

The porpoising effect : wind tunnel testing and reduced-order-modelling.

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UNIVERSITY OF LIÈGE – FACULTY OF APPLIED SCIENCES

The Porpoising Effect: Wind Tunnel Testing and Reduced-Order Modelling

MASTER'S THESIS PRESENTED BY
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Abstract

Ground-effect aerodynamics can generate large amounts of downforce, but it also makes the aerodynamic loading highly sensitive to ride height. When this ride-height sensitivity interacts with the vertical dynamics of a vehicle, self-sustained oscillations may appear. This phenomenon, commonly referred to as porpoising, became a major issue after the reintroduction of ground-effect aerodynamics in Formula One, but its underlying aerodynamic mechanisms remain difficult to isolate on a complete vehicle.

This work investigates porpoising using a simplified wind-tunnel model based on a two-dimensional inverted Tyrrell026S wing operating in ground effect. Static tests were first performed to characterise the evolution of the aerodynamic loads with angle of attack and ground clearance. Dynamic tests were then conducted using two aeroelastic setups: a plunge–pitch setup, allowing both vertical translation and rotation, and a plunge-only setup, designed to isolate the vertical instability mechanism.

The experimental results show that the post-critical response is mainly dominated by plunge motion. Based on this observation, a reduced-order model is developed. The model is first formulated as a two-degree-of-freedom plunge–pitch system and is then reduced to a single-degree-of-freedom plunge formulation. Two aerodynamic descriptions are compared: a quasi-steady model based on the static aerodynamic database, and an unsteady model based on experimentally identified plunge flutter derivatives.

The quasi-steady model predicts the static aeroelastic deflection with reasonable accuracy, but fails to reproduce the measured evolution of modal frequency and effective damping. In contrast, the unsteady model captures the stability trends of the plunge-only system for most tested ground clearances. The results therefore show that static aerodynamic data alone are not sufficient to model porpoising dynamics, and that motion-induced unsteady aerodynamic effects must be included to predict the onset of instability.

Keywords: Porpoising, ground effect, aeroelasticity, inverted wing, wind-tunnel testing, reduced-order modelling, unsteady aerodynamics, flutter derivatives.

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1 Introduction

1.1 Historical Background and Motivation

In the early decades of motor racing, aerodynamic design was mainly focused on reducing drag. The objective was to improve straight-line speed by streamlining the bodywork and reducing the resistance of the vehicle to the incoming airflow. This approach is illustrated in Figure 1.1, which shows the Mercedes-Benz record car driven by Rudolf Caracciola in 1938. At that time, performance gains were primarily sought through engine power and drag reduction, while aerodynamic downforce was not yet a dominant design objective.

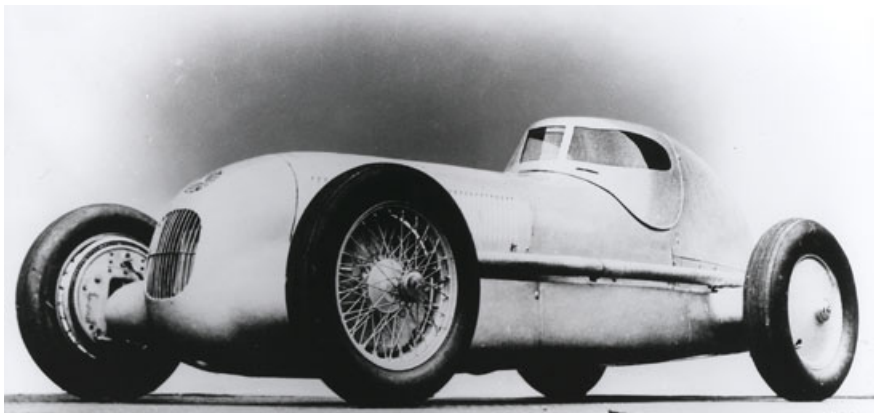


Figure 1.1: Mercedes-Benz record car driven by Rudolf Caracciola during the 1938 speed-record attempts [1].

From the 1960s onwards, racing car designers progressively understood that aerodynamic forces could also be used to improve cornering performance. Instead of only reducing drag, aerodynamic devices such as inverted wings were introduced to generate downforce. This additional vertical load increased the normal force acting on the tyres, thereby increasing the available grip without increasing the vehicle mass [2]. As a result, aerodynamic development became a key performance factor not only on straight lines, but also, and more importantly, during cornering.

A major step in the development of car aerodynamics was achieved by exploiting the presence of the ground, through what is known as ground-effect aerodynamics. This principle was first used by the Chaparral 2J, introduced in the 1970 Can-Am championship. As shown in Figure 1.2, the Chaparral 2J used rubber skirts to seal the region between the underside of the car and the ground. Two auxiliary fans extracted air from beneath the car, creating a low-pressure region under the vehicle and therefore producing a large amount of downforce. A key advantage of this system was that the generated downforce was almost independent of the car speed, since it was produced by the fans rather than by the natural airflow over and under the bodywork. This concept gave the car a significant performance advantage, but it also raised strong opposition from competitors and was eventually banned.

Ground-effect aerodynamics later became a central concept in Formula One with the introduction of the Lotus 78 in 1977. Unlike the Chaparral 2J, the Lotus 78 did not use fans

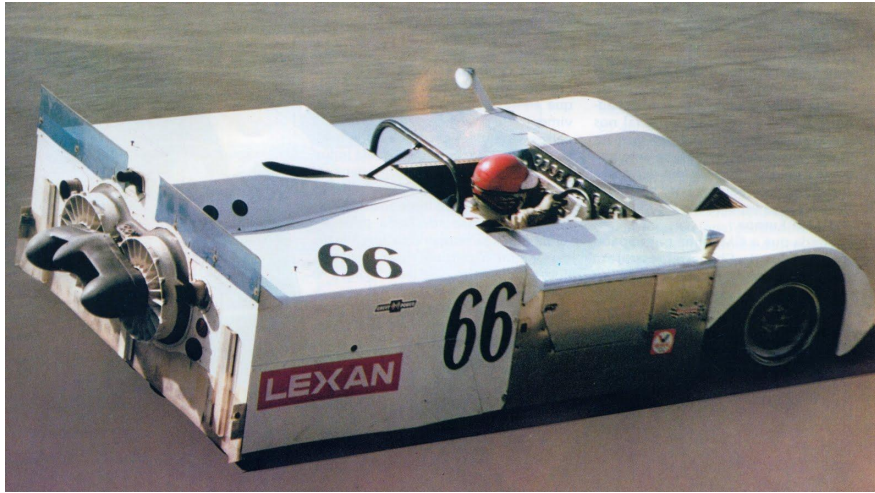


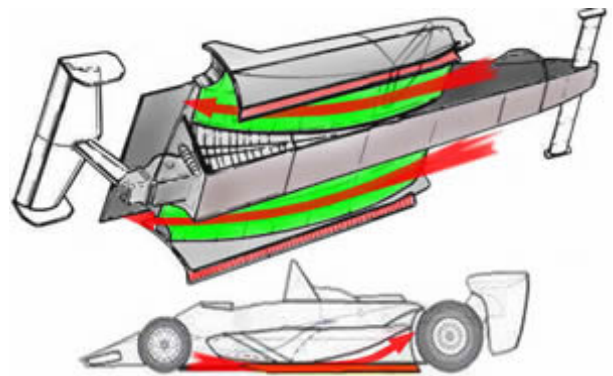
Figure 1.2: Chaparral 2J Can-Am car, one of the first racing cars to exploit ground-effect aerodynamics through fan-generated suction [3].

to extract air from beneath the car. Instead, its sidepods were shaped as inverted Venturi tunnels. The underbody flow was accelerated through these tunnels, which reduced the static pressure beneath the car and generated downforce [4]. This principle is illustrated in Figure 1.3.

The same period also saw other extreme interpretations of ground-effect concepts, such as the Brabham BT46B in 1978. This car used a fan-assisted system and, similarly to the Chaparral 2J, raised significant concerns among rival teams because of its potential performance advantage. These developments showed that ground-effect aerodynamics could provide very large gains in cornering speed, but also that it introduced new safety and regulatory challenges.



(a) Lotus 78 Formula One car [5].



(b) Schematic representation of ground-effect flow beneath a Formula One car [6].

Figure 1.3: Ground-effect aerodynamic concept applied to Formula One.

The main limitation of early ground-effect Formula One cars was their high sensitivity to ride height and sealing. These cars relied on maintaining a strong aerodynamic interaction between the underfloor and the ground. If this interaction was suddenly degraded, for example due to a loss of sealing, excessive ride-height variation or flow separation, the car could lose a large amount of downforce almost instantaneously [7]. Since ground-effect cars were

In 2022, Formula One reintroduced ground-effect aerodynamics as a central element of the technical regulations. The objective was not only to recover downforce from the underfloor, but also to improve the ability of cars to follow each other closely. Before 2022, a large fraction of the downforce was generated by complex external aerodynamic appendages, especially wings, bargeboards and highly sensitive flow-conditioning devices. These elements produced a turbulent wake behind the car, commonly referred to as dirty air, which reduced the aerodynamic performance of a following car and made overtaking more difficult.

A side-by-side comparison of the 2021 and 2022 Mercedes-AMG Petronas Formula 1 cars. The 2022 car (top) features a silver and blue livery with yellow accents, while the 2021 car (bottom) is primarily black with blue and yellow accents. Both cars display various sponsor logos including Petronas, TeamViewer, AMD, and IWC. The year '2022' is written in large black text to the left of the top car, and '2021' is written in large black text to the left of the bottom car. The Mercedes logo is visible on the sidepods of both cars. The background is white.

However, during the first pre-season tests of the 2022 Formula One cars, teams observed strong vertical oscillations on the straights. This phenomenon, commonly referred to as *porpoising*, revealed one of the major limitations of modern ground-effect aerodynamic concepts, being their high sensitivity to ride height and vehicle motion. The issue was not purely aerodynamic and not purely mechanical, but resulted from the coupling between the underfloor flow, the generated aerodynamic loads, the suspension response and the vertical motion of the car [9].

Porpoising rapidly became a critical concern because it directly affected both performance and safety. From a performance perspective, the phenomenon limited the usable operating range of the underfloor. In order to reduce the oscillations, teams often had to increase the static ride height of the car, which weakened the ground-effect mechanism and reduced

the available downforce. As a result, the aerodynamic configuration that could theoretically provide the highest performance was not necessarily the one that could be used safely and consistently on track.

From a safety and reliability perspective, the repeated vertical accelerations affected straight-line stability and generated severe physical discomfort for the drivers. More fundamentally, porpoising demonstrated that ground-effect aerodynamics cannot be assessed only through static aerodynamic performance indicators such as downforce and drag. A configuration producing high downforce may become unsuitable if its aerodynamic response interacts unfavourably with the vehicle dynamics. This makes porpoising a particularly important phenomenon, since it lies at the boundary between aerodynamic efficiency, dynamic stability and practical vehicle performance.

1.2 Objectives

Porpoising is an aeroelastic instability in which the aerodynamic loads generated close to the ground interact with the vertical dynamics of the vehicle. Several studies have already addressed this phenomenon from different perspectives. Gadola et al. [7] showed the importance of combining ride-height-dependent aerodynamic maps with suspension dynamics for high-downforce race cars. More recently, Bauer, Papangelo and Habib [9] proposed a minimal-order model able to reproduce and control porpoising-like oscillations. However, the aerodynamic mechanisms responsible for the loss of stability are still not fully documented. In particular, the role of unsteady aerodynamic effects and their contribution to negative aerodynamic damping remain open points.

The aim of this work is to study porpoising experimentally and to develop a reduced-order model able to reproduce the stability behaviour of a simplified aeroelastic system. The study is based on a two-dimensional inverted wing operating in ground effect. This configuration is not intended to reproduce the full complexity of a Formula One car. Instead, it is used as a simplified test case in which the coupling between ground-effect aerodynamics and vertical structural motion can be isolated. This approach remains consistent with previous studies on inverted wings and ground-effect diffusers, while keeping in mind that three-dimensional effects and complete-vehicle interactions may still play an important role in real applications, as shown by Ehirim, Knowles, and Saddington [10].

The main objectives are therefore to characterise the static aerodynamic behaviour of the tested wing, to observe the onset of porpoising on a dynamic setup, and to identify the aerodynamic contributions associated with the plunge motion. The final objective is to obtain a reduced-order model capable of predicting the static deflection, the modal frequency evolution and the effective damping evolution of the plunge-dominated aeroelastic system. A particular focus is placed on the comparison between quasi-steady and unsteady aerodynamic modelling. The quasi-steady approach relies on static aerodynamic data, whereas the unsteady approach uses *flutter derivatives* identified from the dynamic measurements.

2 Theoretical Framework and Reduced-Order Modelling

This chapter introduces the physical mechanisms required to analyse the porpoising behaviour of a wing operating in ground effect. The phenomenon cannot be described as a purely aerodynamic or purely structural instability. It results from the coupling between aerodynamic loads, which are highly sensitive to ground clearance, and the dynamic response of the elastic system.

The chapter is therefore divided into three main parts. First, the aerodynamic quantities used throughout the work are introduced, together with the notions of boundary layer, flow separation and ground effect. Particular attention is given to the dependence of the aerodynamic coefficients on the ground clearance H , or equivalently on the normalized ground clearance H/c . Secondly, the basic aeroelastic concepts required to study the stability of the system are presented. Finally, the construction of the linear pitch-plunge reduced-order model is presented.

2.1 Aerodynamic Fundamentals

2.1.1 Forces

In a quiescent atmosphere, the air is at rest with respect to the ground. Therefore, a car moving at a velocity U_∞ relative to the ground experiences an incoming airflow of the same magnitude, but in the opposite direction, in the vehicle-fixed reference frame. In the presence of atmospheric wind, the air also has a velocity relative to the ground, and the relative airflow experienced by the car is no longer equal to the vehicle speed alone.

To simplify the aerodynamic analysis, only the quiescent-air case is considered in this work. This means that the air is assumed to have zero velocity relative to the ground, so that the incoming flow velocity seen by the car is directly determined by the vehicle speed U_∞ .

The aerodynamic forces acting on the car are governed by the relative motion of the air with respect to the vehicle. In straight-line motion, and assuming no crosswind or lateral motion, the aerodynamic loading can be represented by a resultant force F_{aero} . This resultant force can be decomposed into two main components: a longitudinal component, called drag D , and a vertical component, called lift L , as illustrated in Figure 2.1.

The drag force D acts in the direction of the relative airflow and opposes the motion of the vehicle in the ground-fixed reference frame. The lift force L is the vertical component of the aerodynamic resultant. For conventional aircraft and most road cars, positive lift is directed upward. For racing cars, however, the objective is generally to generate negative lift, commonly referred to as downforce, in order to increase the normal load on the tyres and improve grip.

The resultant aerodynamic force acts at the centre of pressure, denoted CP . The centre of pressure corresponds to the point at which the distributed aerodynamic pressure and shear stresses acting on the vehicle can be replaced by an equivalent resultant force. Its position is

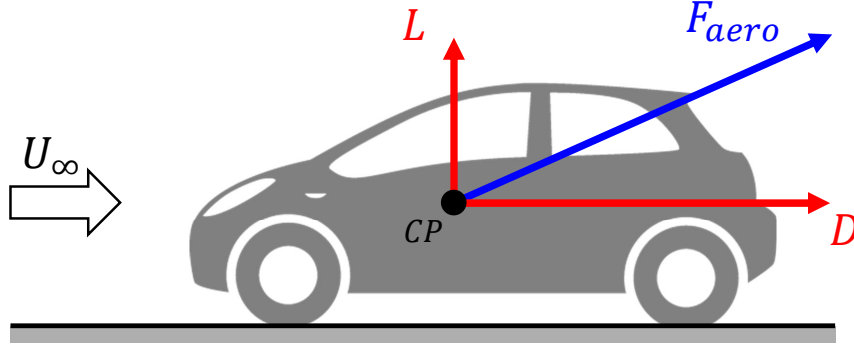


Figure 2.1: Aerodynamic force decomposition in straight-line motion.

not fixed in general and may vary with the vehicle geometry, flow conditions, ... [11]

The lift and drag forces can be expressed as

$$L = \frac{1}{2} \rho U_\infty^2 A C_L, \quad (2.1.1)$$

$$D = \frac{1}{2} \rho U_\infty^2 A C_D, \quad (2.1.2)$$

where ρ is the air density, U_∞ is the relative flow velocity, and A is the reference area. The quantities C_L and C_D are the non-dimensional lift and drag coefficients, respectively.

Equivalently, these coefficients can be obtained from measured or computed force values as

$$C_L = \frac{L}{\frac{1}{2} \rho U_\infty^2 A}, \quad (2.1.3)$$

$$C_D = \frac{D}{\frac{1}{2} \rho U_\infty^2 A}. \quad (2.1.4)$$

2.1.2 Boundary layer and airflow separation

Atmospheric air is a viscous fluid. As a consequence, when a body moves relative to the surrounding air, viscous friction appears between the airflow and the surface of the body. At the wall, the air particles in direct contact with the surface have zero relative velocity with respect to the body. This condition is known as the no-slip condition.

Due to viscosity, the neighbouring layers of air are also slowed down by momentum diffusion. However, this viscous influence progressively decreases with the distance from the wall. Far enough from the surface, the air velocity tends to the external flow velocity U_∞ , and the flow is almost unaffected by the presence of the wall.

The thin region close to the surface where the flow velocity changes from zero at the wall to approximately the freestream velocity is called the boundary layer. Within this region, viscous effects are important and generate wall shear stress. Outside the boundary layer, viscous effects are usually much weaker, and the flow can often be approximated as inviscid [12]. The boundary layer thickness, denoted δ , is commonly defined as the distance from the wall at which the local velocity reaches 99% of the freestream velocity:

$$u(y = \delta) = 0.99 U_\infty. \quad (2.1.5)$$

The wall shear stress is directly related to the velocity gradient at the wall:

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}, \quad (2.1.6)$$

where μ is the dynamic viscosity of air. A steeper velocity gradient at the wall therefore leads to a larger viscous shear stress.

The boundary layer can develop under different flow regimes, mainly laminar or turbulent. A simple reference case is the development of a boundary layer over a flat plate aligned with a uniform freestream. As illustrated in Figure 2.2, the boundary layer initially develops in a laminar regime from the leading edge. Further downstream, disturbances in the flow are amplified and the boundary layer enters a transition region. After this transition, the boundary layer becomes turbulent.

The laminar boundary layer is generally thinner and produces lower wall shear stress than the turbulent boundary layer. However, it is also more prone to separation when exposed to an adverse pressure gradient, because the near-wall flow contains less momentum. In contrast, a turbulent boundary layer is thicker and generates higher skin-friction drag, but it is usually more resistant to separation due to stronger momentum exchange between the outer flow and the near-wall region [12].

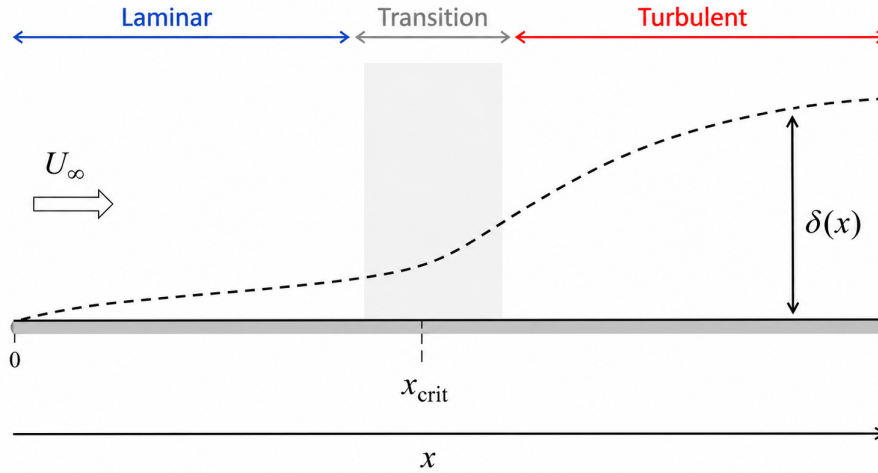


Figure 2.2: Development of a boundary layer over a flat plate aligned with a uniform freestream.

2.1.3 Ground effect

When a car moves close to the ground, boundary layers develop on both the underbody and the ground surface. These viscous regions reduce the effective flow passage beneath the vehicle through the displacement-thickness effect, thereby modifying the pressure distribution and the aerodynamic loads.

The flow around a car is therefore more complex than the flow around an isolated aeronautical airfoil in freestream conditions. The proximity of the ground creates a strong interaction

between the underbody flow and the surrounding flow, while the low-aspect-ratio and geometrically complex shape of the vehicle generates a highly three-dimensional flow field [11].

Although ground effect has been widely studied in aeronautics, particularly for wings during take-off and landing, its application to racing cars involves additional complexities such as confined underbody flow, rotating wheels, moving-ground effects and coupling with vehicle dynamics. Within the scope of this study, the aerodynamics of the complete vehicle is simplified to a two-dimensional airfoil model. This reduction allows the fundamental mechanisms of ground-effect aerodynamics and porpoising to be isolated, while keeping the analysis sufficiently general to be applicable to other ground-effect configurations.

Several studies have investigated the influence of ground proximity on the aerodynamic performance of two-dimensional airfoils. Among them, Ranzenbach and Barlow studied inverted airfoils in ground effect in the context of racing-car front-wing aerodynamics. In their 1994 study [13], the front wing of a Formula One or Indy-type racing car was idealised as a symmetric two-dimensional airfoil operating close to the ground. The authors combined experimental measurements with Reynolds-averaged Navier–Stokes simulations in order to analyse the evolution of the aerodynamic forces as the ground clearance was reduced.

Their results showed that significant negative lift, corresponding to downforce, is generated when the airfoil operates at heights of the order of the airfoil thickness or lower. However, the downforce does not increase indefinitely as the airfoil approaches the ground. Below a sufficiently small clearance, a force-reduction phenomenon appears. The generated downforce decreases, and a force reversal may even occur. This behaviour is attributed to the interaction and possible merging of the boundary layers developing on the airfoil and on the ground, which modifies the effective flow passage beneath the airfoil and reduces the suction responsible for downforce generation.

It should be noted that road conditions, corresponding to the situation in which the ground and the freestream have the same relative velocity, do not imply the complete absence of viscous effects at the ground. In the vehicle-fixed reference frame, an ideal moving ground reduces the upstream ground boundary layer because the ground moves with the freestream velocity. However, as the flow is accelerated through the narrow gap between the airfoil and the ground, velocity gradients can still appear near the ground surface. Consequently, a local viscous region may develop and interact with the boundary layer on the lower surface of the airfoil.

Ranzenbach and Barlow also extended their ground-effect studies to inverted cambered and multi-element airfoils, still in the context of racing-car front-wing aerodynamics [14–16]. These geometries are more representative of race-car front wings than a symmetric airfoil and generate higher downforce levels for comparable ground clearances. Nevertheless, the same qualitative trend is observed. Decreasing the ground clearance initially increases the downforce, but below a critical clearance, the downforce decreases.

Wiedemann proposed a qualitative classification of ground-effect flow regimes based on the ground clearance between the aerodynamic profile and the road surface [17]. This classification is useful to interpret the evolution of the lift coefficient with ground clearance, as illustrated in Figure 2.3. For an inverted airfoil, negative values of C_L correspond to downforce. Three main regimes can be distinguished.

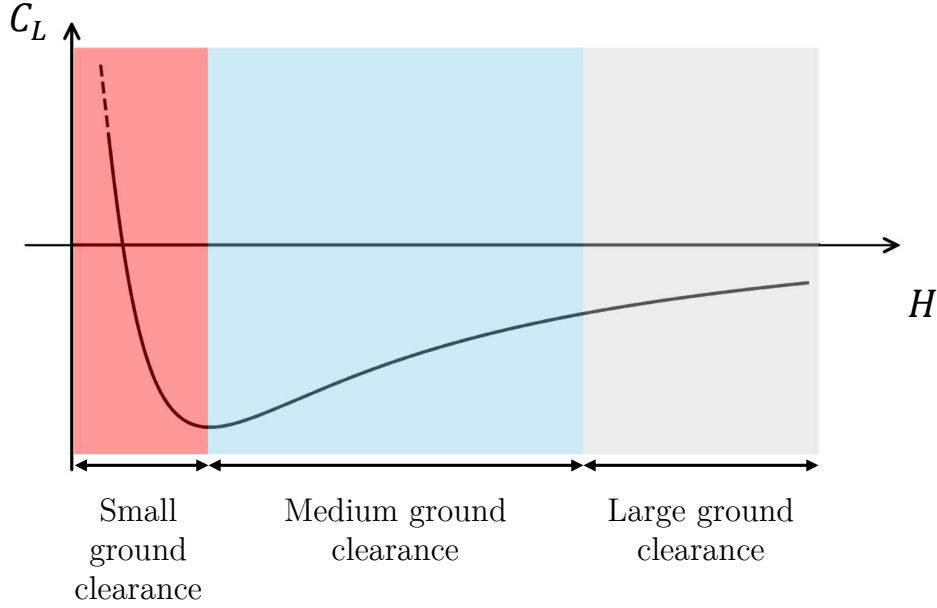


Figure 2.3: Evolution of the lift coefficient C_L with ground clearance H for an inverted airfoil in ground effect.

- **Large ground clearance.** When the clearance is larger than the chord length, the influence of the ground is weak. The airfoil behaves almost as if it were in freestream conditions. The velocity field near the airfoil is only slightly affected by the presence of the ground, and the lift coefficient tends towards its freestream value.
- **Medium ground clearance.** When the clearance becomes smaller than the chord length, the ground significantly modifies the flow field. The flow is accelerated in the gap between the airfoil and the ground, which reduces the pressure on the lower side of the inverted airfoil. This leads to a strong increase in downforce. In this regime, the boundary layers developing on the airfoil and on the ground remain sufficiently separated, so that a potential-flow region still exists between them.
- **Small ground clearance.** At very small clearance, viscous effects become dominant in the narrow gap. The boundary layers developing on the airfoil and on the ground interact strongly and may locally merge. As a result, the effective flow passage is reduced and the potential-flow core can disappear. The velocity beneath the airfoil decreases, the local pressure increases, and the generated downforce is reduced. In extreme cases, force reversal may occur, leading to positive lift.

The amount of downforce generated by an inverted wing in ground effect can be significantly affected by the boundary layer developing on the ground plane. This effect is particularly important in wind-tunnel experiments when a moving-ground system is not available, since a stationary ground plane produces a boundary layer that is not representative of the real road condition.

Marshall, Newman and Williams investigated this effect experimentally by placing an inverted wing above a variable-length splitter plate, allowing different ground boundary-layer thicknesses to be generated at the wing location [18]. Their results showed that the flow beneath the wing is strongly influenced by the thickness of this boundary layer. As the ground boundary layer becomes thicker, the pressure beneath the wing increases, which reduces the suction on the lower surface and therefore decreases the generated downforce. Consequently, when an inverted wing in ground effect is tested above a stationary ground plane,

the measured suction and downforce can be lower than those obtained with a moving-ground configuration.

Zhou et al. [19] experimentally investigated an accelerating inverted wing in ground effect by combining direct force measurements with time-resolved PIV. Their results show that the downforce generated during acceleration can differ significantly from the steady-state behaviour, especially at low ground clearance, where boundary-layer separation, wake development and vortex-ground interactions modify the aerodynamic load. This suggests that ground-effect aerodynamics can be strongly history-dependent and that static aerodynamic maps may not fully describe the force response in dynamic conditions.

2.2 Aeroelasticity

Aeroelasticity refers to the study of the interaction between aerodynamic forces and the deformation of elastic structures exposed to an airflow. It is an interdisciplinary field involving aerodynamics, elasticity and dynamics, as illustrated in Figure 2.4, originating from Professor Collar in 1946 [20]. Aerodynamics determines the forces acting on a body for a given shape, elasticity describes the deformation of the structure under applied loads, and dynamics accounts for inertial effects. Aeroelastic phenomena arise when at least two of these mechanisms are coupled. The main areas are structural dynamics, involving elasticity and inertia. Static aeroelasticity involves aerodynamics and elasticity, while dynamic aeroelasticity involves aerodynamics, elasticity and inertia simultaneously.

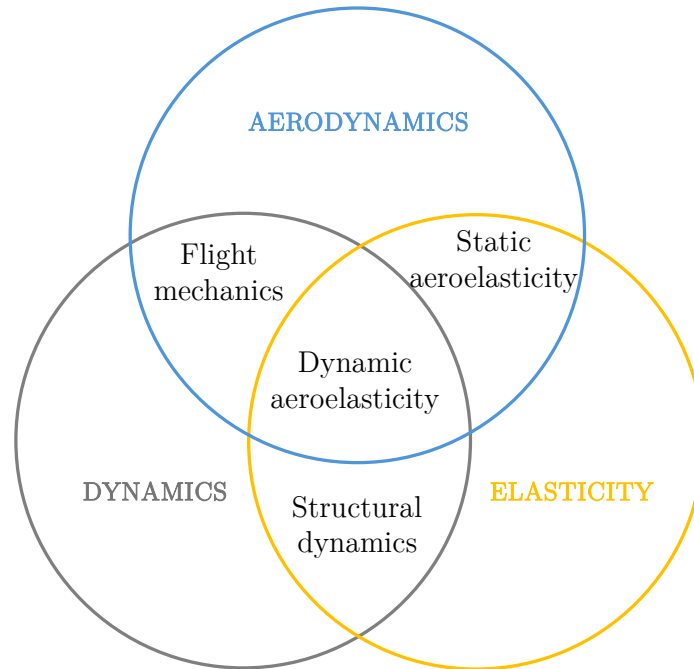


Figure 2.4: Aeroelasticity "triangle" showing the coupling between aerodynamics, elasticity and dynamics, based on Collar's classical representation [20] and its later presentation by Hodges and Pierce [21].

As mentioned earlier, porpoising can be interpreted as an aeroelastic phenomenon, since it results from the coupling between aerodynamic forces and the dynamic response of the

vehicle [9]. Such a system can be described by the following equations of motion:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}_{\text{aero}}(U_\infty, \mathbf{x}(t), \dot{\mathbf{x}}(t)), \quad (2.2.1)$$

where $\mathbf{x}$ is the vector of generalized coordinates describing the degrees of freedom of the system. The matrices $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are respectively the mass, damping, and stiffness matrices. They represent the inertial, dissipative, and restoring structural contributions. The aerodynamic loading appears on the right-hand side through $\mathbf{f}_{\text{aero}}$, which depends on the freestream velocity U_∞ , the system state, and its time derivative.

To study the onset of porpoising, the stability of the aeroelastic system is analysed as a function of the freestream velocity. As illustrated schematically in Figure 2.5, this can be done by tracking the modal damping ratio of the relevant modes as the velocity increases. In an aeroelastic system, the effective modal damping is not constant, it is modified by the aerodynamic forces and can therefore vary with freestream velocity.

A positive modal damping ratio, represented by the green point in Figure 2.5, corresponds to a stable configuration. In this case, if the system is slightly perturbed, the amplitude of the modal response decreases with time. When the modal damping ratio reaches zero, represented by the blue point, the response remains at approximately constant amplitude after a perturbation. This condition defines the neutral stability, boundary between stable and unstable behaviour. Finally, when the modal damping ratio becomes negative, represented by the red point, the perturbation is amplified with time and the corresponding mode becomes unstable.

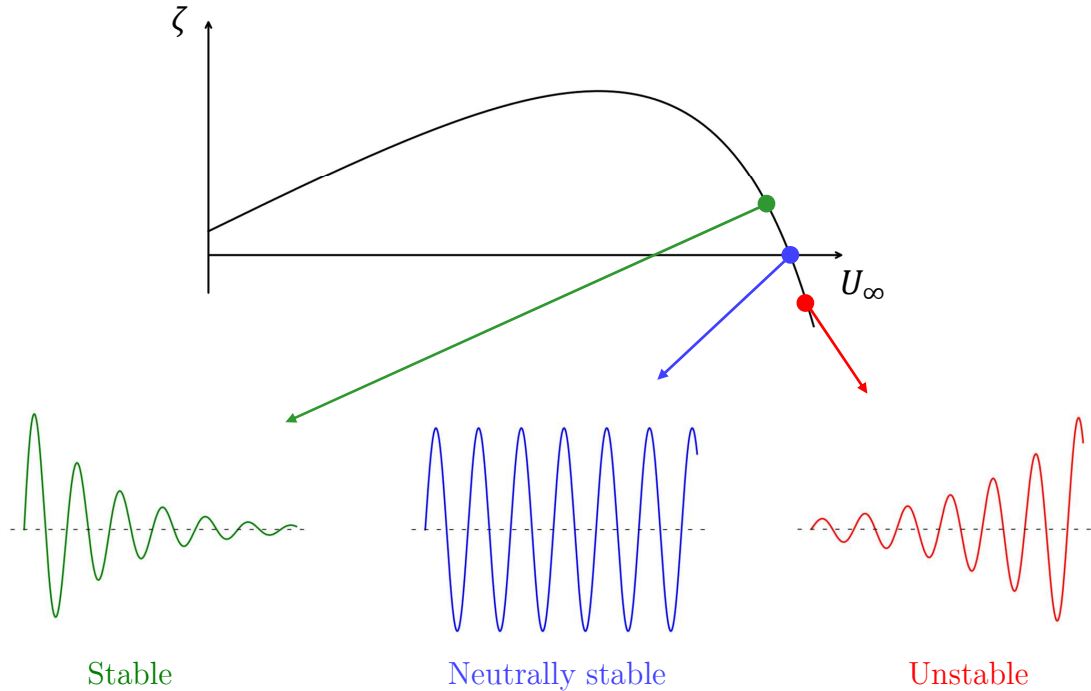


Figure 2.5: Schematic evolution of the modal damping ratio with freestream velocity.

In the context of porpoising, the critical velocity is therefore associated with the freestream velocity at which one of the relevant modes loses stability, such that its effective modal damping becomes negative. As discussed later, the porpoising velocity is identified here as the velocity at which the plunge-dominated response becomes unstable.

2.3 Reduced-Order Modelling

The linear system modelled consists of a rigid wing assembly constrained to translate vertically (along y), and to rotate (along α) in the $x-y$ plane as shown in Figure 2.6. The system is subjected to a vertical aerodynamic force F_L and aerodynamic moment M .

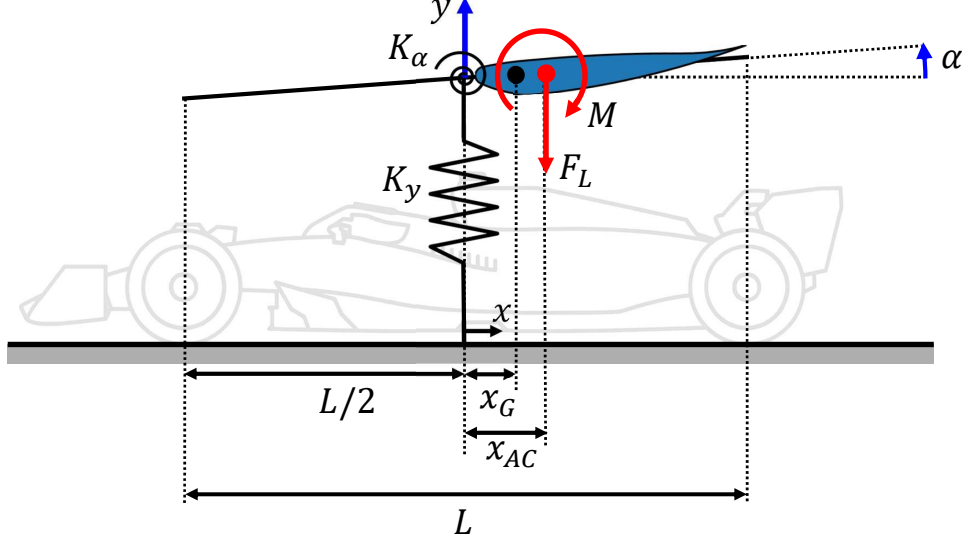


Figure 2.6: Schematic of the pitch-plunge degrees of freedom's system.

The vertical and rotational constraints are represented by two equivalent stiffnesses. The coefficient K_y denotes the linear stiffness associated with the plunge degree of freedom, whereas K_α denotes the torsional stiffness associated with the pitch degree of freedom. The reference frame is attached to the centre of the rigid bar, which is constrained to move only in vertical translation and rotation in the $x-y$ plane.

In the schematic of Figure 2.6, the flexural axis is located at $x = 0$. Consequently, the flexural-axis position is defined as $x_f = 0$ for the configuration considered here.

2.3.1 System of equations of the model

The equations of motion are obtained using the Lagrangian approach. The kinetic and potential energies, and the Rayleigh dissipation function, and external forces are injected into the Lagrange's equation

$$\frac{\partial}{\partial t} \left(\frac{dT}{d\dot{\mathbf{q}}} \right) + \frac{dV}{d\mathbf{q}} + \frac{dD}{d\dot{\mathbf{q}}} = Q, \quad (2.3.1)$$

with $\mathbf{q} = [y \ \alpha]^T$ being the state vector.

The kinetic energy of the system is computed by integrating, over the length of the bar, the kinetic energy of a small element of mass dm . This infinitesimal kinetic energy is therefore written as

$$dT = \frac{1}{2} dm (\dot{y} + (x - x_f) \dot{\alpha})^2, \quad (2.3.2)$$

such that the total kinetic energy of the bar is obtained as follows:

$$T = \frac{1}{2} \int_{\text{bar}} (\dot{y} + (x - x_f)\dot{\alpha})^2 dm \quad (2.3.3)$$

$$= \frac{1}{2} \int_{\text{bar}} [\dot{y}^2 + 2(x - x_f)\dot{y}\dot{\alpha} + (x - x_f)^2\dot{\alpha}^2] dm \quad (2.3.4)$$

$$= \frac{1}{2} \left[\dot{y}^2 \int_{\text{bar}} dm + 2\dot{y}\dot{\alpha} \int_{\text{bar}} (x - x_f) dm + \dot{\alpha}^2 \int_{\text{bar}} (x - x_f)^2 dm \right] \quad (2.3.5)$$

$$= \frac{1}{2} [m\dot{y}^2 + 2m(x_G - x_f)\dot{y}\dot{\alpha} + I_f\dot{\alpha}^2] \quad (2.3.6)$$

$$= \frac{1}{2} [m\dot{y}^2 + 2S\dot{y}\dot{\alpha} + I_f\dot{\alpha}^2]. \quad (2.3.7)$$

In this expression, x_G denotes the position of the centre of gravity of the bar, S is the static unbalance with respect to the flexural axis, and I_f is the mass moment of inertia about the flexural axis. These quantities are defined as

$$x_G = \frac{1}{m} \int_{\text{bar}} x dm, \quad S = m(x_G - x_f) = \int_{\text{bar}} (x - x_f) dm, \quad I_f = \int_{\text{bar}} (x - x_f)^2 dm. \quad (2.3.8)$$

The total potential energy of the bar is directly deduced from the pitch and plunge stiffness.

$$V = \frac{1}{2} K_y y^2 + \frac{1}{2} K_\alpha \alpha^2. \quad (2.3.9)$$

The dissipation terms are introduced in the Rayleigh function, such that:

$$D = \frac{1}{2} C_y \dot{y}^2 + \frac{1}{2} C_\alpha \dot{\alpha}^2. \quad (2.3.10)$$

Since gravity is neglected in the present model, the equations of motion are obtained by substituting the kinetic energy in Equation 2.3.7, the potential energy in Equation 2.3.9, and the Rayleigh dissipation function in Equation 2.3.10 into Lagrange's equation 2.3.1. The generalized aerodynamic loading is also included through the vertical force F_L , defined as positive downward, and through the aerodynamic moment M , acting in the negative α direction. The resulting system of equations of motion is therefore given by

$$\underbrace{\begin{bmatrix} m & S \\ S & I_f \end{bmatrix}}_{\mathbf{M}_{\text{struct}}} \underbrace{\begin{bmatrix} \ddot{y} \\ \ddot{\alpha} \end{bmatrix}}_{\mathbf{C}_{\text{struct}}} + \underbrace{\begin{bmatrix} C_y & 0 \\ 0 & C_\alpha \end{bmatrix}}_{\mathbf{K}_{\text{struct}}} \underbrace{\begin{bmatrix} \dot{y} \\ \dot{\alpha} \end{bmatrix}}_{\mathbf{K}_{\text{struct}}} + \underbrace{\begin{bmatrix} K_y & 0 \\ 0 & K_\alpha \end{bmatrix}}_{\mathbf{K}_{\text{struct}}} \underbrace{\begin{bmatrix} y \\ \alpha \end{bmatrix}}_{\mathbf{F}} = - \underbrace{\begin{bmatrix} F_L \\ M + (x_{AC} - x_f)F_L \end{bmatrix}}_{\mathbf{F}}. \quad (2.3.11)$$

The linearized equations of motion can be written in the standard second-order form, with $\mathbf{x} = \mathbf{q}$

$$\mathbf{M}_{\text{struct}} \ddot{\mathbf{x}} + \mathbf{C}_{\text{struct}} \dot{\mathbf{x}} + \mathbf{K}_{\text{struct}} \mathbf{x} = -\mathbf{F}. \quad (2.3.12)$$

Following the classical decomposition described by Andrianne [22], the aerodynamic force and moment can be separated into three contributions:

$$\mathbf{F} = \mathbf{F}_{\text{st}} + \mathbf{F}_{\text{buf}} + \mathbf{F}_{\text{ae}}, \quad (2.3.13)$$

or, equivalently,

$$\begin{bmatrix} F_L \\ M \end{bmatrix} = \begin{bmatrix} F_{L,\text{st}} + F_{L,\text{buf}} + F_{L,\text{ae}} \\ M_{\text{st}} + M_{\text{buf}} + M_{\text{ae}} \end{bmatrix}. \quad (2.3.14)$$

The three contributions have the following meanings:

- The static contribution, denoted by the subscript 'st', corresponds to the aerodynamic load generated by the mean freestream velocity and the static position of the wing.
- The buffeting contribution, denoted by the subscript 'buf', corresponds to the aerodynamic load induced by external flow disturbances, such as turbulence in the incoming flow.
- The aeroelastic contribution, denoted by the subscript 'ae', corresponds to the motion-induced aerodynamic load. It results from the modification of the relative flow seen by the wing due to its displacement and velocity.

In the present work, buffeting is neglected. The aerodynamic model therefore retains only the static and motion-induced aeroelastic contributions:

$$\mathbf{F} = \mathbf{F}_{\text{st}} + \mathbf{F}_{\text{ae}}. \quad (2.3.15)$$

Two aerodynamic modelling approaches are then considered. In the quasi-steady approach, the aerodynamic load is assumed to follow the instantaneous position of the wing without memory effects. The aerodynamic force and its local gradients are therefore obtained from the static aerodynamic database. In contrast, the unsteady approach accounts for the motion-induced aerodynamic response of the flow, introducing aerodynamic terms that depend explicitly on the dynamic motion of the wing.

For the stability analysis, the equations are linearised around the static equilibrium position. The static aerodynamic contribution determines this equilibrium state, while the linearised aeroelastic contribution provides the aerodynamic stiffness and damping terms governing the stability of the system.

2.3.2 Quasi-steady aerodynamic modelling

In the quasi-steady formulation, the aerodynamic loads are assumed to depend on the instantaneous position and velocity of the wing, but not on the previous history of the flow. The lift force per unit span is therefore written as

$$F_L = \frac{1}{2} \rho c U_\infty^2 C_L(y, \alpha_{\text{eff}}), \quad (2.3.16)$$

where y is the plunge displacement and α_{eff} is the effective angle of attack seen by the wing. This effective angle includes the initial angle of attack, the pitch motion and the apparent incidence induced by the plunge and pitch velocities:

$$\alpha_{\text{eff}} = \alpha_0 + \alpha + \frac{\dot{y}}{U_\infty} + \left(x_{\frac{3c}{4}} - x_f \right) \frac{\dot{\alpha}}{U_\infty}. \quad (2.3.17)$$

The aerodynamic force is thus not a constant external load. It depends on the state of the system through y , α , $\dot{y}$ and $\dot{\alpha}$. Around a given equilibrium state, the lift force can be linearized as

$$F_L = F_{L,\text{eq}} + \delta F_L, \quad (2.3.18)$$

with

$$\delta F_L = \left. \frac{\partial F_L}{\partial y} \right|_{\text{eq}} \delta y + \left. \frac{\partial F_L}{\partial \alpha} \right|_{\text{eq}} \delta \alpha + \left. \frac{\partial F_L}{\partial \dot{y}} \right|_{\text{eq}} \delta \dot{y} + \left. \frac{\partial F_L}{\partial \dot{\alpha}} \right|_{\text{eq}} \delta \dot{\alpha}, \quad (2.3.19)$$

and

$$F_{L,\text{eq}} = \frac{1}{2} \rho c U_\infty^2 C_L(y_{\text{eq}}, \alpha_{\text{eq}}). \quad (2.3.20)$$

Using the chain rule, the lift force derivatives are obtained as

$$\begin{cases} \left. \frac{\partial F_L}{\partial y} \right|_{\text{eq}} = \frac{1}{2} \rho c U_\infty^2 \left. \frac{\partial C_L}{\partial y} \right|_{\text{eq}}, \\ \left. \frac{\partial F_L}{\partial \alpha} \right|_{\text{eq}} = \frac{1}{2} \rho c U_\infty^2 \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}}, \\ \left. \frac{\partial F_L}{\partial \dot{y}} \right|_{\text{eq}} = \frac{1}{2} \rho c U_\infty \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}}, \\ \left. \frac{\partial F_L}{\partial \dot{\alpha}} \right|_{\text{eq}} = \frac{1}{2} \rho c U_\infty \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}}. \end{cases} \quad (2.3.21)$$

The first two derivatives act as aerodynamic stiffness terms, since they multiply the generalized displacements. The last two act as aerodynamic damping terms, since they multiply the generalized velocities.

The aerodynamic lift and moment can therefore finally be expressed as

$$F_L = \frac{1}{2} \rho c U_\infty^2 \left[C_{L,\text{eq}} + \left. \frac{\partial C_L}{\partial y} \right|_{\text{eq}} \delta y + \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \delta \alpha + \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \frac{\delta \dot{y}}{U_\infty} + \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \frac{\delta \dot{\alpha}}{U_\infty} \right], \quad (2.3.22)$$

$$M = \frac{1}{2} \rho c^2 U_\infty^2 \left[C_{M,\text{eq}} + \left. \frac{\partial C_M}{\partial y} \right|_{\text{eq}} \delta y + \left. \frac{\partial C_M}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \delta \alpha + \left. \frac{\partial C_M}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \frac{\delta \dot{y}}{U_\infty} + \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_M}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \frac{\delta \dot{\alpha}}{U_\infty} \right]. \quad (2.3.23)$$

2.3.3 Unsteady aerodynamic modelling

In the unsteady approach, the aerodynamic forces account for the oscillation frequency of the wing, a feature neglected in the quasi-steady formulation. The frequency-dependent aerodynamic response is represented through aeroelastic coupling coefficients, commonly referred to as *flutter derivatives*, introduced by Scanlan. This formulation has been extensively used for bluff-body aeroelasticity, such as bridge decks [23], but can also be applied to the present configuration. Although Theodorsen's theory is generally preferred for streamlined airfoils, the Scanlan formulation was retained in the present work due to its ability to represent unsteady aerodynamic effects directly from experimental data.

Using these aerodynamic coefficients, the aeroelastic lift force and pitching moment become

$$F_{L,\text{ae}} = -\frac{1}{2} \rho c U_\infty^2 \left[k H_1^* \frac{\dot{y}}{U_\infty} + k H_2^* \frac{c \dot{\alpha}}{U_\infty} + k^2 H_3^* \alpha + k^2 H_4^* \frac{y}{c} \right], \quad (2.3.24)$$

$$M_{\text{ae}} = -\frac{1}{2} \rho c^2 U_\infty^2 \left[k A_1^* \frac{\dot{y}}{U_\infty} + k A_2^* \frac{c \dot{\alpha}}{U_\infty} + k^2 A_3^* \alpha + k^2 A_4^* \frac{y}{c} \right]. \quad (2.3.25)$$

The coefficients H_i^* and A_i^* ($i = 1, \dots, 4$) are functions of the reduced frequency

$$k = \frac{c\omega}{U_\infty},$$

where ω denotes the oscillation frequency of the wing.

These dimensionless coefficients depend only on the geometry and the reduced frequency. They provide a compact representation of the unsteady fluid–structure interaction and therefore account for aerodynamic memory effects that cannot be captured by a purely quasi-steady model [22].

As in the quasi-steady formulation, terms proportional to the displacements (y and α) act as aerodynamic stiffness contributions, whereas terms proportional to the velocities ($\dot{y}$ and $\dot{\alpha}$) contribute to the aerodynamic damping.

Identification of *flutter derivatives*

The aeroelastic derivatives are introduced through their non-dimensional form, denoted by $H_i^*(k)$ and $A_i^*(k)$, where k is the reduced frequency.

These derivatives can be separated into direct and indirect coefficients. The direct coefficients are associated with the same degree of freedom as the corresponding equation of motion. In other words, H_1 and H_4 are related to the plunge motion, while A_2 and A_3 are related to the pitch motion. The indirect coefficients describe the coupling between plunge and pitch.

Direct coefficients

Using the measured modal frequency and damping ratio, the non-dimensional direct flutter derivatives are identified as

$$\left\{ \begin{array}{l} H_1^*(\omega) = \frac{4m (\zeta_y \omega_y - \zeta_y' \omega_y')}{\rho c^2 \omega}, \\ H_4^*(\omega) = \frac{2m (\omega_y^2 - \omega_y'^2)}{\rho c^2 \omega^2}, \\ A_2^*(\omega) = \frac{4I_f (\zeta_\alpha \omega_\alpha - \zeta_\alpha' \omega_\alpha')}{\rho c^4 \omega}, \\ A_3^*(\omega) = \frac{2I_f (\omega_\alpha^2 - \omega_\alpha'^2)}{\rho c^4 \omega^2}. \end{array} \right. \quad (2.3.26)$$

In these expressions, ω_y , ω_α , ζ_y , and ζ_α denote the reference structural modal frequencies and damping ratios in plunge and pitch. The primed quantities correspond to the identified values when aerodynamic effects are included.

Indirect coefficients

The indirect coefficients are obtained from the coupled pitch–plunge response. They depend on the relative amplitudes of the plunge and pitch motions and on their phase shift. Denoting by $\hat{y}$ and $\hat{\alpha}$ the oscillation amplitudes in plunge and pitch, and by ϕ their phase difference, the dimensional indirect coefficients are written as

$$\begin{cases} H_2^*(\omega) = \frac{1}{\omega} \frac{\hat{y}}{\hat{\alpha}} [(\omega_y^2 - \omega^2) \sin \phi + (2\zeta_y \omega_y - H_1(\omega)) \omega \cos \phi - H_4^*(\omega) \sin \phi], \\ H_3^*(\omega) = -\frac{\hat{y}}{\hat{\alpha}} \left[(\omega^2 - \omega_y^2) \cos \phi + (2\zeta_y \omega_y - H_1(\omega)) \omega \sin \phi + H_4^*(\omega) \frac{1 + \sin^2 \phi}{\cos \phi} \right], \\ A_1^*(\omega) = \frac{1}{\omega} \frac{\hat{\alpha}}{\hat{y}} [(\omega^2 - \omega_\alpha^2 + A_3(\omega)) \sin \phi + (2\zeta_\alpha \omega_\alpha - A_2(\omega)) \omega \cos \phi], \\ A_4^*(\omega) = \frac{\hat{\alpha}}{\hat{y}} [(\omega^2 - \omega_\alpha^2 - A_3(\omega)) \cos \phi + (2\zeta_\alpha \omega_\alpha - A_2(\omega)) \omega \sin \phi]. \end{cases} \quad (2.3.27)$$

The hats on $\hat{y}$ and $\hat{\alpha}$ are used here to avoid confusion with the initial geometric angle of attack. They represent the oscillation amplitudes used during the identification procedure.

2.3.4 Model solution procedure

Now that the structural equations of motion and the aerodynamic modelling assumptions were established, the objective is to use this model to reproduce and interpret the main quantities, being the static equilibrium positions, the modal frequencies, the effective damping ratios, and the stability limit of the system as a function of the freestream velocity.

Static state equations

The objective is to determine the static aeroelastic deflection of the system. Starting from the equations of motion, the static equilibrium state is obtained by setting the plunge and pitch velocities and accelerations to zero:

$$\dot{\mathbf{x}}_{\text{eq}} = \mathbf{0}, \quad \ddot{\mathbf{x}}_{\text{eq}} = \mathbf{0}, \quad (2.3.28)$$

with

$$\mathbf{x}_{\text{eq}} = \begin{bmatrix} y_{\text{eq}} \\ \alpha_{\text{eq}} \end{bmatrix}. \quad (2.3.29)$$

The equilibrium state must be computed from the static balance of the structural restoring loads and the aerodynamic load.

At static equilibrium, the velocity-dependent contribution to the effective angle of attack vanishes. However, the effective angle of attack still includes the prescribed geometrical incidence α_0 . Therefore,

$$\alpha_{\text{eff,eq}} = \alpha_0 + \alpha_{\text{eq}}. \quad (2.3.30)$$

Since the equilibrium state is static, the aerodynamic lift force can be evaluated using a quasi-steady formulation. Equivalently, this corresponds to the limit of the unsteady formulation for a zero reduced frequency $k = 0$. The lift force at equilibrium is then written as

$$F_{L,\text{eq}} = \frac{1}{2} \rho c U_\infty^2 C_L(y_{\text{eq}}, \alpha_0 + \alpha_{\text{eq}}). \quad (2.3.31)$$

The static balance of the plunge degree of freedom gives

$$K_y y_{\text{eq}} + F_{L,\text{eq}} = 0. \quad (2.3.32)$$

The corresponding pitch equilibrium is obtained from the moment balance about the flexural axis. The aerodynamic resultant is assumed to act at the aerodynamic centre, while the intrinsic pitching moment of the wing about this point is neglected. The aerodynamic moment is therefore reduced to the contribution of the lift force only:

$$M_a = M + (x_{AC} - x_f)F_L \approx (x_{AC} - x_f)F_L. \quad (2.3.33)$$

The static pitch deflection becomes:

$$K_\alpha \alpha_{eq} + (x_{AC} - x_f)F_{L,eq} = 0. \quad (2.3.34)$$

Substituting Equation 2.3.31 into Equations 2.3.32 and 2.3.34 leads to the following implicit equilibrium system:

$$\begin{cases} y_{eq} = -\frac{1}{K_y} \frac{1}{2} \rho c U_\infty^2 C_L(y_{eq}, \alpha_0 + \alpha_{eq}), \\ \alpha_{eq} = -\frac{x_{AC} - x_f}{K_\alpha} \frac{1}{2} \rho c U_\infty^2 C_L(y_{eq}, \alpha_0 + \alpha_{eq}). \end{cases} \quad (2.3.35)$$

The two equilibrium coordinates are therefore related by

$$\alpha_{eq} = -\frac{K_y(x_{AC} - x_f)}{K_\alpha} y_{eq}. \quad (2.3.36)$$

Using this relation, the equilibrium problem can also be reduced to a single nonlinear equation in y_{eq} :

$$y_{eq} = -\frac{1}{K_y} \frac{1}{2} \rho c U_\infty^2 C_L\left(y_{eq}, \alpha_0 - \frac{K_y(x_{AC} - x_f)}{K_\alpha} y_{eq}\right). \quad (2.3.37)$$

Once y_{eq} is obtained, the equilibrium pitch angle follows directly from Equation 2.3.36.

As shown in Equation 2.3.37, the plunge equilibrium position y_{eq} is defined implicitly, since it appears both in the left-hand side and inside the aerodynamic coefficient. The equation can be therefore solved iteratively, starting from the initial guess $y_{eq}^{(0)} = 0$. At each iteration, the previous estimate can be used to update the aerodynamic load and compute a new value of y_{eq} . The process can be repeated until convergence.

Stability analysis

Reintroducing the generalized coordinate vector

$$\mathbf{x} = \begin{bmatrix} y \\ \alpha \end{bmatrix}, \quad (2.3.38)$$

the linearized aerodynamic contribution can be written in matrix form. If the structural equations are written as

$$\mathbf{M}_{struct} \ddot{\mathbf{x}} + \mathbf{C}_{struct} \dot{\mathbf{x}} + \mathbf{K}_{struct} \mathbf{x} = -\mathbf{F}, \quad (2.3.39)$$

then moving the linearized aerodynamic terms to the left-hand side gives

$$\mathbf{M} \ddot{\mathbf{x}} + (\mathbf{C}_{struct} + \mathbf{C}_{aero}) \dot{\mathbf{x}} + (\mathbf{K}_{struct} + \mathbf{K}_{aero}) \mathbf{x} = \mathbf{0}, \quad (2.3.40)$$

with the aerodynamic stiffness and damping matrices computed according to the selected aerodynamic model.

Quasi-steady formulation

Using the quasi-steady aerodynamic model, the aerodynamic stiffness matrix is written as

$$\mathbf{K}_{\text{aero}} = \frac{1}{2}\rho c U_{\infty}^2 \begin{bmatrix} \left. \frac{\partial C_L}{\partial y} \right|_{\text{eq}} & \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \\ (x_{AC} - x_f) \left. \frac{\partial C_L}{\partial y} \right|_{\text{eq}} & (x_{AC} - x_f) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \end{bmatrix}, \quad (2.3.41)$$

and the aerodynamic damping matrix is

$$\mathbf{C}_{\text{aero}} = \frac{1}{2}\rho c U_{\infty} \begin{bmatrix} \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} & \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \\ (x_{AC} - x_f) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} & (x_{AC} - x_f) \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \end{bmatrix}. \quad (2.3.42)$$

Unsteady formulation

Using the unsteady aerodynamic model, based on the *flutter derivatives*, the aerodynamic stiffness matrix is written as

$$\mathbf{K}_{\text{aero}} = -\frac{1}{2}\rho c U_{\infty}^2 \begin{bmatrix} \frac{k^2 H_4^*}{c} & k^2 H_3^* \\ \frac{k^2 A_4^*}{c} & k^2 A_3^* \end{bmatrix}, \quad (2.3.43)$$

and the aerodynamic damping matrix is

$$\mathbf{C}_{\text{aero}} = -\frac{1}{2}\rho c U_{\infty} \begin{bmatrix} \frac{k H_1^*}{U_{\infty}} & \frac{k c H_2^*}{U_{\infty}} \\ \frac{k A_1^*}{U_{\infty}} & \frac{k c A_2^*}{U_{\infty}} \end{bmatrix}. \quad (2.3.44)$$

Once the static equilibrium has been determined, the stability of the system is analysed from the linearized equations of motion written around this equilibrium position. In the following, y and α denote small perturbations around the equilibrium state.

The perturbation displacement vector is defined as

$$\mathbf{x} = \begin{bmatrix} y \\ \alpha \end{bmatrix}. \quad (2.3.45)$$

The linearized equations of motion can be written in the standard second-order form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}, \quad (2.3.46)$$

where

$$\mathbf{M} = \mathbf{M}_{\text{struct}}, \quad \mathbf{C} = \mathbf{C}_{\text{struct}} + \mathbf{C}_{\text{aero}}, \quad \mathbf{K} = \mathbf{K}_{\text{struct}} + \mathbf{K}_{\text{aero}}. \quad (2.3.47)$$

To solve the stability problem, the second-order system is transformed into a first-order state-space form. The state vector is chosen as

$$\mathbf{z} = \begin{bmatrix} \dot{y} \\ \dot{\alpha} \\ y \\ \alpha \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{x} \end{bmatrix}. \quad (2.3.48)$$

From Equation 5.1.2, the acceleration vector is obtained as

$$\ddot{\mathbf{x}} = -\mathbf{M}^{-1}\mathbf{C}\dot{\mathbf{x}} - \mathbf{M}^{-1}\mathbf{K}\mathbf{x}. \quad (2.3.49)$$

Therefore, the first-order system becomes

$$\begin{bmatrix} \ddot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{x} \end{bmatrix}. \quad (2.3.50)$$

Using the state vector $\mathbf{z}$, this system can be written compactly as

$$\dot{\mathbf{z}} = \mathbf{Q}\mathbf{z}, \quad (2.3.51)$$

where the state matrix is

$$\mathbf{Q} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}. \quad (2.3.52)$$

The stability of the system is then obtained by solving the eigenvalue problem associated with the state matrix:

$$\mathbf{Q}\boldsymbol{\phi}_i = \lambda_i\boldsymbol{\phi}_i, \quad (2.3.53)$$

where λ_i is the i th eigenvalue and $\boldsymbol{\phi}_i$ is the corresponding eigenvector.

The solution of the first-order linear system can be written as

$$\mathbf{z}(t) = e^{\mathbf{Q}t}\mathbf{z}(0). \quad (2.3.54)$$

By decomposing the solution on the eigenvectors of $\mathbf{Q}$, one obtains

$$\mathbf{z}(t) = \sum_{i=1}^n c_i \boldsymbol{\phi}_i e^{\lambda_i t}, \quad (2.3.55)$$

where n is the number of states and the constants c_i depend on the initial condition.

Each eigenvalue can be written as

$$\lambda_i = \sigma_i + i\omega_i, \quad (2.3.56)$$

where σ_i is the real part and ω_i is the imaginary part of the eigenvalue.

Following the classical state-space formulation, the natural angular frequency associated with each mode is given by

$$\omega_{n,i} = |\lambda_i|. \quad (2.3.57)$$

The corresponding natural frequency in Hz is

$$f_{n,i} = \frac{\omega_{n,i}}{2\pi}. \quad (2.3.58)$$

The damping ratio of each mode is defined as

$$\zeta_i = -\frac{\text{Re}(\lambda_i)}{\omega_{n,i}}. \quad (2.3.59)$$

The real part of the eigenvalue determines whether the corresponding mode decays or grows with time. If $\text{Re}(\lambda_i) < 0$, the modal contribution decreases exponentially. If $\text{Re}(\lambda_i) > 0$, the modal contribution grows exponentially and the system is unstable.

3 Experimental Setup and Test Campaign

In this chapter, the experimental setups and test campaigns used to investigate the aerodynamic and aeroelastic behaviour of the inverted Tyrrell026S wing in ground effect are presented. All experiments were performed in the wind tunnel of the Université de Liège, described in Appendix C.

The chapter is divided into two main parts. First, the static setup is introduced. Its purpose is to measure the steady aerodynamic coefficients of the wing for different angles of attack and ground clearances, in order to characterise the aerodynamic behaviour of the profile close to the ground. Then, the dynamic aeroelastic setups are presented. These second setups are used to study the response of the wing-structure assembly as the freestream velocity is increased.

As mentioned, the aerodynamic profile selected for the test campaigns corresponds to the Tyrrell026S airfoil, whose geometry was reported by Genua in her CFD study of an inverted airfoil in ground effect [24]. Its corresponding cross-section is shown in Figure 3.1



Figure 3.1: Two-dimensional Tyrrell026S airfoil section used in the experimental setup.

This wing profile was selected because it is commonly used as a benchmark geometry in studies of downforce-generating inverted wings in ground effect, allowing the present results to be compared with previous numerical and experimental investigations. Moreover, this profile is one of the few Formula One airfoil sections for which geometrical data are publicly available. It was used on the front wing of the Tyrrell 026 Formula One car, which competed during the 1998 season, and is derived from a slightly modified NASA Langley LS(1)-0413MOD profile. The normalized coordinates defining the airfoil geometry are reported in Appendix A.

3.1 Static Setup

In this section, the static experimental setup used to characterise the aerodynamic behaviour of the wing is presented. The objective of this setup is to measure the steady aerodynamic forces and moments generated by the inverted wing for different ground clearances and angles of attack.

Design choices

Due to technical constraints, the static and dynamic setups do not use the same wing dimensions. As mentioned later, the wing used in the dynamic setup is relatively large and heavy, which makes load measurements difficult practically. A smaller wing was therefore manufactured for the static setup in order to reduce structural weight and improve the accuracy of the force measurements.

The static wing has a chord of $c = 22.5$ cm and a span of $b = 50$ cm, while keeping the same Tyrrell026S airfoil section. A 2×2 cm aluminium profile passes through the wing and connects it to the support linked to the force sensor, as shown in Figure 3.2.

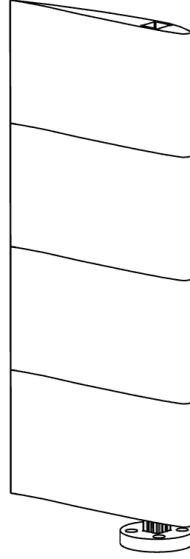


Figure 3.2: CAD view of the static Tyrrell026S wing model and its mounting interface.

Static test campaign

The static test campaign consisted of measuring the aerodynamic loads for several combinations of ride height H and wing incidence α_0 . The objective was to obtain non-dimensional aerodynamic coefficient maps as functions of these two parameters, using the same definitions as in the Dynamic test campaign section, shown later.

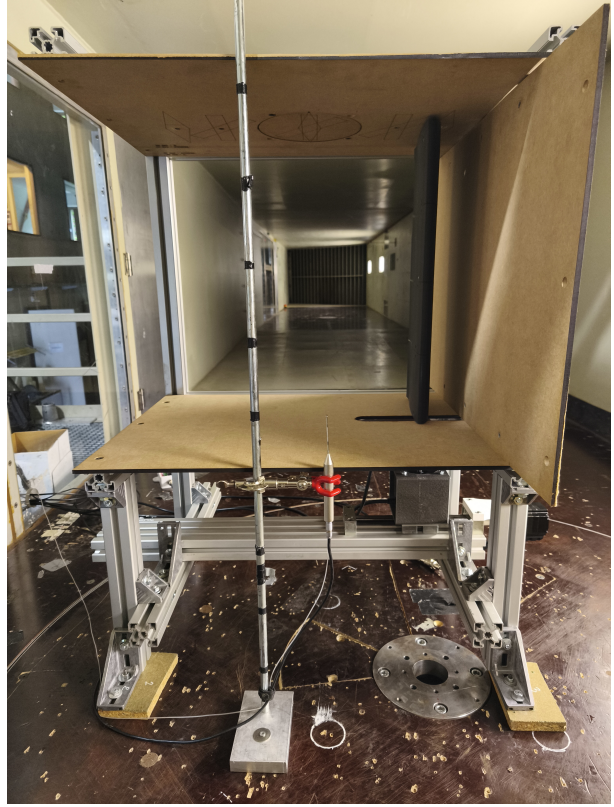
In practice, the wing assembly shown in Figure 3.2 is mounted to the force sensor. The setup includes two stepper motors, one controlling the wing incidence, while the other adjusting the ride height, by translating the wing assembly and the wing incidence controller. Photographs of the static setup are shown in Figure 3.3.

The static aerodynamic tests were performed at a freestream velocity of $U_\infty = 10.5$ m/s. Based on the wing chord $c = 0.225$ m and using the kinematic viscosity of air at ambient conditions, $\nu \approx 1.5 \times 10^{-5}$ m²/s, the corresponding chord-based Reynolds number is approximately

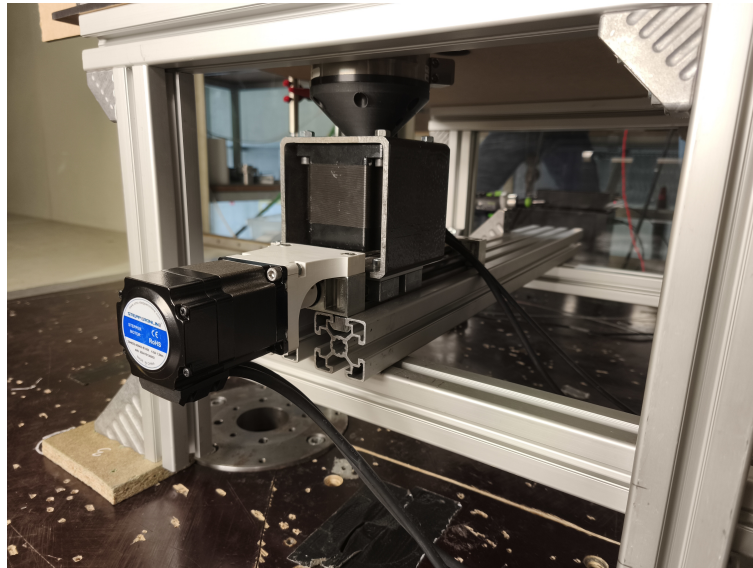
$$Re_c = \frac{U_\infty c}{\nu} \approx 1.6 \times 10^5.$$



(a) Rear view of the static setup.



(b) Front view of the static setup.



(c) Ground-height and wing's incidence adjustment mechanisms.

Figure 3.3: Static experimental setup used to measure the aerodynamic loads of the Tyrrell026S airfoil in ground effect.

3.2 Dynamic Setup

This section presents the dynamic aeroelastic setups used to investigate porpoising instabilities. Unlike the static configuration, the wing is mounted on an elastic support, allowing the system to respond dynamically to the aerodynamic loading. The setups, instrumentation and measurement strategies are described in order to define how the vertical response of the system is recorded and how the stability of the wing–structure assembly is analysed as the freestream velocity is increased.

As will be discussed later, the porpoising response of the system can be reasonably approximated by a single-degree-of-freedom motion. This simplification is based on the analysis of the dynamic results obtained with the first setup, in which the system was free to translate vertically and to rotate in pitch. The results indicated that the instability is mainly associated with the plunge motion. Consequently, the second dynamic setup, restricted to vertical translation only, was developed in order to obtain cleaner experimental data for the modelling of the porpoising instability.

3.2.1 Plunge–pitch dynamic setup

Design choices

The selected airfoil section was used to manufacture a wing with a chord of $c = 0.45$ m and a span of $b = 0.66$ m. These dimensions were chosen to make use of the maximum available space in the setup and to generate aerodynamic loads large enough to excite the dynamic response of the system, and get larger oscillation amplitudes for clearer visualization. A sufficiently large model also reduces the relative influence of manufacturing tolerances and facilitates the observation of ground-effect phenomena at small ride heights. A schematic of the manufactured wing is shown in Figure 3.4.

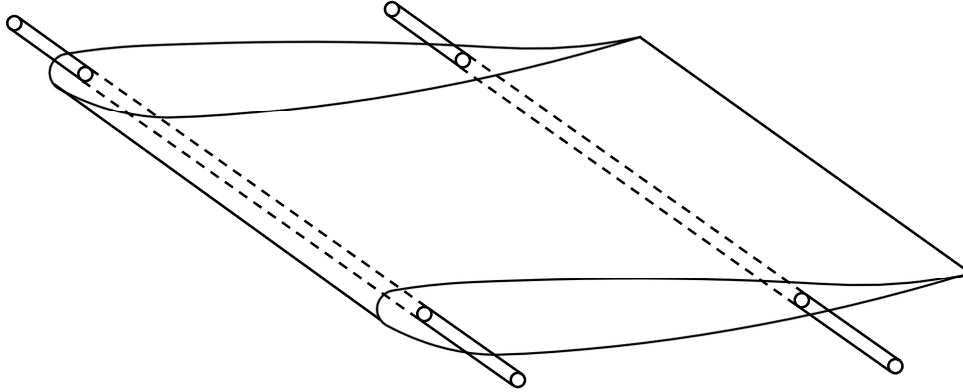


Figure 3.4: Manufactured Tyrrell026S wing model used in the dynamic setup.

As shown in Figure 3.4, two threaded rods pass through the wing. These rods connect the wing to the incidence adjustment support, shown in Figure 3.5. This mechanism allows the wing incidence to be set at discrete angles. The full design provides 13 possible incidence positions, ranging from -9° to 9° , with a step of 1.5° . For technical reasons, the experimental campaign was restricted to incidence angles between 3° and 9° . This corresponds to five

available configurations, highlighted by the red region in Figure 3.5.

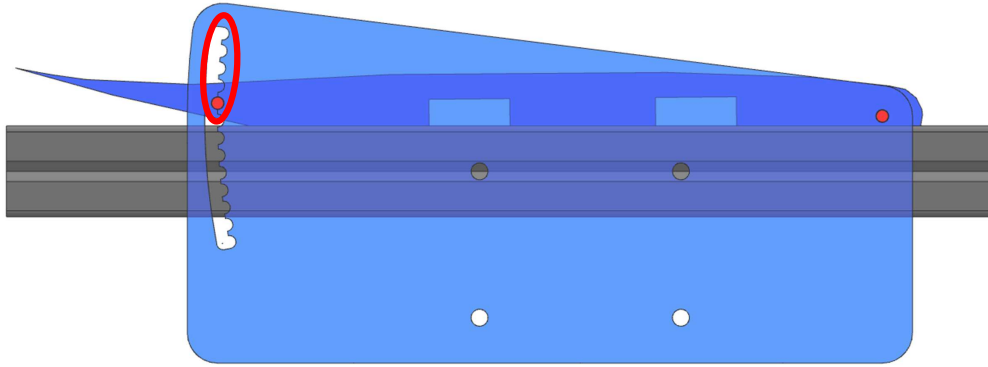


Figure 3.5: Wing incidence adjustment support and selected experimental range.

The complete dynamic setup is shown in Figure 3.6. The wing is mounted on a rigid aluminium support connected to the suspended dynamic assembly. This assembly is supported by springs, allowing vertical and rotational motion. The blue components correspond to the moving assembly, whereas the external aluminium frame remains fixed. The orange plate represents the ground plane. Its vertical position can be adjusted independently in order to impose different ground-clearance configurations.

Two large endplates are placed on both sides of the wing in order to limit spanwise flow and make the flow as close as possible to two-dimensional. These endplates are not directly mounted on the wing, but are fixed to the static structure. This choice avoids adding mass, stiffness or parasitic damping to the moving assembly, which could affect the measured dynamic response. As a consequence of this mounting strategy, a small gap of only a few millimetres remains between the wing and each endplate.

Three laser displacement sensors are used to measure the motion of the moving assembly (brown, purple and light grey boxes). As shown in Figure 3.6, two sensors are placed on the same side of the setup, one near the front part of the suspended bar and one near the rear part. These two measurements provide the vertical displacements at two longitudinal locations, which makes it possible to reconstruct the plunge motion and the pitch rotation of the wing-support assembly. A third laser sensor is installed on the opposite side of the rear part of the structure. Its role is to check whether the response remains essentially two-dimensional or whether an unwanted roll component appears during the tests.

A final design parameter was the longitudinal position of the wing relative to the leading edge of the ground plate. This distance controls the development length of the ground boundary layer before the flow reaches the wing.

In the present setup, no quantitative optimisation of the ground boundary-layer thickness was performed, since imposing a prescribed boundary-layer thickness would require additional dedicated measurements and flow-control devices. The wing was therefore positioned qualitatively so as to leave a significant development length upstream of the wing. As shown in Figure 3.7, the distance between the leading edge of the ground plate and the wing leading edge is approximately 725 mm, allowing the ground boundary layer to develop before interacting with the wing.

Photographs of the actual plunge-pitch dynamic setup used during the experimental campaign are shown in Figure 3.9.

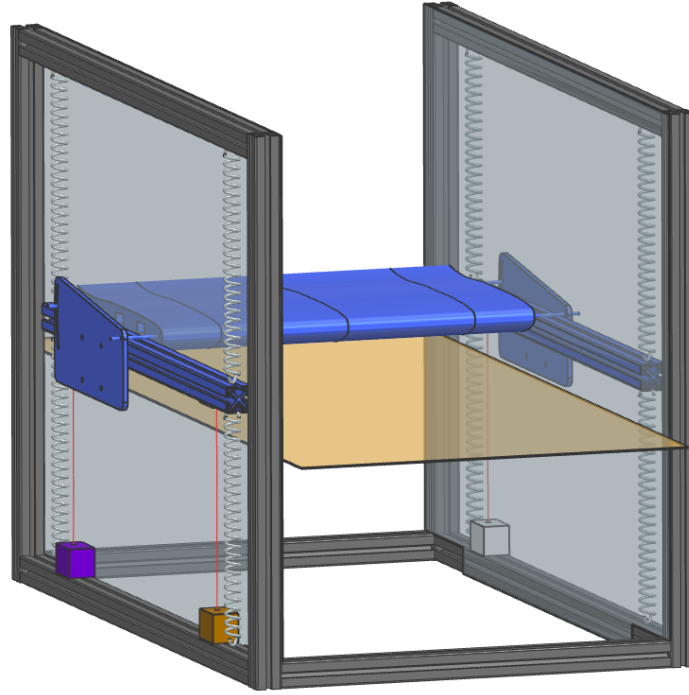


Figure 3.6: CAD side view of the dynamic aeroelastic setup.

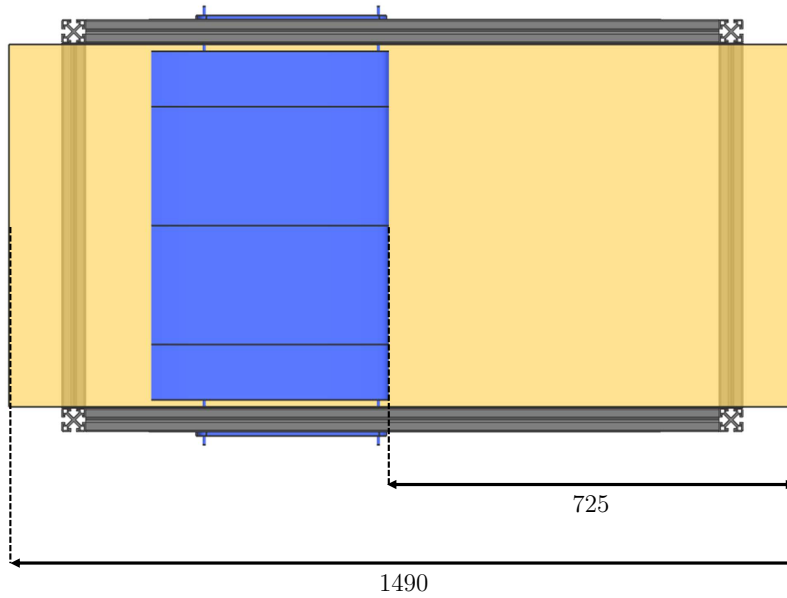


Figure 3.7: Top view of the dynamic setup showing the wing position relative to the ground plate leading edge. Dimensions are given in millimetres.

Dynamic test campaign

The dynamic test campaign consisted of testing several combinations of ride height and wing incidence, while measuring the dynamic response of the system with the laser displacement sensors. For each geometrical configuration, the setup was tested in the wind tunnel over a range of freestream velocities. The two adjustable geometrical parameters are illustrated in Figure 3.8 and defined as follows:

- **Wing incidence** α_0 : angle of the wing relative to the freestream direction. Five incidence values were tested, from 3° to 9° , with a step of 1.5° .
- **Ride height** H : initial ground clearance between the wing and the ground plate, defined at zero incidence, i.e. for $\alpha_0 = 0^\circ$. Three ride heights were considered: $H = 3.2$, 5.2 , and 7.2 cm, corresponding respectively to $H/c = 7.1\%$, 11.6% , and 16.0% , where c denotes the wing chord.

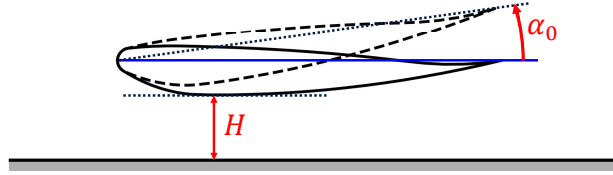


Figure 3.8: Definition of the wing incidence α_0 and ride height H used in the dynamic test campaign.

For each combination of ride height and wing incidence, the dynamic response was measured under both wind-off and wind-on conditions:

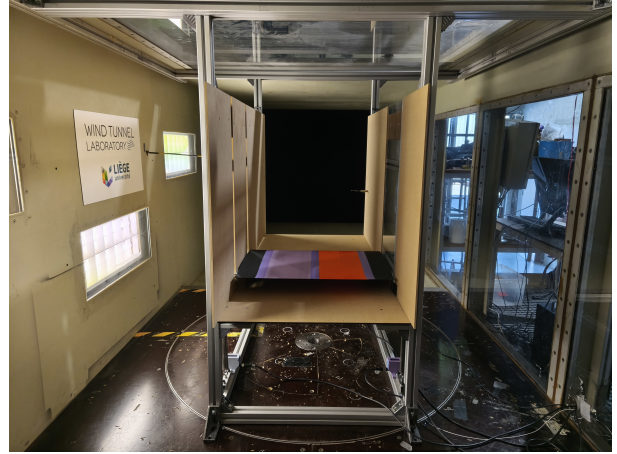
- **Wind-off condition:** $U_\infty = 0$. In this case, the system was manually excited by applying a short vertical impulse to the wing in order to identify its free dynamic response (mainly excited in plunge degree of freedom). This excitation lies between an impulsive excitation and a step-relaxation excitation, as described by Ewins [25], both of which are commonly used for the identification of structural dynamic properties.
- **Wind-on condition:** $U_\infty \neq 0$. The freestream velocity was increased from 8 m s^{-1} up to the critical porpoising velocity U_{porp} of the considered configuration. The velocity step was set to 2 m s^{-1} far from the instability threshold, then progressively reduced to 1 m s^{-1} and 0.5 m s^{-1} when approaching U_{porp} . In this condition, no manual impulse was applied, the system response was excited by the aerodynamic forcing and flow disturbances present in the wind tunnel.

For the dynamic tests, the maximum tested freestream velocity was approximately $U_{\infty, \text{max}} = 40 \text{ m/s}$. Based on the wing chord $c = 0.45 \text{ m}$ and using the kinematic viscosity of air at ambient conditions, $\nu \approx 1.5 \times 10^{-5} \text{ m}^2/\text{s}$, the maximum chord-based Reynolds number was therefore

$$Re_{c, \text{max}} = \frac{U_{\infty, \text{max}} c}{\nu} \approx 1.2 \times 10^6.$$



(a) Perspective view of the plunge-pitch dynamic setup (right rear).



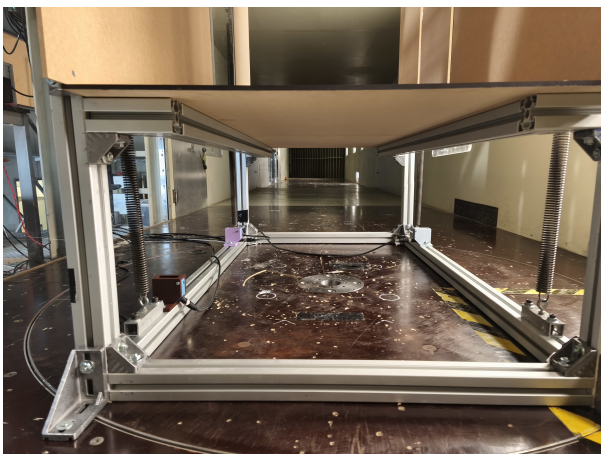
(b) Back view of the plunge-pitch dynamic setup.



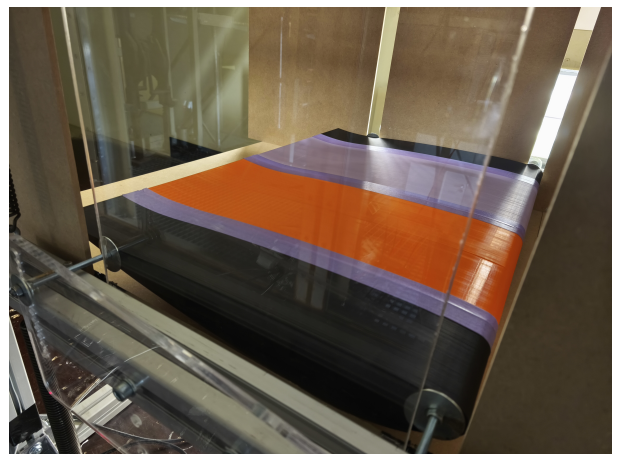
(c) Perspective view of the plunge-pitch dynamic setup (left rear).



(d) Front view of the plunge-pitch dynamic setup.



(e) Location of the laser displacement sensors used to measure the dynamic response of the setup.



(f) Close-up view of the vertical slots in the end-plates, allowing the threaded rods to move with the suspended wing assembly.

Figure 3.9: Photographs of the experimental plunge-pitch dynamic setup installed in the wind tunnel.

3.2.2 Plunge-only dynamic setup

Design choices

The purpose of the plunge-only dynamic setup is to improve the quality of the dynamic measurements used for the modelling of the porpoising instability. With the previous plunge-pitch setup, the extraction of aeroelastic properties was made more difficult by additional motion components, in particular roll contributions contaminating the measured plunge response. The objective of the second setup is therefore to constrain the system so that it can translate vertically only.

The moving assembly is now composed only of the manufactured Tyrrell026S wing and the two threaded rods passing through the wing, as shown in Figure 3.4. This assembly is still assumed to behave as a rigid body. The rigid lateral bars and the incidence-adjustment devices used in the plunge-pitch setup are no longer present.

The wing assembly is connected to an elastic support that provides the structural stiffness and damping in plunge. This support consists of two horizontal aluminium leaf springs fixed to the wing outside the endplates. Since these aluminium strips are allowed to bend vertically, the plunge stiffness of the system is governed by their bending stiffness.

The vertical displacement of the wing is measured by a single laser sensor located on the right-hand side of the setup. Photographs of the plunge-only dynamic setup used during the wind-tunnel tests are shown in Figure 3.10.

Dynamic test campaign

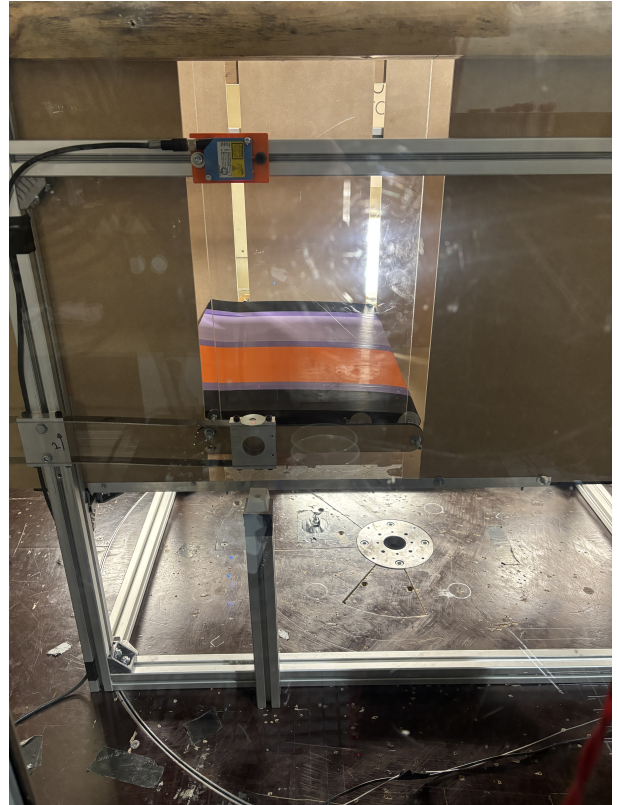
The second dynamic test campaign consisted of testing several initial ground-clearance configurations. These configurations were obtained by vertically adjusting the position of the wing assembly and its elastic supports. For each configuration, the dynamic response of the system was recorded using the laser sensor under both wind-off and wind-on conditions, following the same general procedure as for the plunge-pitch dynamic tests described in the previous section.

During this campaign, porpoising instabilities were observed for freestream velocities up to approximately $U_{\infty, \max} \approx 20$ m/s. Using the wing chord of the plunge-only setup and the kinematic viscosity of air at ambient conditions, this corresponds to a maximum chord-based Reynolds number of approximately

$$Re_{c, \max} = \frac{U_{\infty, \max} c}{\nu} \approx 6 \times 10^5.$$



(a) Front view of the plunge-only dynamic setup.



(b) Side view of the plunge-only dynamic setup.



(c) Rear-left perspective view of the plunge-only dynamic setup.



(d) Rear-right perspective view of the plunge-only dynamic setup.

Figure 3.10: Photographs of the plunge-only dynamic setup installed in the wind tunnel.

4 Experimental Results

This chapter presents and analyses the experimental results obtained from the static and dynamic wind-tunnel campaigns.

First, the static results are discussed. The evolution of the aerodynamic coefficients is analysed as a function of angle of attack and normalized ground clearance H/c . These measurements are used to identify the main aerodynamic trends of the wing close to the ground, including the increase of downforce, the influence of ride height and the possible onset of flow separation at low ground clearance.

Then, the dynamic results are presented. The time responses measured by the laser sensors are analysed to identify the dominant motion of the system, including plunge, pitch and roll contributions. The evolution of the static deflection, modal frequencies and modal damping with freestream velocity is then investigated in order to characterise the stability of the system.

4.1 Static Test Results

The objective of the static wind-tunnel campaign is to characterise the aerodynamic behaviour of the Tyrrell026S wing through steady force measurements. The static setup described in Chapter 3 is used to measure the aerodynamic loads for different combinations of wing incidence α_0 and ground clearance H . The resulting database is then used to obtain non-dimensional aerodynamic coefficients, mainly the lift and drag coefficients. The moment coefficient is not retained in the present analysis due to insufficient confidence in the measured moment signal.

The static measurements were performed by fixing the wing incidence α_0 and progressively decreasing the ground clearance from the largest to the smallest tested value. This procedure was repeated for each incidence angle. Additional checks performed with the opposite procedure, namely fixing the ground clearance and varying the incidence, led to consistent results.

The lift and drag coefficients are computed as

$$C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 A}, \quad (4.1.1)$$

$$C_D = \frac{D}{\frac{1}{2}\rho U_\infty^2 A}, \quad (4.1.2)$$

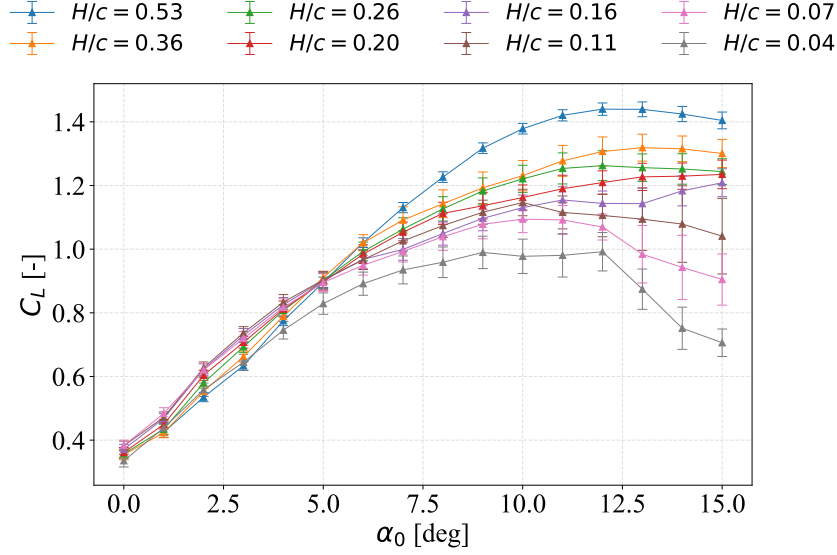
where L and D are respectively the lift and drag forces obtained by projecting the measured force components in the aerodynamic reference frame. In this work, positive lift is defined as a force pointing toward the ground, i.e. positive C_L corresponds to downforce. The drag force is defined positive in the freestream direction.

The static aerodynamic tests were performed at a freestream velocity of $U_\infty = 10.5$ m/s, as

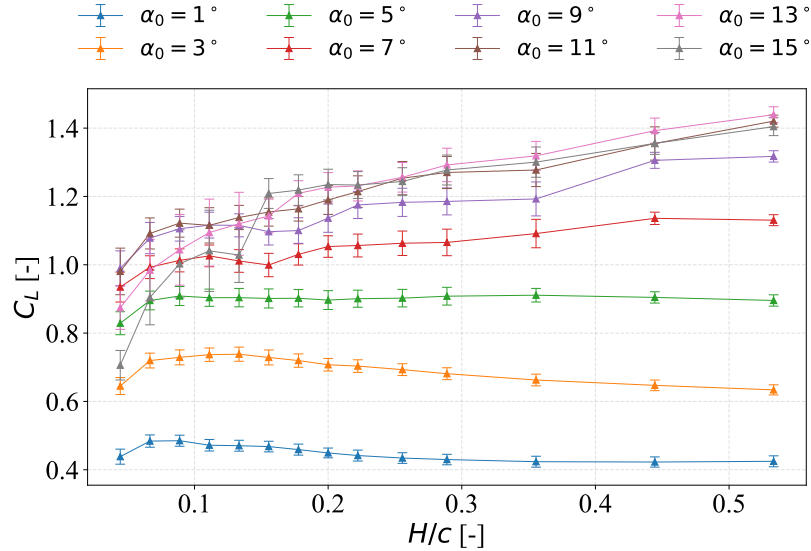
discussed earlier, the corresponding chord-based Reynolds number is approximately

$$Re_c = \frac{U_\infty c}{\nu} \approx 1.6 \times 10^5.$$

Figure 4.1 shows the evolution of the lift coefficient as a function of the two main geometrical parameters of the test campaign: the wing incidence α_0 and the normalized ground clearance H/c .



(a) C_L as a function of wing incidence α_0 for several normalized ground clearances H/c .



(b) C_L as a function of normalized ground clearance H/c for several wing incidences α_0 .

Figure 4.1: Evolution of the lift coefficient of the Tyrrell026S wing measured during the static wind-tunnel campaign.

A positive lift coefficient is obtained even at zero incidence. This is consistent with the cambered and inverted nature of the profile, which is designed to generate downforce. The evolution of C_L with α_0 , shown in Figure 4.1a, is approximately linear over the low-incidence

range, up to about $\alpha_0 = 5^\circ$. In this range, the lift response can be described by an incidence lift-curve slope $dC_L/d\alpha_0$.

To provide a reference, Figure 4.2 compares the measured low-incidence behaviour with a 2π slope. This value corresponds to the classical thin-airfoil lift-curve slope expressed per radian. The reference line is only used to compare slopes, it is vertically shifted and is therefore not intended to predict the absolute value of C_L .

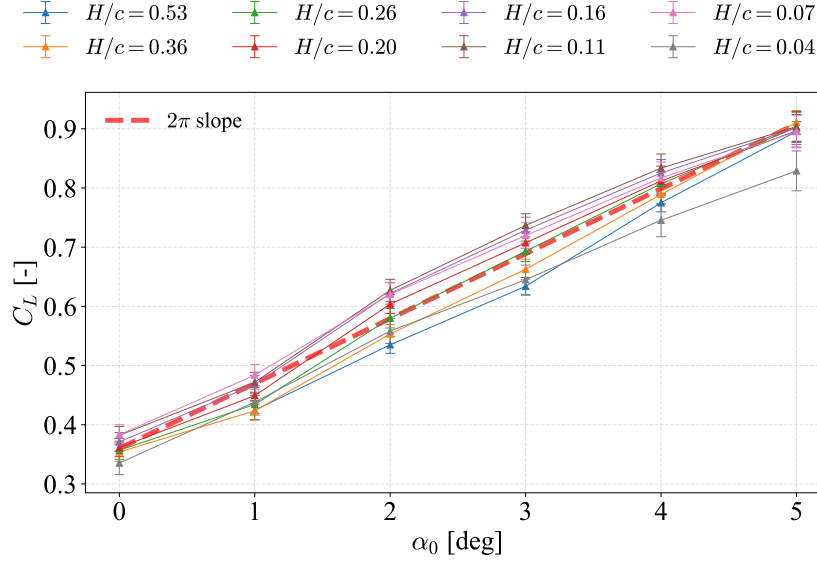


Figure 4.2: Comparison between the measured low-incidence evolution of C_L and a reference 2π lift-curve slope.

For incidences larger than approximately 5° , the curves progressively depart from the linear trend. This departure occurs earlier when the wing is closer to the ground. For example, the curve at low ground clearance, around $H/c = 0.04$, starts to lose linearity at lower incidence than the curve at larger ground clearance, around $H/c = 0.53$. This observation suggests that reducing the ground clearance promotes an earlier loss of lift-curve linearity. A possible physical explanation is the onset of flow separation or ground-induced stall, caused by the stronger interaction between the wing boundary layer and the ground boundary layer. This interpretation is consistent with the ground-effect behaviour reported by Ranzenbach and Barlow [13], although no direct flow-visualisation measurement is available here to confirm the exact separation mechanism.

The incidence lift-curve slope $dC_L/d\alpha_0$ is then extracted from the approximately linear range of the curves, between $\alpha_0 = 0^\circ$ and $\alpha_0 = 5^\circ$. The result is shown in Figure 4.3 as a function of the normalized ground clearance H/c .

The largest lift-curve slope is obtained for an intermediate ground clearance, around $H/c \simeq 0.30$. For larger ground clearances, the slope tends to move back toward the classical 2π reference, which is consistent with a weaker influence of the ground. For smaller clearances, typically below $H/c \simeq 0.15$, the slope decreases significantly. This means that, in very close ground proximity, increasing the incidence becomes less effective at increasing the downforce. This behaviour is consistent with a degradation of the flow in the gap between the wing and the ground.

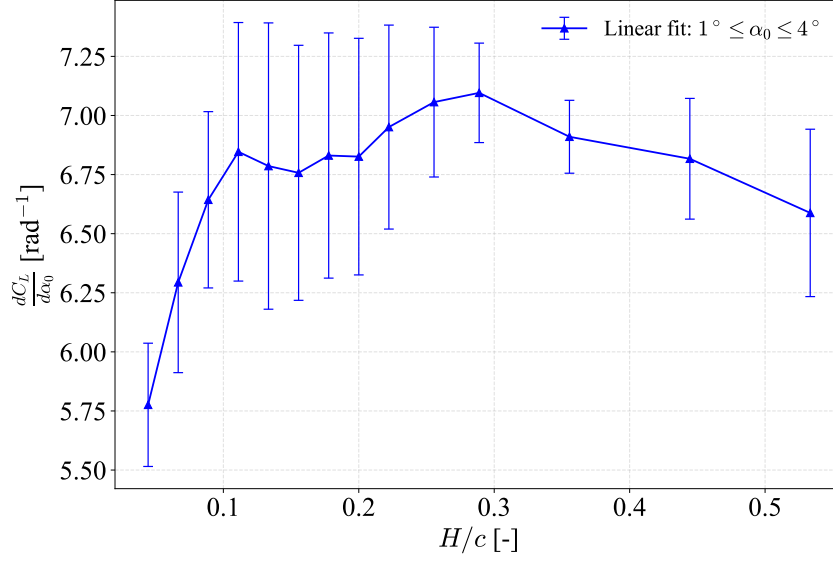


Figure 4.3: Evolution of the incidence lift-curve slope $dC_L/d\alpha_0$ as a function of normalized ground clearance H/c . The slope is obtained from a linear fit of $C_L(\alpha_0)$ over the low-incidence range $1^\circ \leq \alpha_0 \leq 4^\circ$ and is expressed per radian. Error bars indicate the uncertainty associated with the linear fit.

The evolution of C_L with H/c , shown in Figure 4.1b, also confirms that the ground-effect behaviour depends strongly on incidence. At low incidence, especially for $\alpha_0 = 1^\circ$ and $\alpha_0 = 3^\circ$, the expected ground-effect trend is visible: decreasing H/c first increases the downforce, up to an optimum range, before the downforce decreases again at very small clearances. This corresponds to the classical ground-effect behaviour discussed by Wiedemann [17].

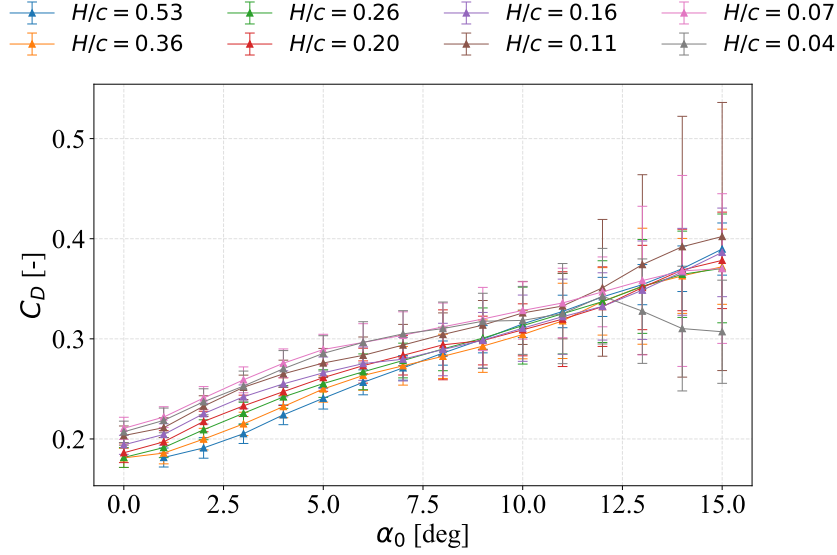
For higher incidences, however, the trend is different. From approximately $\alpha_0 = 5^\circ$ upward, the lift coefficient tends to decrease continuously as the wing approaches the ground. This indicates that the aerodynamic benefit of reducing the ground clearance is lost at high incidence. The most plausible explanation is that the combination of high incidence and small ground clearance promotes premature flow separation, which reduces the suction under the inverted wing and therefore reduces the generated downforce.

Looking now at the drag coefficient, Figure 4.4a shows that C_D increases with the wing incidence α_0 , as expected. Compared with the lift coefficient, the influence of the normalized ground clearance H/c on C_D remains relatively limited over most of the tested range.

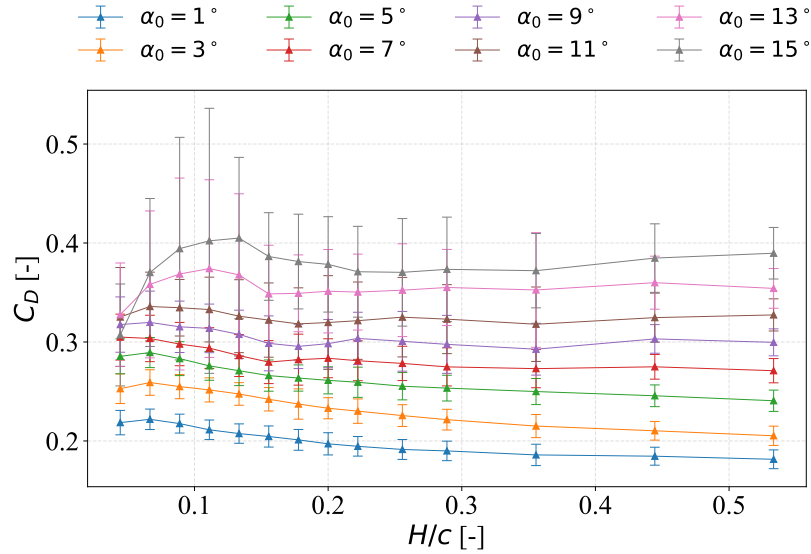
A noticeable deviation is nevertheless observed for the smallest ground clearance, around $H/c = 0.04$, especially at high incidence. This deviation should be interpreted with caution. It may correspond to a real aerodynamic effect caused by the strong interaction between the wing and the ground at very small clearance. However, it may also be partly affected by experimental uncertainty, since these points were measured during a second acquisition attempt. Sensor drift, zero-offset variations, or repeatability issues could therefore have contributed to the observed discrepancy.

The evolution of the drag coefficient as a function of the normalized ground clearance is presented in Figure 4.4b. A common trend can be observed for the different incidence an-

gles. Starting from large ground clearance, decreasing H/c first leads to a gradual increase in C_D . The drag coefficient then reaches a maximum at an intermediate clearance, around $H/c \simeq 0.15$, before decreasing again when the wing is brought very close to the ground.



(a) C_D as a function of incidence α_0 for several ride heights.



(b) C_D as a function of ride height H/c for several incidences.

Figure 4.4: Evolution of the drag coefficient measured during the static wind-tunnel campaign.

This behaviour suggests that the ground effect modifies not only the lift generation but also the drag mechanisms. At intermediate ground clearance, the stronger acceleration of the flow in the gap between the wing and the ground may increase aerodynamic loading and losses, resulting in a higher drag coefficient.

The lift-drag polar shown in Figure 4.5 provides a synthetic view of the aerodynamic compromise of the wing. Since positive C_L corresponds to downforce in the present convention,

the polar indicates the drag penalty associated with a given level of vertical load. The general trend confirms that increasing the downforce is accompanied by an increase in drag. However, the slope of the polar is not constant, which indicates that the aerodynamic efficiency depends on both the incidence angle and the ground clearance. In particular, configurations producing high downforce at moderate drag are the most favourable from an aerodynamic efficiency point of view, whereas points where C_D increases without a significant gain in C_L indicate a less efficient operating condition.

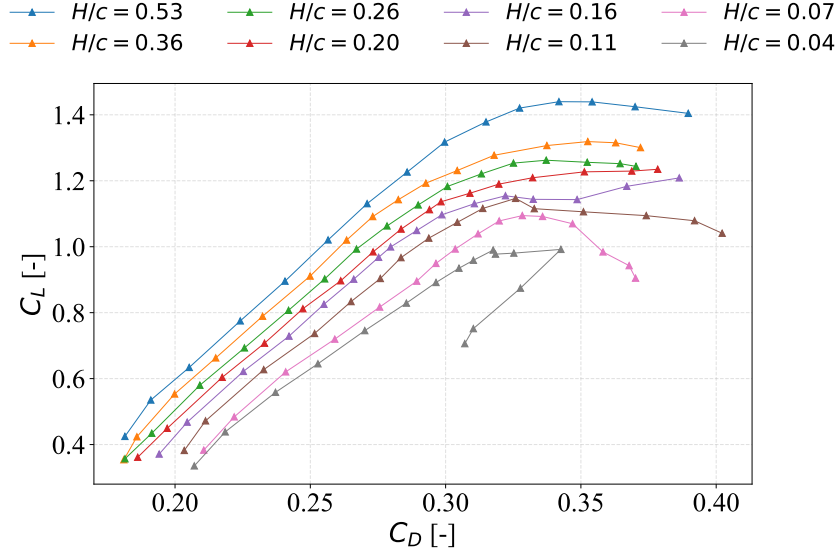


Figure 4.5: Lift coefficient C_L as a function of drag coefficient C_D for the tested ride heights.

4.2 Dynamic Test Results

4.2.1 Plunge-pitch dynamic setup

The objective of the dynamic wind-tunnel campaign is to characterise the response of the plunge-pitch aeroelastic system described in Chapter 3. The dynamic setup consists of a wing connected to two rigid lateral bars, one on each spanwise side. These bars are attached to a fixed rigid frame through springs, allowing the wing-bar assembly to move mainly in plunge, pitch and roll.

The dynamic response is measured using three laser displacement sensors. Each laser records the vertical displacement of a specific point of the wing-bar assembly. The sensor denoted $L1_{RR}$ measures the rear vertical displacement of the right bar, $L2_{FR}$ measures the front vertical displacement of the right bar, and $L3_{RL}$ measures the rear vertical displacement of the left bar. The position of these sensors is shown in Figure 3.6, where $L1_{RR}$ corresponds to the fuchsia laser box, $L2_{FR}$ to the light grey laser box, and $L3_{RL}$ to the brown laser box.

Under the assumption that the wing-bar assembly behaves as a rigid body, these three displacement measurements are sufficient to identify the main rigid-body components of the motion. The common vertical motion of the sensors is associated with plunge, the differential motion between the front and rear sensors on the right side is associated with pitch, and the differential motion between the rear-right and rear-left sensors is associated with roll. Therefore, the measured laser signals can be combined to construct representative plunge, pitch and roll response signals.

The dynamic results are analysed in several steps. First, the raw time histories measured by the three laser sensors are examined in order to assess the amplitude and qualitative behaviour of the response. Then, the laser signals are combined to obtain representative plunge, pitch and roll components. These reconstructed signals are used to evaluate the static deflection of the system, in terms of equilibrium vertical displacement and equilibrium pitch angle, as a function of freestream velocity.

A spectral analysis is then performed by computing the Fast Fourier Transform of the reconstructed response components. This allows the dominant frequencies of the plunge and pitch responses to be identified. Around each dominant spectral peak, a local single-degree-of-freedom fit is used to estimate the corresponding modal frequency and an effective damping ratio. Since the analysis is based on output-only experimental signals rather than measured frequency response functions, the identified damping values should be interpreted as effective damping estimates associated with the observed response peaks [25].

Finally, a bifurcation analysis is carried out by tracking the minimum and maximum measured displacements as a function of freestream velocity. This representation makes it possible to identify the onset of large-amplitude oscillations and to distinguish bounded responses from dynamically unstable regimes.

The results presented in this section correspond first to one specific aeroelastic configuration, defined by a fixed ground clearance H and a fixed initial wing incidence α_0 . Then, the influence of these two parameters is discussed in at the end of this section. Here, the selected reference configuration is

$$H = 52 \text{ mm} \quad \text{and} \quad \alpha_0 = 7.5^\circ.$$

Three response regimes are analysed for this configuration: the wind-off response, the sub-

critical response, and the post-critical response. The wind-off case is used to observe the structural free-decay behaviour of the system without aerodynamic excitation. The sub-critical case corresponds to a bounded response under flow conditions below the instability onset. The post-critical case corresponds to a dynamically unstable response, for which the oscillation amplitude grows with time.

Time-domain response of the laser signals

The raw time histories measured by the three laser displacement sensors are first analysed. Each acquisition lasts 60 s, and the measured quantities correspond to the vertical displacement of the wing–bar assembly at the sensor locations.

Figure 4.6 shows the wind-off response, obtained for a freestream velocity $U_\infty = 0$ m/s. In this case, the three laser signals are centred around zero vertical displacement, since no aerodynamic load is applied to the system. At the beginning of the acquisition, a non-zero displacement of the order of one centimetre is observed. This displacement results from a manual excitation applied to the structure in order to trigger a free response of the system. After the initial excitation, the amplitude of the vertical displacement decreases with time. The decay of the response indicates the presence of structural damping in the system. The response is therefore stable in wind-off conditions, and the observed decay can be used to characterise the structural dynamic behaviour of the setup.

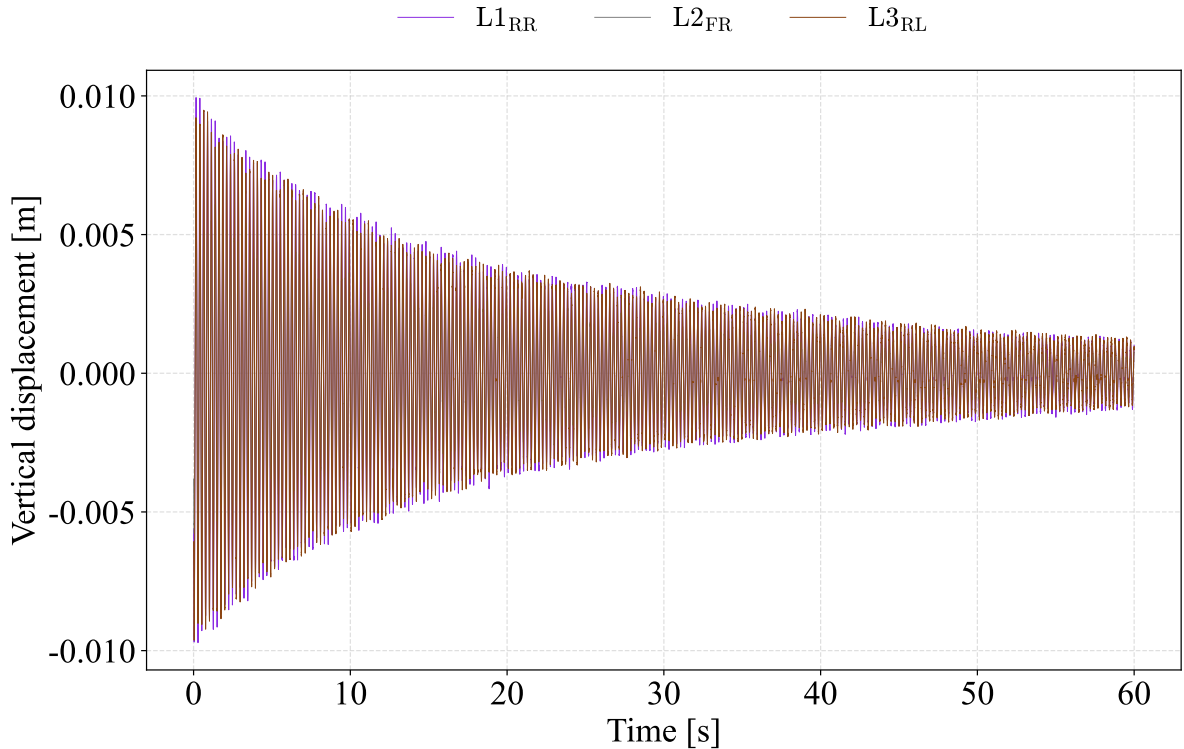


Figure 4.6: Time evolution of the laser displacement signals for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 0.0$ m/s.

The sub-critical response is shown in Figure 4.7 for a freestream velocity $U_\infty = 8.1$ m/s. In this case, the signals are no longer centred around zero displacement. This indicates the presence of a static aeroelastic deflection caused by the aerodynamic loads. The two rear

laser signals remain centred around similar displacement levels, which suggests that the static roll deflection is small. This is expected because the setup is approximately symmetric in the spanwise direction, although perfect symmetry cannot be guaranteed experimentally.

In addition, the front-right and rear-right laser signals are not centred around the same value. This difference indicates the presence of a static pitch deflection of the wing–bar assembly.

From a dynamic point of view, the sub-critical response remains bounded. No growth of the oscillation amplitude is observed over the acquisition time. Small fluctuations are nevertheless visible, with amplitudes of the order of a fraction of a millimetre. These fluctuations are mainly attributed to flow turbulence exciting the structure, with a smaller contribution from laser measurement noise. The response is therefore considered dynamically stable for this freestream velocity.

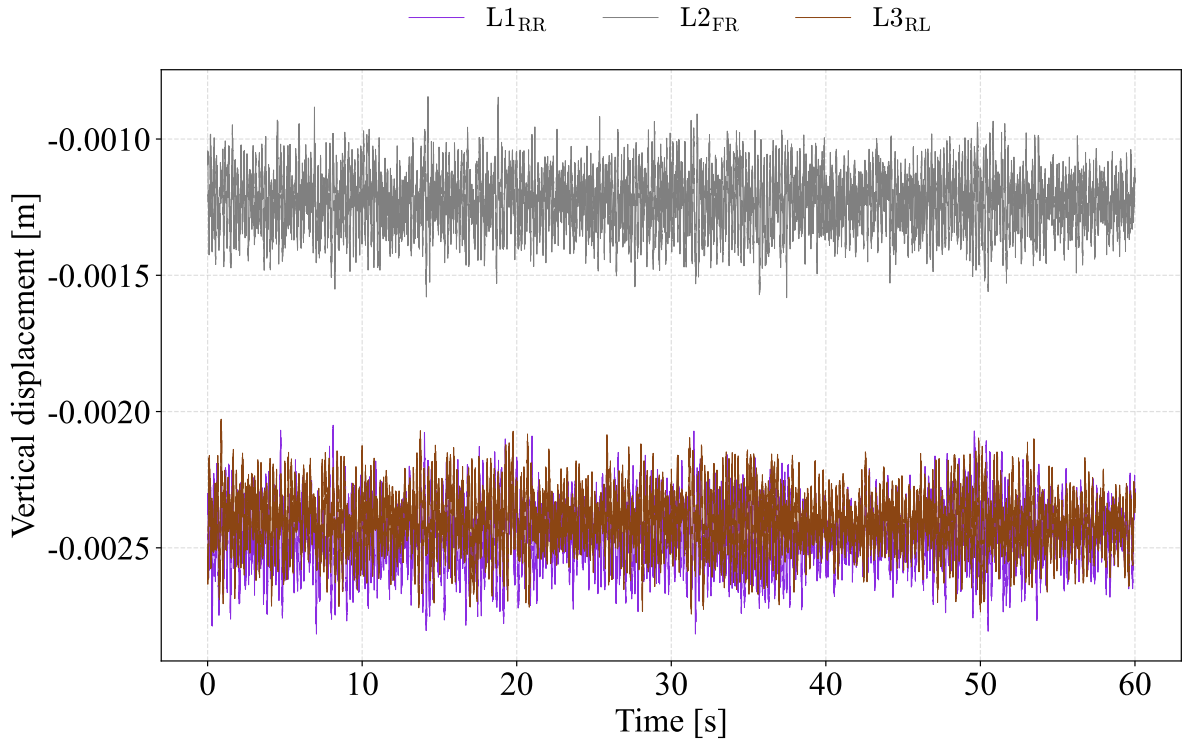


Figure 4.7: Time evolution of the laser displacement signals for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 8.1$ m/s.

Finally, Figure 4.8 shows the post-critical response obtained at $U_\infty = 28.4$ m/s. In this case, the amplitude of the measured vertical displacements increases clearly with time. This growth indicates a dynamically unstable aeroelastic response. For the considered configuration, this velocity is therefore associated with the onset of the porpoising instability.

However, the raw time histories alone do not allow the unstable motion to be identified unambiguously. In particular, it is difficult from this figure alone to determine whether the rear-right and rear-left signals are mainly in phase or out of phase, which would distinguish plunge from roll. Similarly, comparing the front-right and rear-right signals is required to distinguish plunge from pitch.

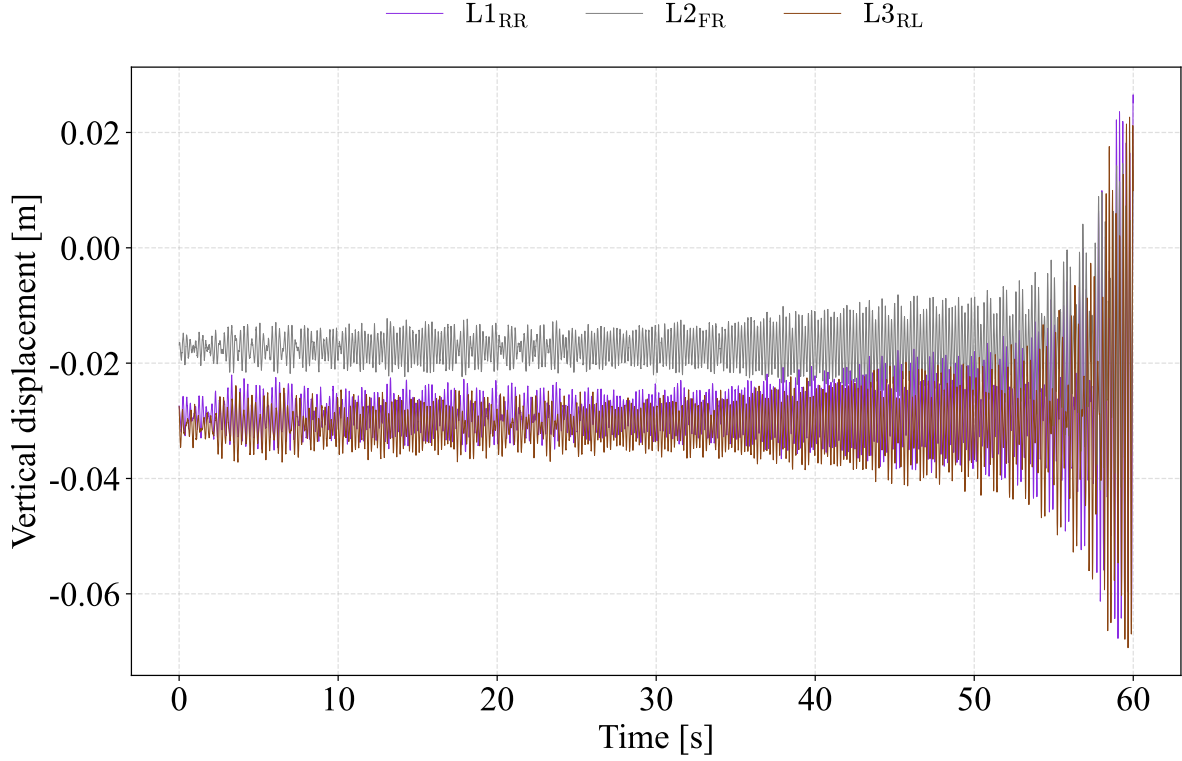


Figure 4.8: Time evolution of the laser displacement signals for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 28.4$ m/s.

Laser-to-laser phase analysis

As discussed previously, the raw laser time histories do not always allow the dominant motion of the system to be identified unambiguously. For this reason, the laser signals are compared directly through laser-to-laser displacement plots. These plots provide a qualitative indication of the relative phase between two measured points of the wing-bar assembly and are used to distinguish plunge, pitch, and roll contributions.

The pitch contribution is assessed by comparing the rear-right laser $L1_{RR}$ with the front-right laser $L2_{FR}$. If both signals move in phase, the front and rear parts of the right bar move together, which is associated with a dominant plunge motion. If both signals move in opposite phase, the front and rear parts move in opposite directions, which indicates a pitch contribution. Similarly, the roll contribution is assessed by comparing the two rear lasers, $L1_{RR}$ and $L3_{RL}$. In this case, an in-phase motion indicates a common vertical displacement of both rear points, whereas an out-of-phase motion indicates a roll contribution.

In a laser-to-laser displacement plot, an approximately straight line with a positive slope indicates that the two signals are mainly in phase. A straight line with a negative slope indicates that the two signals are mainly out of phase. A thicker line, an elliptical cloud, or a more circular distribution indicates that the motion is not purely described by one component only, but contains a combination of motions, possible phase lag, flow-induced fluctuations, or measurement noise.

For the wind-off response, shown in Figure 4.9, the comparison between $L1_{RR}$ and $L2_{FR}$ shows a clear in-phase behaviour. The same observation is made when comparing $L1_{RR}$ and $L3_{RL}$. This indicates that the structural free response is dominated by plunge in the current measurements. If the motion were purely plunge, both laser-to-laser plots would collapse

onto a thin straight line with positive slope. In practice, the lines have a small but finite thickness, which indicates the presence of very small pitch and roll contributions, together with measurement noise. These secondary components remain negligible compared with the plunge motion. This result is consistent with the type of manual excitation applied during the wind-off test, which was mainly a vertical excitation close to the geometrical centre of the system.

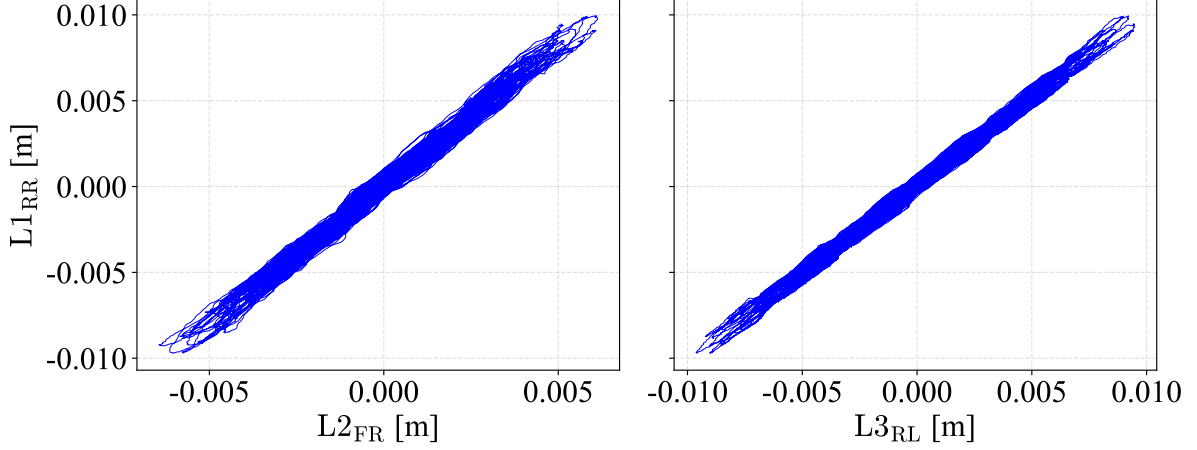


Figure 4.9: Laser-to-laser displacement plots for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 0.0$ m/s.

The sub-critical response is shown in Figure 4.10. In this case, the system is no longer excited manually, but by the unsteady aerodynamic loads and turbulence present in the wind-tunnel flow. The response is therefore qualitatively different from the wind-off case. The first important observation is that the amplitude of the fluctuations remains small, which confirms that the system is dynamically stable at this freestream velocity. The motion corresponds to small oscillations around the static equilibrium position.

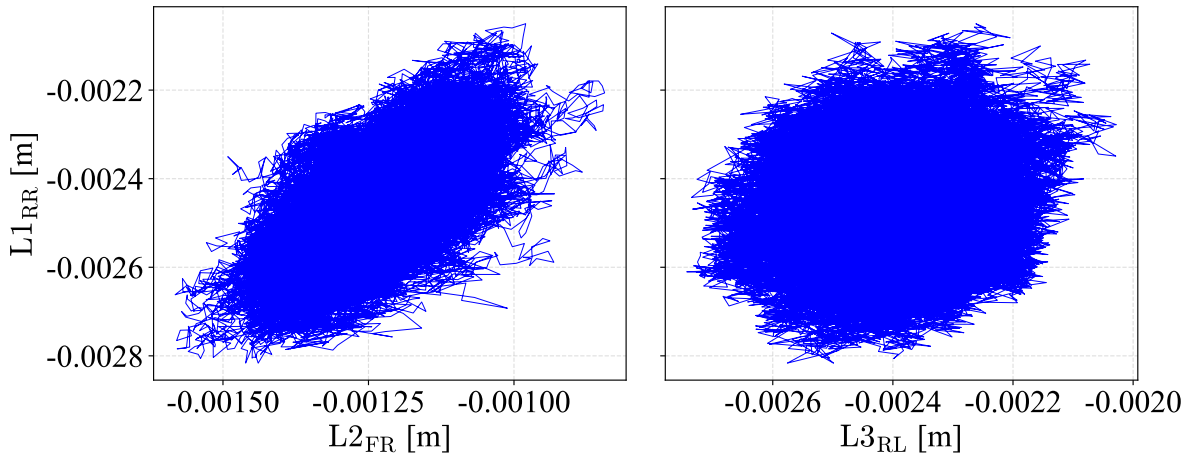


Figure 4.10: Laser-to-laser displacement plots for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 8.1$ m/s.

Looking first at the comparison between the two rear lasers, no clear dominant in-phase or out-of-phase trend is observed. The data form a compact cloud around the equilibrium

position, which indicates that neither pure plunge nor pure roll dominates this part of the response. The rear motion therefore remains weak and mixed. Same observation is made for the comparison between the rear-right and front-right lasers.

The post-critical response, shown in Figure 4.11, exhibits a much clearer dynamic behaviour. The comparison between the front-right and rear-right lasers shows that both signals are mainly in phase and reach large amplitudes. This indicates that the unstable motion is dominated by plunge. However, the comparison between the two rear lasers shows that the motion is not perfectly in phase over the full acquisition. At small amplitudes, near the dense central region of the plot, the response contains a visible mixed component, with some roll contribution. As the oscillation amplitude grows, the dominant trend becomes more clearly in phase, indicating that the unstable response is mainly governed by plunge. Roll remains present as a secondary component, but it does not appear to be the dominant unstable motion.

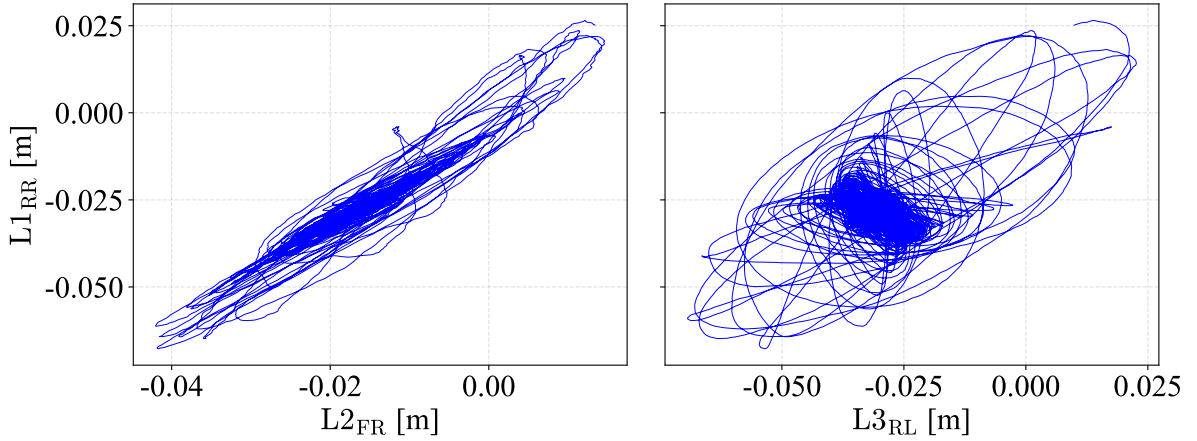


Figure 4.11: Laser-to-laser displacement plots for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 28.4$ m/s.

Static aeroelastic deflection

In the present analysis, only the static plunge and pitch deflections are considered. The roll deflection is not retained here, since it is not of interest.

Let $\bar{L}_{1,RR}(U_\infty)$ and $\bar{L}_{2,FR}(U_\infty)$ denote the time-averaged, wind-off-corrected displacements measured by the rear-right and front-right lasers at a given freestream velocity U_∞ . Assuming that the right bar behaves as a fully rigid body, the static plunge displacement at the mid-point between the two sensors is computed as

$$y_{eq}(U_\infty) = \frac{\bar{L}_{1,RR}(U_\infty) + \bar{L}_{2,FR}(U_\infty)}{2}. \quad (4.2.1)$$

The corresponding static pitch angle is obtained from the difference between the front and rear vertical displacements:

$$\alpha_{eq}(U_\infty) = \tan^{-1} \left(\frac{\bar{L}_{2,FR}(U_\infty) - \bar{L}_{1,RR}(U_\infty)}{L_{12}} \right), \quad (4.2.2)$$

where L_{12} is the longitudinal distance between the two right-side laser measurement points. For small pitch angles, this expression can be linearised as

$$\alpha_{\text{eq}}(V) \simeq \frac{\bar{L}_{2,FR}(V) - \bar{L}_{1,RR}(V)}{L_{12}}. \quad (4.2.3)$$

The resulting static plunge and pitch deflections are shown in Figure 4.12. For the reference configuration, defined by an initial ground clearance $H = 52$ mm and an initial wing incidence $\alpha_0 = 7.5^\circ$, the system progressively deflects as the freestream velocity increases. Around $U_\infty = 20$ m/s, the mid-point vertical displacement reaches approximately -1 cm, while the pitch angle reaches approximately -0.5° .

The last point of the curves should be interpreted with caution. At this velocity, the system is no longer in a purely static or bounded dynamic regime, since the response becomes unstable. Consequently, the time-averaged displacement at this point does not represent a well-defined static equilibrium in the same sense as for the sub-critical cases.

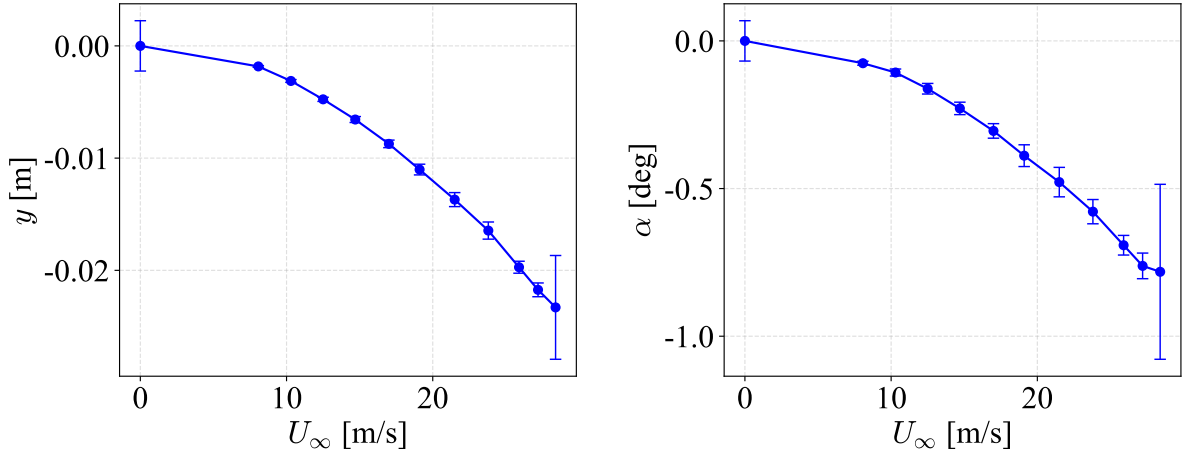


Figure 4.12: Static equilibrium evolution of the system as a function of freestream velocity for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° .

Frequency-domain modal identification

The objective of this section is to analyse the frequency content of the measured response in order to identify the dominant modal frequencies and the associated effective damping ratios. The analysis focuses mainly on the plunge and pitch components of the motion as a function of freestream velocity. The roll component is also reconstructed and inspected, but its detailed modal identification is not retained in the present study.

Before computing the frequency spectra, the three laser signals are combined into representative motion components. Since the laser signals measure local vertical displacements, their common and differential components can be used to isolate the main rigid-body contributions of the wing–bar assembly.

The reconstructed plunge signal is defined as

$$z_{(a)}(t) = L1_{RR}(t) + L2_{FR}(t), \quad (4.2.4)$$

A factor $1/2$ could be introduced to obtain the mean vertical displacement, but it does not affect the frequency content nor the normalized FFT amplitude. Therefore, the proportional signal of Equation 4.2.4 is sufficient for modal identification.

The reconstructed pitch signal is obtained from the differential motion between the rear-right and front-right sensors:

$$z_{(b)}(t) = L1_{RR}(t) - L2_{FR}(t). \quad (4.2.5)$$

This signal is proportional to the pitch rotation of the right bar. The sign convention has no influence on the FFT magnitude and therefore does not affect the modal frequency or damping identification.

The reconstructed roll signal is obtained from the differential motion between the rear-right and rear-left sensors:

$$z_{(c)}(t) = L1_{RR}(t) - L3_{RL}(t). \quad (4.2.6)$$

This signal is proportional to the roll motion of the assembly.

The Fast Fourier Transform is then applied to the reconstructed signals $z_{(a)}$, $z_{(b)}$, and $z_{(c)}$. In the following figures, the corresponding spectra are denoted respectively by $\text{FFT}_{(a)}$, $\text{FFT}_{(b)}$, and $\text{FFT}_{(c)}$. The spectra are shown for the wind-off condition, the sub-critical response, and the post-critical response in Figures 4.13, 4.14, and 4.15.

Figure 4.13 shows the spectra obtained in wind-off conditions, which correspond to the structural response of the system without aerodynamic loading. The upper plot, associated with the reconstructed plunge signal $\text{FFT}_{(a)}$, shows a dominant peak at approximately 4.09 Hz. This peak is identified as the structural plunge mode. The corresponding effective damping ratio is estimated as

$$\zeta_{y,\text{struct}} \simeq 0.22\%. \quad (4.2.7)$$

The middle plot corresponds to the reconstructed pitch signal $\text{FFT}_{(b)}$. The plunge mode remains visible in this spectrum because the manual excitation applied during the wind-off test was mainly vertical and therefore strongly excited the plunge component. Nevertheless, a second peak is observed at approximately 7.61 Hz and is identified as the structural pitch mode. The associated effective damping ratio is estimated as

$$\zeta_{\alpha,\text{struct}} \simeq 0.08\%. \quad (4.2.8)$$

Finally, the lower plot corresponds to the reconstructed roll signal $\text{FFT}_{(c)}$. A peak is observed around 5 Hz, which indicates the presence of a roll mode. However, the detailed identification of the roll modal damping is not performed here, since the present analysis focuses on the plunge and pitch dynamics involved in the porpoising response.

The frequencies and damping ratios identified in wind-off conditions are therefore considered as the structural modal properties of the setup. They provide the reference values against which the evolution of the aeroelastic response under increasing freestream velocity can be compared.

In sub-critical conditions, at $U_\infty = 8.1$ m/s, the spectral content of the reconstructed signals is shown in Figure 4.14. Compared with the wind-off case, the spectra contain more broad-band content. This is expected, since the system is now excited by the unsteady aerodynamic loads and by the residual turbulence of the wind-tunnel flow, rather than by a single manual initial excitation.

Looking first at the reconstructed plunge spectrum $\text{FFT}_{(a)}$, both the plunge and roll contributions are visible. The plunge peak remains located close to the structural plunge frequency

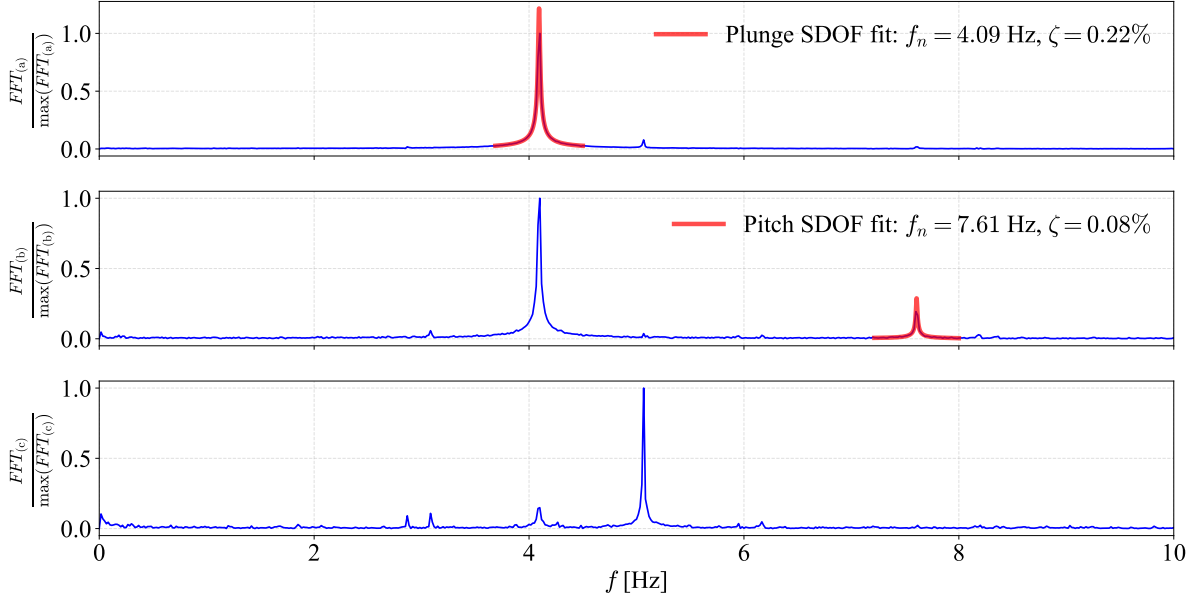


Figure 4.13: Normalized FFT spectra of the dynamic response for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 0.0$ m/s.

identified in wind-off conditions, although a slight frequency shift is observed due to the presence of aerodynamic loading. The effective damping ratio associated with the plunge peak increases to approximately

$$\zeta_y \simeq 1.66\%. \quad (4.2.9)$$

This increase indicates that, at this velocity, the aerodynamic contribution still has a stabilising effect on the plunge response.

The roll contribution is also visible in the plunge spectrum. Its peak appears sharper than the plunge peak, which suggests a lower effective damping ratio for the roll component.

The reconstructed pitch spectrum $\text{FFT}_{(b)}$ shows that the pitch frequency remains almost unchanged compared with the wind-off case. The pitch effective damping ratio also increases compared with the structural value, reaching approximately

$$\zeta_\alpha \simeq 0.27\%. \quad (4.2.10)$$

Therefore, both the plunge and pitch modes remain stable at $U_\infty = 8.1$ m/s, with positive effective damping ratios.

An additional spectral contribution is also observed around 3 Hz, especially in the reconstructed pitch spectrum. This peak is not associated with the main plunge, pitch, or roll components considered in the present analysis. It is interpreted as a possible lateral or sliding motion of the dynamic setup. This interpretation remains a hypothesis, since no direct horizontal displacement measurement is available.

A plausible explanation is the interaction between the airflow and the exposed moving components of the dynamic rig. As described in the setup section, endplates were used around the wing to promote a nearly two-dimensional flow in the region of interest. However, the other moving parts of the setup, such as the lateral rigid bars and the springs, are not fully enclosed by aerodynamic fairings. These exposed components may therefore interact with a more three-dimensional and unsteady flow, which could excite secondary lateral motions of the assembly. This additional contribution is not retained in the subsequent modal tracking.

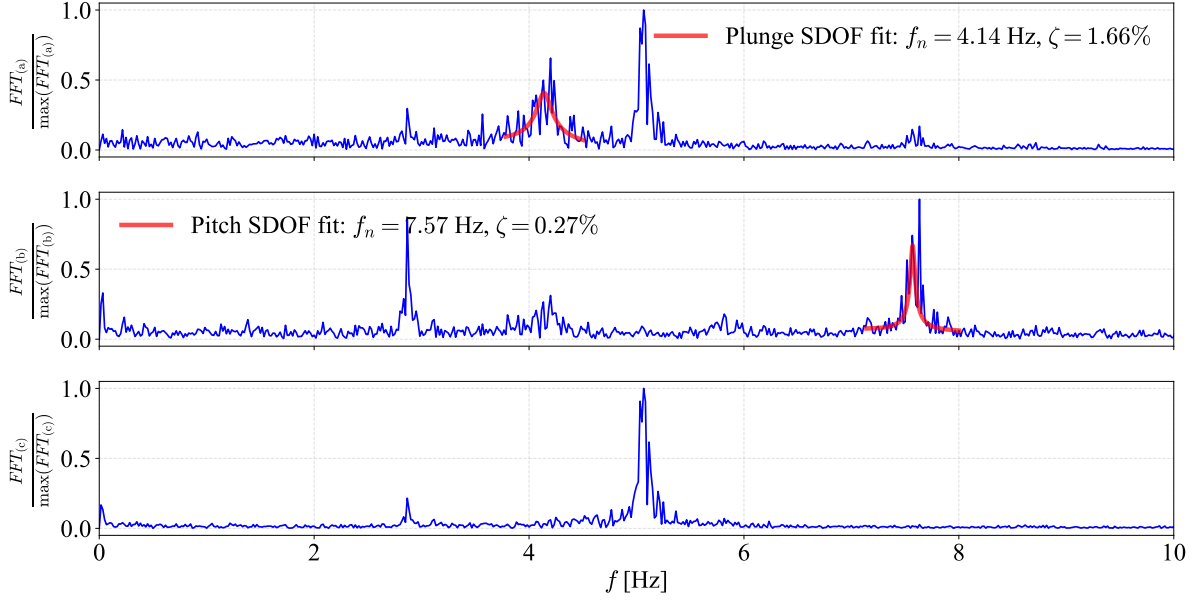


Figure 4.14: Normalized FFT spectra of the dynamic response for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 8.1$ m/s.

In post-critical conditions, shown in Figure 4.15, the interpretation of the spectra becomes less straightforward than in the wind-off and sub-critical cases. The response is no longer stationary, since the oscillation amplitude grows with time. As a result, the FFT peaks are broader and the modal identification becomes less precise.

The pitch and plunge frequencies remain close to the values identified at lower freestream velocities. No abrupt frequency shift is observed when the system enters the post-critical regime. This indicates that the instability is mainly associated with a change in effective damping rather than with a sudden change in modal frequency.

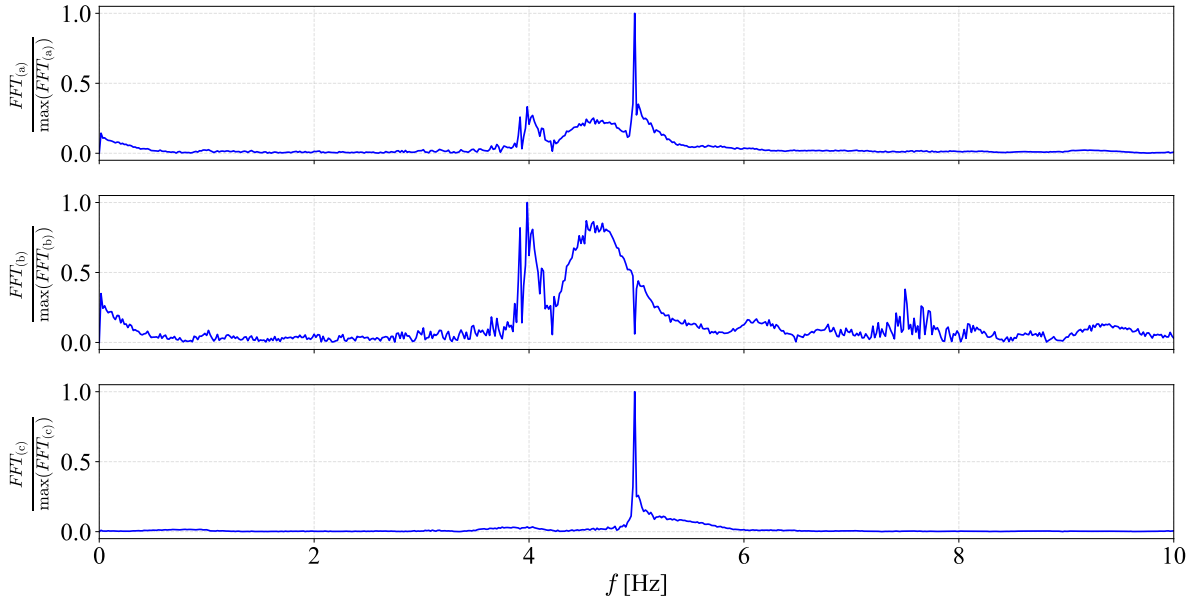


Figure 4.15: Normalized FFT spectra of the dynamic response for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° , at $U_\infty = 28.4$ m/s.

The roll contribution is also clearly visible in the post-critical spectra. In this specific config-

uration, roll appears to be the contributor to the early development of the unstable response. This roll component is interpreted as an undesired experimental contribution, likely caused by small spanwise asymmetries, imperfect mechanical alignment, or aerodynamic interaction with the exposed lateral moving parts of the setup. It may therefore anticipate or contaminate the measured onset of instability.

Nevertheless, the large-amplitude post-critical motion remains mainly associated with plunge. The porpoising instability is interpreted as a plunge-dominated instability, with secondary roll contamination in this particular test case. Cleaner configurations, discussed later, support this interpretation more directly.

Stability analysis

The evolution of the identified plunge and pitch modal properties with freestream velocity is shown in Figure 4.16. The left plot presents the evolution of the modal frequencies, while the right plot presents the corresponding effective damping ratios obtained from the local spectral peak fitting procedure.

The first important observation is that both the plunge and pitch frequencies remain almost constant over the tested velocity range. The plunge frequency stays close to 4 Hz, while the pitch frequency remains close to 7.6 Hz. Looking at the damping evolution, the pitch mode remains positively damped over the analysed velocity range. Its effective damping ratio increases gradually with freestream velocity, which indicates that the pitch response does not appear to be the destabilising component for this configuration.

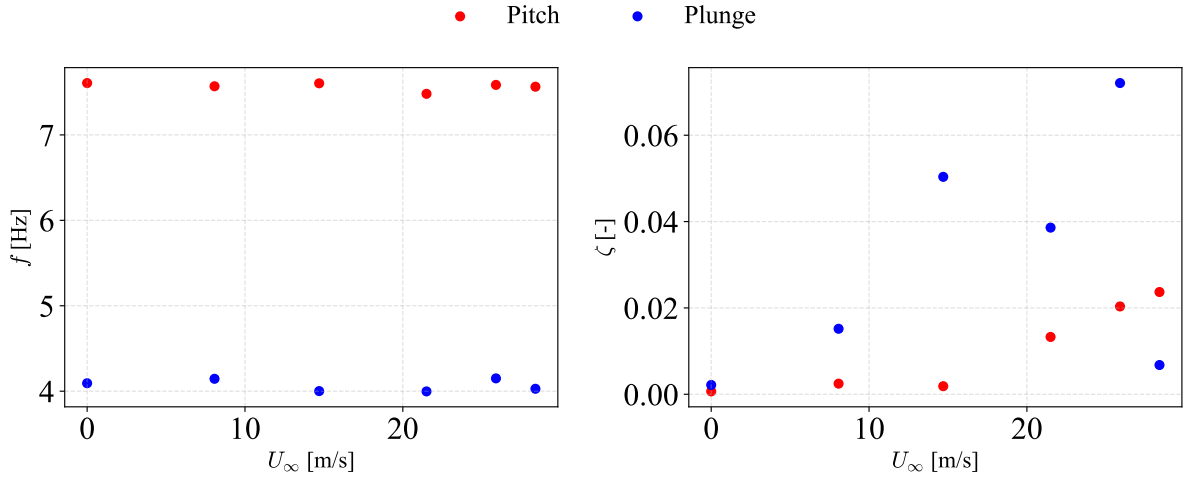


Figure 4.16: Evolution of the identified modal frequencies and effective damping ratios as a function of freestream velocity for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° .

The plunge mode shows a different behaviour. In the sub-critical regime, its effective damping ratio increases significantly with velocity, reaching values of the order of a few percent. This indicates that, at moderate velocities, the aerodynamic contribution still has a stabilising effect on the plunge response. However, close to the onset of instability, a marked drop of the identified plunge damping is observed.

Bifurcation analysis

The bifurcation diagram shown in Figures 4.17 and is used as qualitative indicator of the nonlinear behaviour of the aeroelastic system. For each freestream velocity, the measured response is represented by the averaged minimum and maximum vertical displacements, so that the plotted curves correspond to an amplitude envelope of the measured motion rather than to equilibrium branches in the strict dynamical-systems sense, as is usually done in nonlinear vibration theory when plotting bifurcation diagrams [26].

At low freestream velocity, the minimum and maximum displacement curves remain close to each other, indicating a bounded small-amplitude response around the static equilibrium position. It should be noted that, under wind-off conditions, i.e. when the freestream velocity is zero, the amplitudes are non-zero due to the manual excitation applied to the system, as discussed earlier. When the freestream velocity is increased, a sudden transition towards a large-amplitude oscillatory response is observed. The separate bifurcation runs also show that the response does not necessarily follow the same path as the basic velocity sweep. This path dependence suggests a hysteresis-like nonlinear behaviour, with possible coexistence of small- and large-amplitude responses over a finite range of freestream velocities [26].

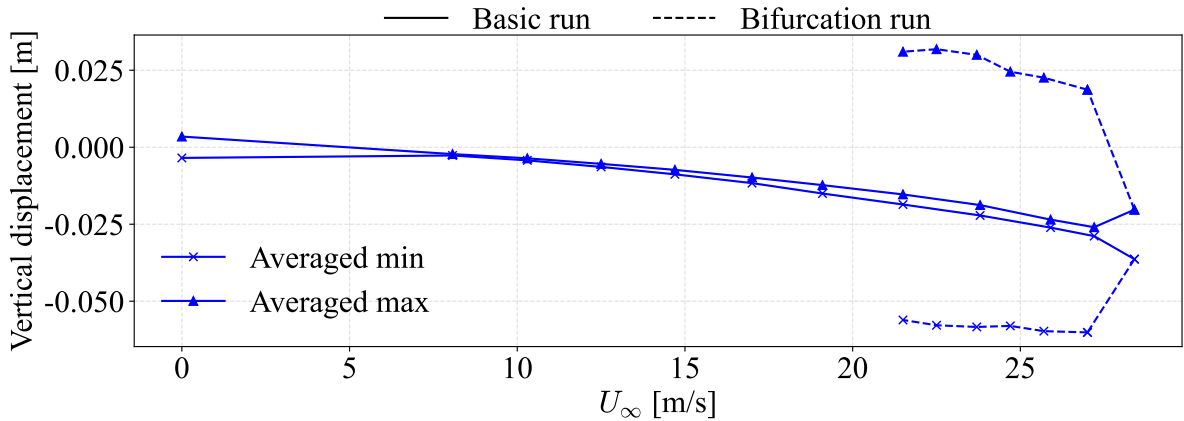


Figure 4.17: Bifurcation diagram of the measured vertical displacement response as a function of freestream velocity for an initial ground clearance of 52 mm and an initial wing angle of attack of 7.5° .

Effect of the initial wing incidence

The effect of the initial wing incidence α_0 is now investigated. For this purpose, several dynamic tests performed with the same initial ground clearance, $H = 52$ mm, but with different values of α_0 , are compared. The objective is to evaluate how the initial aerodynamic loading of the wing influences the onset of porpoising.

The first comparison is based on the porpoising velocity U_{porp} , defined here as the first freestream velocity at which a clear post-critical response is observed during the dynamic test campaign. This velocity therefore corresponds to an experimentally identified instability threshold for the considered configuration.

Figure 4.18 shows the evolution of U_{porp} as a function of the initial wing incidence α_0 , for the fixed initial ground clearance $H = 52$ mm. A clear decrease of the porpoising velocity is observed when α_0 increases. This means that higher initial wing incidences lead to the onset

of instability at lower freestream velocities.

This trend is consistent with the static aerodynamic results discussed previously. Increasing α_0 increases the aerodynamic loading of the wing and brings the flow closer to non-linear or separated conditions, especially in ground effect. As a consequence, the aeroelastic equilibrium becomes more sensitive to changes in ride height and pitch angle. This can promote the destabilising coupling between aerodynamic forces and structural motion, leading to porpoising at lower velocities.

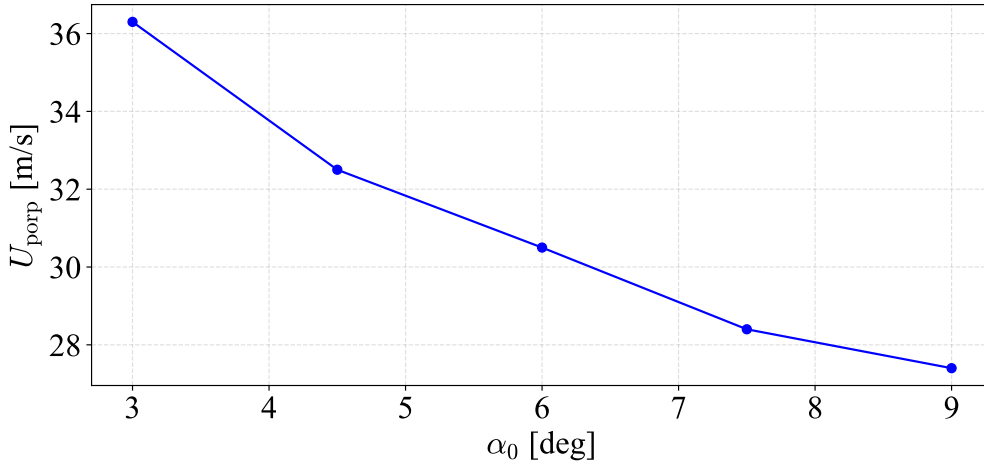


Figure 4.18: Evolution of the experimentally identified porpoising velocity U_{porp} as a function of the initial wing incidence α_0 , for a fixed initial ground clearance $H = 52$ mm.

Another relevant result is the influence of the initial wing incidence α_0 on the static aeroelastic deflection of the system. Figure 4.19 shows the evolution of the equilibrium plunge displacement y and pitch angle α as functions of the freestream velocity, for several values of α_0 and for the same initial ground clearance configuration, $H = 52$ mm.

The left plot shows that the magnitude of the static plunge deflection increases with freestream velocity for all tested incidences. Moreover, for a given freestream velocity, a larger initial incidence leads to a larger static deflection. Increasing α_0 increases the aerodynamic loading of the wing, so that the system reaches the same level of plunge deflection at a lower freestream velocity than for a smaller initial incidence.

The right plot shows the corresponding static pitch deflection. As for the plunge displacement, the magnitude of the pitch angle increases with freestream velocity and with the initial wing incidence.

Overall, Figure 4.19 confirms that increasing the initial wing incidence leads to a stronger static aeroelastic deformation. This contributes to the reduction of the porpoising velocity observed previously, since higher incidences bring the system more rapidly toward highly loaded and low-clearance operating conditions.

Finally, the bifurcation diagrams obtained for the different initial wing incidences can be superposed in order to assess the influence of α_0 on the system response, as shown in Figure 4.20. When the initial wing incidence is increased, the large-amplitude branch appears at lower freestream velocities, which is consistent with the previous observations.

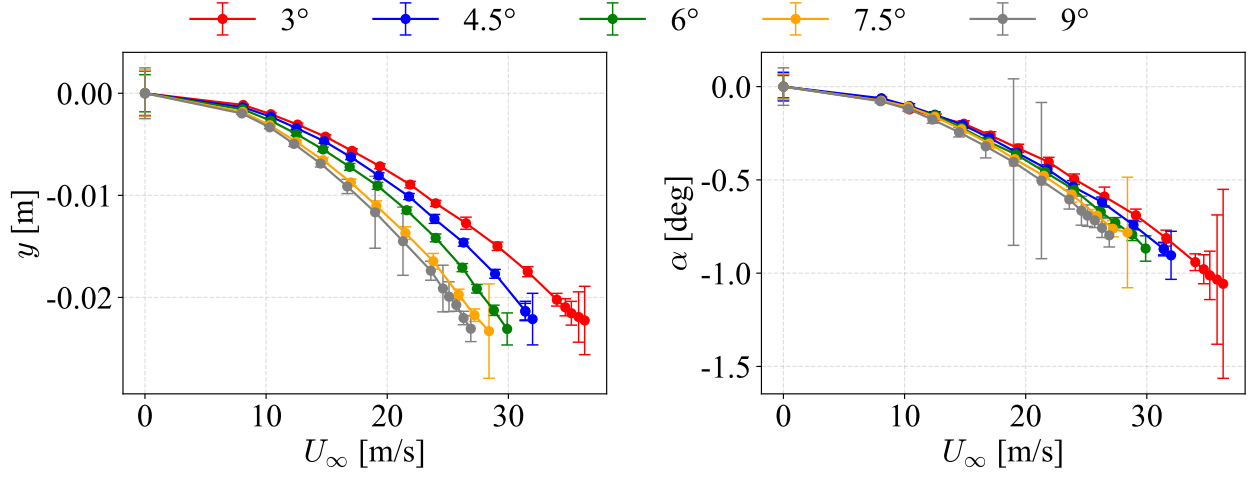


Figure 4.19: Evolution of the static aeroelastic equilibrium as a function of freestream velocity for different initial wing incidences α_0 , at a fixed initial ground clearance $H = 52$ mm. The left plot shows the equilibrium plunge displacement y , while the right plot shows the equilibrium pitch angle α . Error bars correspond to the standard deviation of the measured signals.

Another interesting result from this figure concerns the curve corresponding to $\alpha_0 = 9^\circ$. Large amplitudes are observed between $U_\infty = 17$ and $U_\infty = 25$ m/s. This corresponds to an experimentally observed unstable roll response, which eventually stabilises around $U_\infty = 25$ m/s, just before the onset of the unstable plunge response of the system.

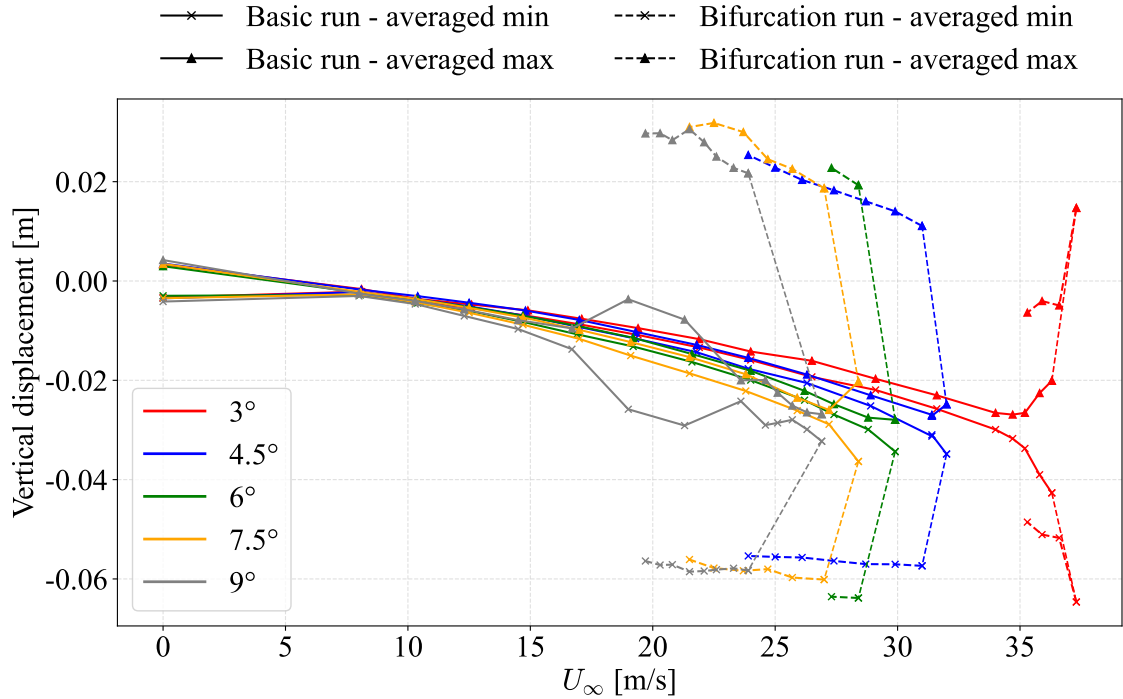


Figure 4.20: Bifurcation diagram of the vertical displacement response as a function of freestream velocity for several initial wing incidences α_0 , at a fixed initial ground clearance $H = 52$ mm. Solid lines represent the basic velocity sweep, while dashed lines represent the bifurcation runs.

Effect of the initial ground clearance

After analysing the influence of the initial wing incidence α_0 , the effect of the initial ground clearance H on the porpoising behaviour is now investigated. In this section, the discussion is limited to the evolution of the porpoising velocity U_{porp} , defined as the first freestream velocity at which a clear post-critical response is observed.

Figure 4.21 gathers the experimentally identified porpoising velocities for the three tested initial ground clearances H , combined with the different initial wing incidences α_0 . The main trend is that increasing the initial ground clearance leads to higher porpoising velocities. This behaviour is physically consistent with the previous observations. Starting from a larger ground clearance, the wing needs a larger aerodynamic load, and therefore a higher freestream velocity, to reach the low-clearance operating region where the aeroelastic instability develops.

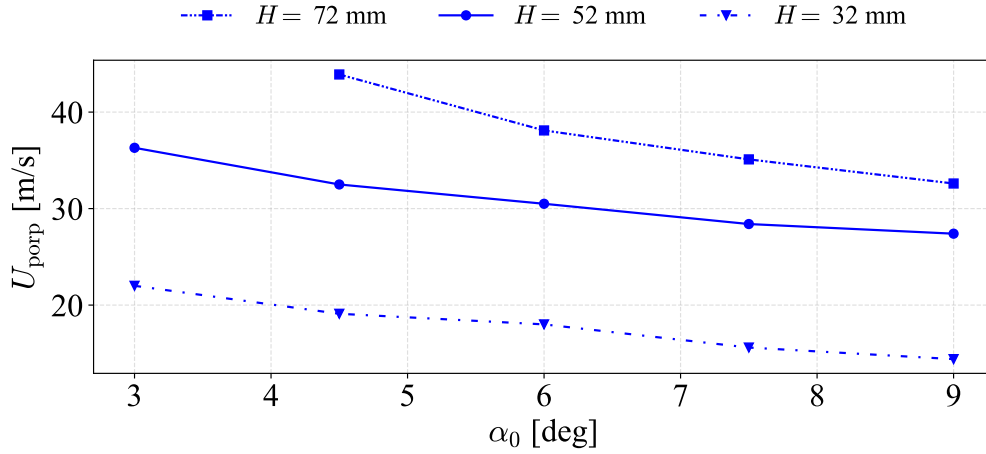


Figure 4.21: Evolution of the experimentally identified porpoising velocity U_{porp} as a function of the initial wing incidence α_0 for the different tested initial ground clearances H .

A complementary way to analyse the onset of porpoising is to look at the effective ground clearance reached when the instability appears. Until now, the results have shown that the porpoising velocity depends on both the initial wing incidence α_0 and the initial ground clearance H . However, this does not directly indicate whether porpoising occurs at a unique critical ground clearance, or whether the instability threshold also depends on the freestream velocity and on the complete aeroelastic operating point.

For this reason, Figure 4.22 shows the ground clearance at the onset of porpoising, denoted here as H_{porp} , for the different tested initial ground clearances H and initial wing incidences α_0 . This value corresponds to the estimated clearance reached by the wing when the first post-critical response is observed.

The figure shows that porpoising does not occur at a unique value of H_{porp} for all configurations. Instead, the ground clearance at instability onset remains grouped according to the initial ground-clearance configuration. The configurations starting from $H = 72$ mm reach porpoising at larger final clearances than those starting from $H = 52$ mm, while the configurations starting from $H = 32$ mm reach porpoising at even smaller final clearances.

This result indicates that porpoising cannot be explained only by reaching a single critical ground clearance. If this were the case, all configurations would collapse toward the same value of H_{porp} , independently of H and α_0 .

The influence of the initial wing incidence α_0 on H_{porp} appears weaker than the influence of the initial ground clearance. For a given H , the final ground clearance at porpoising remains relatively close over the tested incidence range. This suggests that increasing α_0 mainly reduces the freestream velocity required to reach the unstable operating condition, rather than imposing a strongly different final ground clearance.

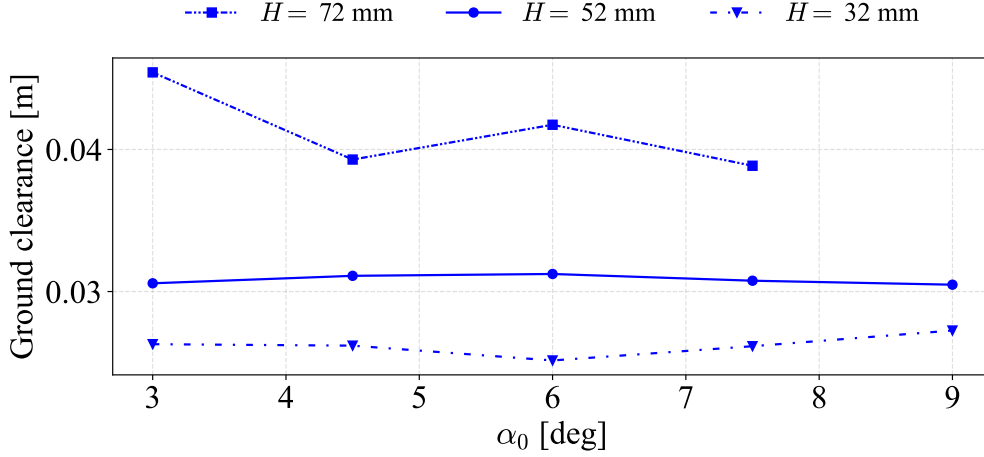


Figure 4.22: Ground clearance at the onset of porpoising H_{porp} as a function of the initial wing incidence α_0 , for the three tested initial ground clearances H .

These observations are still valid with the idea that porpoising is linked to the low-clearance aerodynamic behaviour of the wing, but they also show that ground clearance alone is not sufficient to define the instability threshold. Possible explanations include Reynolds-number effects on the static aerodynamic properties, changes in the aerodynamic gradients with velocity, and unsteady aerodynamic effects associated with the oscillatory motion of the wing. The present measurements do not allow these mechanisms to be separated quantitatively, but they show that the onset of porpoising must be interpreted as a coupled aeroelastic instability rather than as the direct consequence of a fixed critical ride height.

In addition to the ground clearance reached at instability onset, it is also useful to examine the effective wing incidence when porpoising appears. This quantity is denoted here as α_{porp} and corresponds to the actual wing incidence at the first post-critical response. It is obtained by combining the initial wing incidence α_0 with the static pitch deflection of the system at the porpoising velocity:

$$\alpha_{\text{porp}} = \alpha_0 + \alpha_{\text{eq}}(U_{\text{porp}}). \quad (4.2.11)$$

Figure 4.23 shows the evolution of α_{porp} as a function of the initial wing incidence α_0 , for the three tested initial ground clearances H . A linear trend is observed for all configurations. As expected, increasing the initial incidence leads to an increase of the effective incidence at the onset of porpoising.

For a given value of α_0 , the final incidence at porpoising is not exactly the same for all initial ground clearances. The configurations starting from the smallest ground clearance, $H = 32$ mm, generally reach porpoising with a slightly larger effective incidence than the configurations starting from $H = 52$ mm and $H = 72$ mm.

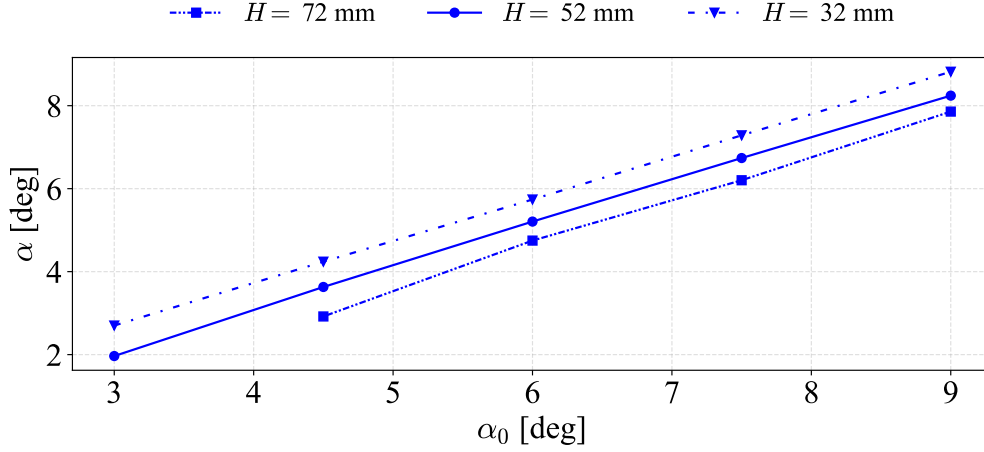


Figure 4.23: Effective wing incidence at the onset of porpoising α_{porp} as a function of the initial wing incidence α_0 , for the three tested initial ground clearances H .

Plunge-dominated porpoising response

As discussed previously, the porpoising instability is mainly associated with an unstable plunge motion. However, in the results presented earlier, this behaviour was partly masked by secondary roll contributions. Those cases were useful to illustrate the different dynamic features encountered during the test campaign, but they did not provide the clearest visual evidence of a plunge-dominated instability.

The purpose of this section is therefore to present a cleaner porpoising case, for which the dominant plunge nature of the instability is more clearly visible.

The last sub-critical response is shown in Figures 4.24 and 4.25. The laser-to-laser displacement plots in Figure 4.24 show that both signal comparisons are mainly in phase. As discussed earlier, an in-phase motion between the rear-right and front-right sensors indicates that the front and rear parts of the right bar move together, which is characteristic of plunge. Similarly, the in-phase motion between the rear-right and rear-left sensors indicates that the rear part of the assembly moves almost uniformly in the spanwise direction, with no dominant roll contribution.

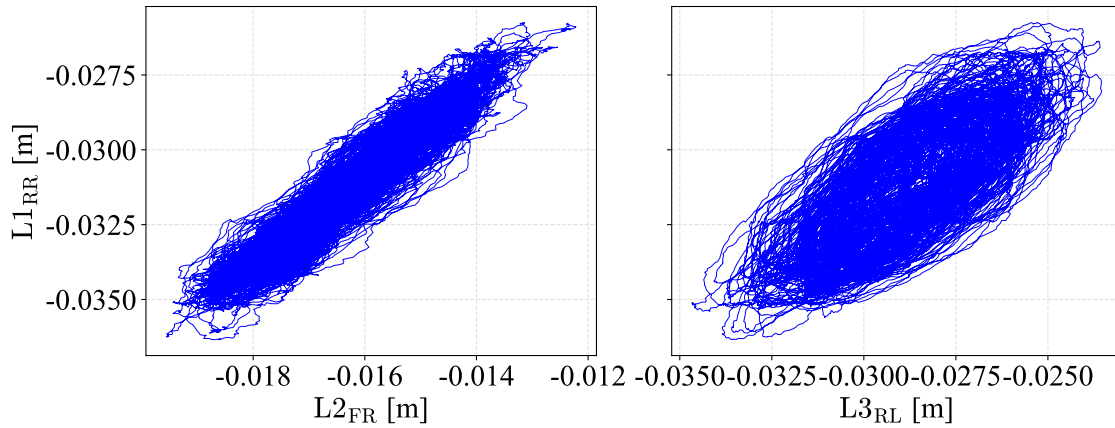


Figure 4.24: Laser-to-laser displacement plots for a zero-incidence ride height of 52 mm and an initial wing angle of attack of 6° , at $U_\infty = 29.9$ m/s.

Although plunge is clearly dominant, the oscillation amplitude remains bounded and relatively small. The response therefore remains dynamically stable at this velocity. Furthermore, when looking at the FFT spectra of the three reconstructed signals (as developed earlier) shown in Figure 4.25, the dominant peak is located slightly below 4 Hz, corresponding to the plunge mode. Small pitch and roll contributions are still visible, but their amplitudes remain much lower than the plunge contribution.

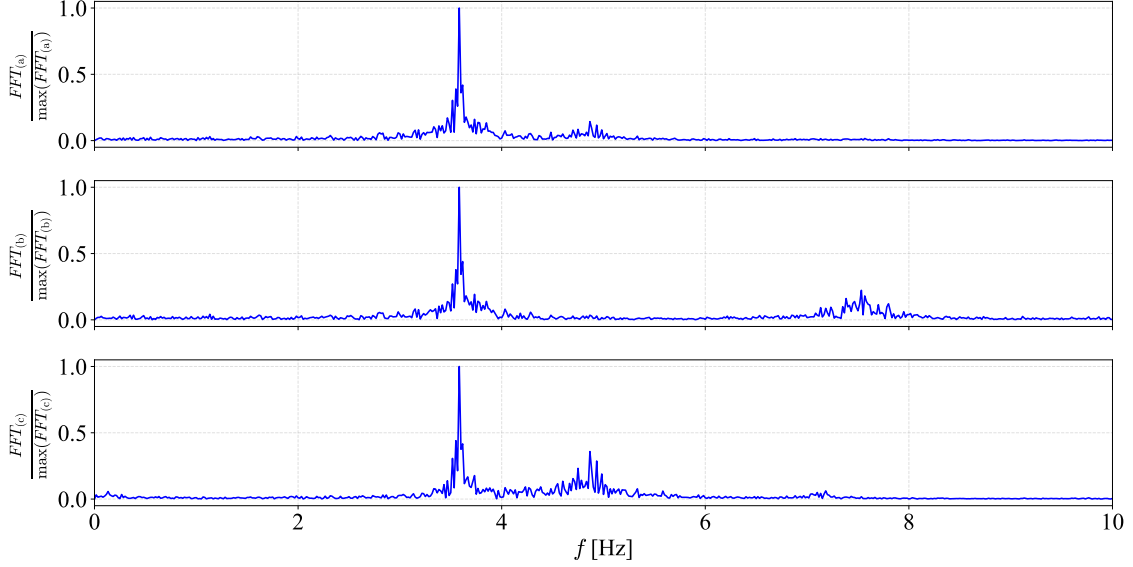


Figure 4.25: Normalized FFT spectra of the reconstructed dynamic response components for a zero-incidence ride height of 52 mm and an initial wing angle of attack of 6° , at $U_\infty = 29.9$ m/s.

The following tested velocity corresponds to the post-critical response. The laser-to-laser displacement plots are shown in Figure 4.26. Compared with the previous sub-critical case, the measured amplitudes become much larger, reaching values of the order of several centimetres. This clear amplitude increase indicates that the system has entered an unstable dynamic regime.

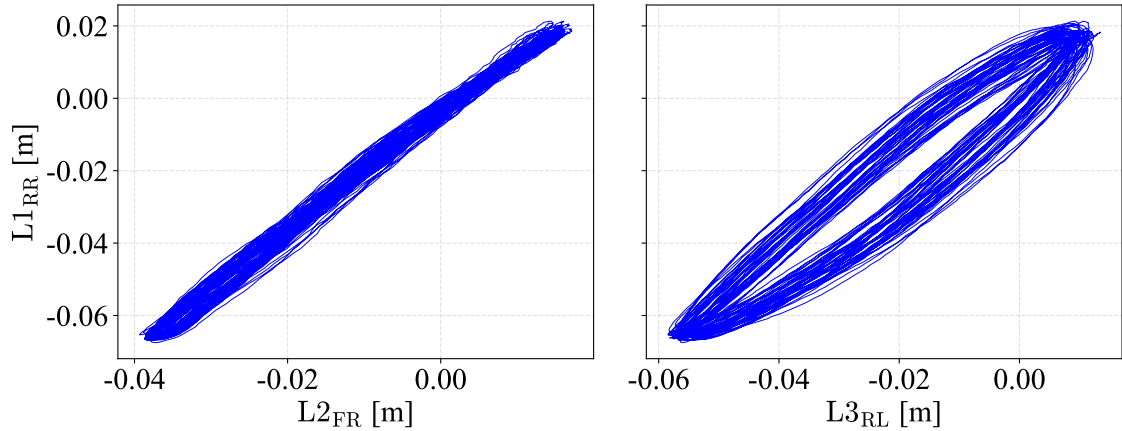


Figure 4.26: Laser-to-laser displacement plots for a zero-incidence ride height of 52 mm and an initial wing angle of attack of 6° , at $U_\infty = 30.55$ m/s.

Both laser-to-laser comparisons remain mainly in phase during the large-amplitude response. This means that the instability is dominated by a common vertical motion of the measured

points, and therefore by plunge.

This cleaner case therefore supports the interpretation made previously, such that the porpoising instability observed in the present setup is primarily a plunge-dominated instability. Roll may appear in some configurations because of experimental imperfections or secondary coupling effects, but it is not the main mechanism of the clean porpoising response.

4.2.2 Plunge-only dynamic setup

In this section, the results from the second dynamic test campaign are presented. These tests were performed with the plunge-only dynamic setup, following the conclusion of the previous section that the porpoising response can be reasonably reduced to a single-degree-of-freedom plunge motion. Therefore, only the vertical response is analysed here.

The analysis first focuses on a reference configuration with an initial ground clearance $H = 67$ mm. The influence of H is then studied by comparing the results obtained for all tested ground clearances. As in the previous section, the time signals are used to compute the static deflection, while spectral analysis is used to identify the modal frequency and effective damping ratio as functions of freestream velocity. Finally, the effect of the initial ground clearance on the post-critical clearance H_{porp} is briefly discussed. In this test campaign, H is the only varied geometrical parameter, while the initial wing incidence is fixed at $\alpha_0 = 4.5^\circ$.

Static aeroelastic deflection

As shown in Figure 4.27, for the configuration considered here, corresponding to an initial ground clearance $H_0 = 67$ mm and an initial wing angle of attack $\alpha_0 = 4.5^\circ$, the static displacement progressively increases in magnitude with freestream velocity. At $U_\infty \approx 10$ m/s, the equilibrium position is approximately $y_{\text{eq}} = -4$ mm. According to the sign convention used in this work, this negative value indicates a downward displacement of the wing, and therefore a reduction of the current ground clearance.

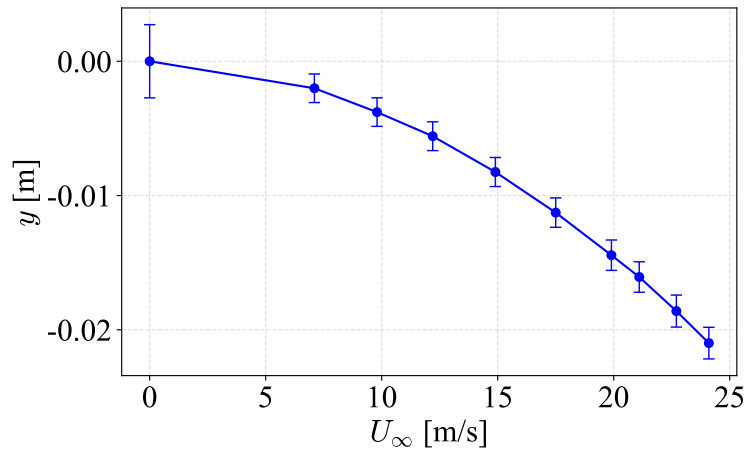


Figure 4.27: Evolution of the static aeroelastic plunge deflection as a function of freestream velocity for an initial ground clearance $H_0 = 67$ mm and an initial wing angle of attack $\alpha_0 = 4.5^\circ$.

Stability analysis

The stability of the plunge-only system can be assessed by tracking the evolution of the identified plunge-mode frequency and effective damping ratio as a function of the freestream velocity. Compared with the previous plunge-pitch setup, the main advantage of the plunge-only configuration is that the measured response is no longer significantly contaminated by pitch or roll contributions. The frequency-domain identification of the plunge mode is therefore cleaner and more reliable.

The evolution of the plunge-mode frequency and effective damping ratio is shown in Figure 4.28. The damping ratio first increases with freestream velocity, indicating a stabilising aerodynamic contribution in this velocity range. However, near $U_\infty \approx 20$ m/s, it starts to decrease, showing that the system is moving towards a less stable condition. Although the test was stopped before negative damping was reached, similar configurations showed that this trend can lead to porpoising instability.

The identified plunge frequency follows a similar two-phase behaviour. It first increases slightly up to approximately $U_\infty = 10$ m/s, indicating an increase in effective stiffness. At higher velocities, it decreases significantly, showing a reduction of the effective stiffness of the aeroelastic system.

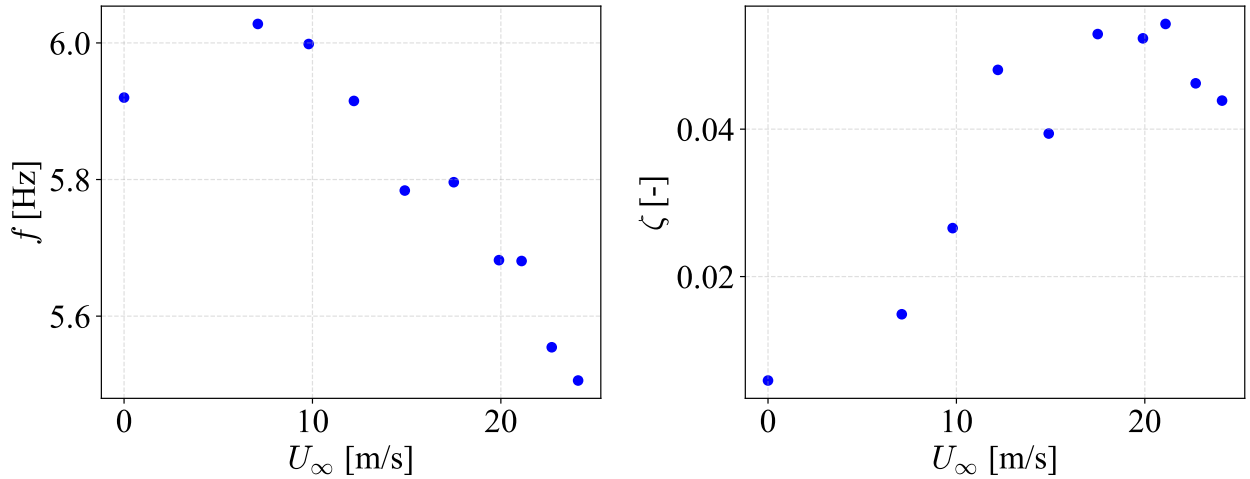


Figure 4.28: Evolution of the identified plunge-mode frequency and effective damping ratio as a function of freestream velocity for an initial ground clearance $H_0 = 67$ mm and an initial wing angle of attack $\alpha_0 = 4.5^\circ$.

A first interpretation of the observed stiffness variation can be attempted using a quasi-steady aerodynamic argument. In such an approach, a vertical displacement of the wing modifies the aerodynamic load through the dependence of the lift coefficient on the ground clearance. This dependency acts as an aerodynamic stiffness term. More specifically, if the aerodynamic force varies with the vertical position y , the linearised force contribution around the equilibrium position can be written as proportional to $\partial C_L / \partial y$. The sign of this gradient therefore determines whether the aerodynamic contribution increases or decreases the effective stiffness of the plunge mode.

For the configuration shown in Figure 4.28, the frequency initially increases up to approximately $U_\infty = 10$ m/s. At this velocity, the measured static deflection is approximately

$y_{\text{eq}} = -4$ mm. The corresponding current normalized ground clearance is therefore

$$\frac{H_{\text{current}}}{c} = \frac{H_0 + y_{\text{eq}}}{c} = \frac{67 - 4}{450} \approx 0.14. \quad (4.2.12)$$

The static aerodynamic measurements presented in Figure 4.1b can then be compared with this operating range. For $\alpha_0 = 4.5^\circ$, no direct static curve is available, so the neighbouring curves at $\alpha_0 = 3^\circ$ and $\alpha_0 = 5^\circ$ must be used as an indication. Around $H/c \approx 0.14$, the static data do not show a clear change of slope that would directly explain the frequency increase followed by the frequency decrease observed in the dynamic test. In fact, for lower ground clearances, the sign of the apparent static slope may even suggest the opposite trend.

This mismatch indicates that the quasi-steady interpretation based only on the static $C_L(H)$ curves may be not sufficient to fully explain the dynamic behaviour observed in the plunge-only setup. Several reasons may explain this limitation. First, the static and dynamic setups are not strictly identical, and ground-effect aerodynamics is highly sensitive to small geometrical differences, leakage, blockage, endplate effects, and the effective sealing of the region beneath the wing. Second, the aerodynamic response may include significant unsteady effects that are not captured by static measurements.

This second point is particularly relevant here. For the present dynamic response, the reduced frequency can be estimated as

$$k = \frac{\omega c}{U_\infty}. \quad (4.2.13)$$

Using the identified plunge frequency and the chord of the plunge-only setup, values of the order of

$$k \approx 0.7 \quad (4.2.14)$$

are obtained in the velocity range of interest. Such values are not small from an unsteady-aerodynamics point of view. Although quasi-steady aerodynamics is commonly assumed to be valid for low reduced frequencies (based on half-chord length), typically $k < 0.05$, Brunton et al. [27] showed that, for a sinusoidally plunging flat plate at low Reynolds number, the quasi-steady assumption can remain accurate up to approximately $k = 0.25$. Motions involving larger reduced frequencies are therefore generally expected to require an unsteady aerodynamic formulation [28]. This could supports that the dynamic behaviour observed here cannot be fully interpreted using only the static aerodynamic database.

Effect of the initial ground clearance

The effect of the initial ground clearance H is now investigated. The comparison is first performed on the static aeroelastic deflection, shown in Figure 4.29. Overall, all configurations exhibit the same qualitative behaviour as the one discussed previously, such that when the freestream velocity increases, the aerodynamic load increases and the wing is progressively displaced downwards, reducing the current ground clearance.

However, noticeable differences appear between the tested configurations. The largest initial ground clearances, $H = 86$ mm and $H = 109$ mm, lead to smaller static deflections (in magnitude) over the tested velocity range. This is consistent with the fact that, far from the ground, the ground-effect contribution to the aerodynamic load is weaker. Conversely, the lowest initial ground-clearance configuration, $H = 20$ mm, exhibits a much larger deflection (in magnitude) for a given freestream velocity. This suggests that the aerodynamic loading

is significantly stronger when the wing starts very close to the ground. This trend is qualitatively expected from ground-effect aerodynamics, although the comparison with the static aerodynamic measurements of Figure 4.1b is not fully straightforward once again.

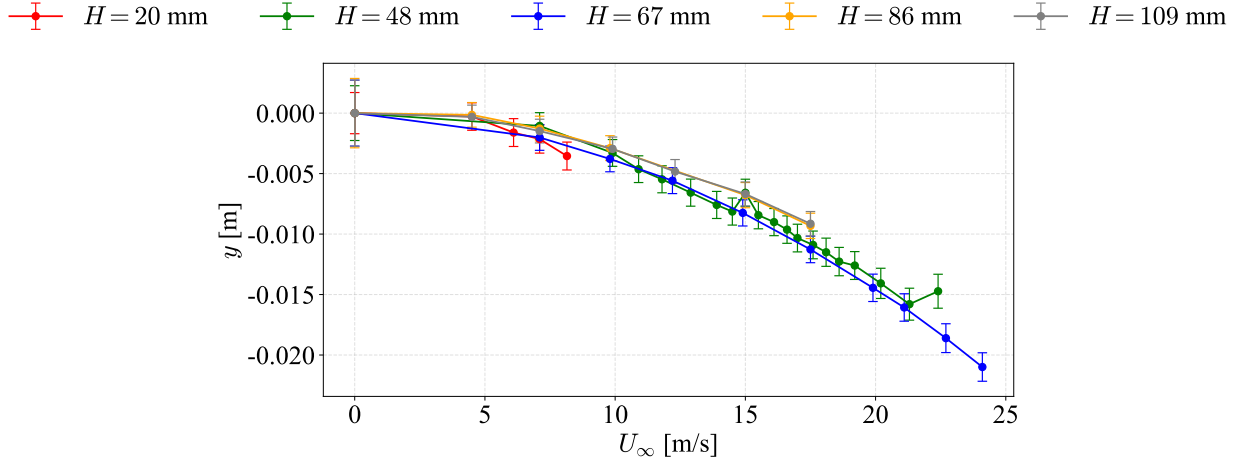


Figure 4.29: Evolution of the static aeroelastic plunge deflection as a function of freestream velocity for different initial ground clearances H .

The effect of the initial ground clearance on the plunge-mode stability is then analysed through the evolution of the identified modal frequency and effective damping ratio, shown in Figure 4.30. The frequency plot shows that the same general trend as previously observed is recovered for the different ground-clearance configurations. The plunge frequency first increases slightly with freestream velocity, before decreasing at higher velocity. This behaviour indicates that the effective stiffness of the aeroelastic system is modified by the aerodynamic contribution.

The influence of the initial ground clearance on the frequency evolution remains moderate for most configurations. The high-clearance cases, $H = 86$ mm and $H = 109$ mm, give very similar results. The intermediate configurations, $H = 48$ mm and $H = 67$ mm, also remain relatively close to each other. The lowest configuration, $H = 20$ mm, behaves more distinctly, which is consistent with the stronger static deflection observed in Figure 4.29. This indicates that the aerodynamic stiffness contribution becomes more significant when the wing starts very close to the ground.

The damping-ratio evolution provides clearer information about the stability limit. For the low and intermediate ground-clearance configurations, the effective damping ratio first increases with freestream velocity and then drops rapidly when the system approaches the instability. Increasing the initial ground clearance shifts this drop towards higher freestream velocities, meaning that the onset of porpoising is delayed. The largest ground-clearance configurations do not reach the decreasing branch within the available velocity range. Therefore, for these cases, only the initial stabilising trend can be observed from the present data.

Finally, the damping ratio is represented as a function of the current ground clearance in Figure 4.31. The current ground clearance is obtained from the initial ground clearance and the measured static deflection. This representation is useful because porpoising is expected to be strongly related to the proximity of the wing to the ground.

If the instability were governed only by a critical current ground clearance, all damping curves

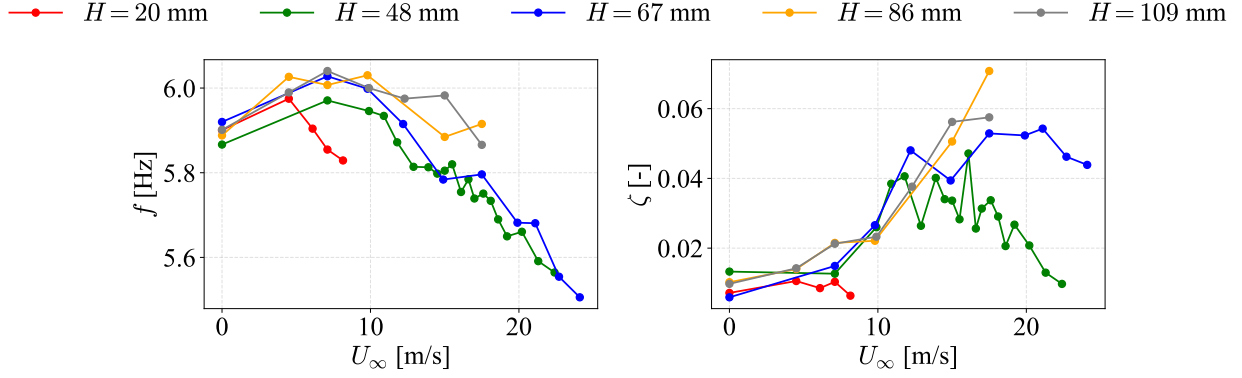


Figure 4.30: Evolution of the identified plunge-mode frequency and effective damping ratio as a function of freestream velocity for different initial ground clearances H .

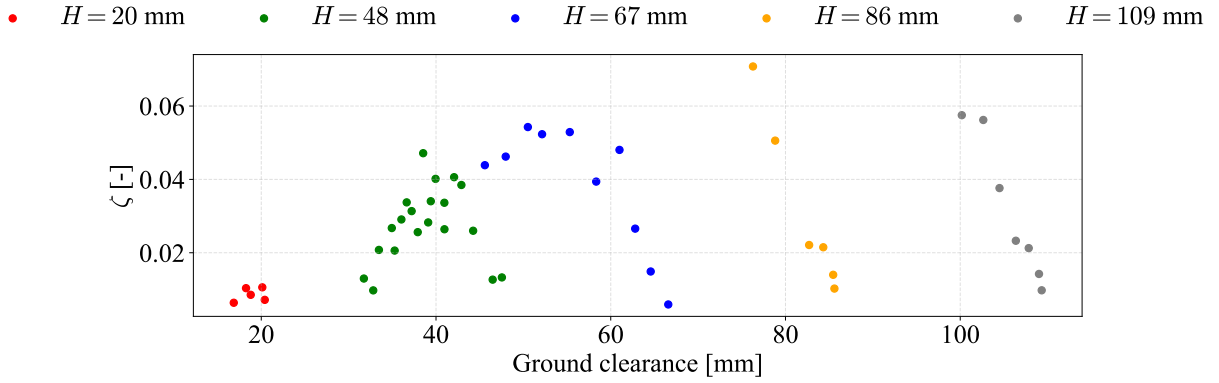


Figure 4.31: Evolution of the identified effective damping ratio as a function of the current ground clearance for different initial ground clearances H .

would be expected to approach zero at approximately the same value of ground clearance. This is not observed here. Instead, the damping evolution depends on the initial configuration and on the corresponding freestream velocity. This indicates that the onset of porpoising cannot be described by a purely static criterion based only on the current ground clearance.

4.2.3 Influence of pitch freedom on static plunge deflection

This section compares the static plunge deflection measured with the plunge–pitch dynamic setup and with the plunge-only dynamic setup. Only the plunge component is considered in this comparison. Indeed, in the plunge-only setup, the wing is constrained to vertical translation and its incidence remains fixed at the initial value α_0 . Therefore, only plunge–pitch configurations with the same initial wing incidence, $\alpha_0 = 4.5^\circ$, are retained for comparison.

Figure 4.32 shows the static restoring force associated with the plunge displacement. This force is obtained by multiplying the measured static plunge deflection by the structural plunge stiffness. Under static equilibrium, this restoring force balances the aerodynamic vertical load, up to the sign convention. It can therefore be interpreted as an indirect estimate of the aerodynamic force acting on the wing as a function of freestream velocity.

For both setups, the influence of the initial ground clearance remains relatively limited in the tested range, as the different configurations collapse approximately onto a single trend.

The comparison also shows that allowing pitch motion in the two-degree-of-freedom setup has only a moderate influence on the static aerodynamic loading. Although the wing can rotate slightly in pitch, which tends to reduce the effective incidence, the resulting change in static plunge deflection remains limited.

The isolated red point away from the main trend is treated as an outlier and is not used in the physical interpretation of the comparison.

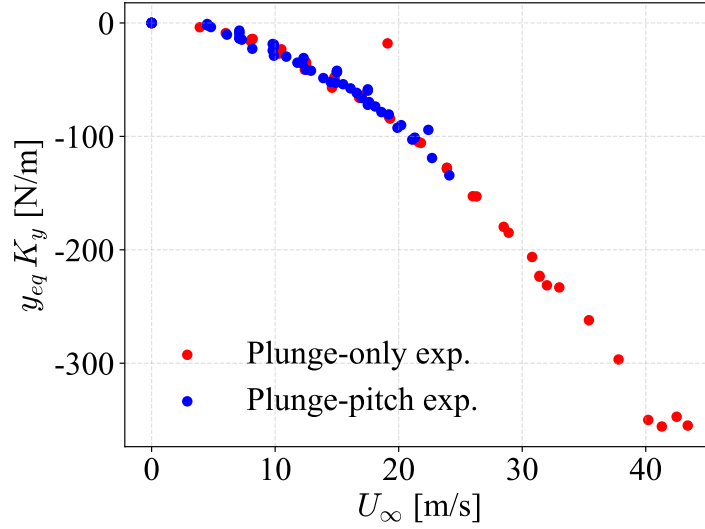


Figure 4.32: Comparison of the static plunge deflection measured with the plunge-pitch and plunge-only dynamic setups for an initial wing incidence $\alpha_0 = 4.5^\circ$. The plotted data include the different tested initial ground clearances.

4.3 Identification of Plunge Flutter Derivatives

4.3.1 Identification from dynamic measurements

As introduced in Chapter 2, the unsteady aerodynamic formulation used in the reduced-order model is based on aerodynamic coefficients referred to as *flutter derivatives*. The objective of this section is to identify these coefficients from the dynamic experimental results.

Only the plunge-related coefficients are considered here, for two reasons. First, the previous experimental analysis showed that the porpoising response can be reasonably represented as a plunge-dominated instability. Second, the plunge-pitch measurements are significantly affected by undesired motion components, especially roll contributions, which makes the experimental identification of the coupled pitch-plunge derivatives unreliable. The identification is therefore restricted to the plunge-only setup and to the direct plunge derivatives H_1^* and H_4^* .

The identified coefficients are computed from the measured evolution of the plunge-mode

frequency and damping ratio according to

$$\begin{cases} H_1^*(\omega) = \frac{4m (\zeta_y \omega_y - \zeta'_y \omega'_y)}{\rho c^2 \omega}, \\ H_4^*(\omega) = \frac{2m (\omega_y^2 - \omega'^2_y)}{\rho c^2 \omega^2}. \end{cases}$$

In these expressions, the unprimed quantities correspond to the reference structural properties, while the primed quantities correspond to the values identified under aerodynamic loading.

The coefficients are identified separately for each tested initial ground clearance H . Figures 4.33 and 4.34 respectively show the identified evolution of H_1^* and H_4^* as functions of the reduced velocity

$$U_{\text{red}} = \frac{U_\infty}{\omega H}.$$

Polynomial fits are added only to make the trends easier to visualize. They are not intended to represent a physical law at this stage.

Figure 4.33 shows the damping-related coefficient H_1^* . Different behaviours are observed depending on the initial ground clearance. For the lowest tested clearance, $H = 20$ mm, H_1^* increases continuously with the reduced velocity. For the intermediate clearances, $H = 35$, 48, and 67 mm, a similar curved trend is observed, H_1^* is initially negative and then increases towards less negative, or even positive, values. This transition is directly related to the reduction of effective damping and therefore to the approach of the porpoising instability.

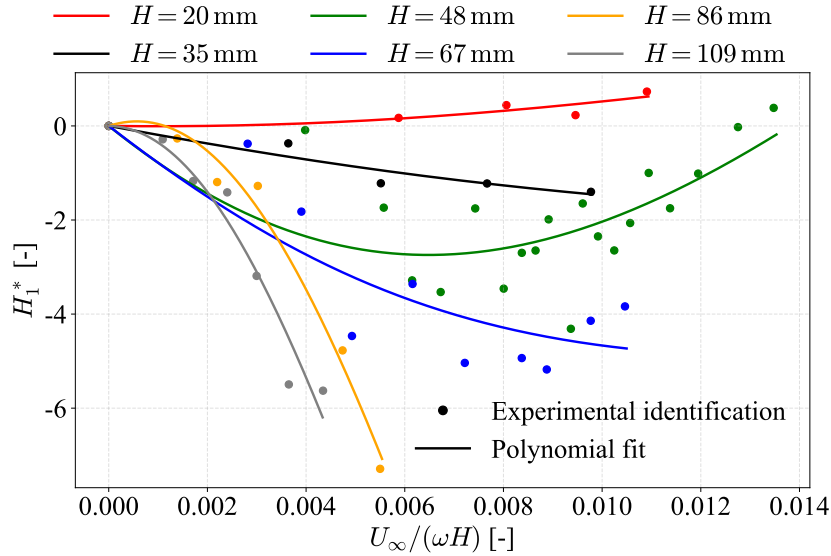


Figure 4.33: Identified evolution of the plunge aerodynamic damping derivative H_1^* as a function of the reduced velocity $U_{\text{red}} = U_\infty/(\omega H)$ for the different tested initial ground clearances H .

For the largest clearances, $H = 86$ and 109 mm, a different behaviour is observed. The coefficient remains negative and tends to decrease with the reduced velocity, indicating an

increasing stabilizing aerodynamic damping contribution. This is consistent with the experimental stability results presented previously, where the high-clearance configurations did not reach the damping drop associated with porpoising within the tested velocity range.

Concerning the stiffness-related aerodynamic coefficient H_4^* , Figure 4.34 shows that all tested configurations exhibit a similar qualitative trend. For low reduced velocities, H_4^* first decreases, before increasing again at higher reduced velocities.

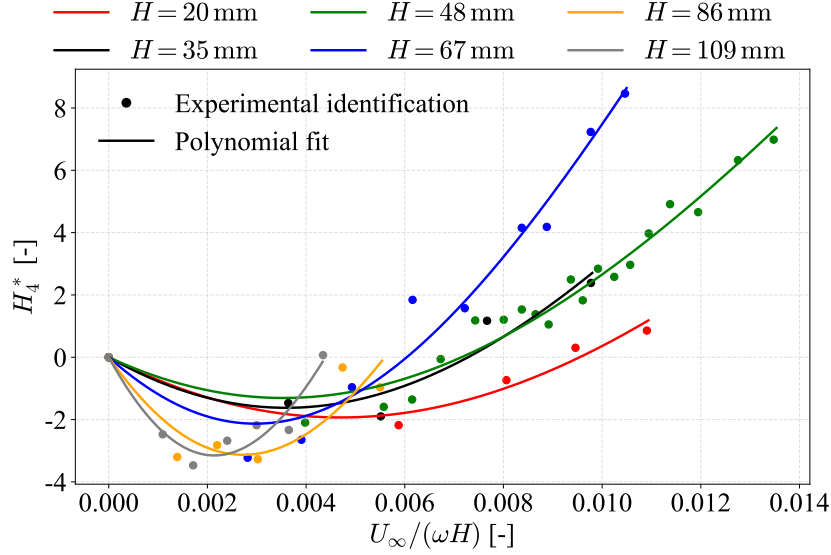


Figure 4.34: Identified evolution of the plunge aerodynamic stiffness derivative H_4^* as a function of the reduced velocity $U_{\text{red}} = U_{\infty}/(\omega H)$ for the different tested initial ground clearances H .

With the sign convention used in this work, H_4^* is directly associated with the aerodynamic stiffness contribution in plunge. Therefore, a change in its value modifies the total effective stiffness of the aeroelastic system. In particular, the increase of H_4^* towards positive values corresponds to a reduction of the total effective plunge stiffness, which is consistent with the decrease of the identified plunge frequency observed experimentally.

4.3.2 Scaled representation of the identified derivatives

The next objective is to obtain a common representation of the identified plunge aerodynamic derivatives H_1^* and H_4^* , independently of the initial ground clearance H . Since H is the main parameter varied between configurations, a scaling based on this quantity is used to collapse the identified data onto a common trend.

Figures 4.35 and 4.36 show the result obtained by scaling the coefficients with c^2/H^2 . This empirical scaling significantly improves the collapse of the data, suggesting that the magnitude of the plunge aerodynamic derivatives is strongly governed by the initial ground clearance. The resulting common trend is then used as a master curve for the reduced-order model.

The main exception is the lowest ground-clearance configuration, $H = 20$ mm, which does not collapse with the other data. This behaviour could be explained by the such small

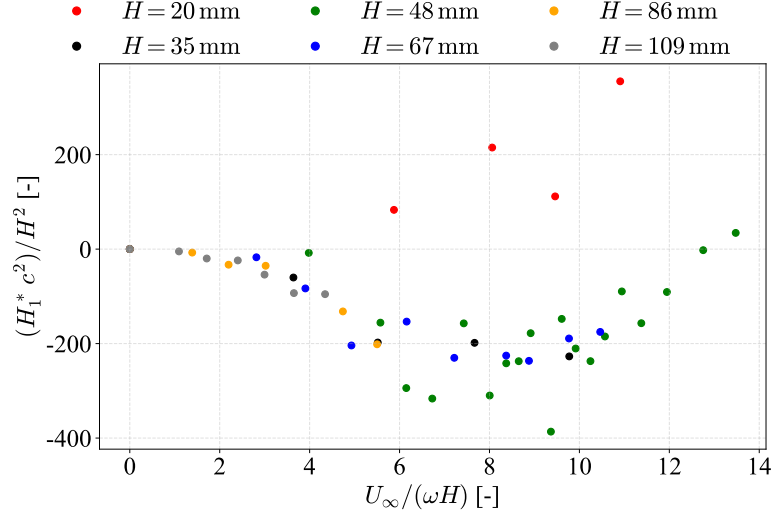


Figure 4.35: Scaled evolution of the plunge aerodynamic damping derivative H_1^* as a function of the reduced velocity $U_{\text{red}} = U_\infty/(\omega H)$.

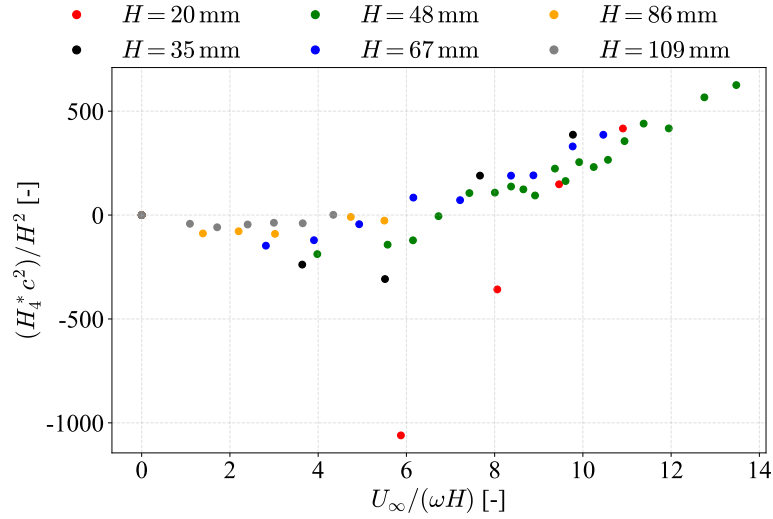


Figure 4.36: Scaled evolution of the plunge aerodynamic stiffness derivative H_4^* as a function of the reduced velocity $U_{\text{red}} = U_\infty/(\omega H)$.

ground clearance, where the flow is expected to be highly sensitive to local effects such as leakage, endplate gaps, boundary-layer interaction and small geometrical uncertainties. This configuration is therefore treated separately and is not considered representative of the common scaled trend.

5 Reduced-Order Model Validation and Parametric Analysis

In this chapter, the model developed in the previous chapter is used. More specifically, as mentioned earlier, the model is first reduced to a single degree of freedom, corresponding to plunge motion, and is then validated using the experimental results from the plunge-only dynamic test campaign.

5.1 Reduction to a Plunge-Only Model

The objective of this section is use the mathematical model, described in Chapter 2, able to predict the dynamic response of the aeroelastic system. As shown in the previous chapter, the porpoising instability observed experimentally is mainly associated with the plunge motion. The contribution of pitch remains secondary for the configurations retained for the modelling. Therefore, the porpoising response can be reasonably represented by a single-degree-of-freedom plunge model.

The two-degree-of-freedom pitch-plunge formulation is therefore reduced to a single plunge degree of freedom. This reduction is performed by neglecting the pitch equation and the coupling terms associated with the pitch coordinate α . Starting from Equation 2.3.11, the reduction of the structural equations is illustrated as follows:

$$\underbrace{\begin{bmatrix} m & S \\ S & I_y \end{bmatrix}}_{\mathbf{M}_{\text{struct}}} \underbrace{\begin{bmatrix} \ddot{y} \\ \ddot{\alpha} \end{bmatrix}} + \underbrace{\begin{bmatrix} C_y & \phi \\ 0 & C_\alpha \end{bmatrix}}_{\mathbf{C}_{\text{struct}}} \underbrace{\begin{bmatrix} \dot{y} \\ \dot{\alpha} \end{bmatrix}} + \underbrace{\begin{bmatrix} K_y & \phi \\ 0 & K_\alpha \end{bmatrix}}_{\mathbf{K}_{\text{struct}}} \underbrace{\begin{bmatrix} y \\ \alpha \end{bmatrix}} = - \underbrace{\begin{bmatrix} F_L \\ M + (x_{AC} - x_f)F_L \end{bmatrix}}_{\mathbf{F}} \quad (5.1.1)$$

The remaining equation therefore describes only the plunge motion of the system.

The same reduction must be applied to the aerodynamic contribution. In the linearized formulation, the aerodynamic force vector is expressed through equivalent aerodynamic stiffness and damping matrices, leading to the following second-order system:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}, \quad (5.1.2)$$

where

$$\mathbf{M} = \mathbf{M}_{\text{struct}}, \quad \mathbf{C} = \mathbf{C}_{\text{struct}} + \mathbf{C}_{\text{aero}}, \quad \mathbf{K} = \mathbf{K}_{\text{struct}} + \mathbf{K}_{\text{aero}}, \quad (5.1.3)$$

The aerodynamic stiffness and damping matrices are then reduced in the same way as the structural matrices, such that the pitch equation is removed, and only the terms associated with the plunge degree of freedom are retained. This reduction is shown below for both the quasi-steady and unsteady aerodynamic formulations.

Quasi-steady formulation

In the quasi-steady formulation, the aerodynamic stiffness and damping matrices are obtained from the local derivatives of the lift coefficient around the equilibrium position. After reduction to the plunge-only degree of freedom, only the terms associated with y and $\dot{y}$ are retained:

$$\mathbf{K}_{\text{aero}} = \frac{1}{2}\rho c U_{\infty}^2 \begin{bmatrix} \left. \frac{\partial C_L}{\partial y} \right|_{\text{eq}} & \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \\ (x_{AC} - x_f) \left. \frac{\partial C_L}{\partial y} \right|_{\text{eq}} & (x_{AC} - x_f) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \end{bmatrix}, \quad (5.1.4)$$

$$\mathbf{C}_{\text{aero}} = \frac{1}{2}\rho c U_{\infty} \begin{bmatrix} \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} & \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \\ (x_{AC} - x_f) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} & (x_{AC} - x_f) \left(x_{\frac{3c}{4}} - x_f \right) \left. \frac{\partial C_L}{\partial \alpha_{\text{eff}}} \right|_{\text{eq}} \end{bmatrix}. \quad (5.1.5)$$

Unsteady formulation

In the unsteady formulation, the aerodynamic contribution is expressed through the flutter derivatives. Applying the same plunge-only reduction removes the pitch-related coefficients and the coupling terms. The remaining terms are the direct plunge aerodynamic stiffness and damping contributions:

$$\mathbf{K}_{\text{aero}} = -\frac{1}{2}\rho c U_{\infty}^2 \begin{bmatrix} \frac{k^2 H_4^*}{c} & k^2 H_3^* \\ \frac{k^2 A_4^*}{c} & k^2 A_3^* \end{bmatrix}, \quad (5.1.6)$$

$$\mathbf{C}_{\text{aero}} = -\frac{1}{2}\rho c U_{\infty} \begin{bmatrix} \frac{k H_1^*}{U_{\infty}} & \frac{k c H_2^*}{U_{\infty}} \\ \frac{k A_1^*}{U_{\infty}} & \frac{k c A_2^*}{U_{\infty}} \end{bmatrix}. \quad (5.1.7)$$

Once the mass matrix $\mathbf{M}$, damping matrix $\mathbf{C}$, stiffness matrix $\mathbf{K}$, and degree-of-freedom vector $\mathbf{x}$ have been reduced to the plunge-only formulation, the same procedure as the one introduced previously can be used to predict the static deflection and the stability properties of the system.

For clarity, the stability formulation is recalled here. The reduced equation of motion is first rewritten as a first-order state-space system:

$$\begin{bmatrix} \ddot{\mathbf{x}} \\ \dot{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I}_{1 \times 1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{x} \end{bmatrix}. \quad (5.1.8)$$

In the plunge-only model, $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are 1×1 matrices, and the displacement vector reduces to

$$\mathbf{x} = [y]. \quad (5.1.9)$$

The corresponding state vector is therefore defined as

$$\mathbf{q} = \begin{bmatrix} \dot{\mathbf{x}} \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ y \end{bmatrix}. \quad (5.1.10)$$

The system can then be written compactly as

$$\dot{\mathbf{q}} = \mathbf{Q}\mathbf{q}, \quad (5.1.11)$$

where the state matrix is given by

$$\mathbf{Q} = \begin{bmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{K} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}. \quad (5.1.12)$$

The stability of the plunge-only system is then determined by solving the eigenvalue problem associated with $\mathbf{Q}$. The eigenvalues provide the modal frequency and damping ratio of the reduced aeroelastic system for each freestream velocity.

Consequently, the final reduced model keeps only the plunge structural properties and the direct plunge aerodynamic contributions. This one-degree-of-freedom formulation is the model used in the following sections to predict the static deflection, modal frequency and effective damping ratio.

5.2 Parametric Modelling of the Plunge Flutter Derivatives

The unsteady aerodynamic formulation used in the reduced-order model relies on the plunge flutter derivatives H_1^* and H_4^* . As shown previously, the directly identified values of these coefficients depend on the initial ground clearance H . However, the scaled representation introduced in Figures 4.35 and 4.36 showed that most of this ground-clearance dependency can be collapsed by using the scaling factor c^2/H^2 .

The objective of the present section is therefore to define a parametric model for the scaled aerodynamic coefficients. This model provides a continuous expression of the plunge flutter derivatives that can be directly used in the reduced-order model.

For the reasons discussed previously, the lowest ground-clearance configuration, $H = 20$ mm, is excluded from the fit. This configuration was shown to behave differently from the other cases and is therefore not considered representative of the common scaled trend.

The scaled damping-related derivative is fitted using a cubic polynomial, while the scaled stiffness-related derivative is fitted using a quadratic polynomial. Reintroducing the reduced velocity

$$U_{\text{red}} = \frac{U_{\infty}}{\omega H}, \quad (5.2.1)$$

the fitted scaled coefficients are written as

$$\frac{c^2}{H^2} H_1^* = a_3 U_{\text{red}}^3 + a_2 U_{\text{red}}^2 + a_1 U_{\text{red}}, \quad (5.2.2)$$

$$\frac{c^2}{H^2} H_4^* = b_2 U_{\text{red}}^2 + b_1 U_{\text{red}}, \quad (5.2.3)$$

with the parameters of these fitting polynomials being:

$$\begin{aligned}
a_1 &= 4.766, \\
a_2 &= -10.711, \\
a_3 &= 0.8048, \\
b_1 &= -49.778, \\
b_2 &= 7.559.
\end{aligned}$$

The actual plunge flutter derivatives used in the model are then recovered as

$$H_1^* = \frac{H^2}{c^2} (a_3 U_{\text{red}}^3 + a_2 U_{\text{red}}^2 + a_1 U_{\text{red}}), \quad (5.2.4)$$

$$H_4^* = \frac{H^2}{c^2} (b_2 U_{\text{red}}^2 + b_1 U_{\text{red}}). \quad (5.2.5)$$

Figures 5.1 and 5.2 compare these polynomial fits with the scaled identified coefficients. Except for the excluded $H = 20$ mm configuration, the fitted curves provide a satisfactory approximation of the common trend followed by the experimental data. These parametric expressions are therefore retained for the unsteady aerodynamic model used in the following stability predictions.

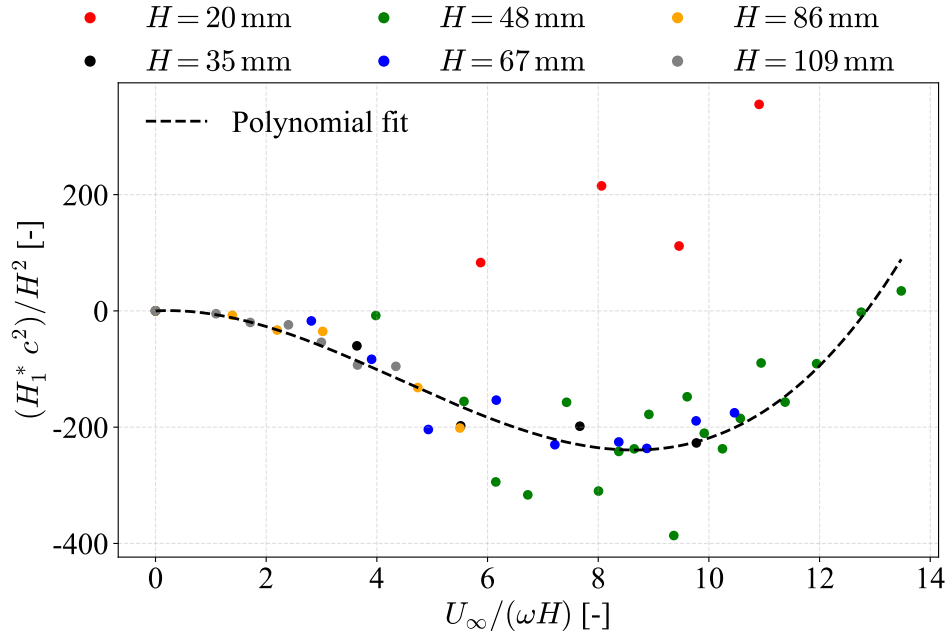


Figure 5.1: Parametric fit of the scaled plunge aerodynamic damping derivative $(c^2/H^2)H_1^*$ as a function of the reduced velocity $U_{\text{red}} = U_{\infty}/(\omega H)$.

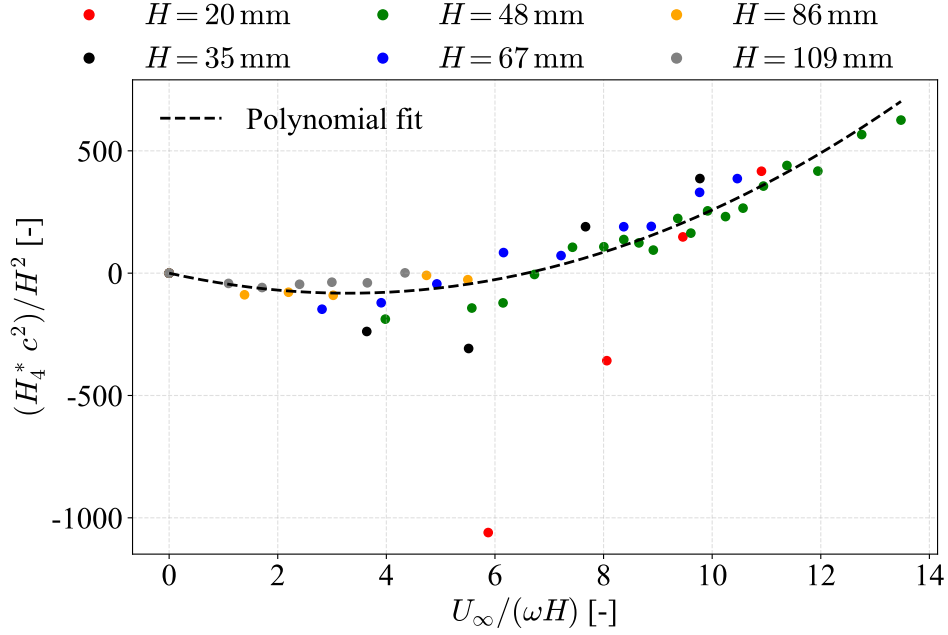


Figure 5.2: Parametric fit of the scaled plunge aerodynamic stiffness derivative $(c^2/H^2)H_4^*$ as a function of the reduced velocity $U_{\text{red}} = U_{\infty}/(\omega H)$.

5.3 Comparison with Experiment

In this section, the reduced-order model is run using the structural parameters of the plunge-only dynamic setup. The objective is to compare the model predictions with the experimental measurements in order to assess the ability of the model to reproduce the observed behaviour. Two types of results are considered: the static aeroelastic deflection and the stability evolution of the plunge mode. The static deflection is computed using the quasi-steady aerodynamic model (Q.S. model), since the unsteady formulation based on flutter derivatives only describes the motion-induced aerodynamic stiffness and damping terms. It does not provide the static aerodynamic load required to determine the equilibrium position.

For the stability analysis, both aerodynamic formulations are tested. The quasi-steady model (Q.S. model) and the unsteady aerodynamic-derivative model (AD model) are used to compute the evolution of the plunge frequency and effective damping ratio with freestream velocity. The corresponding predictions are then compared with the experimentally identified values.

The model parameters corresponding to the plunge-only dynamic setup are summarized in Table 5.1.

Category	Parameter	Value
Configuration	H	$[0.02 - 0.109] \text{ m}$
	α_0	4.5°
Environment	ρ	1.225 kg m^{-3}
	g	9.81 m s^{-2}
Wing geometry	c	0.45 m
	b	0.66 m
Structural model	K_y	$1.045 \times 10^4 \text{ N m}^{-2}$
	f_y	5.865 Hz
	$\omega_y = 2\pi f_y$	36.85 rad s^{-1}
	m	7.70 kg m^{-1}
	C_y	6.81 N s m^{-2}
Aerodynamic-derivative model	a_1	4.766
	a_2	-10.711
	a_3	0.8048
	b_1	-49.778
	b_2	7.559

Table 5.1: Model parameters used for the comparison with the plunge-only experimental setup.

Although the original two-degree-of-freedom model is not used in the final stability predictions, the plunge–pitch structural parameters required to define this model are summarized in Appendix B.

5.3.1 Static aeroelastic deflection

The static aeroelastic deflection is computed for the six experimentally tested configurations using the quasi-steady aerodynamic model. The comparison between the model predictions and the experimental measurements is shown in Figure 5.3.

Overall, the model predicts the static plunge deflection with reasonable accuracy. Small discrepancies are still observed, but their magnitude remains limited. These differences may be caused by uncertainties in the measured structural stiffness, as well as by inaccuracies in the static lift-coefficient map C_L . As discussed previously, this map was obtained with a separate static setup, and ground-effect aerodynamics is highly sensitive to small geometrical and environmental differences between setups.

Despite these limitations, the agreement between the model and the experimental data is satisfactory. The quasi-steady model can therefore be considered adequate for predicting the static equilibrium deflection of the plunge-only system.

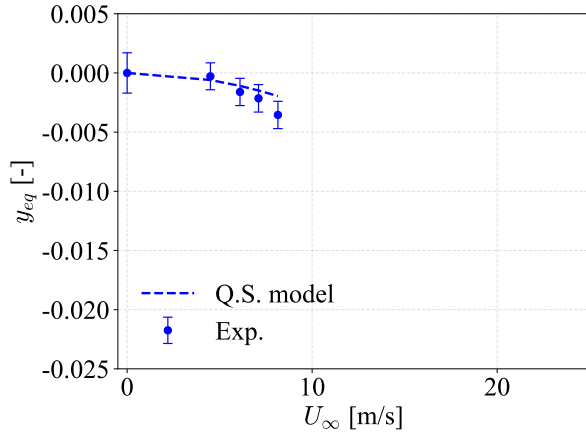
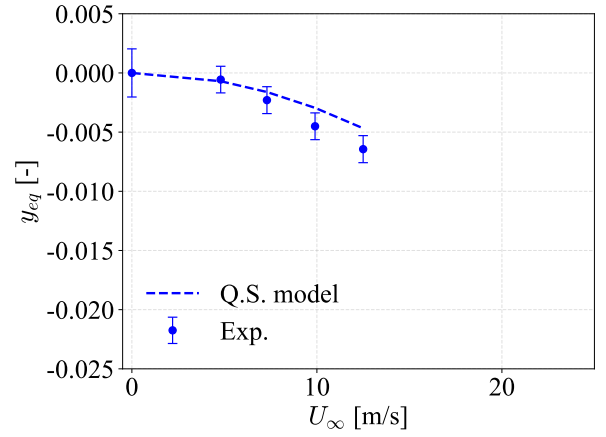
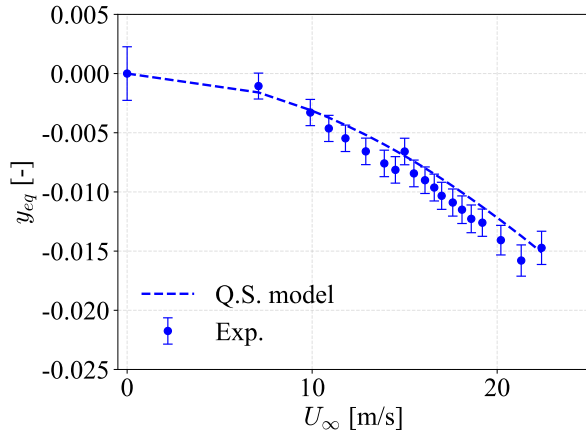
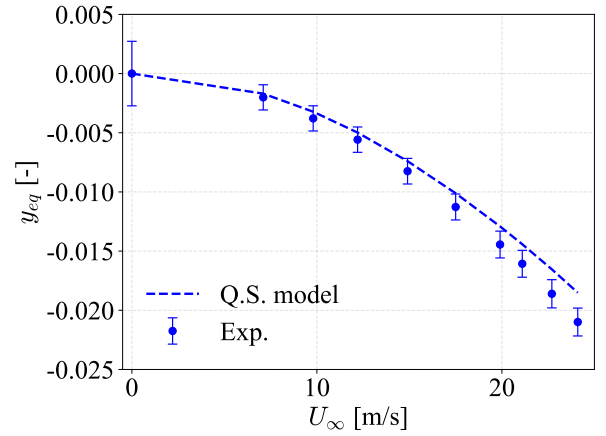
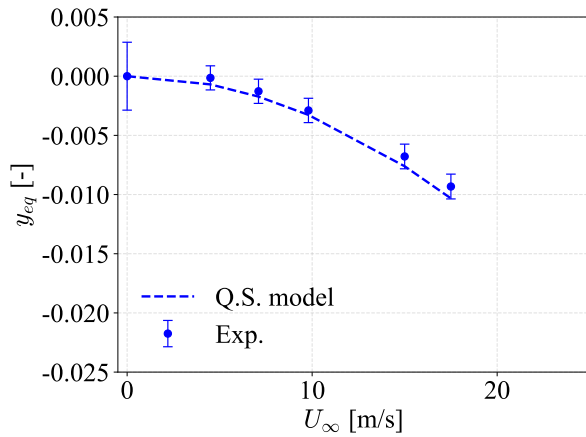
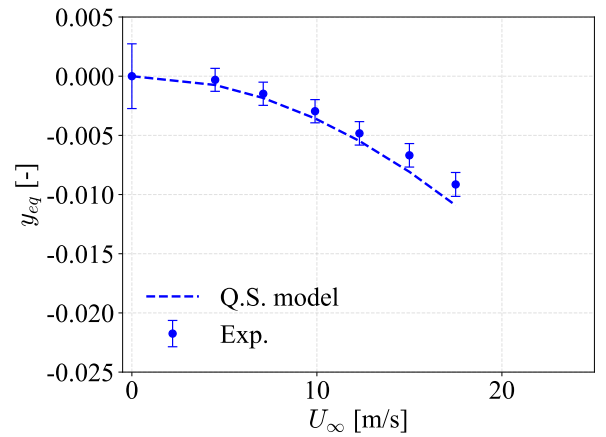
(a) $H = 20$ mm.(b) $H = 35$ mm.(c) $H = 48$ mm.(d) $H = 67$ mm.(e) $H = 86$ mm.(f) $H = 109$ mm.

Figure 5.3: Comparison between the measured and modelled static equilibrium displacement (using static tests measurements) of the plunge-only aeroelastic system for different initial ground clearances H .

5.3.2 Stability prediction

The stability evolution of the plunge-only system is computed using both the quasi-steady and unsteady aerodynamic models. The predicted modal frequency and effective damping ratio are compared with the experimental values identified from the dynamic tests. The objective is to assess the validity of the quasi-steady approximation and to evaluate the accuracy of the parametrized flutter-derivative model.

For the lowest initial ground clearance, $H = 20$ mm, shown in Figure 5.4a, both models fail to reproduce the experimental frequency and damping evolutions. The poor quasi-steady prediction confirms that the plunge dynamics cannot be captured from static aerodynamic data only. The unsteady model also performs poorly for this case, which is consistent with the previous observation that the $H = 20$ mm configuration does not follow the common scaled trend of the identified flutter derivatives, used for H_1^* modelling. This very-low-clearance case is therefore considered separately.

For the intermediate clearances, $H = 35$, 48, and 67 mm, shown in Figures 5.4b, 5.4c, and 5.5a, the unsteady model gives a good prediction of both the modal frequency and the effective damping ratio. It captures the main experimental trends, including the frequency decrease at higher velocities and the damping drop associated with the approach to porpoising.

In contrast, the quasi-steady model generally fails to reproduce the stability evolution. It predicts an almost constant or slightly increasing plunge frequency, whereas the experiments show a significant frequency reduction at higher velocity. Its damping predictions are also inconsistent: for $H = 35$ mm, it predicts a monotonic decrease from low velocities, while the experimental damping first increases before decreasing. The best agreement is obtained for $H = 48$ mm, although the porpoising velocity is still underpredicted.

For the largest clearances, $H = 86$ and 109 mm, the unsteady model again captures the main stability trends. The main discrepancy is observed for $H = 109$ mm, where the frequency is slightly overpredicted. The quasi-steady model captures the qualitative increase of damping for these high-clearance cases, but its frequency prediction remains inaccurate.

Overall, the parametrized unsteady model provides satisfactory stability predictions for all configurations except the very-low-clearance case $H = 20$ mm, corresponding to $H/c < 0.05$. The quasi-steady model is adequate for predicting the static equilibrium, but not the dynamic stability. This confirms that unsteady aerodynamic effects are required to model the porpoising dynamics of the present system.

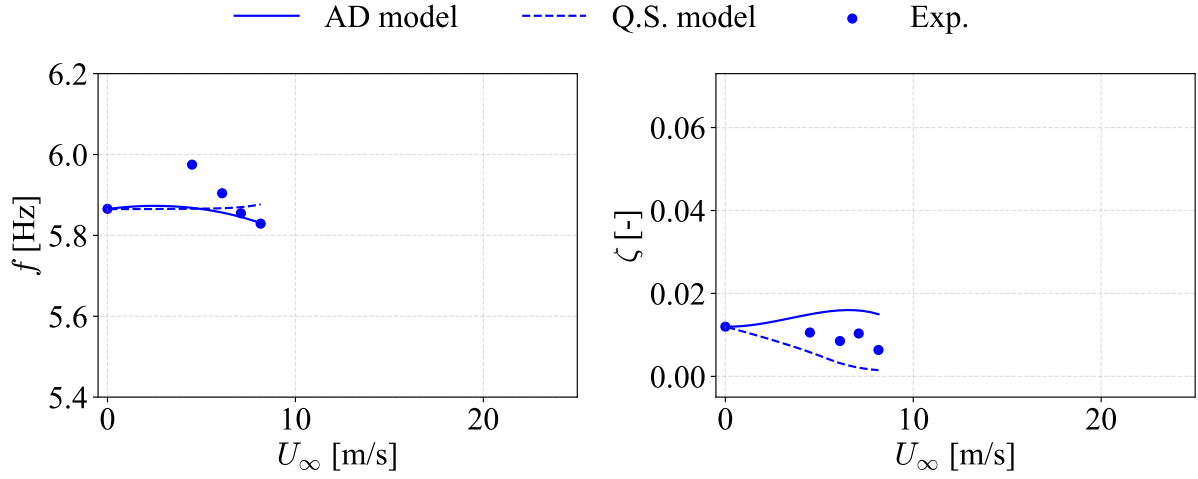
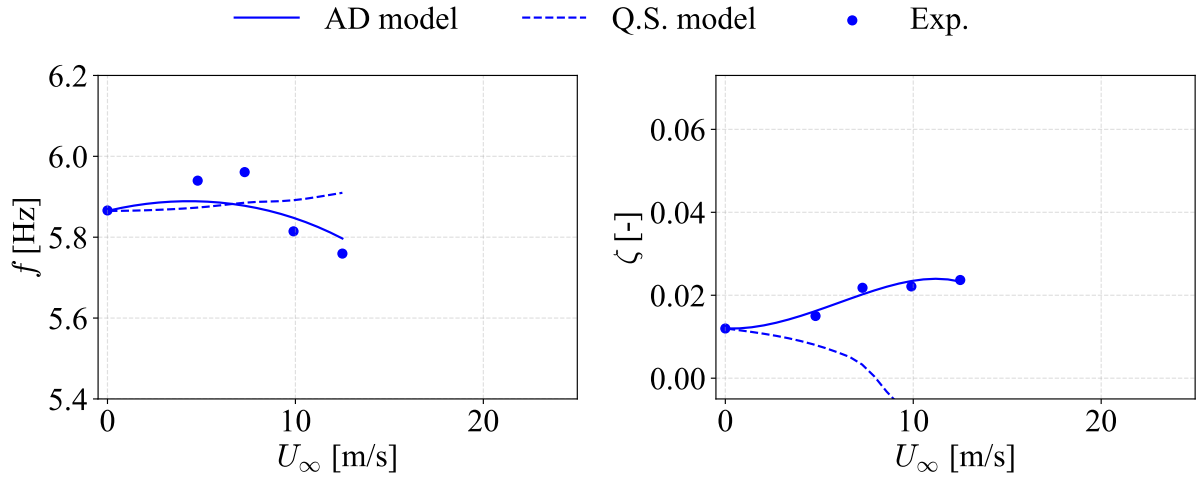
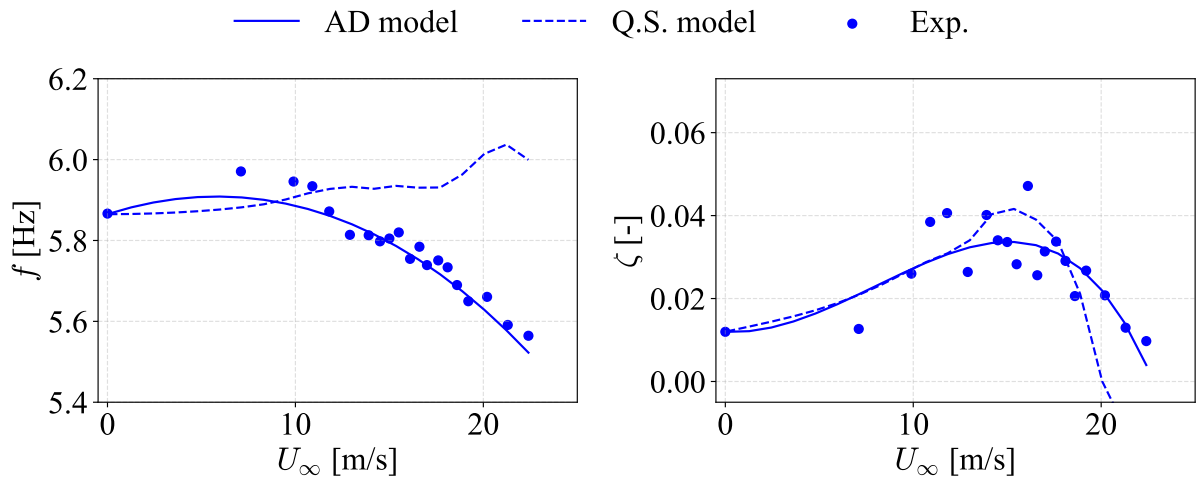
(a) $H = 20$ mm.(b) $H = 35$ mm.(c) $H = 48$ mm.

Figure 5.4: Comparison between the experimental and modelled plunge-mode stability results of the plunge-only aeroelastic system for the initial ground clearances $H = 20$, 35 , and 48 mm.

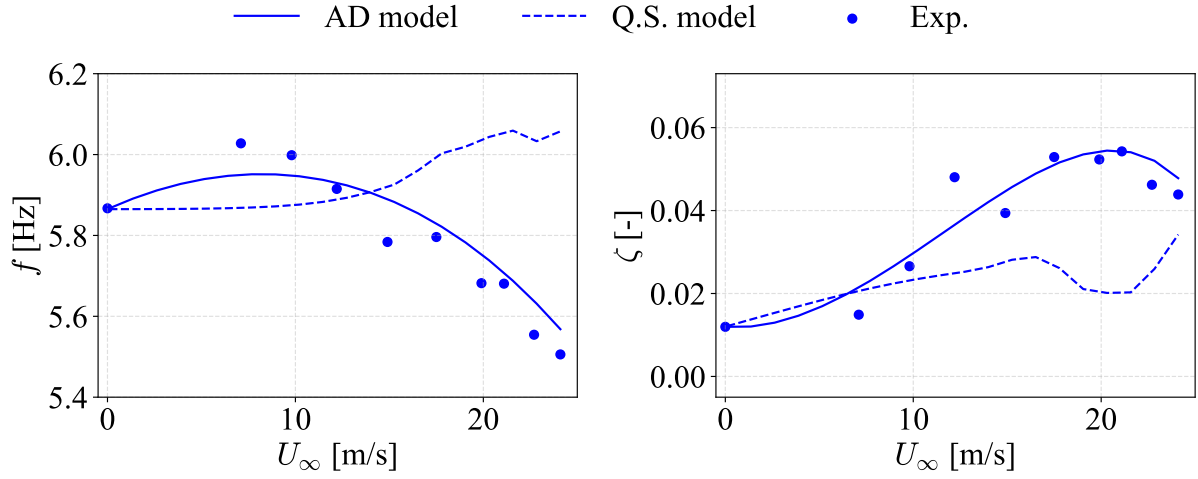
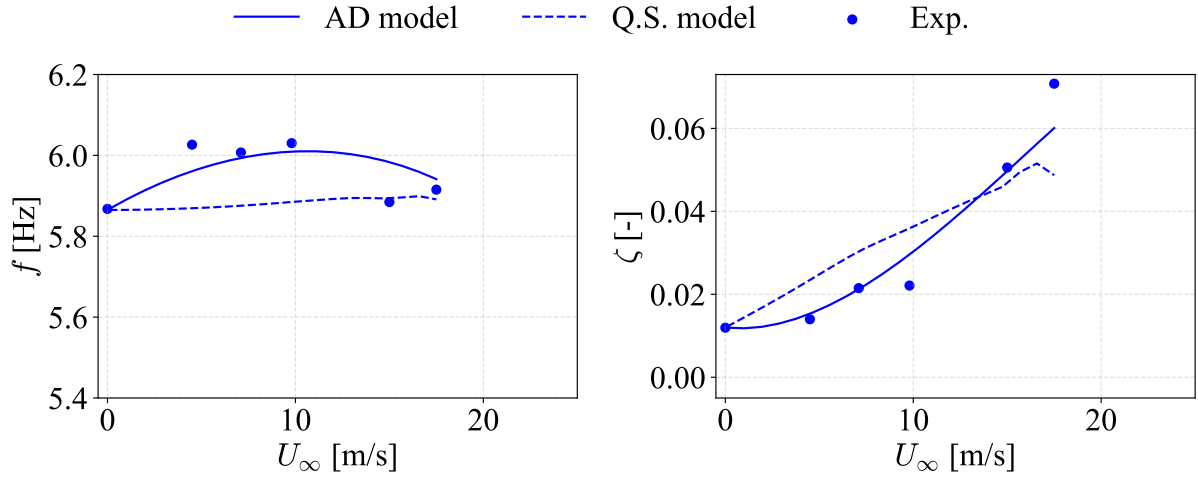
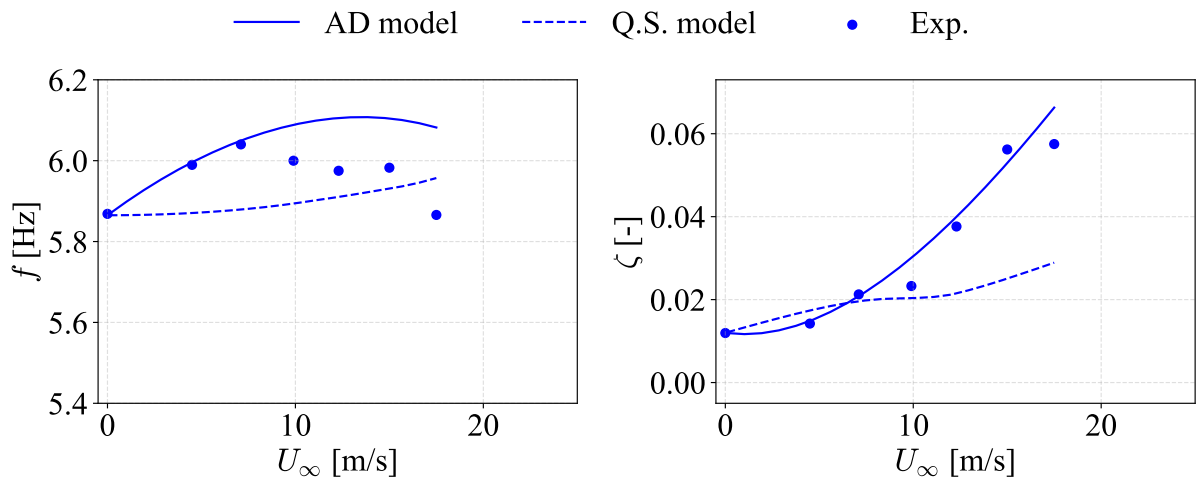
(a) $H = 67$ mm.(b) $H = 86$ mm.(c) $H = 109$ mm.

Figure 5.5: Comparison between the experimental and modelled plunge-mode stability results of the plunge-only aeroelastic system for the initial ground clearances $H = 67$, 86, and 109 mm.

5.4 Parametric Stability Analysis

In this section, the reduced-order model is used to study the influence of selected parameters on the stability of the plunge-only system. Two parameters are considered: the initial ground clearance H and the structural plunge stiffness K . For each study, the selected parameter is varied over twenty equally spaced values, while the remaining model parameters are kept constant.

The stability results are presented in non-dimensional form. The freestream velocity is scaled using the plunge frequency and the wing chord:

$$U^* = \frac{U_\infty}{\omega'_y c}.$$

This definition differs from the reduced velocity used for the flutter-derivative identification, where the characteristic length was the initial ground clearance H . The modal frequency is scaled as

$$f^* = \frac{f'_y}{f_y} = \frac{\omega'_y}{\omega_y}.$$

For each parametric study, the lowest tested value of the varied parameter is represented by the blue curve, while the highest value is represented by the red curve. The intermediate curves correspond to increasing values of the parameter from blue to red.

Figure 5.6 shows the influence of the initial ground clearance H on the stability of the system. The frequency plot shows a common trend, the non-dimensional frequency first increases with U^* , then decreases at higher velocities. Increasing H gives higher maximum frequency, and shifts the maximum frequency towards higher values of U^* . The damping plot shows a similar behaviour, the effective damping first increases and then decreases as the system approaches instability. Increasing the initial ground clearance delays the damping drop, and therefore increases the porpoising velocity, as shown in Figure 5.7.

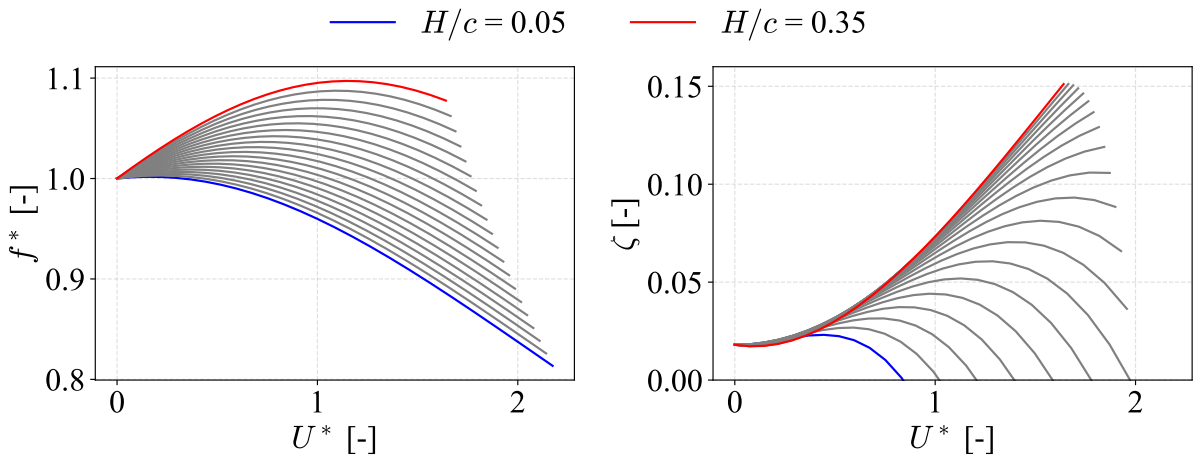


Figure 5.6: Influence of the initial ground clearance H on the non-dimensional stability evolution of the plunge-only system.

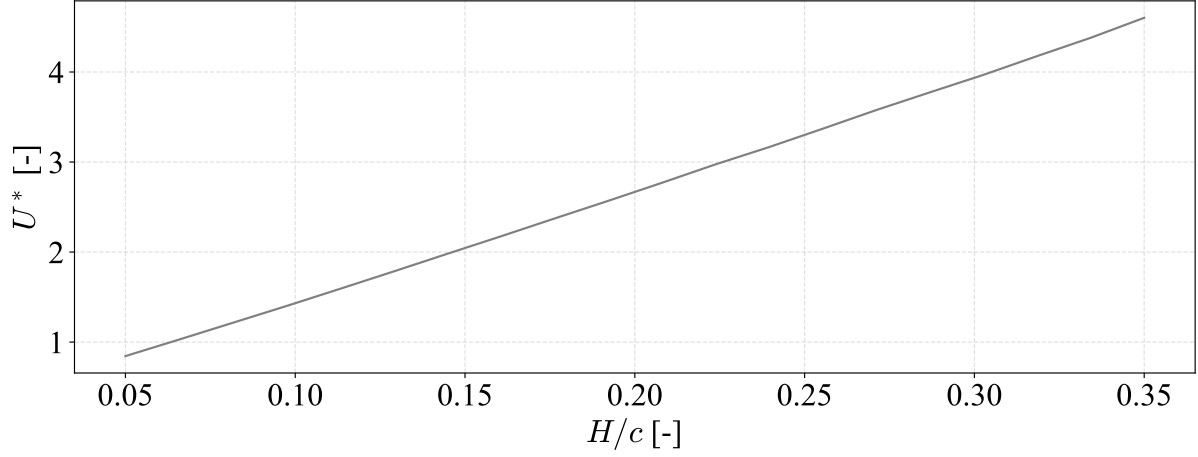


Figure 5.7: Evolution of the predicted porpoising velocity as a function of the initial ground clearance H .

The influence of the structural plunge stiffness K is shown in Figure 5.8. In the selected non-dimensional representation (where K_y corresponds to the value given in Table 5.1), changing K mainly shifts the frequency level, while the damping curves collapse onto a nearly common trend. This suggests that, once the velocity is scaled by $\omega'_y c$, the structural stiffness has a limited influence on the shape of the damping evolution. Its main effect is therefore to modify the dimensional velocity scale through the structural frequency.

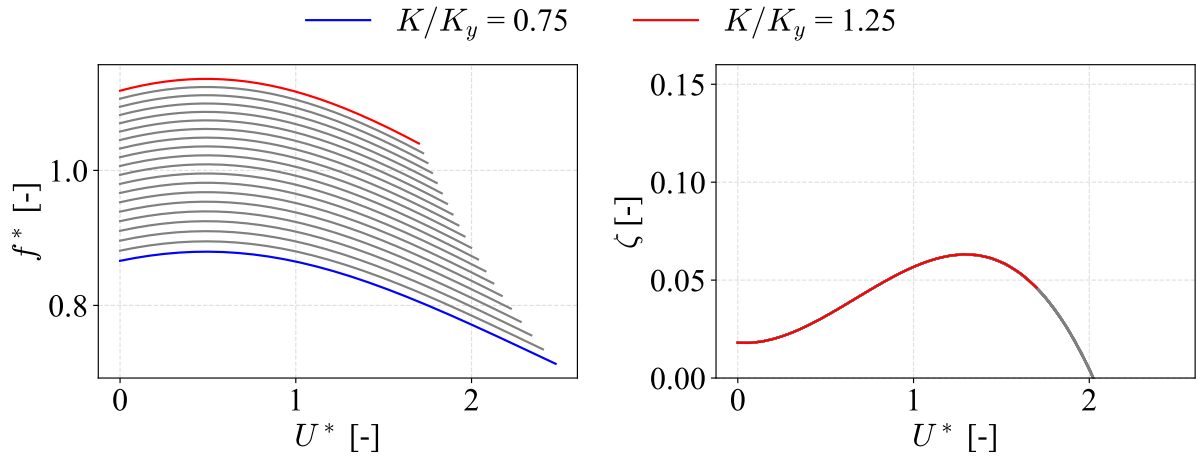


Figure 5.8: Influence of the structural plunge stiffness K on the non-dimensional stability evolution of the plunge-only system.

6 Conclusion and Perspectives

6.1 Conclusion

The objective of this work was to investigate porpoising on a simplified aeroelastic system and to develop a reduced-order model able to reproduce the main stability trends observed experimentally. The study focused on an inverted Tyrrell026S wing operating in ground effect, in order to isolate the coupling between ride-height-sensitive aerodynamics and vertical structural dynamics.

The static wind-tunnel campaign confirmed the strong dependence of the aerodynamic loads on ground clearance and wing incidence. Decreasing the ground clearance generally increased the generated downforce, but the aerodynamic behaviour became more sensitive at very low clearances. These measurements were then used as the static aerodynamic database for the quasi-steady model.

The dynamic tests showed that the system response evolves significantly with freestream velocity. The plunge-pitch setup made it possible to observe the coupled motion of the wing, while the plunge-only setup provided cleaner data for the analysis of the vertical instability mechanism. The post-critical porpoising response was found to be mainly dominated by plunge motion, which justified the reduction of the model to a single vertical degree of freedom.

The stability analysis of the plunge-only setup showed that both the modal frequency and the effective damping ratio vary with freestream velocity. The frequency first increases slightly and then decreases at higher velocities, indicating a modification of the effective aerodynamic stiffness. The damping ratio also evolves with the same trend as the frequency.

The direct plunge *flutter derivatives* H_1^* and H_4^* were identified from the dynamic measurements. These coefficients describe the aerodynamic damping and stiffness contributions associated with plunge motion. Their values depend on the initial ground clearance, but an empirical scaling based on c^2/H^2 allowed most configurations to collapse onto a common trend.

The developed reduced-order model was then compared with the experimental results. The quasi-steady model predicted the static aeroelastic deflection with reasonable accuracy, showing that the static aerodynamic database correctly represents the mean loading. However, it failed to reproduce the measured frequency and damping evolutions. In contrast, the unsteady model based on the identified flutter derivatives captured the main stability trends for most tested ground clearances. This is the main conclusion of the work:

- Static aerodynamic data alone are not sufficient to model porpoising dynamics, and porpoising instability does not occur at a unique particular ground clearance.
- Unsteady aerodynamic contribution must be included in porpoising modelling to take the history of the flow into account.

The validated model was finally used for a parametric stability analysis. Increasing the initial ground clearance delayed the onset of porpoising, consistently with the experiments. The influence of structural stiffness was also studied in non-dimensional form. When the freestream

velocity was scaled by the structural frequency and the chord, the damping curves approximately collapsed, suggesting that the stiffness mainly changes the dimensional velocity scale of the instability.

6.2 Perspectives

The present work combined wind-tunnel experiments and reduced-order modelling to investigate the porpoising instability of an inverted wing in ground effect. Although the experimental results and the unsteady plunge model provided consistent stability predictions, several points remain open. Future work should therefore focus both on improving the experimental characterisation of the phenomenon and on extending the modelling framework.

6.2.1 Experimental perspectives

- **Cleaner investigation of pitch effects.** The plunge–pitch setup discussed in Chapter 4 made it possible to observe the coupled response of the system, but the extraction of clean pitch-related aeroelastic quantities remains difficult. A refined two-degree-of-freedom setup, with reduced parasitic roll motion and more accurate pitch measurement, would allow the influence of pitch on porpoising to be quantified more reliably. This would also make it possible to assess whether the plunge-only reduction used later in the work remains valid over a wider range of configurations.
- **Further investigation of very-low-clearance configurations.** As observed in Chapter 4, the lowest tested ground clearance does not follow the same trends as the other configurations, both in the experimental stability results and in the identified flutter derivatives. This range should be investigated with additional repeated measurements.
- **Validation of the static aerodynamic database.** The static lift-coefficient trends obtained in Chapter 4 do not fully match the idealized behaviour presented in Chapter 2 or some trends reported in the literature for similar inverted wings. Additional static measurements would therefore be useful to confirm the measured $C_L(H, \alpha)$ map. This would increase confidence in the quasi-steady model and would help distinguish modelling errors from uncertainties in the static aerodynamic database.
- **Measurement of the aerodynamic moment.** The present model approximates the aerodynamic moment from the lift force and a lever arm, due to the absence of direct moment measurements. Measuring the moment coefficient C_M would allow the two-degree-of-freedom plunge–pitch model to be fed more consistently. This is necessary if future work aims to identify pitch-related and coupled flutter derivatives.
- **Direct flow-field characterisation.** In this study, the aerodynamic damping and stiffness contributions were inferred from the structural response. This identifies the global aeroelastic effect, but not the flow mechanism responsible for it. Pressure measurements, flow visualization or time-resolved PIV could be used to determine whether the damping loss is associated with separation, vortex dynamics, ground-boundary-layer interaction, or another unsteady mechanism.
- **Use of a moving-ground system.** The experiments were performed with a fixed ground plane, so a boundary layer develops upstream of the wing. This differs from

the real road condition, where the ground moves with the vehicle. A moving-ground setup would reduce this discrepancy and would allow the sensitivity of the static loads and flutter derivatives to the ground-condition modelling to be assessed.

6.2.2 Modelling perspectives

- **Extension to a full plunge–pitch unsteady model.** The final model used in Chapter 5 was reduced to a single plunge degree of freedom. A natural extension would be to retain the full plunge–pitch formulation and include the pitch-related flutter derivatives A_i^* and the coupling derivatives H_2^* , H_3^* , A_1^* , and A_4^* . This requires cleaner pitch data and direct aerodynamic moment information.
- **Improved quasi-steady formulation.** The quasi-steady model used in Chapter 5 was based mainly on the lift contribution. Including the drag component and its projection on the generalized coordinates could improve the static and linearized aerodynamic force prediction, especially when the wing incidence or pitch angle becomes non-negligible.
- **Link between static aerodynamic maps and flutter derivatives.** A useful extension would be to compare the identified flutter derivatives with quasi-steady derivatives obtained from the static map $C_L(H, \alpha)$. In particular, H_4^* can be compared with the local derivative $\partial C_L / \partial H$, while H_1^* can be compared with the derivative $\partial C_L / \partial \alpha_{\text{eff}}$. This would help quantify which part of the identified aerodynamic stiffness and damping can be explained by static data, and which part is genuinely unsteady.
- **Nonlinear modelling of the post-critical response.** The present model focuses on the linear stability boundary, while the post-critical oscillations are mainly discussed qualitatively in Chapter 4. A nonlinear model could be developed to predict the finite-amplitude limit-cycle oscillations observed after the onset of porpoising.
- **Bifurcation analysis.** The bifurcation diagrams introduced in Chapter 4 suggest path dependence and possible hysteresis-like behaviour. A more systematic bifurcation analysis could be performed to determine whether multiple stable and unstable response branches coexist. This would provide a more complete description of the transition from stable motion to fully developed porpoising.

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A Airfoil geometry

The airfoil geometry is defined by a set of normalized coordinates. The coordinates are given as x/c and y/c , where c is the chord length. The lower and upper surfaces are both ordered from the leading edge to the trailing edge.

Table A.1: Normalized coordinates of the Tyrrell026S airfoil section.

Lower surface				Upper surface			
x/c	y/c	x/c	y/c	x/c	y/c	x/c	y/c
0.0000	0.0000	0.1796	-0.0763	0.0000	0.0000	0.1802	0.0465
0.0010	-0.0076	0.1996	-0.0771	0.0010	0.0079	0.2002	0.0473
0.0020	-0.0107	0.2496	-0.0753	0.0020	0.0109	0.2501	0.0489
0.0049	-0.0168	0.2996	-0.0721	0.0051	0.0173	0.3001	0.0504
0.0099	-0.0228	0.3496	-0.0679	0.0101	0.0232	0.3501	0.0514
0.0149	-0.0266	0.3996	-0.0631	0.0151	0.0270	0.4001	0.0519
0.0199	-0.0293	0.4496	-0.0574	0.0201	0.0299	0.4501	0.0521
0.0249	-0.0319	0.4996	-0.0508	0.0251	0.0312	0.5001	0.0517
0.0298	-0.0344	0.5497	-0.0434	0.0301	0.0321	0.5501	0.0509
0.0348	-0.0368	0.5997	-0.0351	0.0351	0.0329	0.6001	0.0496
0.0398	-0.0392	0.6497	-0.0261	0.0401	0.0336	0.6500	0.0479
0.0448	-0.0414	0.6997	-0.0163	0.0451	0.0344	0.7000	0.0456
0.0498	-0.0436	0.7498	-0.0056	0.0501	0.0352	0.7500	0.0429
0.0548	-0.0458	0.7998	0.0060	0.0551	0.0359	0.8000	0.0409
0.0598	-0.0479	0.8498	0.0183	0.0601	0.0367	0.8500	0.0412
0.0698	-0.0518	0.8999	0.0315	0.0701	0.0379	0.9000	0.0446
0.0797	-0.0554	0.9199	0.0370	0.0801	0.0392	0.9200	0.0468
0.0897	-0.0588	0.9399	0.0426	0.0902	0.0404	0.9400	0.0496
0.0997	-0.0619	0.9599	0.0484	0.1002	0.0413	0.9600	0.0527
0.1197	-0.0672	0.9799	0.0543	0.1202	0.0432	0.9800	0.0563
0.1396	-0.0713	0.9900	0.0573	0.1402	0.0446	0.9900	0.0583
0.1596	-0.0744	1.0000	0.0603	0.1602	0.0457	1.0000	0.0603

B Plunge–Pitch Model Parameters

This appendix summarizes the structural and geometrical parameters of the plunge–pitch dynamic setup used in the two-degree-of-freedom model. The corresponding schematic is shown in Figure B.1, and the parameter values used for the model are listed in Table B.1.

Category	Parameter	Value
Configuration	H_0	0.032, 0.052, 0.072 m
	α_0	3°, 4.5°, 6°, 7.5°, 9°
Environment	ρ	1.225 kg m ⁻³
	g	9.81 m s ⁻²
Wing geometry	c	0.45 m
	b	0.66 m
	x_{AC}	0.1475 m
Structural model	x_f	0 m
	x_G	0.115 m
	l	0.91 m
	m	12.12 kg m ⁻¹
	I_{CG}	1.016 kg m
	K_h	9.697×10^3 N m ⁻²
	ζ_h	1.705×10^{-3}
	ζ_α	2.62×10^{-4}

Table B.1: Model parameters used for the plunge–pitch dynamic setup.

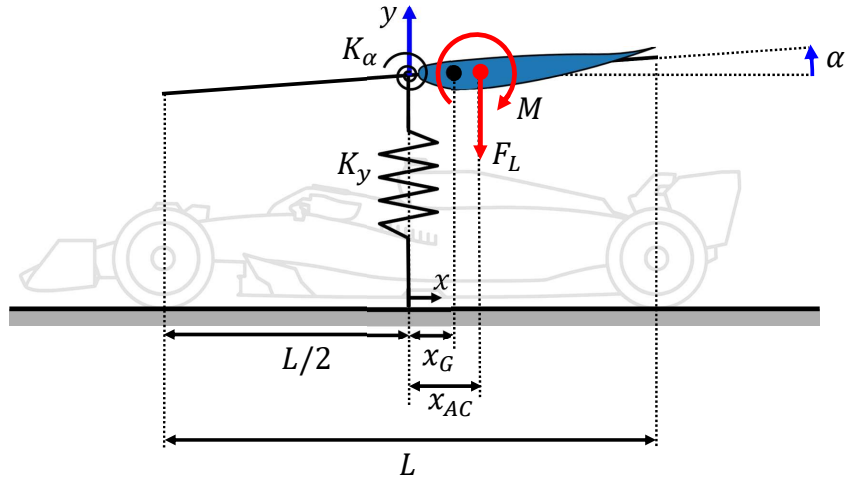


Figure B.1: Schematic of the pitch-plunge degrees of freedom's system.

C Wind Tunnel Facility

The wind tunnel of University of Liège was built on the Sart-Tilman site at the end of 1999. It is a subsonic closed-loop facility, with a maximum Mach number below 0.15. The tunnel can also operate in open-loop configuration, depending on the test requirements. Its modular design allows two different test sections to be used, such as the aeronautic/automobile test section, denoted TS1, and the wind-engineering test section, denoted TS2.

The main characteristics of both test sections are summarized in Table C.1. TS1 is mainly intended for aeronautical and automotive applications, with models of frontal area lower than approximately 0.3 m^2 . This section is equipped with glass walls, allowing optical flow visualization techniques such as Particle Image Velocimetry. TS2 is larger and is mainly dedicated to civil engineering and atmospheric boundary layer studies.

For the present work, the aeronautic/automobile test section TS1 was used for both static and dynamic test campaigns.

Characteristic	TS1	TS2
Dimensions, width $\times$ height $\times$ length	$2.0 \times 1.5 \times 5.0\text{ m}$	$2.5 \times 1.8 \times 2.0\text{ m}$
Airspeed range, closed loop	2–60 m/s	2–40 m/s
Airspeed range, open loop	2–40 m/s	2–30 m/s
Test-section turntable diameter	1.5 m	2.0 m
Turntable rotation	$\pm 90^\circ$	$\pm 180^\circ$
Thermal stability	$\sim 1^\circ\text{C}$	$\sim 1^\circ\text{C}$
Streamwise static-pressure gradient	$0.3\% \text{ m}^{-1}$	$0.5\% \text{ m}^{-1}$
Mean angle divergence	$< 0.2\%$	$< 0.2\%$
Speed non-uniformity	$< 0.5\%$	$< 0.8\%$
Turbulence level	0.15%	$< 0.23\%$

Table C.1: Main characteristics of the Université de Liège wind tunnel test sections, adapted from [29].

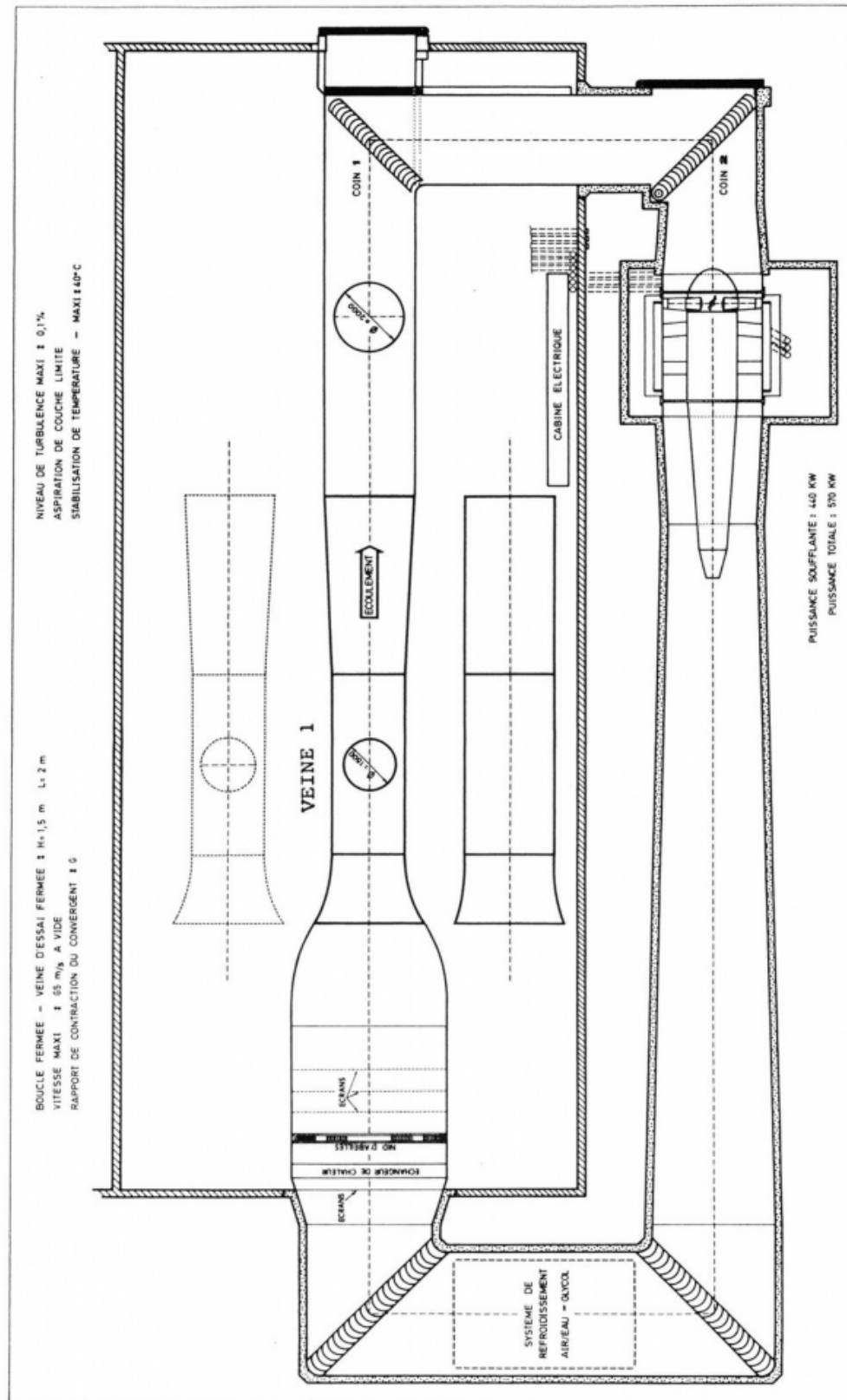


FIGURE 1. - VEINE 1 - BOUCLE POUR ETUDES D'AERONEFS ET DE VEHICULES AUTOMOBILES

Figure C.1: General layout of the TS1 closed-loop wind-tunnel test section.