redefinition of the second, the CGPM has developed a series of criteria to be respected.

3.6.1 Uncertainty budget for optical standards

This criterion, dubbed "criterion I.1," requires that intended as primary standards have an uncertainty below 2×10^{-18}, corresponding to an improvement in the precision of cesium fountain clocks, whose current uncertainty is estimated at approximately 3×10^{-16} [88, 95]. Furthermore, to achieve this uncertainty, it is necessary to list all the systematic effects already explored previously and to evaluate their impact on the transitions studied. In addition, this criterion can be refined by two additional sub-criteria: at least three identical clocks, i.e., the same type of clock experiencing the same transition, in three different laboratories must demonstrate the ability to achieve this uncertainty. It is also required that at least three uncertainty measurements have been published on three different transitions [38].

3.6.2 Validation of uncertainty budgets in relation to frequencies

This criterion, dubbed "criterion I.2.", complements the previous one. Indeed, this criterion requires that two clocks be compared by measuring their frequency ratio with an uncertainty of less than 5×10^{-18} [95]. As before, this point is divided into two parts. First, this measurement must be performed on two clocks of the same type, and must be performed at least three times between different laboratories. Second, at least five cross-ratios must be performed on clocks with different transitions, the purpose being to determine whether errors may be present in the clock calculations. Technologies must also be implemented to meet this criterion. Indeed, it is necessary to achieve high precision in the comparison between these clocks. To achieve this, portable clocks or optical links are designed to allow for highly accurate comparisons of clocks and transitions [96].

3.6.3 Continuity with the cesium-based definition

Criterion I.3. is a criterion that requires the new definition to maintain continuity with the current definition of the second, based on cesium standards. Thus, it is necessary to maintain a link between the future definition and the current definition. In practice, this means that it must be possible to make three measurements of the frequency of each transition based on the hyperfine transition frequency of the cesium atom [88]. Thus, the principle is to make comparisons between cesium clocks and optical clocks. To do this, an optical clock is compared several times with cesium clocks from the same institute, or at TAI.

3.6.4 Regular contributions to TAI

Criterion I.4. is a fundamental criterion since it requires integrating optical clocks into the TAI calculation, by providing at least 3 calibrations per month with uncertainties lower than 2×10^{-16}, to remain in agreement with the uncertainty of the cesium fountains.

These calibrations must come from 5 different clocks, over a full period of one year. The purpose of the maneuver is to verify whether TAI would remain consistent if optical clocks were considered as primary standards and cesium beam clocks became secondary [90].

3.6.5 High reliability of optical standards

Criterion I.5. complements criterion I.4. by focusing on the robustness of clocks, in the sense that the clock must operate continuously [95].

3.6.6 Durable techniques for optical comparisons

Criterion II.1. concerns the availability of means for comparing clocks, as well as their accessibility and durability. This criterion particularly concerns techniques that allow clocks to be compared with each other, as discussed above. For example, we have optical fibers, which are links that allow frequencies to be transferred over long distances without loss of information or accuracy, but also transportable clocks, also called TOCs, which are cesium or strontium clocks, which are transported in vehicles or containers and are of an adequate size to be moved and make useful comparisons between different clocks from different laboratories [96, 97].

3.6.7 Knowledge of local geopotential

Criterion II.2. requires knowledge of the difference in geopotential between laboratories with an uncertainty comparable to that desired for the clocks used in the definition. Indeed, as previously explained, the effects of gravity, through general relativity, will play an important role in systematic effects, generating an uncertainty given by

$$\frac{\Delta\nu}{\nu} = \frac{g\Delta h}{c^2} \tag{3.40}$$

If we want a frequency imprecision given by 10^{-18}, namely the one desired in the context of the redefinition of the second, then it is necessary to be able to determine this geopotential very precisely [97, 98].

3.6.8 Reliability of time-frequency links

This last criterion, II.3., complements II.1., by requiring that the optical or microwave links can operate continuously for periods sufficient for the exercise that we wish to implement. We therefore want the installed optical fibers to be able to remain operational and operate continuously in order to be able to adequately compare the clocks with each other, and evaluate their contribution to TAI [96].

3.6.9 Possibility of using the clock as secondary reference

Criterion III.1. requires that clocks currently being studied to redefine the second be eligible to participate in the definition of UTC as a secondary standard. This possibility must be included on the list of frequency standards authorized to define the current second [90]. Therefore, the transition frequency can be measured relative to the cesium definition with an uncertainty of the order of 10^{-16}, as required for the current definition. Moreover, this criterion is fundamental because it ensures continuity with the current definition, but also attests to a certain reliability of the candidates used in the redefinition, since the frequency of such a clock is reproducible and its uncertainty is optimally controlled.

3.6.10 Ability to align with TAI

Criterion III.2., which is fundamental in the redefinition of a second, implies that the redefined second must be compatible with the TAI, and that there must be no break in the current time scale. This means that once the second defined based on optical clocks has been determined, it must have a duration identical to the second based on cesium clocks, with an uncertainty of around 10^{-18}, so that the TAI does not diverge from the currently established definition [88]. In the event that the newly defined second has a difference with the current second greater than the uncertainty, it will be necessary to be able to correct for this uncertainty by adding corrections (such as leap seconds, for example, within the framework of UTC, or by modifying the weighting of one or more optical clocks used). It will therefore be essential to avoid any sudden discontinuity in the definition of TAI, given the use of TAI and UTC as a reference in many systems used on a human scale and requiring high precision and stability, particularly communication and navigation systems. A discontinuity in the definition of the temporal unit could thus lead to significant disruptions in these systems.

3.6.11 Frequency access for industry

Criterion III.3. highlights the need for the optical second to be accessible not only to metrology laboratories, but also from an industrial and commercial perspective, so that companies, industries, and laboratories not involved in fundamental research can still have access to the new second. This criterion requires several sub-criteria to be met. First, the clocks involved in the definition must be commercially reproducible, while maintaining acceptable dimensions for industrial propagation [95]. Furthermore, it must be possible to set up time or frequency transfer chains to distribute the signal that will define the second within these commercially viable clocks. This means that the resulting signal must be transferable by physical means, such as satellites or optical fiber, to these clocks, allowing its widest possible dissemination [96]. Regardless of any technical interest from the point of view of redefinition, this criterion will be fundamental for the practical acceptance of the second in the scientific community, but also outside it, and to find a use for it other than purely in the context of research. This will allow the new second to be used to develop tools based on the use of the time scale. It is therefore fundamental that the new second be accessible to the widest possible audience, and that its dissemination be as wide as possible. Its propagation will also allow for greater robustness, since the second will not depend solely on a few standards scattered around the world, but on a much larger number of clocks.

3.6.12 Continued use of cesium

Criterion III.4. stipulates that once the second is redefined, the cesium clocks that are currently the primary standards should not disappear, but will lose their status as primary standards. They will then become the secondary standards for the realization of the new definition of the second, as some optical clocks currently are [38]. This allows the new second to be expressed based on the cesium definition. In this case, the frequency of the hyperfine transition of the cesium atom will no longer be fixed exactly, but experimentally with a known uncertainty. This criterion has the same value as retaining the definition of UT. It allows for the preservation of a record of a known second, and its potential use as a comparison between data sets. Moreover, widely adopted today, the cesium fountain clock is a very reliable standard. Hundreds of them can be found around the world, used for all sorts of applications. It will also be possible to keep them in case of problems with optical clocks, their usefulness and operability being widely accepted.

3.6.13 Comprehensive documentation

Criterion III.5. ensures the provision of comprehensive documentation detailing the development of the new second, including the frequencies of the standards used, the comparison protocols and the recommendations for disseminating this second. This documentation must be understandable to the entire scientific community and validated within metrology institutes to ensure global consistency on the redefined second [88].

3.6.14 Statuts of redefinition criteria

Let's now look at the progress of each of the criteria for redefining the second. A complete representation is proposed in Figure 3.4.

- Criterion I.1. Uncertainty budget of optical standards: After testing several clocks, some, whether lattice or ion, have managed to exceed this threshold. For example, the ^{87}Sr lattice clock or the $^{27}A^{+}$ ion clock have achieved this performance, with, for example, an uncertainty of the order 2.1×10^{-18} for the former and 9.4×10^{-19} for the latter, both listed at the University of Colorado. Strontium or ytterbium clocks are also found, notably in France, Germany, and Japan, with similar uncertainties. The CGPM considers that this criterion is met by a percentage of between 30 and 50% [38, 99].

- Criterion I.2. Validation of uncertainty budgets by frequency ratios: This criterion concerns the comparison between clocks of the same type, but also with regard to different transitions on an external basis through optical links or transportable clocks. For example, comparisons between clocks have already been made thanks to these links over significant distances, of around 1 thousand kilometers, notably between Paris and Braunschweig with a comparison between two strontium clocks with an uncertainty of 5×10^{-17}. Currently, no progress has demonstrated performance with an uncertainty of 5×10^{-18} over a long distance. The CGPM considers that this criterion is met with less than 30 % [96, 97].

- Criterion I.3. This criterion is well advanced in most laboratories, given that most optical clocks that indicate an interesting transition have had their frequency measured based on the frequency of cesium beam clocks and most optical clocks, such as the strontium clocks at SYRTE and PTB, have been calibrated based on their own cesium fountain clocks. Based on this calibration, the uncertainty is of the order of 10^{-16}, given that this is the limiting value of the frequency uncertainty that can be achieved with cesium beam clocks. In fact, each transition mentioned above has already been evaluated three times with a frequency uncertainty of less than 3×10^{-16}. The BIPM therefore considers this criterion to be almost fulfilled (between 90 and 100 %) [90, 95].

- Criterion I.4. Regular contribution of optical clocks to the TAI: Several of the major laboratories that have designed an optical clock already regularly contribute to the TAI, as secondary standards, of course. However, the exact criterion requiring the contribution to the TAI to be made by 3 calibrations per month, for 1 year, from 5 different clocks, has not yet been met, as the number of reliable optical clocks is still very low [88, 95].

- Criterion I.5. High reliability of optical clocks: This criterion establishes the robustness of the clocks and their continuous operation in order to be fully operational. For example, an ytterbium clock at the PTB in Germany operated for 2 weeks, for 99.8% of the time. A compact strontium clock also operated for 6 months at 80.3%. This operating time of close to 100% makes these clocks almost usable. Based on the tests carried out, it is estimated that very satisfactory results will be expected in the coming weeks or months [95, 97].

- Criterion II.1. Sustainable techniques for optical comparison: This criterion is the one that allows us to highlight techniques for comparing optical clocks with each other, particularly based on optical fibers or transportable clocks. In this respect, it has already been shown that it has been possible, over distances of approximately 1,000 km, that comparisons were entirely possible between clocks based on optical fiber networks. However, although this criterion is satisfied for intracontinental distances, namely that it is possible to compare European clocks with each other, for example, it is not yet possible to do so for a comparison between a European clock and an American clock. Recent progress, particularly with regard to geographical coverage and the repeatability of comparisons, allows us to indicate to the BIPM that this criterion is met to between 50 and 70%, with transoceanic links still having to be materialized [96, 97].

- Criterion II.2. Knowledge of the local geopotential: It is necessary that the difference in local geopotential between laboratories using the same clock be known with an uncertainty compatible with the requirements of the redefinition, namely an uncertainty of the order of 10^{-18}. As seen previously, the gravitational shift is given by

$$\frac{\Delta\nu}{\nu} = \frac{\Delta W}{c^2} \tag{3.41}$$

  with ΔW being the known potential difference between the two clocks. With the desired uncertainty for optical clocks, this corresponds to a difference of the order of a centimeter. The height difference within the same country is known, at a distance of about 5 to 10 cm. Furthermore, given that the mass of a laboratory atomic clock is in the order of ten kilograms, the uncertainty is in the order of 10^{-17}. This criterion can thus be considered to be met in a proportion of 70 to 90% and is well on the way to being complete [97].

- Criterion II.3. Reliability of highly stable time-frequency links: This criterion consists of having the most stable time-frequency links possible, with continuous operation over sufficient durations, without experiencing any disruption. It is therefore necessary for the implemented optical link to be efficient, to enable the most accurate comparisons possible between atomic clocks. Currently installed links operate on average for durations of a few weeks to a few months, without interruption. However, there is a distinction to be made between intracontinental and extracontinental links. Indeed, in the former case, the criterion is considered met. However, this is absolutely not the case for the latter, where reliable links do not yet exist. For example, in the former case, there is a link connecting two French cities and one German city: Paris-Strasbourg-Braunschweig, which operated independently for several weeks [96].

- Criterion III.1. Use of optical clocks as secondary standards: A growing number of optical clocks are involved in the current definition of the second. For example, the NIST and PTB clocks, currently among the most advanced in research, regularly contribute to the definition of the cesium second. This criterion is thus met since approximately ten optical clocks frequently contribute to the definition of the cesium second and are therefore secondary representations of the second, with measured frequencies and desired uncertainties for the cesium second, of the order of 10^{-16} [90, 95].

- Criterion III.2. TAI alignment and correction capability: Metrology laboratories are attempting to synchronize optical clocks with TAI as they progress, with the aim of having at least 3 per month. Thus, currently, the scientific community agrees that continuity between the two types of standards is ensured with an uncertainty of less than 10^{-16}, ensuring that no divergence exists between atomic time and optical time. In addition, the active participation of optical clocks in TAI for several years makes it possible to verify that no major jumps occur using these clocks [38, 88].

- Criterion III.3. Industrial Access to Frequency: Technological progress has been made in this area, particularly in the design of transportable clocks (which can be used, for example, for comparisons between clocks). However, no complete optical clock exists that can be marketed. However, systems exist to efficiently create and distribute time or frequency. The existence of intracontinental links for connecting and transferring time or frequency between different locations has already been mentioned several times previously. Again, these links do not exist on a global scale, but only on a continental scale, with an uncertainty of around 10^{-19}. Furthermore, all of this has already been developed to enable cesium-based second-based links. Technological advances, particularly in lasers or based on the optimization of optical links, open the door to improving these devices in the coming years [95, 96].

- Criterion III.4. Continuity of Cesium Use: It is already established that a dozen cesium clocks in metrology laboratories will retain an important role in the implementation of the optical seconds. This criterion is therefore considered met, but it will be necessary to continue funding these devices to ensure their commercial development and maintain continuity with the current definition. This criterion therefore acts as a precautionary principle [38].

- Criterion III.5. Documentation: This criterion will certainly take the form of a report published by the BIPM, in the same way as Circulars T, which will include all the knowledge necessary for the practical and most stable and accurate implementation of the new seconds. This criterion is currently at an early stage, given that no direction has yet been established regarding the definition to be used [88].

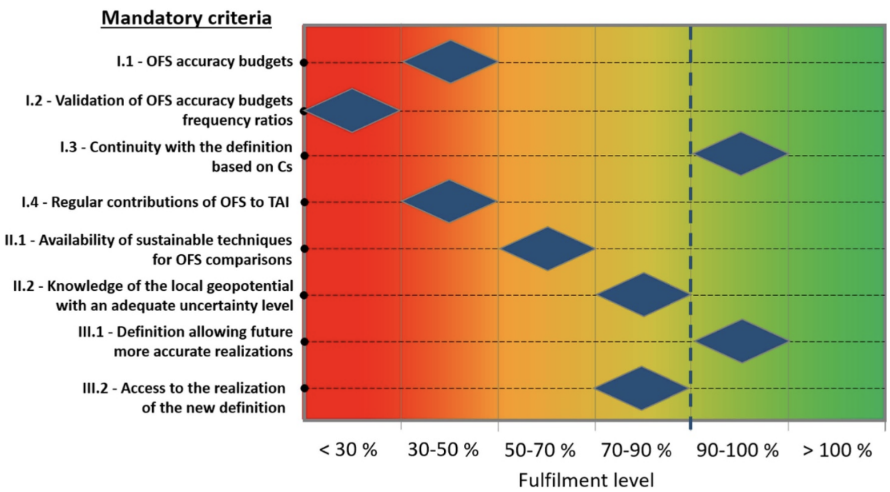

Figure 3.5: Fulfilment levels of criteria for the redefinition of second in 2022 [50]

3.7 Possible redefinitions options

Several scenarios are being considered for redefining the second by 2030. There are three of them. First, either a redefinition based on a single transition, as is the case with the cesium hyperfine transition, is being considered. Then, the second option is to use a set of transitions and average them. Finally, the last one consists of fixing the value of a constant that will be used to redefine the second based on other fundamental constants, such as the speed of light, or Planck's constant.

3.7.1 Definition by a single transition

For a candidate optical clock to be adopted as a new reference, its reliability must exceed that of the cesium clock. In practice, the CIPM imposes an uncertainty of around 10^{-18} for the same transition, as demonstrated by several independent laboratories [90]. Stability must also be high, i.e., approximately 10^{-16} per second, as is the case for lattice clocks in particular. Furthermore, systematic effects must be controlled [99, 100].

Among the transitions already studied, some have achieved significant performance. For example, the strontium clock achieved an uncertainty of 8.1×10^{-19}, roughly comparable to the aluminum ion clock. Among the transitions, some candidates stand out: neutral clocks use many particles to optimize stability, but require fine control of systematic effects, while ion clocks have lower stability, since only a few ions are used, but their systematic shifts are generally smaller [38].

In the case where only one transition is chosen, the second is then set as $t = \frac{1}{\nu}$. The clock realizing this second would then be chosen as the primary measurement standard. The other clocks would then become secondary standards. Compared to current systems, cesium fountain clocks are the primary standards, while other clocks, such as hydrogen or rubidium clocks, are secondary standards. This model is therefore very close to current systems, but it gives a significant role to a single atomic species. Continuity with the current system is therefore ensured and can be accessed very quickly through conversion tables.

3.7.2 Averaging over several transitions

Instead of fixing a single frequency, the definition would be based on a weighted geometric mean of the frequencies of several selected optical transitions [88]. Mathematically, this definition would be written as

$$\prod_{i=1}^{M} \nu_i^{\omega_i} = N\,\mathrm{Hz} \tag{3.42}$$

with

$$\sum_{i=1}^{M} \omega_i = 1 \tag{3.43}$$

where ν_i are the frequencies of the different transitions selected and ω_i are the relative weights. This definition ensures that any clock based on one of the selected transitions can realize the second via known frequency ratios. In practice, the realization relies on an adjustment of the inter-species ratios [95]. The weights ω_i must reflect the quality of each transition. A natural rule would be to choose the weights based on the uncertainty σ_i associated with each frequency, such that

$$\omega_i \propto \frac{1}{\sigma_i^2} \tag{3.44}$$

Thus, on this basis, the propensity of the most precise frequencies is greater than that of the less precise transitions. For example, the most studied and most accomplished transitions, which can be realized with the uncertainties sought by the CGPM, namely of the order 10^{-18}, will have a greater weight than the others, which are little known and little studied, for the moment [88]. The advantage of this definition is that the weights can be modified periodically to ensure the reliability, stability, and continuity of the definition. Furthermore, it would be possible to adapt the definition to technological progress, by increasing the weight of a species according to the evolution of research.

3.7.3 Using fundamentals constants

The principle of this definition is to use physical constants and to fix the numerical value of a constant, other than c, h, e, k_B, N_A, and K_{cd}, which are the constants related to the SI units (with the exception of the cesium hyperfine transition, which is used for the definition of the second). The three constants that currently stand out are the electron mass m_e, the fine structure constant α, or the Compton frequency [38, 101].

$$\nu_c = \frac{mc^2}{h} \tag{3.45}$$

where m is the mass of the particle, in this case, that of the electron [101] The idea in this case would actually be to fix the value of the electron mass. Indeed, the speed of light c and Planck's constant h are already perfectly determined on the basis of general relativity and quantum mechanics. Fixing the mass of the electron would therefore make it possible to fix exactly the Compton frequency ν_c. It would then be possible, on the basis of this frequency, to determine a unit of time which would simply be the inverse of this unit of frequency.

3.8 Schedule and projections

By around 2020, three target windows were under discussion : 2026, 2030 and 2034 [88].

The three proposed dates first required the validation of a clear plan to define the criteria that needed to be met in order to redefine the second. In all three projections, the plan had to be approved by the 27th CGPM (2022) [102] The scientific community quickly realized that the first redefinition date was unrealistic, given that, in addition to proposing a plan, the 2022 CGPM would approve the plan, and a concrete definition proposal would have to satisfy the criteria by this year. Currently, however, it has still not been decided which option would be used to define the second, and not all criteria have been fully met [88, 102].

The roadmap endorsed by the CGPM in 2022 was followed up in early 2023 with progress update.

The deadline set by the two remaining timeline is the 2026 CGPM. During this CGPM, a decision will determine whether a has consensus and whether the criteria are sufficiently met to allow the redefinition of the second. The work for this deadline therefore had to be carried out during 2024 and 2025, during which metrology laboratories must attempt to meet the mandatory criteria as much as possible and propose a redefinition option for the second [102]. This work thus leads to the CGPM, where the decision will be made on whether or not to redefine the second in 2030. As previously stated, a redefinition in 2026 is not anticipated, as the criteria have not been sufficiently met to initiate the process. If the CGPM considers that technical advances are sufficiently advanced, it will then validate the roadmap that will allow the redefinition of the second in 2030.

It will also have to decide which species and transitions constitute the definition, depending on the chosen direction, and on the nature of the transitions. In the event of disagreement or failure to complete the redefinition criteria, the decision would then be postponed to the CGPM in 2030, with a view to a redefinition in 2034 [88], as represented in Figure 3.5.

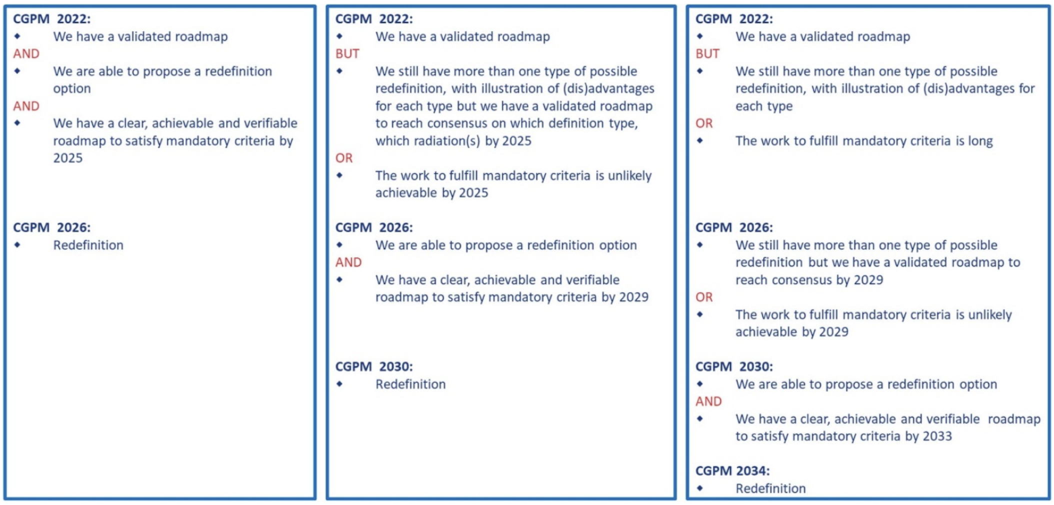

Figure 3.6: Scenarios for the redefinition of the second [50]

Once the definition is validated, the BIPM will then have to implement and monitor the new second, notably by integrating the new definition into the TAI and UTC. In this case, the standards validated by the CGPM will become the primary standards, while the cesium clocks will become the secondary standards. Metrology laboratories will then have to adjust to the new definition.

3.9 Opinion on the question of redefinition

The scientific community's desire to propose a new definition of the second by 2030, based on optical clocks, opens the way to essential questions, particularly regarding the choice of clock, transitions, and the methods that will enable its implementation. After examining in detail the hypotheses for this redefinition, this section is devoted to a personal scientific opinion on what I believe to be the best path forward. In this opinion, I will propose identifying the most appropriate clock to achieve the new definition, notably by comparing their performances. I will then discuss what an ideal transition might be to obtain a new time reference before examining the different options available to us: a definition based on one or more transitions, or a purely theoretical definition.

3.9.1 Optical clocks

To choose the optical clock that will serve as the basis for the future definition, it is necessary to compare the candidates according to several criteria. As previously presented, state-of-the-art optical clocks are divided into two categories: clocks with neutral atoms trapped in an optical lattice, and those with a trapped single ion.

Optical clocks today surpass by several orders of magnitude the stability of cesium atomic fountains, which are currently the functional clocks officially used as the primary standards defining the second [38, 58, 63]. In particular, neutral atom lattice clocks benefit from the simultaneous interrogation of thousands of ultra-cold atoms, which allows for greater stability, with an uncertainty of up to $10^{-}16$ over a second, this uncertainty being given by the Allan deviation of such a clock [67]. They are therefore, at the outset, as efficient as cesium clocks, which have been in development for about fifty years [63]. In contrast, single-ion clocks, which have only a single reference atom at a time, exhibit lower short-term stability. It is typically around 10^{-15} over a second. A longer integration time is therefore required to achieve the same level of stability as a lattice clock. Thus, in the case of short-term stability, optical lattice clocks have the advantage due to the presence of a large number of atoms in the trap, which can be an advantage for a time unit that will be broadcast and compared frequently [69].

Then, the accuracy of a clock is another important factor. This is measured by its total systematic uncertainty, namely the ability to know the frequency of the transition without bias, with the uncertainty caused by these systematic effects optimally controlled. In this case, the best optical clocks today achieve relative uncertainties of the order of 10^{-18}, or even slightly lower in some cases [51, 66, 75]. This corresponds to an error of less than one second on the age of the Universe. It would therefore take several more billion years before reaching the relative uncertainty of one second. Controlling systematic effects, such as the Autler-Townes effect, blackbody radiation, and gravitational shift, among others, and technological developments, such as the implementation of a so-called magic wavelength optical lattice or more efficient and ultrastable lasers, can improve the accuracy of these clocks [6, 56]. Ultimately, over a full run of an optical clock, with multiple operating cycles and interrogation times longer than one second, the total systematic uncertainty generally reaches the order of 10^{-18} s per second, whether for a lattice clock or a trapped-ion clock. Thus, there is a set of transitions capable of defining the second without introducing significant error for billions of years. In this case, neither category of clock has a significant advantage over the other, as systems for minimizing systematic effects have been implemented for both types [93]. However, the two types of clocks present differences in the challenges they face. For example, lattice clocks require control over the density of atoms and any collisions that might occur in the trap, while ion clocks require control mechanisms for the electric potential or magnetic field.

It is also essential that the physical phenomenon that will define the second be easily reproducible. Otherwise, it cannot be used to define a time scale. Thus, it is important that a single laboratory not once propose a transition that can redefine the second. This phenomenon must be observable again and reproducible in several independent laboratories, with consistent results, even in different locations. For example, currently, it seems that strontium lattice clocks are one of the most mature technologies in this area, used by several laboratories around the world (notably NIST in the United States, PTB in Germany, LNE in France, and even NICT in Japan), and can therefore be compared between laboratories [100]. Ytterbium lattice clocks also seem to be popular, with a few laboratories studying this transition. In the case of ion clocks, there are a few laboratories studying the octupole transition, particularly in Germany and the United Kingdom [87]. However, they are currently less widespread than lattice clocks. Thus, in terms of availability, the advantage lies with lattice clocks, particularly those based on strontium and ytterbium, which have been reproduced and compared internationally. Thus, this seems a solid basis for redefining the second, as it does not depend on a prototype that worked only once, in a single laboratory [96].

It is also necessary for the clock to operate continuously, with significant stability, and without repeated human intervention. Currently, optical clocks remain complex devices, difficult to move and market. However, optical clocks include cooling systems, which have already been extensively tested and developed over the past few decades. A few transportable clocks have also emerged to facilitate comparisons. However, the technology surrounding optical clocks, including laser, confinement, vacuum, and systematic protection systems, is still extremely complex, and it appears difficult to arrive at optimal solutions for their transport in the coming months, as may be the case for other clocks, such as hydrogen masers, for example [96, 100]. Furthermore, it is quite difficult to imagine leaving an optical clock running on its own for several months. Some tests show that some clocks can operate stably for a few weeks, but this is far from the results achieved by cesium clocks, which can run automatically for a significant period of time.

Finally, it is essential, and this goes hand in hand with what was previously discussed, that optical clocks be easy to implement and compatible for use by metrology institutes around the world. It must therefore be possible to implement them in these laboratories without major technical difficulties, with the aim of making the second achievable in all these institutes, to participate in international comparisons and integrate the UTC calculation. Currently, each laboratory builds a clock that it explores, agreeing to incur significant financial, human, and technical costs. The future definition of the second will therefore have to take into account the efforts already made by each laboratory and respect the sensitivities and discoveries already highlighted. For example, the strontium clock is a clock that is being developed in several laboratories. Therefore, documentation and software are shared, as well as commercial components, such as lasers. This will make it easier for different laboratories to accept that strontium lattice clocks can be used within the framework of the new definition, as the vast majority of them already use them routinely [32]. It will also be necessary to ensure that clocks of the same type are comparable with each other, either via optical links or via satellite, which is already the case for the French and German strontium optical clocks (although intercontinental links have not yet been developed and a comparison with American clocks, for example, is not yet possible based on these links).

Therefore, on this basis, I believe that the optical clock that represents the best experimental choice, if only one had to be chosen, would be the optical lattice clock, with the strontium-87 transition. This clock achieves the stability and accuracy sought by the scientific community with a view to redefining the second. It is also technologically mature, since several functional examples are present in laboratories around the world. It is relatively pragmatically feasible, as although the infrastructure is cumbersome, it is manageable and the related documentation is shared. Ytterbium clocks are also candidates worth considering, as they are almost as efficient as strontium clocks. The fundamental difference between the two is that strontium clocks are more widely used in laboratories, and comparisons between them show similar results. At the same time, it is obviously crucial not to rule out trapped-ion clocks, as they can verify and complement measurements by offering additional guarantees, particularly regarding systematic effects, which can be better controlled. However, they are disadvantaged by their slower measurement rate and more limited distribution. I therefore believe they could serve as secondary standards to stabilize the second, rather than being a basis for its definition.

3.9.2 Ideal transitions

The choice of clock described above must translate concretely into the selection of a specific atomic transition whose frequency will serve as a new reference for the second. Of the three definition options considered, two approaches are favored by the scientific community. The first is the definition based on a single atomic transition (a single species of atom or ion serving as the primary standard, as cesium is currently). The second is based on a combination of several atomic transitions, which is actually a weighted average of several individual standards.

In the scenario where a single atomic transition is used to define the second, the $^1S_0 \rightarrow {}^3P_0$ transition of the strontium-87 atom is, in my opinion, the ideal choice [66]. This is the transition used in strontium optical lattice clocks, corresponding to a frequency of approximately 4.3×10^{14} Hz (in the red, at 698 nm). This transition is said to be electrically forbidden, which gives it an extremely narrow line and therefore a very large quality factor Q, conducive to significant stability and precision. First of all, as mentioned previously, the strontium-87 optical transition has been measured with a relative uncertainty of the order of 2×10^{-18} in several laboratories. It has been on the list of frequencies recommended by the CGPM for several years as a secondary representation of the second [31]. This means that even before the official redefinition, this transition is internationally recognized and some laboratories already use it to calibrate the TAI. Furthermore, strontium-87 is one of the fermionic isotopes of strontium that does not have an electronic magnetic moment, as evidenced by the $^1S_0$ state. It is therefore an isotope that is not very sensitive to external magnetic fields. In addition, its shift due to thermal radiation is very small (of the order of 10^{-17} by kelvin around 300K) [101]. This therefore facilitates the reproduction of the same frequency in different laboratories. The correction for the effect of blackbody radiation, while maintaining standard conditions in the trap cavity, makes it possible to achieve identical frequencies of the strontium transition in several laboratories, with an uncertainty of 10^{-17}, or better, which is already beyond what is currently feasible with cesium. This also demonstrates a certain consistency at the international level. In terms of definition, defining the second based on a single transition is the closest thing to the current scheme with the cesium definition [4]. Given the value of the transition, we could say that: **The second is the duration of** $42\,9XX\,XXX\,XXX\ ,XXX$ **periods of the radiation corresponding to the transition between the $^1S_0$ and $^3P_0$ states of the ^{87}Sr atom**, with the number of periods calibrated to correspond to the absolute frequency of this transition. This sentence then fixes the strontium frequency as the reference for the second [102]. The advantage of this definition is that it is simple to communicate and understand, even outside the scientific community, since a single reference atom and a single line are used, in the same way as what we did for 50 years with cesium-133. It should be noted that other transitions could have been considered for a single definition, in particular the $^1S_0 \rightarrow {}^3P_0$ transition of ytterbium 171 (at 578 nm), which offers comparable performances [32, 51, 99]. It has the advantage of having a higher absolute frequency (of the order of 5×10^{14} Hz), which gives a higher number of cycles per second. The performances are thus comparable. The major difference is, as for clocks, due to the relatively low deployment of this transition internationally. It would, however, be a very honorable candidate, if an obstacle were to arise with strontium. As for ion transitions, one could imagine defining the second via the octupole transition of ionized ytterbium (467 nm) or via the hyperfine transition of ionized aluminum (267 nm) [30, 98]. However, they are currently poorly reproduced and require additional scientific and technological advances in order to achieve the desired stabilities. This makes them less suitable as an immediate primary standard.

The other possible approach to redefining the second is not to limit ourselves to a single reference atom, but to rely on several high-performance atomic transitions simultaneously, combining their frequencies in a coherent way. Concretely, this would involve defining the second so that a set of chosen transitions have known fixed frequencies, and ensuring that the official value of the second is linked to an average of these different frequencies [4, 44]. The idea is that by including several atomic species in the definition, we obtain a standard that is more robust against future evolution: if a new, even better clock appears (based on another transition), it could be incorporated into the average without the need to redefine the unit again. Moreover, this pools the strengths and weaknesses of each transition, reducing the risk that an unknown bias specific to a given species will distort the realization of the unit in the long term. If this option is chosen, the choice of transitions to be included in the reference set must be judicious. Ideally, several complementary transitions, originating from both neutral atoms and trapped ions, should be retained to diversify the systems exploited. Based on current progress, this set could contain:

- transition $^1S_0 \to {}^3P_0$ of ^{87}Sr: This is the same transition as previously mentioned. It would bring stability and accuracy to the definition, which are in the order of what is sought. It would serve as the primary transition of the average;
- transition $^1S_0 \to {}^3P_0$ of ^{171}Yb: This would be a secondary trans ition, bringing performance comparable to that of strontium but with a different species. Since they are different species, the systematic errors are not correlated and an average of the two would therefore be more reliable than either separately;
- Octupole transition of $^{171}Yb^+$: This is one of the most studied ion transitions currently. It has the advantage of exhibiting different characteristics since it is an ionized species. It therefore does not undergo the same systematic errors and is not trapped in a light potential like the two previous species. It therefore has a sensitivity to the Autler-Townes effect that is nonexistent, compared to the previous species.
- transition $^1S_0 \to {}^3P_0$ of $^{27}Al^+$: This transition could be useful since it has the smallest uncertainty ever achieved, of the order of 9×10^{-19}.

This list is obviously non-exhaustive and non-evolving, but it presents the most prominent transitions for a definition of the second in the current state of scientific research. In practice, each transition would be assigned a weight proportional to the confidence we have in its occurrence. For example, and this is typically what is envisaged, each transition would be weighted according to its systematic uncertainty. The absolute frequency values would thus be fixed at the time of the redefinition, as is the case for the frequency of the hyperfine transition of caesium 133. The definition could then be stated as: **The second is defined so that the transition frequency of ^{87}Sr is X Hz, that of**

^{171}Yb is *Y* Hz, that of ^{171}Yb^{+} is *Z* Hz, ..., that is to say by assigning to each of these references an exact value, within the limits of the uncertainties known at that time [32]. No laboratory would use all the transitions at the same time, but each laboratory would provide the BIPM with its frequencies, and the BIPM would take a weighted average to establish the TAI and create the timescale. The advantage of this option is therefore its flexibility and sustainability: adding a new revolutionary clock to the list could be done without changing the formal definition, by being included in the average as a contributing standard. Moreover, the multitude of transitions used makes it possible to limit biases due to undetected systematic errors, errors being linked to the nature of the atoms. This would also be closer to what is done in practice, given that, although the second is defined based on cesium-133, the TAI is calculated by averaging over 400 clocks, including hydrogen masers, but also optical clocks as secondary standards, in order to smooth out any potential errors [4]. This approach also has drawbacks, notably through the complexity of the definition. Indeed, the definition of the second in this case is no longer just a sentence that links the second to the frequency of an atom, but involves several references. For a scientist, this should not pose a problem, but for the understanding of the public at large, it can make the unit abstract, while it is already unnatural to define it on the basis of an atomic transition and the vast majority of the non-scientific community still bases itself on the movement of the Earth around the Sun.

3.9.3 Redefinition scenarios

After identifying the clock and transitions, it is appropriate to examine how, in practice, the realization of the second would be achieved, implemented, and maintained on a daily basis.

In the case of the first option, the second is officially redefined based on a single atomic transition. Realization would then be achieved at each location using an optical clock of this type, or by comparing a local clock to a reference clock of this type. At the time of the redefinition, only a limited number of laboratories will have the clock chosen as a reference and functional to achieve the new definition [31]. These laboratories will serve as primary laboratories to achieve the second. The data from these laboratories will be used to feed the TAI on a regular basis, as is currently the case with cesium clocks, but with a precision one hundred times better than today. Metrology institutes that do not have the chosen clock will then be able to undertake to obtain or manufacture one and will be able to take advantage of this to allow their microwave atomic clocks (hydrogen, rubidium, cesium) but also optical clocks for which the transition has not been chosen to also integrate the TAI calculation, as secondary standards. Moreover, while waiting for the development of clocks in the various institutes worldwide, it is a safe bet that the cesium definition will continue to play an important role in order to ensure continuity of the definition. As also described previously, it will be necessary to propose comparisons between clocks from different laboratories in order to maintain the uniqueness of the definition. The comparison will be made through different links, such as fiber optics or satellite links, and will be among the simplest since it will consist of comparing the same transition in several different locations. As discussed previously, comparisons can be made through transportable clocks, as soon as such transportability becomes possible, in order to achieve a direct comparison [102]. From a reliability perspective, it will be essential to ensure that the chosen clocks can operate stably over a long period of time, without prolonged intervention. It will therefore be necessary to ensure that a clock is always functional. This condition may be difficult to meet at first if only one clock is chosen and if few laboratories around the world possess said clock. In summary, the single-transition scenario is consistent with the current organization of metrology: a few laboratories maintain a primary standard, and the others align through the TAI [30]. Personally, I believe this scenario is feasible by 2030 and would allow for a smooth and understandable transition.

The scenario of a definition based on an average of several transitions differs in that it requires several clocks of different types. Implementing such a definition would be more complex, as it requires synchronizing several different clocks to construct the second. If the previous example were to be implemented, it seems almost impossible for a laboratory to possess the complete set of clocks that would be used for the definition. Thus, achieving the second requires the establishment of a collaborative network: each laboratory will perform the definition with the atomic species they are studying, and the results from each laboratory, provided that the species appears in the definition, will be pooled to implement TAI. The criteria set by the CGPM also specify that a minimum of three laboratories effectively implementing the transition is required. This will also be a more complex mechanism, since the BIPM will have to frequently modify the weighting of the different species based on the deviations that the different clocks exhibit from the fixed value of the transition frequency. Increased cooperation between the different laboratories and the BIPM will be necessary to maintain an acceptable implementation of the second. Furthermore, clock comparisons are still mandatory and essential for this redefinition, with the difference that it will no longer be necessary to compare two identical clocks, but two different clocks, thus measuring all possible frequency ratios and regularly verifying that these ratios are maintained. The idea could be to design a portable clock that will travel from laboratory to laboratory to measure these frequency ratios. The comparison system is therefore much more cumbersome and redundant than in the case of a single transition [95, 96]. From a metrological perspective, this option can be seen as more inclusive than the first. Indeed, an institute that develops a clock with a particular transition and achieves remarkable performance will be able to contribute to the TAI without having to develop all types of clocks and transitions [44]. The redefinition thus does not favor a single type of expertise to the detriment of others, but rather highlights the diversity of scientific advances. This definition also has the advantage of being more robust, since a set of transitions makes it possible to smooth out any systematic effects that might not have been detected, as these depend on the nature of the atomic species. Thus, to satisfy this definition, it would be necessary to implement a global network of clocks. Establishing such a network seems feasible, but would require increased commitment and coordination, which seems to me to be highly ambitious for a redefinition by 2030.

Finally, the third option is to define the second based on the numerical value of a constant, such as the electron mass m_e, or the Compton frequency associated with this mass ν_e, instead of relying on an atomic transition [101]. This choice would aim to align the second with the other units of the International System, which are all fundamental physical constants, such as the speed of light c, or Planck's constant h. This method would allow the second to be embedded within fundamental physics. This second would thus be universal since it would not depend on a particular isotope of any atom and would be applicable at all times, without needing to be modified again. It would also not be subject to systematic effects since it would not be based on any experimental protocol. It would also be consistent with the International System by treating time as the same thing as other units, such as mass or length. However, it would suffer from several major challenges. First, the physical constants considered are currently not fixed exactly, but with an uncertainty, which can be much larger than that provided by optical clocks. There is also no experimental hardware precise enough to implement this experimentally, nor a comparison system to compare these tools.

3.9.4 Final opinion

At the end of this analysis, my personal position regarding the redefinition of the second by 2030 is becoming clearer. In my opinion, it should be based on the first and second options; the third does not seem adequate, as explained previously. I believe this redefinition should be carried out in two stages [102]. First, I believe the new definition of the second should be based on the optical lattice clock, using the $^1S_0 \rightarrow {}^3P_0$ transition of strontium-87 as the primary standard, while also allowing for a longer-term multi-transition approach. In practice, this means initially opting for the first option when voting on the redefinition, as it provides simplicity and speed, before gradually evolving by integrating other standards as primary standards to create this new second and ensure its sustainability.

This gradual strategy reconciles the advantages of both options under consideration. The strontium transition has the advantage of being sufficiently studied and tested by laboratories, which know which standard to work with, with a definition that can be stated simply, proposing a smooth transition to a new atom, in line with what is currently being done for cesium-133. However, the initial choice of a single main standard should not overshadow the achievements achieved by other clocks, which, in some cases, are just as excellent as those using strontium.Thus, in parallel with a pre-definition using exclusively strontium, it would be desirable for the scientific community to continue developing other clocks, or to pursue the efforts already in place in order to propose a complete system, on a global scale, for realizing the second based on an average of several optical clocks. Thus, the official definition of the second would be that based on the strontium-87 transition, but the practical implementation of TAI could already use a weighted average including the other species studied in this framework, as secondary standards. This gradual addition would immediately guarantee the benefits of robustness and inclusivity without changing the definition. This approach is comparable to what we are experiencing today, given that cesium defines the second, but TAI is in practice realized on the basis of clocks that are not all cesium [30, 102].

Thus, in 2030, I believe that cesium will indeed become a secondary realization of the second, with the establishment of an evolving definition: strontium would become the temporary official reference for this definition, before technological and scientific advances allow the definition to be modulated by adding other standards to the weighted average [98, 66]. The complexity of this definition of the second will lie in immediately anticipating that other transitions may occur, even if they are not initially present. This method, in my opinion, would allow for an optimal realization of the time scale. This opinion is based both on current reality and on the possibilities of technological advances: it is clear that the strontium clock could quickly become the standard carrying the new definition.

However, it seems to me that currently, a multi-standard definition would be difficult to implement and that it is necessary to wait a few more years before they can be incorporated into a definition.

In conclusion, I argue that the definition of the second is likely for 2030, based on strontium-87 clocks, while maintaining the scientific community's clear desire to add, once sufficient progress has been made, other standards, which will already be used for the implementation of the TAI. This will ensure that the second of 2030 will be at least 100 times more accurate than that of 1967, while maintaining efforts to modernize the metrological engineering currently under development.

Conclusion

The measurement of time is a fundamental issue, both from a physical perspective and in the technological and institutional aspects of the contemporary world. It is based on historical, astronomical, and then physical foundations, which have evolved with advances in observation and growing demands for timekeeping accuracy. This thesis examined this evolution, from Universal Time (UT), based on Earth's rotation to the possibility of a redefinition achieved by optical clocks in the coming years.

The first chapter reconstructed the history of time measurement based on astronomical observations. The Earth's rotation, relative to the fixed stars or the Sun, was for centuries the absolute reference for the flow of time, particularly based on concepts such as sidereal time and mean solar time. Universal Time, based on this rotation, gradually evolved to account for the irregularities observed in the Earth's rotation [2, 3].

The second chapter presented the foundations of the current definition of the second, adopted in 1967 by the General Conference on Weights and Measures, based on the hyperfine transition of the ground state of the cesium-133 atom. The operation of atomic clocks, from thermal jet devices to cold atom fountains, showed how quantum physics is used to produce highly stable reference oscillators. International Atomic Time, defined as a weighted average of globally distributed atomic clocks, now constitutes the backbone of modern metrology. This chapter also introduced essential concepts in metrology, such as Allan deviation, accuracy, stability, and the sources of these uncertainties. It also showed that, although robust and reliable, cesium clocks are reaching a performance limit [4, 5].

The third chapter focused on the future. It explored in depth the operation of optical clocks, distinguishing between optical lattice and trapped-ion clocks. The purpose was to study advances enabled by the use of optical frequencies. These are much higher than the microwave frequency used in cesium clocks. The role of components, their tuning, and the principal systematic effects that limit the performance of these clocks were then discussed. This chapter concluded with an analysis of the criteria proposed by the CGPM for choosing a new standard for the second. The analysis conducted in this thesis confirms that the current definition of the second is now exceeded by the technical capabilities of optical clocks. Indeed, while the best cesium fountains achieve an accuracy of around 10^{-16}, optical clocks regularly demonstrate performance of around 10^{-18}, a gain of two orders of magnitude. This results in a growing gap between the unit and its best practical realizations which could harm the consistency of the International System of Units [7, 8, 99].

However, a redefinition can only be considered within a rigorous framework. It requires worldwide experimental validation, intercomparison of optical clocks, the definition of a unified implementation, and a support strategy for metrology organizations and industries. It is therefore essential to lay the physical, institutional, and technical foundations for this transition now. The major personal contribution of this thesis was to propose an evaluation of candidate transitions for the redefinition of the second, taking into account the stability and accuracy of clocks, but also their reproducibility between laboratories, the maturity of the technologies used, and compatibility with existing infrastructures. Following this analysis, some transitions, such as that of strontium-87, appear particularly promising, while others, such as that of mercury ion, still pose significant challenges [7, 8, 30]. Finally, an implementation timeline is discussed.

These contributions aim to enrich the debate by providing decision-making tools that are not based solely on raw metrological performance, but that also integrate dimensions of governance, equity between laboratories, and practical implementation.

Redefining the second is not simply a matter of improving the accuracy of our clocks. This definition will indeed have significant impacts on a large number of everyday tools, such as telecommunications networks and navigation systems. The integration of optical clocks on a large scale will require the construction of an interconnected global network and a partial overhaul of metrology infrastructures. It will also invite other fundamental reflections: is a single standard necessary? Can the second be defined based on a universal constant?

This study does not aim to definitively resolve these questions, but it hopes to have made a rigorous, structured, and coherent contribution to a major debate that will have a lasting impact on history and metrology. It emphasizes that the redefinition of the second is both inevitable and highly strategic, because it conditions our ability to measure, synchronize, and understand the subtle phenomena of our Universe.

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