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Parallel Harmonic Balance Method for the Analysis of Nonlinear Mechanical Systems

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Abstract

The accurate prediction of nonlinear vibration phenomena remains a key challenge in modern turbomachinery and aerospace design, as lightweight and highly integrated structures are increasingly affected by geometric nonlinearities, modal interactions, and amplitude-dependent dynamics that cannot be captured by classical linear vibration models. Robust frequency-domain formulations combined with scalable numerical solvers are therefore required to address industrial-scale finite element analyses.

This work presents a parallel Harmonic Balance framework for the analysis of geometrically nonlinear elastic systems. Large deformations are modelled using a finite element formulation based on the Green-Lagrange strain tensor. The application of the Harmonic Balance Method leads to a nonlinear algebraic system solved using a Newton–Raphson algorithm embedded within a continuation strategy, allowing nonlinear frequency responses (NLFRs) and nonlinear normal modes (NNMs) to be investigated in a unified manner. For the computation of nonlinear normal modes, an additional phase condition and a relaxation parameter are introduced to ensure uniqueness and convergence.

In parallel, a rotating-frame formulation is developed and integrated into the framework to account for rotational effects, enabling the computation of Campbell diagrams as well as their consistent extension to the evaluation of NNMs and NLFRs.

The framework is implemented in the in-house pyHarm solver and subsequently deployed on Quanscient's cloud-native multiphysics platform Allsolve, which relies on domain decomposition and distributed-memory linear solvers (MUMPS, PETSc) for large-scale parallel finite element simulations. Comprehensive numerical studies are carried out on academic benchmark structures as well as on an industrial fan-blade configuration. Nonlinear frequency responses, nonlinear normal modes, and Campbell diagrams are analysed, and an excellent agreement between the different descriptions is observed, confirming the physical consistency of the proposed methodology. Scalability analyses demonstrate efficient strong scaling behaviour up to 64 MPI processes, with computational performance primarily governed by matrix assembly and factorisation. The Alternating Frequency–Time formulation is compared with a direct frequency-domain evaluation of the nonlinear terms, and the impact of quadruple-precision arithmetic is investigated, highlighting a trade-off between numerical robustness and computational cost.

Overall, the proposed framework enables predictive and scalable nonlinear vibration analysis in the frequency domain, while also providing a consistent nonlinear rotating-frame formulation for the investigation of rotating aerospace structures.

Keywords: Harmonic Balance Method, Nonlinear Frequency Response, Nonlinear Normal Modes, Geometric Nonlinearities, Continuation, Rotating Frame, Campbell Diagram, High-Performance Computing, Quanscient Allsolve.