

Thermo-mechanical topology optimization of detector support: application to the VIGIL JEDI instrument Centre Spatial de Liège

Auteur : Heine, Clément

Promoteur(s) : Jacques, Lionel

Faculté : Faculté des Sciences appliquées

Diplôme : Master en ingénieur civil en aérospatiale, à finalité spécialisée en "aerospace engineering"

Année académique : 2025-2026

URI/URL : <http://hdl.handle.net/2268.2/26166>

Avertissement à l'attention des usagers :

Tous les documents placés en accès ouvert sur le site le site MatheO sont protégés par le droit d'auteur. Conformément aux principes énoncés par la "Budapest Open Access Initiative"(BOAI, 2002), l'utilisateur du site peut lire, télécharger, copier, transmettre, imprimer, chercher ou faire un lien vers le texte intégral de ces documents, les disséquer pour les indexer, s'en servir de données pour un logiciel, ou s'en servir à toute autre fin légale (ou prévue par la réglementation relative au droit d'auteur). Toute utilisation du document à des fins commerciales est strictement interdite.

Par ailleurs, l'utilisateur s'engage à respecter les droits moraux de l'auteur, principalement le droit à l'intégrité de l'oeuvre et le droit de paternité et ce dans toute utilisation que l'utilisateur entreprend. Ainsi, à titre d'exemple, lorsqu'il reproduira un document par extrait ou dans son intégralité, l'utilisateur citera de manière complète les sources telles que mentionnées ci-dessus. Toute utilisation non explicitement autorisée ci-avant (telle que par exemple, la modification du document ou son résumé) nécessite l'autorisation préalable et expresse des auteurs ou de leurs ayants droit.



Thermo-mechanical topology optimization of detector support: application to the VIGIL JEDI instrument

Heine Clement

Thesis presented to obtain the degree of :
Master of Science in aerospace Engineering

Thesis supervisor :
Jacques Lionel

Academic year: **2025 - 2026**

Acknowledgments

I would like to express my sincere gratitude to Lionel Jacques, my supervisor, for his invaluable advice, guidance, and the time he devoted to answering my numerous questions throughout this work. I would also like to thank the staff of the CSL for their warm welcome and hospitality during my time at their facilities.

I am equally grateful to Jean-Marc Vaassen, Stéphane Paquay, and Pascal De Vincenzo for their assistance and for the time they generously dedicated to supporting me.

Finally, I would like to thank my family for their moral support and encouragement. In particular, I wish to express my deepest gratitude to Alix Dawans for the constant support provided throughout these four months of work.

Summary

The JEDI instrument onboard the VIGIL mission consists of two telescopes, each equipped with a detector. These detectors are supported by dedicated structures that must satisfy conflicting thermo-mechanical requirements. As a result, an optimal geometry maximizing the natural frequency while meeting thermal constraints can be sought. This objective constitutes the main focus of the present study.

The first part of this work is devoted to the development and validation of a parametric simulation framework. The validation process was carried out through comparison with the analytical Euler–Bernoulli beam theory, a mesh convergence study, and a comparison with experimental vibration data. Once validated, the model was used to investigate the influence of geometric parameters on the structural dynamics, first by neglecting heat conduction through the support and then by including thermal conduction effects in the simulations. Finally, the parametric study was used to develop a regression model capable of predicting the resonance frequency directly from the design parameters, thereby eliminating the need for computationally expensive numerical simulations during preliminary design iterations.

The second part of this study focuses on topology optimization with the objective of identifying optimal structural layouts. The optimization process was performed using the *OOFELIE* software developed by *Open-Engineering*. As in the parametric analysis, thermal constraints were first neglected and subsequently included in order to evaluate their influence on the optimization results. This approach made it possible to assess the impact of the thermal constraint on the generated topologies and to explore potential design solutions for detector support structures.

Overall, this work provides both a parametric design methodology and a topology optimization framework for the preliminary development of lightweight detector support structures subjected to stringent thermo-mechanical requirements.

Declaration on the use of Artificial Intelligence

Artificial intelligence was used in the preparation of this thesis to assist with paraphrasing and proofreading, including the correction of grammatical and spelling errors. This tool was also used to support the debugging and correction of computer scripts.

All AI-generated content and suggestions were carefully reviewed and verified by the author of this thesis before being incorporated into the final document.

Contents

1	Introduction	1
1.1	Mission context	1
1.2	Detector support conflicting requirements	2
1.2.1	Mechanical requirement	2
1.2.2	Thermal requirement	3
1.2.3	Conflict between the two requirements	3
1.3	State of the art	4
1.3.1	Flying design	4
1.4	Theoretical concepts	9
1.4.1	Analytical solution for a modal of a beam	9
1.4.2	Topology optimization	11
1.5	Objectives	16
2	Parametric analysis of the standard 3 blades isostatic design	17
2.1	Verification of the model	21
2.1.1	Assumptions of the model	22
2.1.2	Convergence of the model	22
2.1.3	Comparison with Euler-Bernoulli analytical solution of hollow triangular section beam	23
2.1.4	Comparison with Extreme UV Imager (EUI) vibration test	26
2.2	Methodology	30
2.3	Parameter's influence	31
2.3.1	Excluding thermal limitation	31
2.3.2	Including thermal limitation	40
2.4	Frequency prediction	47
2.4.1	Latin Hypercube Sampling (LHS)	48
2.4.2	Frequency regression	48
2.4.3	Convergence study	49
2.4.4	Results	50
2.4.5	Regression improvement	51
2.4.6	Comparison of the three regression	55
2.5	Discussion	59
3	Topology optimization	60
3.1	Without thermal limitation	62
3.1.1	Parallel fixation	63
3.1.2	Perpendicular fixation	68
3.2	With thermal limitation	72
3.2.1	Parallel fixation	74
3.2.2	Perpendicular fixation	77
3.3	Conclusion	80

TABLE OF CONTENTS

4	Conclusions and perspectives	81
4.1	Conclusions	81
4.2	Perspectives	83
4.2.1	Three blades design parametrical simulation	83
4.2.2	Thermo-mechanical topology optimization	83
A	Convergence study of mode 1-6	85
B	Fourth mode variation	87
	Bibliography	89

List of Figures

1.1	Possible impact by intense solar activity.	1
1.2	JEDI layout.	2
1.3	Kepler FPA cell design	4
1.4	Kepler entire FPA	5
1.5	3MI FPA design	5
1.6	Euclid FPA design	6
1.7	Simbol-X FPA design	6
1.8	NIRCam FPA	7
1.9	MIRI pick-off mirror support	8
1.10	PLATO FPA design	8
1.11	EUI FPA design	9
1.12	Euler-Bernoulli displacement assumption	10
1.13	Euler-Bernoulli rotation assumption	10
1.14	Representation of the checkerboard with a scheme and an application example	14
1.15	Effect of the two parameters β and η on the Heaviside projection	15
1.16	Thermo-mechanical topology optimization algorithm	15
2.1	Geometrical parameter representation	18
2.2	First mode shape representation, frequency = 398 [Hz]	19
2.3	Second mode shape representation, frequency = 409 [Hz]	19
2.4	Third mode shape representation, frequency = 2476 [Hz]	20
2.5	Fourth mode shape representation, frequency = 3150 [Hz]	20
2.6	Fifth mode shape representation, frequency = 3509 [Hz]	21
2.7	Sixth mode shape representation, frequency = 3509 [Hz]	21
2.8	Convergence of the relative error for sixth first modes	23
2.9	Hollow triangular beam cross-section	23
2.10	Comparison of the analytical and numerical solution with a length's variation	24
2.11	Comparison of the analytical and numerical solution with a width's variation	25
2.12	Comparison of the analytical and numerical solution with a thickness's variation	25
2.13	Comparison of the original model and the simplified	26
2.14	Accelerometer placement during the EUI shaker experiment	27
2.15	Power spectral density comparison with the experimental and numerical responses to a excitation along the X-axis	28
2.16	Power spectral density comparison with the experimental and numerical responses to a excitation along the Y-axis	29
2.17	Power spectral density comparison with the experimental and numerical responses to a excitation along the Y-axis	30
2.18	Blades' parameters variation	32
2.19	Representation of the boundaries of the equilateral ratio	36
2.20	Variation of the equilateral ratio	36

2.21	Fixation optimization representation	37
2.22	Variation of the assembly triangle's size	38
2.23	Variation of the detector's mass	39
2.24	Width-length interaction with different thermal conductance	41
2.25	Width-length interaction with different blade's thickness	42
2.26	Width-length interaction with different detector's mass	42
2.27	Thickness-width interaction with different thermal conduction	43
2.28	Thickness-width interaction with different length	44
2.29	Thickness-width interaction with different detector's mass	45
2.30	Thickness-length interaction with different thermal conductance	45
2.31	Thickness-length interaction with different width	46
2.32	Thickness-length interaction with different detector's mass	47
2.33	Comparison of two different sampling method in a 2D domain	48
2.34	Convergence study of the error from the regression	50
2.35	Convergence study of the enhanced regression with 28 coefficients	52
2.36	Relative error from a 2000 testing dataset with a regression of 100 training samples	53
2.37	Histogram of the number of error in the 20 % worst case for each parameter	54
2.38	Convergence study of the regression with a reduced parameter domain	55
2.39	Regressions' histogram compared with a theoretical normal distribution	57
2.40	Comparison of the prediction with numerically computed frequency	58
3.1	Side view (left) and rear view (right) of the two boundary conditions	61
3.2	Meshed geometry with parallel imposed fixation points	63
3.3	Meshed geometry with parallel free fixation	64
3.4	Mechanical topology optimization result for a parallel fixation with fixed attachment point with a minimum frequency constraint of 300 [Hz]	64
3.5	Mechanical topology optimization result for a parallel fixation with fixed attachment point with a minimum frequency constraint of 500 [Hz]	65
3.6	Convergence of the frequency and the NDR for the parallel imposed fixation	65
3.7	Mechanical topology optimization result for a parallel fixation with free attachment point with a minimum frequency constraint of 300 [Hz]	66
3.8	Mechanical topology optimization result for a parallel fixation with free attachment point with a minimum frequency constraint of 500 [Hz]	67
3.9	Convergence of the frequency and the NDR for the parallel free fixation	67
3.10	Meshed geometry with perpendicular imposed fixation points	68
3.11	Meshed geometry with perpendicular free fixation	68
3.12	Mechanical topology optimization result for a perpendicular fixation with imposed attachment point with a minimum frequency constraint of 300 [Hz]	69
3.13	Mechanical topology optimization result for a perpendicular fixation with imposed attachment point with a minimum frequency constraint of 500 [Hz]	69
3.14	Convergence of the frequency and the NDR for the perpendicular imposed fixation	70
3.15	Mechanical topology optimization result for a perpendicular fixation with free attachment point with a minimum frequency constraint of 300 [Hz]	71
3.16	Mechanical topology optimization result for a perpendicular fixation with free attachment point with a minimum frequency constraint of 500 [Hz]	71
3.17	Convergence of the frequency and the NDR for the perpendicular free fixation	72
3.18	Thermo-mechanical optimization scheme	73
3.19	Thermo-mechanical topology optimization result for a parallel fixation with imposed attachment point with a minimum frequency constraint of 500 [Hz]	75
3.20	Convergence of the frequency and the NDR for the perpendicular free fixation	76

3.21	Thermo-mechanical topology optimization result for a parallel fixation with free attachment with a minimum frequency constraint of 500 [Hz]	76
3.22	Convergence of the frequency and the NDR for the perpendicular free fixation	77
3.23	Thermo-mechanical topology optimization result for a perpendicular fixation with imposed attachment point with a minimum frequency constraint of 500 [Hz]	78
3.24	Convergence of the frequency and the NDR for the perpendicular imposed fixation	78
3.25	Thermo-mechanical topology optimization result for a perpendicular fixation with free attachment point with a minimum frequency constraint of 500 [Hz]	79
3.26	Convergence of the frequency and the NDR for the perpendicular free fixation	79
A.1	Convergence study of the three last modes	86
B.1	Fourth natural frequency with blade geometry variation	87

List of Tables

1.1	Beta coefficient values for different boundary condition	11
2.1	List of parameters	17
2.2	Standard value for the parameter's variation	18
2.3	Ti6Al4V properties	22
2.4	Blade geometry parameter ranges	24
2.5	Sine sweep test characteristics	26
2.6	Comparison of the numerical and experimental mode with the MAC	27
2.7	Frequency comparison shaker and model results X-axis	28
2.8	Frequency comparison shaker and model results Y-axis	29
2.9	Frequency comparison shaker and model results 2-axis	30
2.10	Standard value for the parameter's variation	31
2.11	Blades' parameters variation results	33
2.12	Parameter's values for the ANOVA method	34
2.13	ANOVA method results	34
2.14	Optimization process summary for a support without thermal consideration	40
2.15	Optimization process on the blades' dimensions including thermal limitation	47
2.16	Regression coefficients results	50
2.17	Neglected terms for the enhanced regression	52
2.18	Enhanced regression coefficients results	52
2.19	Mean parameter values for the worst 20% predictions compared to the overall ranges . . .	53
2.20	New parameter's range for the enhanced regression	55
2.21	Comparison of the three regressions	55
2.22	68-95-99.7 % confidence interval for the regression	56
3.1	Thermal properties of materials	72

List of abbreviations

ANOVA ANalysis Of VAriance.

AR Aspect-Ratio.

CCD Charge Coupled devices.

CME Coronal Mass Ejection.

CSL Centre Spatial de Liege.

DoF Degree of Freedom.

ESA European Space agency.

EUI Extrem Ultraviolet Imager.

EUV Extreme Ultra Violet.

EWOC Enhanced Wide-angle Observation of the Corona.

FEM Finite Element Method.

FPA Focal Plane Assembly.

ISS International Space Station.

JEDI Joint EUV coronal Diagnostic Investigation.

JWST James Webb Space Telescope.

LHS Latin Hypercube Sampling.

MAC Modal Assurance Criterion.

MMA Method of Moving Asymptotes.

NASA National Aeronautics and Space Administration.

NDR Non-Discreteness Rate.

NLOpt Non-Linear Optimization.

NOAA National Oceanic and Atmosphere Administration.

PLATO PLAnetary Transits and Oscillations of stars.

RBE Rigid Body Element.

RBM Rigid Body Mode.

SIMP Solid Isotropic Material with penalization.

SWOC Space Weather Operational Coronal imager.

Nomenclature

A	Area of the cross-section of the beam [m^2]
$\mathbf{b}$	Body force vector
ρ	Density of the material [$\frac{kg}{m^3}$]
$\mathbf{D}$	Elasticity matrix
f_n	Natural frequency associated to the nth mode [Hz]
$\mathbf{I}_n$	Identity matrix of size "n"
Δ	Laplacian
L	Length [m]
$\mathbf{g}$	Mechanical load vector
I	Moment of inertia [m^4]
$\mathbf{K}_{mec}$	Mechanical stiffness matrix
$\mathbf{K}_{th}$	Thermal stiffness matrix
σ	Stress tensor
$\mathbf{T}$	Temperature field [K]
k	Thermal conductivity [$\frac{W}{mK}$]
α	Coefficient of thermal expansion [K^{-1}]
$\mathbf{B}$	Thermal gradient matrix
$\mathbf{q}$	Volumetric heat generation vector
E	Young Modulus of the material [$\frac{N}{m^2}$]

Chapter 1

Introduction

1.1 Mission context

The understanding and prediction of space weather phenomena have become essential for ensuring the reliability and safety of modern space missions. To improve space weather monitoring capabilities, the European Space agency (ESA) has initiated the Vigil mission. This name is not an acronym unlike many other missions. It originates from the Latin *vigilis* and *vigilia* which means "sentry" or "keeping a devoted" watch. This mission was approved on May,5, 2022 [1]. This statement establishes the objectives and payload of the mission. The spacecraft will carry six instruments, four contracted by Airbus Defense and Space. The remaining two were contracted by the National Oceanic and Atmosphere Administration (NOAA) and the National Aeronautics and Space Administration (NASA) in the United States of America [2]. The launch of the mission is planned in 2031.

The spacecraft will be positioned at the fifth Lagrange point (L5) and will directly observe the Sun and its corona in the Earth's direction. Its observations will be useful by monitoring the Sun's activity before it reaches the Earth due to the Sun's rotation. It will deliver warning of an incoming Sun's activity four to five days before it reaches the Earth. Additional observation can be done in prevention of the Coronal Mass Ejections (CMEs). These valuable information can be used to prevent damages to satellite in orbit or radiation risks for astronauts in the International Space Station (ISS). Moreover, intense solar activity can lead to impacts on Earth's surface, causing disruptions to telecommunications systems and positioning errors.

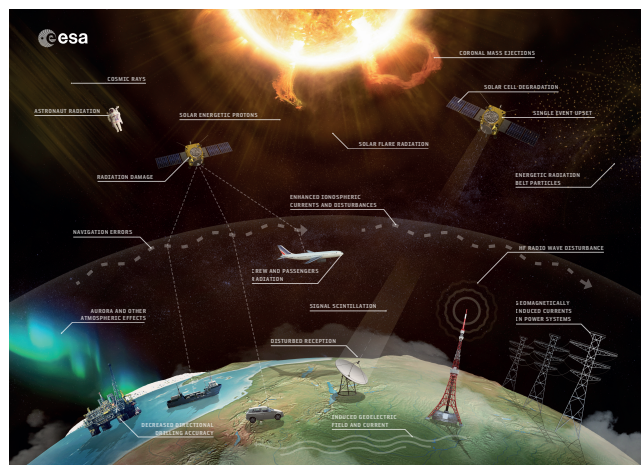


Figure 1.1: Possible impact by intense solar activity.

This thesis focuses on the Joint EUV coronal Diagnostic Investigation (JEDI) instrument, contracted by NASA and announced on May 21, 2024 [3]. As its name implies, this instrument will observe the Sun

in the Extreme Ultra Violet (EUV) domain. Observations in this wavelength range impose constraints on the Focal Plane Assembly (FPA) to ensure suitable operating conditions for the detector. These constraints may concern the structural stiffness of its support, for example by imposing a minimum first natural frequency [4], or by limiting the heat transfer between the detector and the rest of the satellite. Thermal requirements exist to facilitate detector cooling, as the detector operates at a lower temperature than the rest of the instrument. These two constraints conflict with each other. This conflict is detailed in Section 1.2, which explains why structural engineers seek an optimum solution for each application. The JEDI instrument is divided into two telescopes, the Enhanced Wide-angle Observation of the Corona (EWOC) and the Space Weather Operational Coronal imager (SWOC) [5]. Figure 1.2 shows the layout of the JEDI instrument, including the two separate optical paths and, consequently, two FPAs equipped with identical detectors inherited from the Extrem Ultraviolet Imager (EUI) instrument. Results from this study can be applied to both of the FPAs of these instruments.

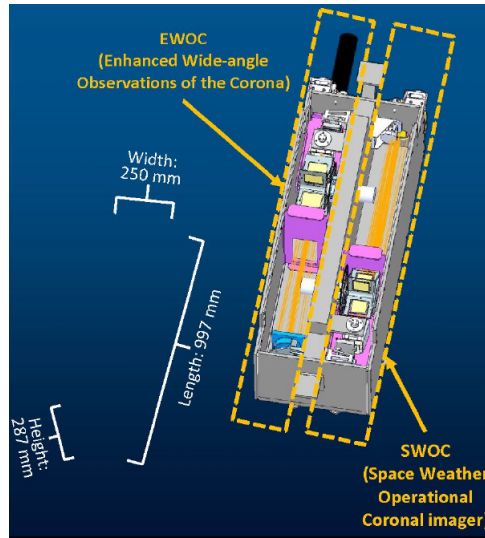


Figure 1.2: JEDI layout.

1.2 Detector support conflicting requirements

In this section, the difficulty of designing a detector support is explained. Designing a detector support structure is challenging because it must satisfy a set of conflicting requirements. There are generally two main types of conflicting requirements: one mechanical and the other thermal.

1.2.1 Mechanical requirement

The structural integrity of the instrument is essential for the proper functioning of the mission. The structure must withstand the applied loads and not fail during instrument operation. The launch phase is usually considered to impose the highest loads on the spacecraft. The mechanical requirements exist to ensure a minimum structural stiffness.

It usually takes the form of a minimum first natural frequency or a maximum allowable displacement under a given load. A minimum natural frequency is defined to avoid resonance induced by the launcher on the spacecraft. The displacement constraint prevents optical misalignment in the optical system. These mechanical requirements generally lead to an increase in the volume of the support structure. Taking the example of a support beam with a length L and a cross-sectional area A , a larger cross-section distributes the stress more effectively and prevents the beam from failing or undergoing excessive displacement. The first natural frequency can be computed using the equation below [6, 7].

$$f = \frac{3.52}{2\pi} \sqrt{\frac{EI}{mL^4}} = \frac{3.52}{2\pi} \sqrt{\frac{EI}{\rho AL^4}} \quad (1.1)$$

Equation 1.1 seems to contradict what was stated previously, as the frequency is computed using the inverse square root of the cross-sectional area. Increasing the cross-sectional area would therefore decrease the natural frequency. Nevertheless, the moment of inertia is related to the square of the cross-section. The ratio of the moment of inertia to the cross-sectional area scales with the cross-section. As a result, the natural frequency increases with the square root of the cross-sectional area. Increasing the first natural frequency therefore requires increasing the volume of the support. It can be stated that mechanical requirements tend to promote an increase in the cross-sectional area of the support.

1.2.2 Thermal requirement

In a space telescope, the operating temperature of the detector is crucial. The detector temperature has several impacts on the quality of observations.

For observations in the far-infrared domain, the relative emission temperature according to blackbody radiation laws is close to the spacecraft temperature [8]. The surroundings of the detector emit radiation in the observable domain. This phenomenon is called background noise.

Cooling the detector also affects the dark current. Dark current refers to false-positive signals caused by the thermal excitation of electrons in the detector. Dark current increases with integration time, which is necessary due to the low intensity of some sources. If the detector temperature decreases, the dark level, the lowest measured signal under completely dark conditions, also decreases [9–11].

These are the main reasons why the detector is kept at a low temperature, usually achieved through passive cooling with a radiator radiating into deep space or through active cooling to reach cryogenic temperatures. Thermal constraints are therefore important in the design of a detector support. In this thesis, radiation exchange between the support and its surroundings is neglected. This assumption is made because the software can not currently handle radiative heat transfer. In addition, the thermal environment of the detector surroundings is unknown, which affects the radiative transfer. The consideration of this heat exchange should be addressed in future studies. The limiting mechanism is thermal conduction through the support. For a structure with a constant cross-sectional area A and length L , the thermal conduction resistance can be computed using Equation 1.2.

$$R_{th} = \frac{L}{kA} \quad (1.2)$$

Where k is the thermal conductivity of the material. The thermal conductance can be expressed as the inverse of the thermal resistance as formulated in Equation 1.3.

$$GL_{th} = \frac{1}{R_{th}} = \frac{kA}{L} \quad (1.3)$$

The conductance expresses the ability to conduct heat through the structure. For a given temperature difference between the two sides, the conductance is directly related to the heat transfer. In this study, this quantity is minimized or subjected to an upper bound in order to prevent thermal coupling between the detector and its surroundings.

Equation 1.3 shows that the thermal conductance scales linearly with the cross-sectional area. Minimizing the thermal conductance therefore implies minimizing the cross-sectional area.

1.2.3 Conflict between the two requirements

As seen in the previous section, the two requirements are conflicting. While the mechanical requirement implies an increase in the cross-sectional area, the thermal requirement promotes its minimization. There-

fore, an optimum solution can be determined. This thesis aims to explore the optimization process in the design phase of a thermally decoupled space-based detector support.

1.3 State of the art

1.3.1 Flying design

This section aims to provide an overview of existing and previously studied detector support designs. The objective is not to present an exhaustive review of all possible configurations, but rather to identify key trends, typical architectures, and relevant examples that can serve as references for the development and optimization of the support structure considered in this study.

Kepler telescope

The Kepler telescope was launched in 2009 with the goal of detecting Earth-like exoplanets. Its FPA consists of an array of 42 Charge Coupled devices (CCDs) and four fine pointing sensors. The mechanical requirement for the support is that the CCDs must have a maximum displacement of $2.7 \text{ } [\mu\text{m}]$. This maximum displacement requirement implies that each support holding a pair of CCDs must be sufficiently stiff and must not bend under high loads. The thermal requirement for the detector is that the operating temperature is $-84 \text{ } [^{\circ}\text{C}]$. Achieving this temperature requires thermal decoupling between the CCD array and its surroundings. This decoupling is achieved through insulating supports holding the FPA. The design of each cell with two CCDs is shown in Figure 1.3.

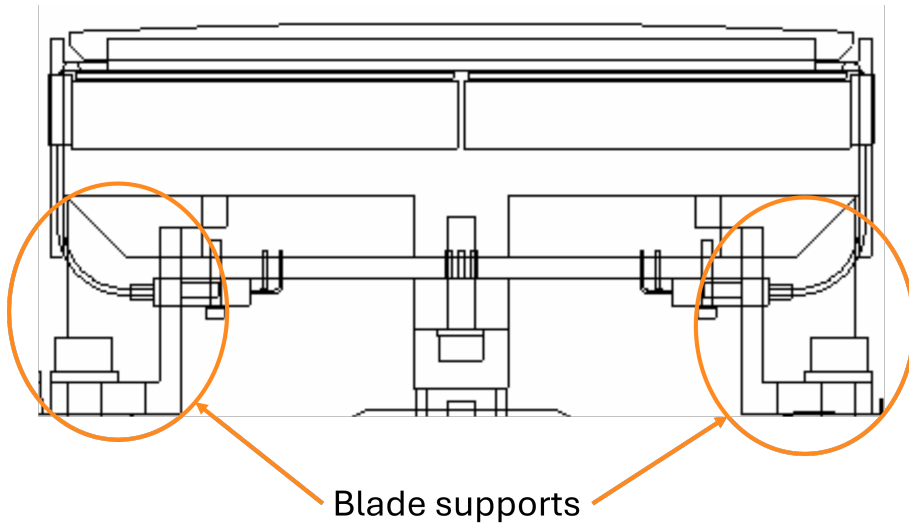
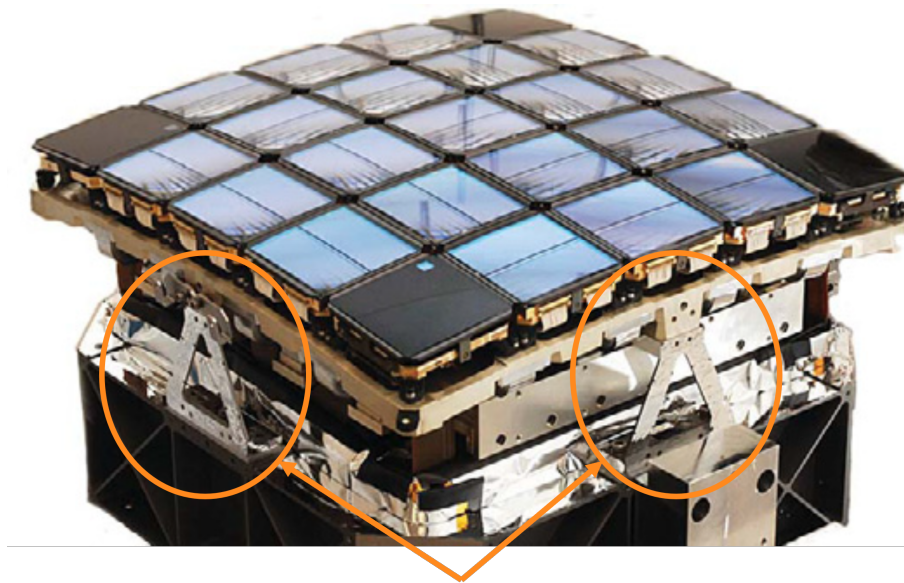


Figure 1.3: Kepler FPA cell design

Each pair of CCDs is supported by four blades, as can be seen in Figure 1.3. The array of 42 CCDs is also supported by four bipods to increase the thermal path to the surroundings and to thermally decouple the FPA. These bipods can be seen in Figure 1.4.

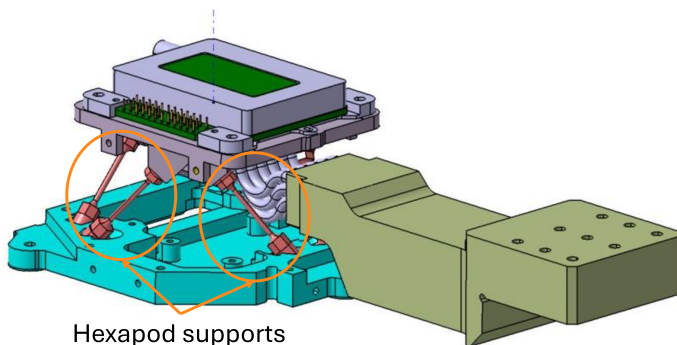


Bipod supports

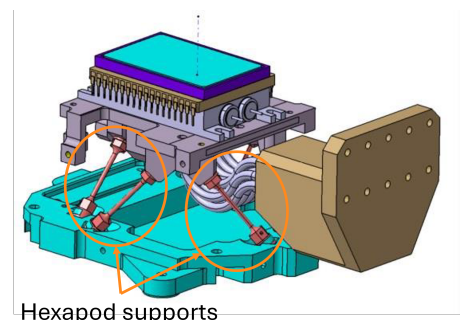
Figure 1.4: Kepler entire FPA

3MI instrument

The 3MI instrument, aboard the METOP-SG satellite, was launched in 2025, and similar satellites are currently under development. Its purpose is to study Earth's meteorology. It is composed of two telescopes, VNIR and SWIR. The mechanical requirement is that the first eigen frequency of each FPA is greater than 500 [Hz]. The operating temperatures of the detectors are -20 [°C] for the VNIR and -88.15 [°C] for the SWIR. The thermal conduction through the support is limited to a maximum of 0.005 [W/K]. The design of the supports meeting these requirements is shown in Figure 1.5.



Hexapod supports



Hexapod supports

Figure 1.5: 3MI FPA design

The FPA of each telescope is supported by a hexapod structure, ensuring its structural rigidity and low thermal conduction through each shaft of the support. These shafts are made of a titanium alloy because it offers high structural rigidity and low thermal conductivity [12].

Euclid telescope

The Euclid telescope was launched in 2023. It is designed to study the acceleration of the Universe and the three-dimensional distribution of matter. The FPA is composed of an array of 16 CCDs arranged in a four-

by-four layout. The thermal requirement limits heat conduction through the FPA support. The maximum allowable conduction between the FPA and the optical bench is 0.3 [W]. The operating temperature of the detectors is -120 [°C]. This thermal constraint is intended to prevent heat from increasing the temperature of the CCDs and, consequently, the need for a larger cooling system. The support must withstand the mechanical loads encountered during launch and be sufficiently stiff to prevent structural failure. The adopted design is shown in Figure 1.6.

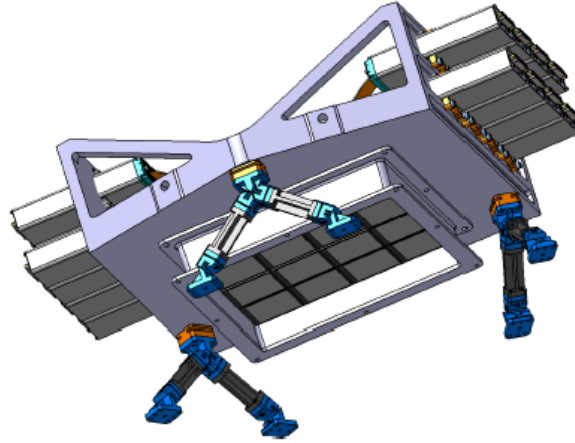


Figure 1.6: Euclid FPA design

The detector array is supported by three bipods made of titanium. This material is chosen due to its high Young's modulus, which allows it to withstand mechanical stresses, and its low thermal conductivity, which ensures thermal decoupling between the optical bench and the FPA [13].

Simbol-X spacecraft

Launched in 2013, the Simbol-X satellite is designed to observe and identify black holes. It is also useful for studying cosmic particle acceleration. It is designed to operate in the X-ray domain. This domain involves the detection of photons with energies above 17 [keV]. The detection of these short-wavelength photons does not require cooling of the detector. In the case of Simbol-X, the operating temperature of the detector is 45 [°C]. However, the FPA is thermally decoupled to avoid thermo-elastic stresses and optical misalignment. The FPA design is shown in Figure 1.7.

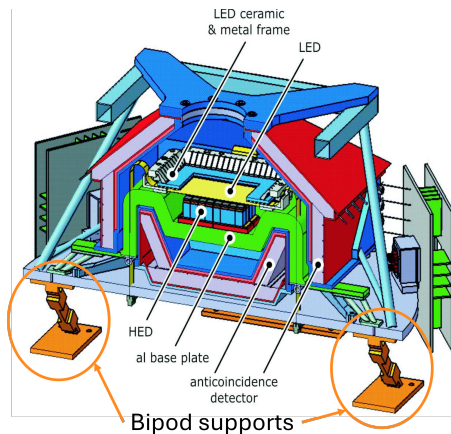


Figure 1.7: Simbol-X FPA design

The FPA is supported by four bipods with a varying cross-section to ensure mechanical stiffness and thermal decoupling [14].

James Webb Space Telescope

The James Webb Space Telescope (JWST) is the largest telescope sent into space. It orbits the second Lagrange point (L2) to be shielded from the Sun by the Earth. Onboard, the NIRCam instrument observes in the near-infrared domain. This domain requires the detector to be cooled to reduce background noise. The operating temperature of the detector is $-233\text{ }^{\circ}\text{C}$. The thermal dissipation of the optical assembly is required to be below 69 [mW] . This involves thermal decoupling between the FPA and the optical bench. This requirement prevents misalignment of the optical components. Its FPA is composed of a two-by-two array of CCDs, the design is shown in Figure 1.8 [15, 16].

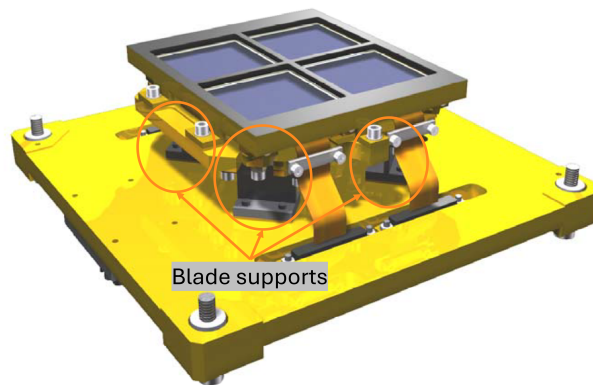


Figure 1.8: NIRCam FPA

The plane is supported by four blades similar to those used in the Kepler telescope. The blades ensure thermal decoupling through their thin cross-section and structural stiffness through their width in different orientations.

Another instrument carried by the JWST is MIRI. The MIRI instrument's pick-off mirror is mounted on a thermally decoupled support to avoid heating the structure. Thermal decoupling of a mirror is uncommon because mirrors are usually maintained at the same temperature as the surrounding structure. In the case of MIRI, the mirror is subject to decontamination using a heater bonded to its back surface. The optical system is highly sensitive to temperature variations and could become misaligned due to thermal changes. To avoid heat transfer into the structure, the mirror is therefore thermally decoupled. The final design of the pick-off mirror support is shown in Figure 1.9 [17].

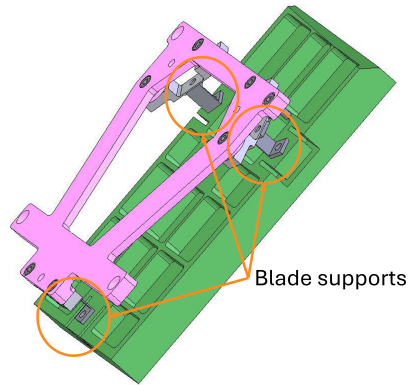


Figure 1.9: MIRI pick-off mirror support

Figure 1.9 shows that the mirror is held by three blades arranged in a triangular configuration. This geometry ensures a stiff isostatic mounting and provides thermal decoupling through the use of thin blades.

PLATO telescope

The PLANetary Transits and Oscillations of stars (PLATO) is a mission designed to detect exoplanets using the transit method. Its launch is planned for December 2026. It is composed of 26 cameras, each with its own FPA. The detector operating temperature is -80 [°C]. Excess heat is evacuated through copper braids. To ensure that the detector remains cold, the support must thermally decouple the detector from the optical bench. The instrument must withstand the mechanical loads experienced during launch without failure. The support must be sufficiently stiff to maintain optical alignment under these high loads. The final design is shown in Figure 1.10.

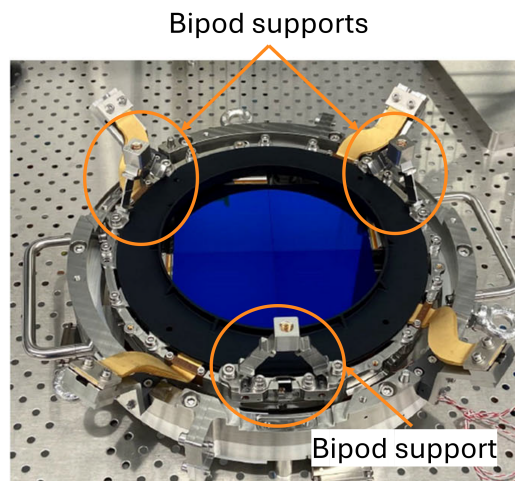


Figure 1.10: PLATO FPA design

To ensure the rigidity of the detector assembly, it is held by three bipods with varying cross-sections to achieve high thermal resistance. These bipods are made of titanium, chosen for the same reasons as previously explained: high stiffness and low thermal conductivity [18].

EUI instrument

On the Solar Orbiter mission, the EUI instrument observes the Sun, particularly its chromosphere and inner corona. The spacecraft was launched in 2020. Its orbit around the Sun makes it one of the closest satellites to the Sun, with a perihelion of 42 million kilometers. Its detector is cooled to -40 [°C], while the whole structure is exposed to the Sun. This exposition to solar radiation heats the structure. Thermal decoupling is necessary to maintain the low temperature of the detector.

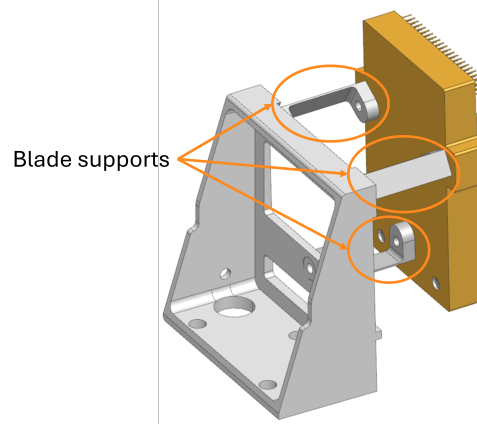


Figure 1.11: EUI FPA design

The FPA is supported by four titanium blades, ensuring both rigidity and thermal decoupling, as can be seen in Figure 1.11 [19, 20].

Conclusion

In general, two main designs stand out: the first is a blade-based design, as used in the Kepler telescope, the NIRCам and MIRI instruments onboard the JWST, and the EUI instrument. The second is a bipod design with thin cross-sections, as used in the Euclid mission, the Simbol-X telescope, and the PLATO mission. In this thesis, the focus is on the blade-based design with three blades arranged in a triangular configuration, as in the EUI instrument, shown in Figure 1.11.

1.4 Theoretical concepts

This section aims to provide a theoretical background on two concepts used in this study. The first is the Euler-Bernoulli beam equation as applied to modal analysis, and the second is thermo-mechanical topology optimization.

1.4.1 Analytical solution for a modal of a beam

A beam is a structure where one dimension is much greater than the other two. In this beam, a displacement at a given point can be defined as: let $u(u_1, u_2, u_3)$ be the displacement in a beam as a function of the coordinates x_1, x_2, x_3 . Each component of the displacement depend on the three coordinates: $u_1(x_1, x_2, x_3)$, $u_2(x_1, x_2, x_3)$, $u_3(x_1, x_2, x_3)$. Those coordinates are relative to the orthogonal Cartesian basis defining the Cartesian space $\vec{i}_1, \vec{i}_2, \vec{i}_3$ where $\vec{i}_1$ is relative to the aligned with the beam axis and $\vec{i}_2, \vec{i}_3$ define the cross-section plane of the beam.

An analytical solution of a beam displacement has been proposed by Euler-Bernoulli. As it is an analytical solution, it assumes 3 hypothesis to simplify the formulation. These assumptions are the following [21]:

- The cross-section is infinitely rigid in its own plane.

- The cross-section of a beam remains plane after deformation.
- The cross-section remains normal to the deformed axis of the beam.

The first assumption implies that there can not be a displacement in the cross section plane depending of x_2, x_3 . Then, the cross-section remains undeformed in its own plane. The displacement can be written as in Equation 1.4.

$$u_1 = u_1(x_1, x_2, x_3) \quad (1.4)$$

$$u_2 = u_2(x_1) \quad (1.5)$$

$$u_3 = u_3(x_1) \quad (1.6)$$

The second hypothesis states that the cross-section can only move according to one rigid-body translation $\bar{u}_1(x_1)$ and two rigid-body rotations $\Phi_2(x_1)$ and $\Phi_3(x_1)$. These motions are represented in Figure 1.12.

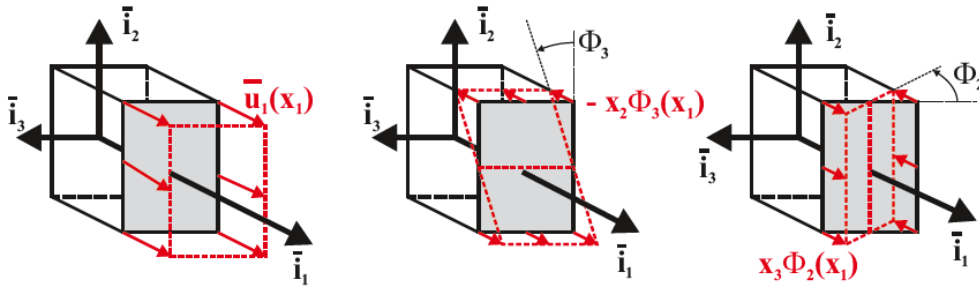


Figure 1.12: Euler-Bernoulli displacement assumption

The first component of the displacement can be rewritten depending only on x_1 :

$$u_1 = \bar{u}_1(x_1) + x_3\Phi_2(x_1) - x_2\Phi_3(x_1) \quad (1.7)$$

Note that the sign convention defining the rotation angle (Φ_2, Φ_3) is chosen such that positive rotations follow the directions of the corresponding axes $(\bar{i}_2, \bar{i}_3)$. This convention explains the negative sign on the third term in Equation 1.7.

The third assumption states that the cross-section remains normal to the deformed axis of the beam. The consequence is that the rotation angles Φ_2, Φ_3 are equal to the slope of the deformed axis. This relationship is represented in Figure 1.13.

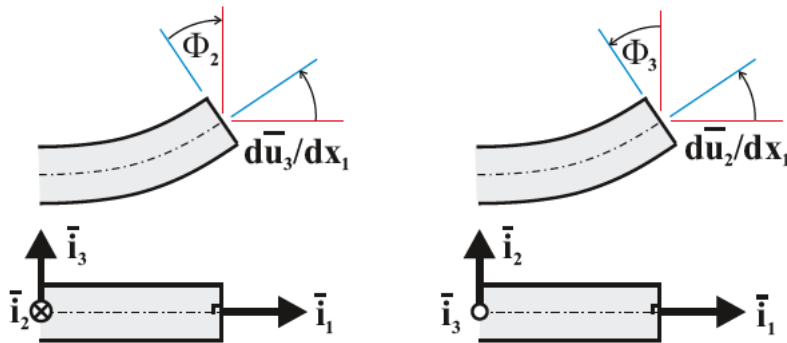


Figure 1.13: Euler-Bernoulli rotation assumption

As the slope of the beam deformation and the rigid-body rotation of the cross-section are two angles with perpendicular sides, the two angles are equal. The first component of the displacement can be written, using Equation 1.7, as:

$$u_1 = \bar{u}_1(x_1) + x_3 \frac{d\bar{u}_3}{dx_1} - x_2 \frac{d\bar{u}_2}{dx_1} \quad (1.8)$$

Equation 1.8 can be obtained because the third Euler-Bernoulli assumption states that:

$$\Phi_2 = \frac{d\bar{u}_3}{dx_1} \quad (1.9)$$

$$\Phi_3 = \frac{d\bar{u}_2}{dx_1} \quad (1.10)$$

With these assumptions, a formula for the natural frequencies of a beam with constant cross-section made of an isotropic material can be derived. From the displacement field in a beam:

$$EI \frac{\delta^4 x_2}{\delta x_1^4} + \rho A \frac{\delta^2 x_2}{\delta t^2} = 0 \quad (1.11)$$

Assuming harmonic motion and separating variables into temporal and spatial parts, the expression for the first natural frequency can be obtained [7]:

$$f_n = \frac{\beta_n^2}{2\pi} \sqrt{\frac{EI}{\rho AL^4}} \quad (1.12)$$

The coefficient β_n is relative to the n -th mode and depends on the boundary conditions of the beam. Table 1.1 summarizes values of these coefficients for common boundary conditions [6]. In this study, the boundary condition chosen from Table 1.1 is the cantilever configuration, which consists of a beam clamped at one end and free at the other.

Configuration	β_1^2	β_2^2	β_3^2
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free-free	22.4	61.7	61.7
Clamped-Clamped	22.4	61.7	61.7

Table 1.1: Beta coefficient values for different boundary condition

1.4.2 Topology optimization

In this section, topology optimization theory is presented. The purpose is to provide sufficient understanding of the algorithm used to solve a topology optimization problem. The explanation is divided into five parts: thermo-mechanical problem formulation, material interpolation laws, sensitivity analysis, filters and the algorithm used to solve the topology optimization problem.

The thermo-mechanical problem is solved by first solving the thermal problem and then computing the thermally induced strain. This strain is then introduced into the mechanical problem, which is subsequently solved. The displacement field over the entire domain is obtained.

Thermal problem

The governing equation of the thermal problem, neglecting the thermal radiation, is represented by Equation 1.13.

$$k\Delta T = \mathbf{q} \quad (1.13)$$

The equation above can be discretized and used to solve the 3D thermal problem using the Finite Element Method (FEM). Equation 1.13 becomes:

$$\mathbf{K}_{th}\mathbf{T} = \mathbf{q} \quad (1.14)$$

Equation 1.14 has the form of a standard FEM problem, where a stiffness matrix multiplies the state variable vector, in this case the temperature field, and equals a source vector corresponding to volumetric heat generation. The problem is solved using a lower-upper (LU) matrix factorization, as the matrices are sparse. Directly inverting such matrices would fill them and increase the computational cost.

Thermal induced strain

The thermally induced strain is governed by the thermal expansion coefficient α . The strain vector due to temperature distribution in an isotropic material can be written in Voigt notation:

$$\boldsymbol{\epsilon}_{th} = \begin{bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ \alpha\Delta T \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1.15)$$

Where ΔT is the temperature difference, computed by averaging the temperature at the nodes of the element and subtracting a reference temperature. There is no shear stress due to the fact that the deformation is the same in all directions because the material is isotropic. The load vector can be computed using the following equation:

$$\mathbf{g}_{th} = \int_{Volume} \mathbf{B}^T \mathbf{D} \boldsymbol{\epsilon}_{th} dVolume \quad (1.16)$$

The thermally induced load vector can be added to the mechanical problem to solve the thermo-mechanical problem.

Mechanical problem

The equation governing the static linear mechanical problem is the following:

$$\nabla^T \boldsymbol{\sigma} = \mathbf{b} \quad (1.17)$$

Under the small-displacement assumption, for a linear elastic and isotropic material, Hooke's law can be formulated. The problem can be discretized using the FEM and written in matrix form, similar to Equation 1.14.

$$\mathbf{K}\mathbf{q} = \mathbf{g} \quad (1.18)$$

This equation can be solved using an LU matrix decomposition, taking into account that the matrix is sparse and reducing the computational cost. The displacement can then be computed over the global domain.

Material interpolation

The goal of the topology optimization problem is to determine where it is necessary to have material to ensure the structural integrity under constraints. To do so, each mesh is associated to a material density μ going from zero to one. A material density of one means that the material is necessary and a zero density means that the material can be removed. A material law is applied to the thermo-mechanical properties of the material. These law are called Solid Isotropic Material with penalization (SIMP), they are used to the convergence of the optimization problem. In this thesis, two will be used the general SIMP and the SIMP-alpha laws. The second law is a mixed SIMP method which is exponential with a RAMP method which is linear. It was introduced because problem including self-weight or any volumetric acceleration do not converge with the SIMP law [22].

The goal of topology optimization is to determine where material is required to ensure structural integrity under given constraints. To do so, each mesh element is assigned a material density μ ranging from zero to one. A material density of one indicates that the material is present, while a density of zero indicates that the material can be removed. A material interpolation law is then applied to the thermo-mechanical properties of the material. These laws are called SIMP and are used to ensure convergence of the optimization problem. In this thesis, the used approach is the SIMP-alpha method. It is a mixed approach combining the exponential SIMP formulation with the linear RAMP method. It was introduced because problems involving self-weight or other volumetric acceleration effects do not converge properly with the standard SIMP method [22]. The law is formulated in Equation 1.19.

$$f_{SIMP-\alpha}(\mu) = \frac{q-1}{q} \mu^p + \frac{1}{q} \mu \text{ with } q \in [15; 20] \quad (1.19)$$

The term p is called the penalization factor and affects the convergence of the algorithm. The value of this parameter is usually chosen between three, five, and seven. Note that in the density interpolation law, the penalization factor is set to one. The material interpolation law is used to scale the material properties. In a thermo-mechanical problem, these laws are applied to the density, Young's modulus and thermal conductivity when considering thermally induced loads. The effective properties are defined by Equation 1.20.

$$\rho(\mathbf{x}) = \rho_{min} + f(\mu)(\rho_0 - \rho_{min}) \quad (1.20)$$

$$E(\mathbf{x}) = E_{min} + f(\mu)(E_0 - E_{min}) \quad (1.21)$$

$$k(\mathbf{x}) = k_{min} + f(\mu)(k_0 - k_{min}) \quad (1.22)$$

$$(1.23)$$

Where a property with a zero index denotes the nominal value of the property, and the "min" index refers to a lower bound ensuring numerical stability. This lower bound ensures that the mass, mechanical, and thermal stiffness matrices are not singular and that the problem remains solvable. The material laws are used to compute the sensitivities of the objective function with respect to the design variables.

Sensitivity analysis

Sensitivity analysis is essential for solving topology optimization problems. The goal of this part is to compute the first derivative of the objective function and the constraints with respect to the design variables. These sensitivities are obtained using Lagrange multipliers. This method avoids computing the derivatives of the displacement and temperature fields with respect to the design variables.

The sensitivity analysis drives the optimization process of the solver. In *OOFELIE*, two solvers are available. These solvers are the Method of Moving Asymptotes (MMA) and a solver from the Non-Linear Optimization (NLOpt) library. In this thesis, the second solver is used.

Filters on density and Heaviside projection

This section aims to explain two filters used in the remainder of this study. The first is the density filter, and the second is the Heaviside projection filter.

The density filter was introduced to avoid checkerboard patterns in solutions obtained from topology optimization. A checkerboard pattern is characterized by an alternating arrangement of solid and void elements with densities of one and zero, respectively. The issue with checkerboard patterns is that they correspond to non-manufacturable topologies. An example of a checkerboard pattern is shown in Figure 1.14.

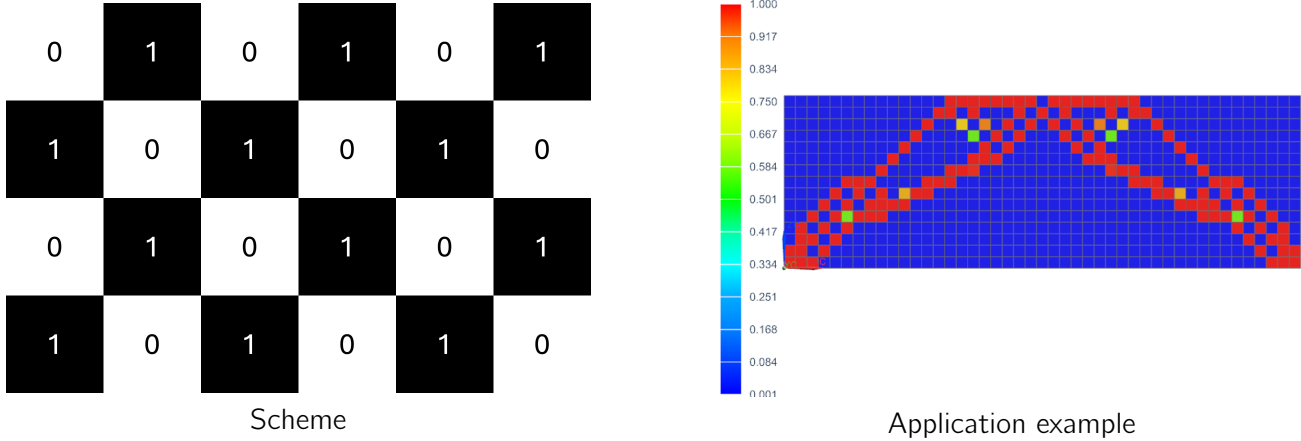


Figure 1.14: Representation of the checkerboard with a scheme and an application example

The density filter solves this problem by averaging the density of each mesh element with the densities of its neighbors within a given radius. The filter parameter is its radius. A large radius smooths a wider region but prevents a fully black-and-white converged solution. A smaller radius allows the solution to be more localized in terms of density but may permit checkerboard patterns.

The goal of the Heaviside filter is to force intermediate density values to converge toward either void or solid material, corresponding to zero and one, respectively. The filter performs a projection of the density of each mesh element using Equation 1.24.

$$\mu_{Projected} = \frac{\tanh(\beta\eta) + \tanh(\beta(\mu - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))} \quad (1.24)$$

The two parameters, β and η , in Equation 1.24 characterize the sharpness of the projection and the threshold for material definition. In other words, densities below η are projected toward zero, while densities above η are projected toward one. The parameter β controls how strongly the projected density deviates from the original density. A large β induces a significant change, whereas a small β results in a more gradual modification. The effect of these parameters on the projection can be observed in Figure 1.15.

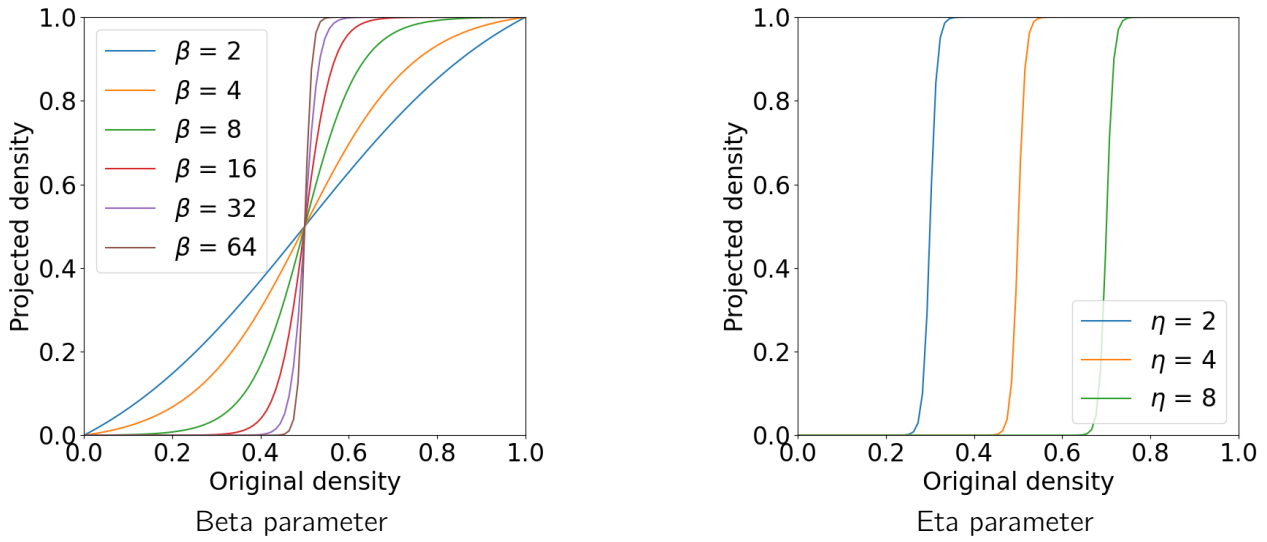


Figure 1.15: Effect of the two parameters β and η on the Heaviside projection

Figure 1.15 illustrates what has been stated above: the β parameter influences the steepness of the projection, while the parameter η shifts the projection threshold.

The Heaviside projection is purely mathematical and not part of the optimization process. Its only purpose is to force intermediate-density elements to converge toward either void or solid material. In this study, this filter is mainly used in thermo-mechanical topology optimization to improve convergence of the optimization process.

Thermo-mechanical topology optimization problem

With the thermal FEM solution, the coupling term, the mechanical FEM solution, and the sensitivity analysis, the topology optimization algorithm can be described. The method and its steps are represented in Figure 1.16.

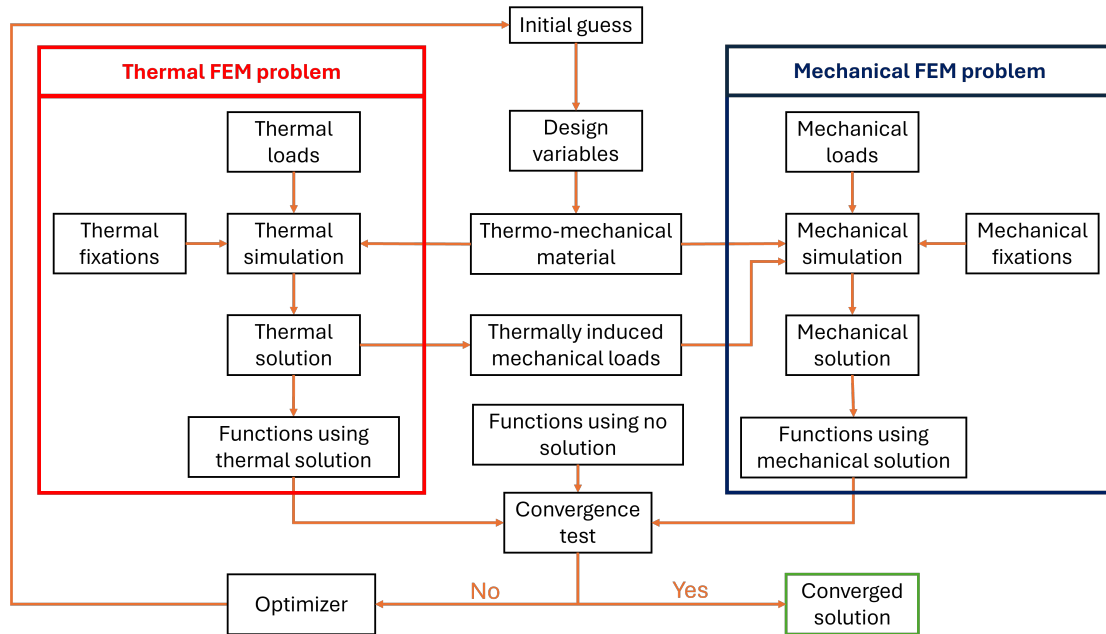


Figure 1.16: Thermo-mechanical topology optimization algorithm

The algorithm is as follows: first, an initial guess is made for the design variables over the entire domain. Then the thermal solution is computed, and the temperature field is used to derive the thermally induced load, which is then applied to the mechanical FEM problem. Next, the mechanical solution is computed, and the displacement field over the entire domain is obtained. The displacement and temperature fields are then used to compute the sensitivity analysis. The last step of the iteration is the convergence check, which can be performed using multiple criteria, in this study, it is based on the Non-Discreteness Rate (NDR). This rate is computed according to Equation 1.25.

$$NDR = \frac{4}{N} \sum_{e=1}^N \mu_e (1 - \mu_e) \quad (1.25)$$

This quantity describes the ratio of the domain that has converged. The goal of topology optimization is to determine where material should be kept or removed, if the material density is zero or one everywhere, the NDR is equal to zero. The worst case occurs when the entire domain has a material density of 0.5, in this case, the NDR takes a value of one, meaning that the solution is not converged. If the solution does not pass the convergence check, the sensitivity analysis is performed and a new guess for the design variables is generated for the next iteration. The algorithm then repeats this process until convergence is reached or the maximum number of iterations is exceeded.

1.5 Objectives

The objective of this thesis is to develop a parametric finite element model of a detector mechanical support based on designs previously used in space applications. The design is the three blades in a triangle assembly described in Section 1.3. The simulations are performed using the *NX Siemens* software. The developed model is qualitatively verified using vibration test data from the EUI detector support tested at the Centre Spatial de Liege (CSL). A mesh convergence study is conducted to ensure the reliability of the numerical model, and the results are compared with analytical predictions based on the Euler-Bernoulli beam theory.

The model is parameterized through geometric and meshing parameters. The influence of each parameter on the structural behaviour is investigated individually. Thermal constraints are subsequently incorporated into the simulations, and the design parameters are adjusted to satisfy prescribed thermal conductance requirements.

Based on these analyses, the objective is to establish a design guideline enabling the rapid identification of geometries exhibiting specific thermo-mechanical characteristics. Such a guideline aims to support the preliminary design phase by reducing the need for extensive modeling and simulation efforts.

A surrogate model is implemented based on this parametric study. Its goal is to compute the first natural frequency without any numerical computation. Its results are then compared with numerical results to characterize its accuracy and limitations.

A second objective of this work is to investigate alternative detector support concepts using topology optimization techniques. The *OOFELIE* software developed by *Open-Engineering* is employed to generate optimized topologies under mechanical and thermal objectives and constraints. Several multiphysics optimization cases are studied to evaluate potential designs for future detector support structures in space instruments.

Chapter 2

Parametric analysis of the standard 3 blades isostatic design

The first analysis of this thesis is a parametric study based on a blade support structure as presented in Figure 1.11. In this study, an isosceles triangular structure made of three blades is considered. This design is chosen because it is an isostatic mounting. Isostatic mounts are used to constrain a plane, in optical applications, these supports are useful to avoid misalignment by constraining the orientation of the surfaces. The blade structure is composed of two parts: the blade itself and the feet as can be seen in Figure 2.1. The blade is the thin part connecting the feet, which are the thicker parts where the screws hold the structure. Several parameters are varied to evaluate their influence on the modal response of the structure. The list of all parameters considered is summarized in Table 2.1.

Name	Symbol	Range	Units
Length	P_1	[20-100]	[mm]
Width	P_2	[5-50]	[mm]
Thickness	P_3	[0.5-2]	[mm]
Assembly triangle Base	P_4	[20-100]	[mm]
Assembly triangle height	P_5	[20-100]	[mm]
Detector's mass	P_6	[0.05-0.5]	[kg]

Table 2.1: List of parameters

The geometrical parameters are represented in Figure 2.1. The first three are related to the geometry of the blades, and the next two are related to the geometry of the assembly. The base of the triangle is the side that does not have equal-length sides. The height is defined as perpendicular to the base of the triangle. The last parameter characterizes the mass of the supported object, in this case a detector. However, the results of this study can be applied to any object supported by a similar structure.

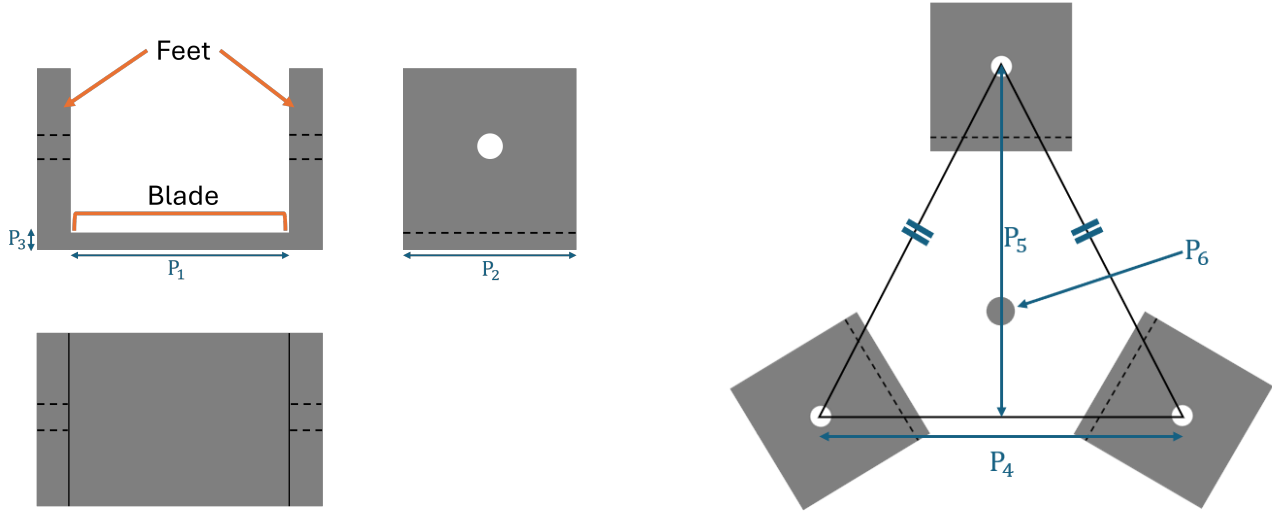


Figure 2.1: Geometrical parameter representation

The boundary condition applied to the model to avoid Rigid Body Mode (RBM) is that the structure is clamped on one side. The constraint is applied to a 0D node located at the center of the hole intended for the screws connecting each blade to the bracket on the opposite plane of the detector attachment. These boundary conditions constrain the structure while allowing displacement at the clamped feet of the support. The bracket is the component connecting the blades to the optical bench, it is represented in Figure 2.13. This model is the closest representation of a screw connection that can be achieved. The constraint is assumed to be perfectly rigid, and the node is connected to the hole surface using Rigid Body Element (RBE).

The displacement fields of the first six mode shapes obtained from this simulation are represented in Figures 2.2 to 2.7. For each figure, two views are shown: the front view and the side view. In the side view, the fixation plane is on the left and the detector plane is on the right. The parameters used in the simulation are summarized in Table 2.2. The first three parameters are chosen to be consistent with the blade geometry of the EUI instrument. The last three are chosen arbitrarily with physically consistent values.

P_1	P_2	P_3	P_4	P_5	P_6
25 [mm]	7 [mm]	0.4 [mm]	40 [mm]	40 [mm]	0.1 [kg]

Table 2.2: Standard value for the parameter's variation

The first mode shape is represented in Figure 2.2. This mode corresponds to a translation of the detector plane relative to the blades along the height direction (vertical). The fixation plane and the detector plane remain parallel, and the strain is located only in the blades and not in the feet. The frequency of the first resonant mode is 398 [Hz]. The first mode shape corresponds to a global bending of the structure.

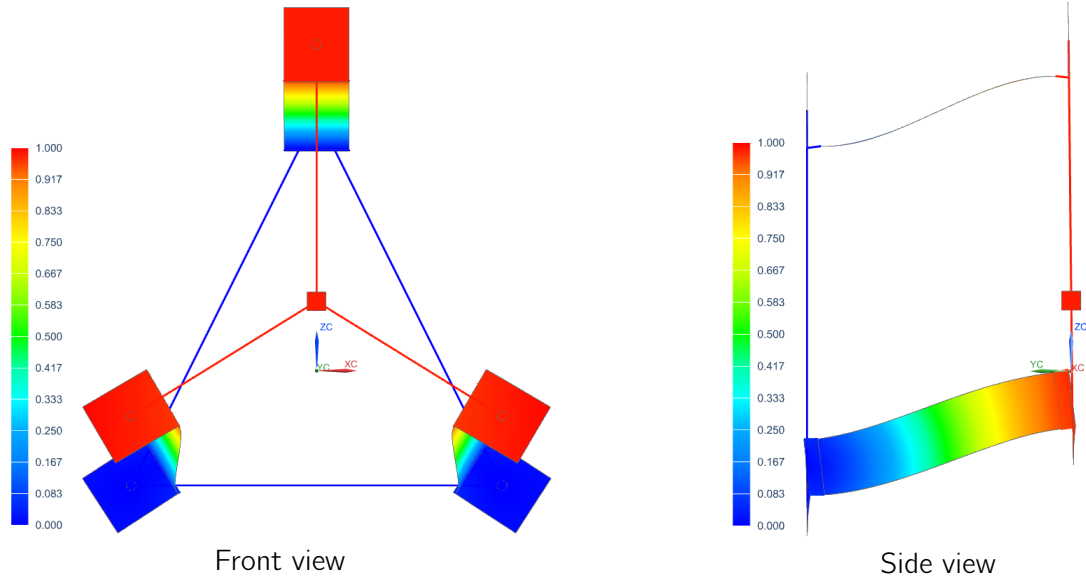


Figure 2.2: First mode shape representation, frequency = 398 [Hz]

The second mode is shown in Figure 2.3. It is similar to the first mode and corresponds to a pure bending of the structure, but the displacement occurs along the base direction (horizontal). The feet remain undeformed and undergo a RBM. The displacement is located in the blades, as in the first mode shape. The second natural frequency is close to the first resonant frequency due to the similarity of the deformation pattern, it is equal to 409 [Hz].

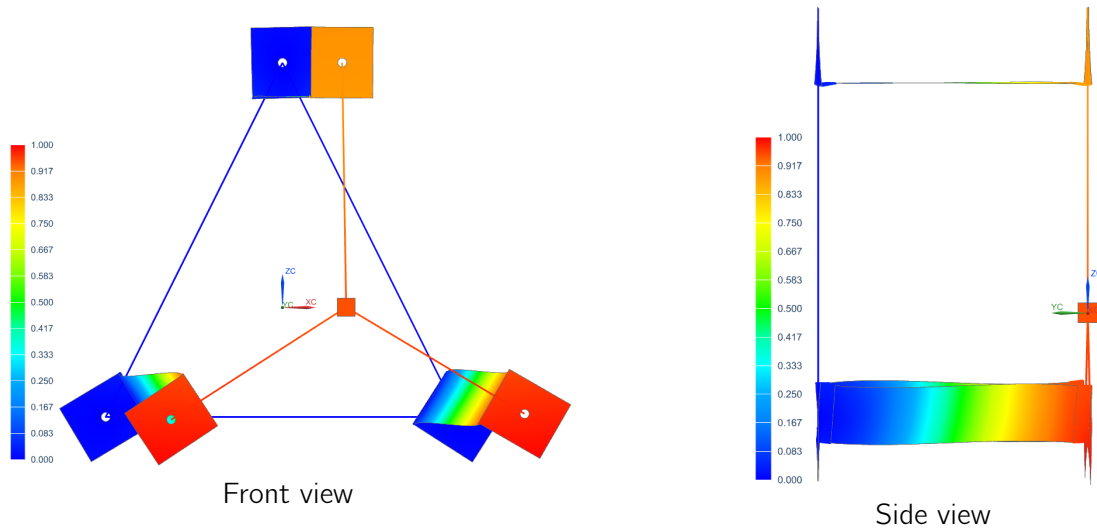


Figure 2.3: Second mode shape representation, frequency = 409 [Hz]

The third mode shape corresponds to a compression of the structure combined with bending of the blades. The structure becomes shorter along the longitudinal direction of the blades. The frequency of this mode is much higher than the first two modes, at 2476 [Hz]. The maximum displacement occurs at the ends of the lower feet.

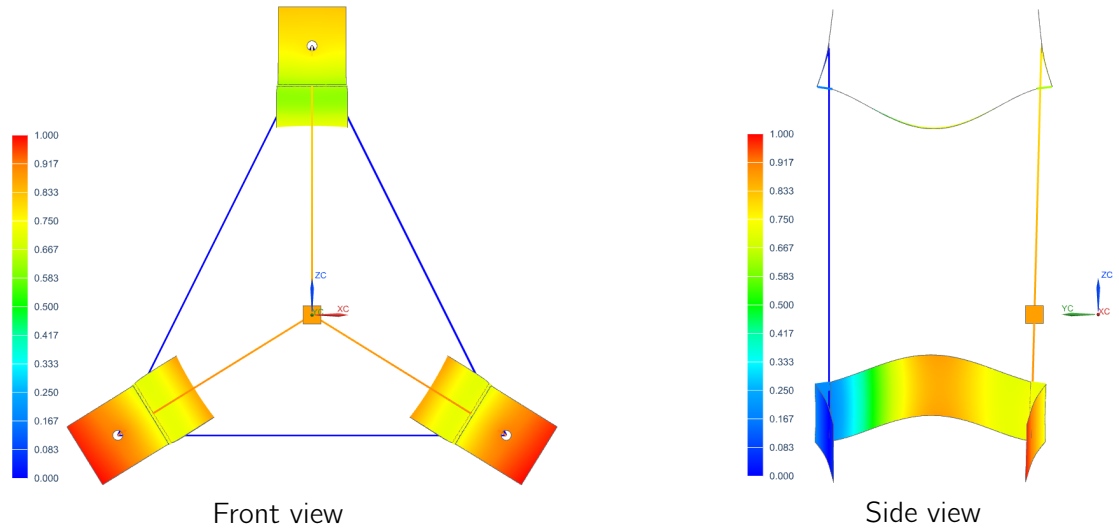


Figure 2.4: Third mode shape representation, frequency = 2476 [Hz]

The fourth mode, represented in Figure 2.5, corresponds to a torsion of the support. The two planes remain parallel to each other, and the detector plane rotates around the detector's 0D node. The maximum displacement occurs at the points farthest from the detector node. The oscillation frequency of this mode is 3150 [Hz].

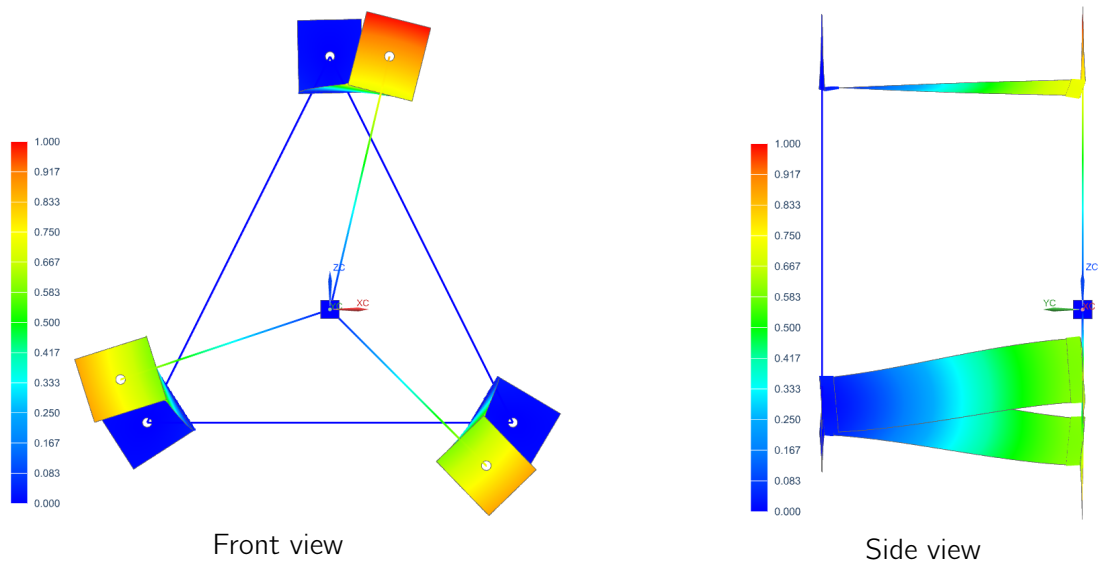


Figure 2.5: Fourth mode shape representation, frequency = 3150 [Hz]

The fifth mode shape is displayed in Figure 2.6. It corresponds to the bending of the two lower blades across the base of the triangular assembly, while the top blade remains straight. The maximum displacement occurs at the middle of the lower blades. This mode oscillates at a frequency of 3509 [Hz].

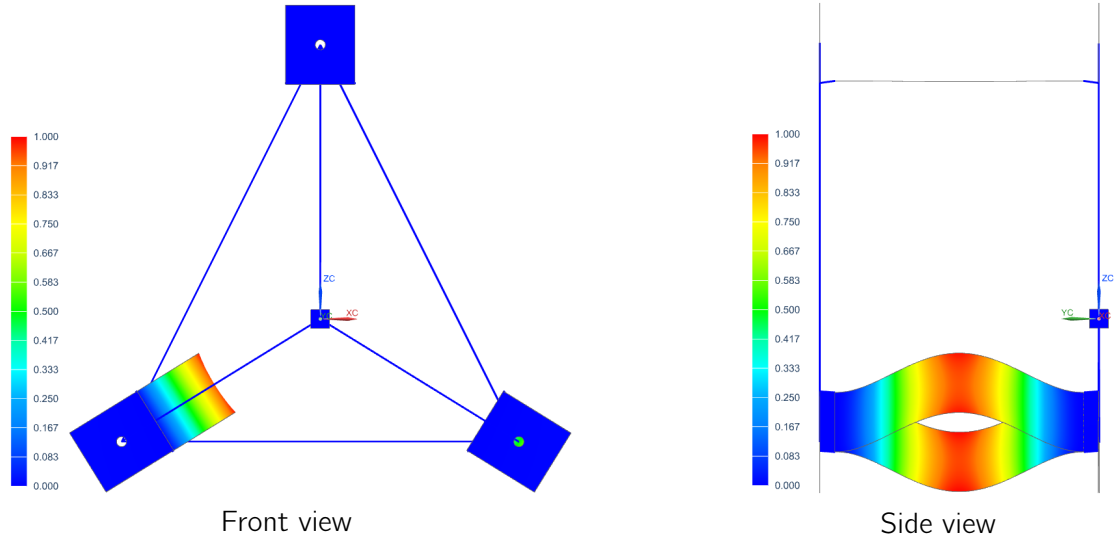


Figure 2.6: Fifth mode shape representation, frequency = 3509 [Hz]

The sixth mode shape, represented in Figure 2.7, is similar to the fifth mode shape. The frequencies of the last two mode shapes are identical, equal to 3509 [Hz]. However, the magnitude of the bending in the blades is inverted: the lower blades exhibit smaller bending displacement compared to the top blade, which shows the largest displacement of this mode shape at its midpoint. The two planes at the ends of the blades remain parallel and do not exhibit relative motion with respect to each other.

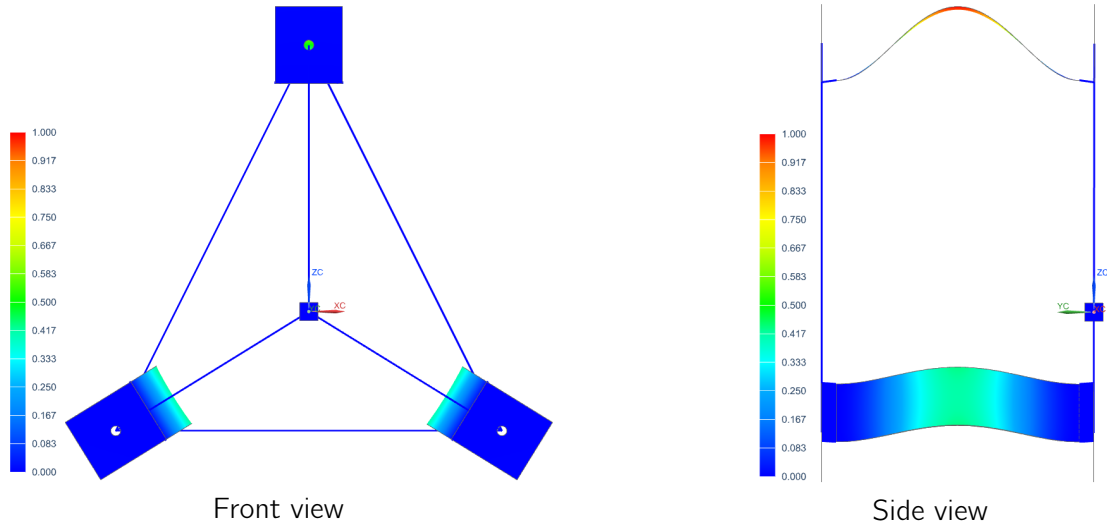


Figure 2.7: Sixth mode shape representation, frequency = 3509 [Hz]

Based on this model, a parametric study is conducted. It is structured into five parts: model validation, methodology for result generation, presentation of results, and a discussion.

2.1 Verification of the model

The verification of the model is very important. This step ensures that the results computed numerically are close to those that can be obtained in reality. The verification is carried out in four parts. First, the assumptions of the model are stated in order to identify its limitations. Then, a mesh convergence study is performed to determine the minimum number of elements per side required to ensure simulation accuracy.

After that, the model is compared with the analytical solution presented in Section 1.4.1. Finally, the model is compared with vibration test data obtained by CSL on the EUI instrument.

2.1.1 Assumptions of the model

Several assumptions are made during the parametric simulation. Solving a numerical model involves simplifying the physics and introducing modeling assumptions. Future modal analyses are performed under the hypothesis of linear elasticity, neglecting plasticity and other non-linear effects. Small displacements are also considered, their order of magnitude is not comparable to the dimensions of the geometry. The damping of the structure is also neglected, and the simulations solve an undamped free vibration eigenvalue problem. Perfect boundary conditions are assumed at the screw connections. The material is Ti-6Al-4V, also known as titanium grade five, and is considered isotropic with the properties listed in Table 2.3 [20]. The Young's modulus is higher than that reported in the cited sources, but this is discussed in Section 2.1.4. This value is still acceptable and has been experimentally demonstrated as possible [23].

Density	ρ	4430 [kg/m ³]
Young's modulus	E	120 [GPa]
Poisson's ratio	ν	0.34 [-]
Thermal conductivity	k	7 [W/mK]

Table 2.3: Ti6Al4V properties

The mesh is made of linear quadrilateral elements (*Quad4*). To reduce computation time, the detector is considered to be much stiffer than the support. The entire detector is then modeled as a lumped mass (P_6) located at the incenter of the assembly triangle. This lumped mass has no rotational inertia, this choice is made because the detector mass may vary, and computing its inertia would be difficult due to this variability. This node is connected to the screw fixation using RBE. These assumptions may limit the accuracy of the results, particularly regarding boundary condition modelling and potential nonlinear effects.

2.1.2 Convergence of the model

In this section, a convergence test is performed to determine the minimum number of elements per side and to avoid numerical shear locking. This effect makes the model artificially stiffer than reality and is due to an insufficient number of finite elements in a given dimension. In this case, the thin parts are meshed with 2D elements, but the thicker part where the screws are fixed was initially meshed with 3D tetrahedral elements. The convergence study is carried out by varying the mesh size. It revealed that the first six natural frequencies do not converge with a reasonable number of 3D elements.

The solution was to model each blade using three planar surfaces discretized with 2D shell elements. Natural frequencies as a function of the number of elements across the blade width are shown in Figure 2.8. For clarity, only the error of the first sixth modes are presented in this section, the frequency convergence graphs are included in Appendix A. As expected, the relative error, computed with respect to the results obtained with 50 elements, decreases as the number of elements increases. Shear locking appears at low mesh densities, as the frequency converges from above, a low number of elements makes the structure artificially too stiff. In the remainder of this thesis, a discretization of 40 elements is chosen to ensure a relative error below 0.5 percent. This value was also used by Hongan Xu et al. [24]. A dynamic meshing strategy is used to ensure at least 40 elements per side. To achieve this, the mesh size on the blade is set equal to the smallest dimension between the length and width divided by 40.

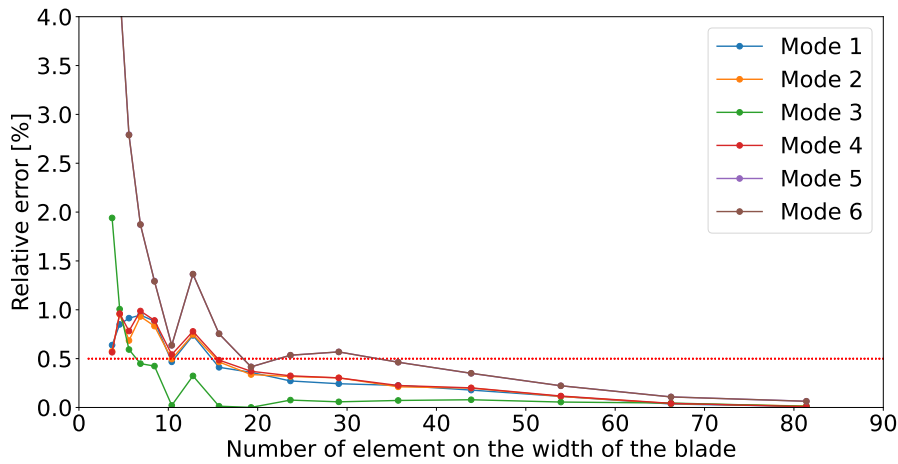


Figure 2.8: Convergence of the relative error for sixth first modes

2.1.3 Comparison with Euler-Bernoulli analytical solution of hollow triangular section beam

The purpose of this section is to compare the numerical results obtained with *NX Siemens* to analytical results computed using the Euler–Bernoulli beam theory. This theory is applied to a hollow triangular cross-section beam. Since the first mode corresponds to bending of the blade structure, only pure bending is considered in the analytical model. The second moment of area of the triangular cross-section is computed as [7]:

$$I_{yy} = \frac{\sqrt{3}}{96} a^4 \quad (2.1)$$

Where a is the side length of the triangular cross-section. The moment of inertia of the hollow triangular cross-section is computed by subtracting the inertia of the inner triangle from that of the outer triangle. From Equation 1.12, and using the β parameter for a clamped-free beam given in Table 1.1, the first natural frequency can be computed and expressed as a function of the beam length. The side length of the triangle can be compared with the width (P_2) of the numerical model. The wall thickness of the hollow beam is compared with the thickness (P_3) of the structural model. The schematic of the beam cross-section is shown in Figure 2.9.

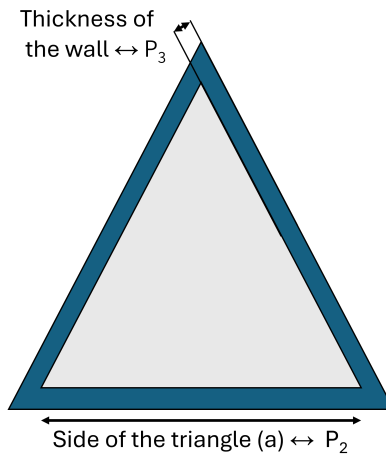


Figure 2.9: Hollow triangular beam cross-section

The results are represented in the figure below. The variation of the dimensions is within the range summarized in Table 2.4.

Parameter's name	Range [mm]
Length	30-100
Width	5-50
Thickness	0.5-2

Table 2.4: Blade geometry parameter ranges

These ranges are chosen to explore a wide range of possible designs while remaining physically reasonable. The length variation could be wider, but it would not make sense to consider a design with a length greater than 100 [mm]. A shorter length could also be considered, but it would eventually violate the beam assumption which is that one dimension is greater than the two others. The width has a large variability, but the highest values cannot be implemented due to the thermal decoupling requirement. The thickness is kept small to reduce thermal conduction through the blades. A thickness larger than the width could be considered, but the first mode would then be torsion around the detector, similar to the fourth mode shown in Figure 2.5. Comparing this mode to a pure bending analytical model would therefore be meaningless. Note that some parameter combinations may violate the beam assumption.

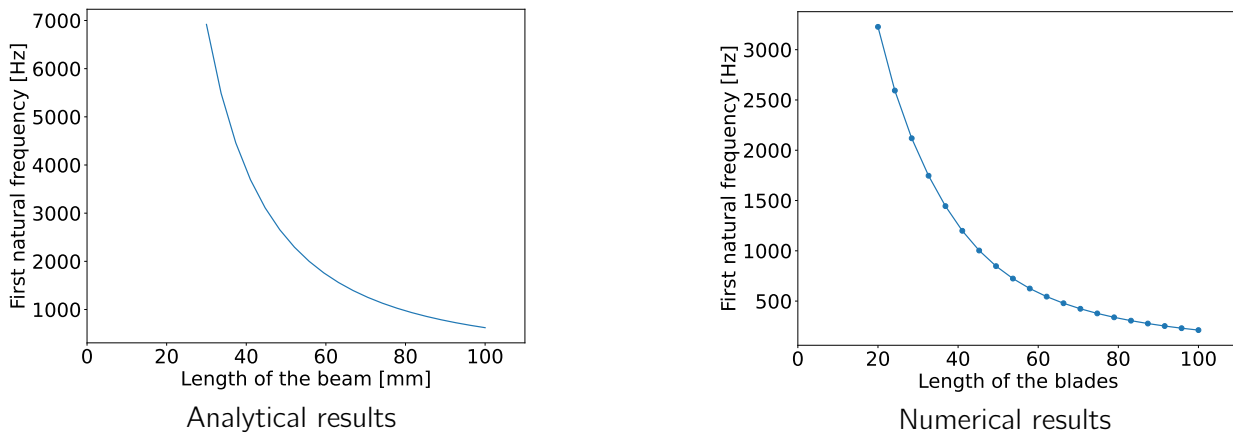


Figure 2.10: Comparison of the analytical and numerical solution with a length's variation

Figure 2.10 shows similar results. These results are obtained by fixing the width at 10 [mm] and the thickness at 0.5 [mm], while varying the length. For short lengths, the analytical solution predicts higher frequencies, due to the beam not having a sufficiently large aspect ratio. The beam assumption is no longer valid when the length becomes small. Qualitatively, both solutions show an exponential decay of the first natural frequency as the length increases.

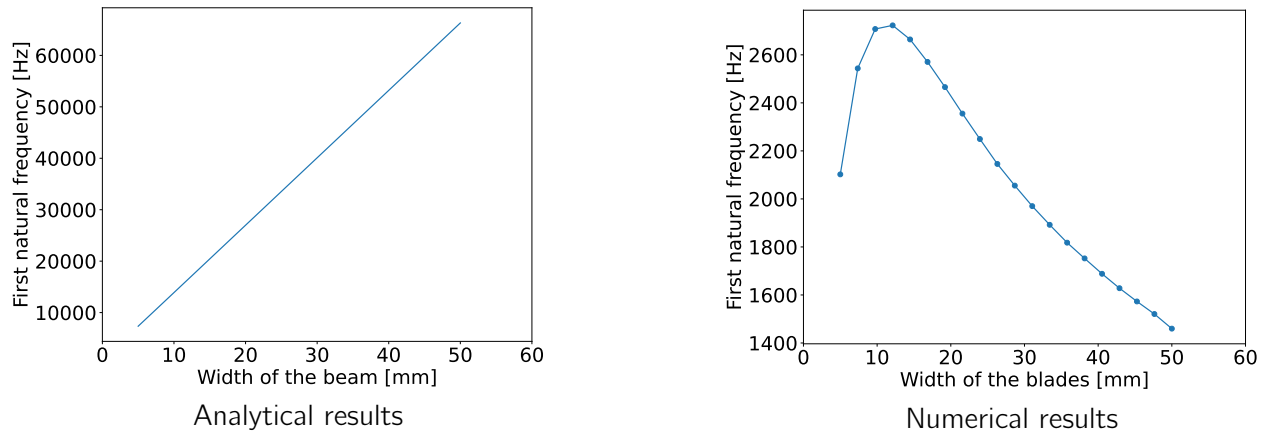


Figure 2.11: Comparison of the analytical and numerical solution with a width's variation

The variation of the width represented in Figure 2.11 shows a clear difference in behaviour between the analytical and numerical results. This is due to the different influence of the width in the two approaches. In the analytical model, the width variation is introduced by modifying the side of the cross-section of the beam. This change affects the second moment of area in Equation 2.1. The Euler-Bernoulli formulation predicts the response of a beam with an increased second moment of inertia, the structure is therefore stiffer and the natural frequency increases. Furthermore, the analytical solution is restricted to bending modes, but as the width increases, torsional effects appear in the beam. The Euler-Bernoulli formulation does not account for this phenomenon, leading to an overestimation of the first natural frequency.

The FEM model captures the torsional behaviour of the blade, and the natural frequency therefore decreases when the width increases beyond 10 [mm]. Another explanation is that the beam hypothesis-one dimension being much larger than the two others-is violated, as the blade length is 25 [mm] and the width becomes greater than the length. The decrease in natural frequency can also be explained by the increase in mass with width. As the mass increases, the natural frequency decreases. This is due to the fact that the resonant frequency is inversely proportional to the square root of the mass, as explained in Section 2.3.1.

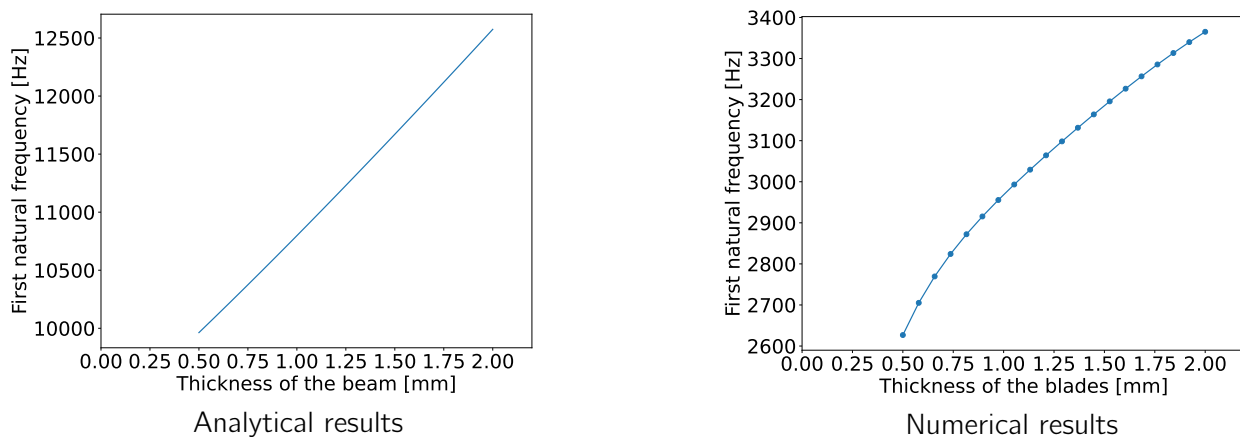


Figure 2.12: Comparison of the analytical and numerical solution with a thickness's variation

Figure 2.12 shows similar behaviour between the analytical and numerical results. Even if the first natural frequency values are not similar, both approaches show a linear, or nearly linear in the case of the numerical results, increase in frequency as the thickness increases. The difference in order of magnitude can be explained by the analytical solution assuming pure bending and by the violation of the beam as-

sumptions.

Overall, the numerical model and the analytical solution show good agreement. The observed discrepancies can be attributed to the limitations of Euler–Bernoulli beam theory and to situations in which its underlying assumptions are not satisfied. The FEM model therefore provides physically consistent results. The next step is to compare these results with experimental data in order to assess their validity.

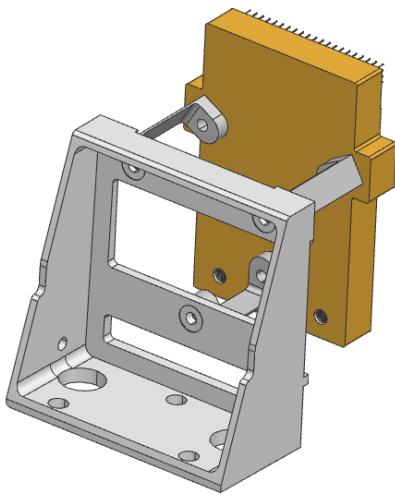
2.1.4 Comparison with Extreme UV Imager (EUI) vibration test

In this section, the model is compared to data obtained from a vibration test performed on the EUI instrument at the CSL. The test is based on a low-level sine sweep and has the following characteristics:

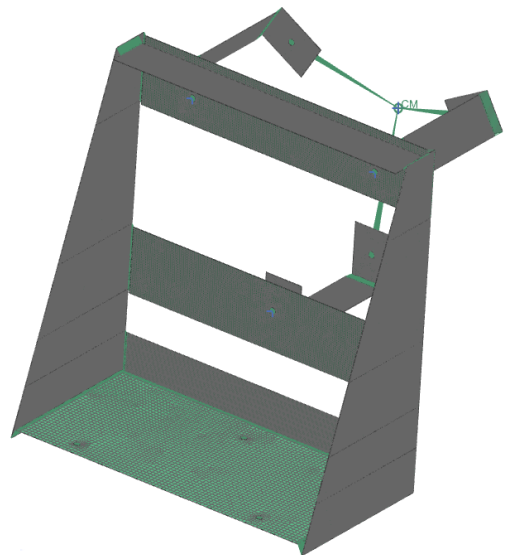
Frequency	Level	Sweep rate
5-2000 [Hz]	0.5 [g]	2 [Oct/min]

Table 2.5: Sine sweep test characteristics

A similar test is performed in *NX Siemens*. The geometry is modeled in the software with several simplifications. As the structure is composed mainly of plates, it is modeled using 2D shell elements, with a mesh density consistent with the number of elements per side obtained in the convergence study in Section 2.1.2. The detector is assumed to be much stiffer than the support and is modeled as a node located at the geometric center of the support. Dimensions are preserved during simplification, although some plate thicknesses are adjusted to account for stiffer elements not represented in the simplified model. The connections between the plates, the bracket, and the blades are made using RBE. Initially, Young's modulus was set to 114 [GPa], but it was tuned to 120 [GPa] to match the first mode frequency more accurately. The simplified and full models are shown in Figure 2.13.



Original model



Simplified and meshed model

Figure 2.13: Comparison of the original model and the simplified

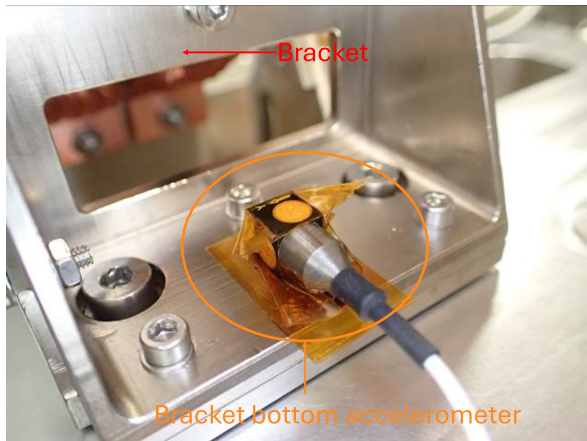
The sine excitation was applied in the direction parallel to the blades. In the shaker test, the accelerometer was placed on the side of the detector, in the model, the acceleration is extracted at the detector node. The directions used in the result comparison are:

- x-axis: parallel to the optical path (excitation direction).
- y-axis: perpendicular to the optical path and in the horizontal plane.
- z-axis: vertical.

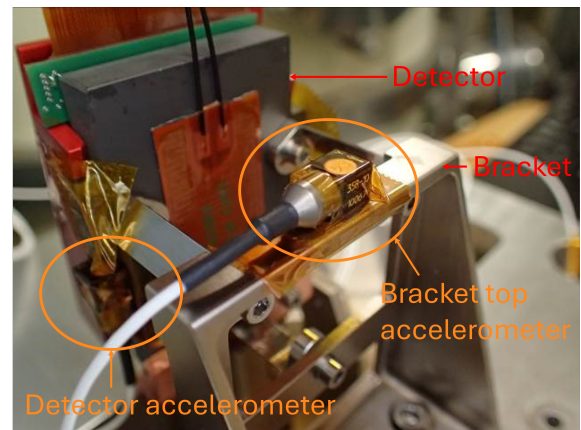
With the model and the data from the vibration test, the modes can be identified using the Modal Assurance Criterion (MAC), by comparing the experimentally and numerically obtained mode shapes. The MAC is based on the following equation:

$$MAC_{ij} = \frac{|\phi_{exp,i}^T \phi_{num,j}|^2}{(\phi_{exp,i}^T \phi_{exp,i})(\phi_{num,j}^T \phi_{num,j})} \quad (2.2)$$

Where ϕ is the mode shape, including the acceleration of the measured Degree of Freedom (DoF). A MAC value close to 100 [%] indicates that the numerical mode is comparable to the experimental mode. During the experiment, three accelerometers were placed: two on the bracket and one on the side of the detector. Figure 2.14 shows the placement of these accelerometers for the vibration test. In the numerical simulation, equivalent measurement node locations are used to track the acceleration at these points and to apply Equation 2.2. As the detector is modeled using a single 0D node, the acceleration measured on the side of the detector corresponds to the displacement of this node itself. The bottom bracket accelerometer cannot be replicated in the numerical model because the bottom of the bracket is assumed to be fixed. The MAC is applied using two DoFs, namely the detector-side and the top-bracket accelerometers.



Bottom of the bracket



Top of the bracket and detector

Figure 2.14: Accelerometer placement during the EUI shaker experiment

The MAC matrix and the modal frequencies are summarized in Table 2.6. The experiment identified three modes at 392, 574, and 726 [Hz], while the numerical model identified four modes at 338, 383, 866, and 1159 [Hz].

	Numerical modes	Mode 1	Mode 2	Mode 3	Mode 4
Experimental modes	Frequency [Hz]	338	383	866	1159
Mode 1	392	20.59 %	0.59 %	0.3 %	1.60 %
Mode 2	574	10.02 %	55.06 %	83.59 %	85.42 %
Mode 3	726	38.83 %	0.28 %	0.16 %	0.42 %

Table 2.6: Comparison of the numerical and experimental mode with the MAC

Table 2.6 shows that the MAC is close to 100 [%] for the association of the third and fourth numerical modes with the second experimental mode, with a correlation close to 85 [%]. The first mode from each

method correlates at 20 [%], which is not sufficient to be significant. The same analysis can be performed for the association of the first numerical mode with the third experimental mode. The correlation of 38 [%] is not significant enough to draw any conclusion. The poor agreement between the two methods is due to the fact that only two useful accelerometers were available during the experiment. The mode shapes are not well identified with two sensors and only six DoFs. For future comparisons of the model, it would be beneficial to include more sensors during the experiment. Multiple tests may also be required to avoid influencing the modal dynamics of the structure through the mass and stiffness of the accelerometers.

As the MAC does not provide a clear modal correspondence between numerical and experimental results, the comparison is instead performed by associating natural frequencies and effective masses. The results are separated into three groups for each acceleration direction. The comparison is presented in Figures 2.15, 2.16, and 2.17.

Acceleration along X-Axis

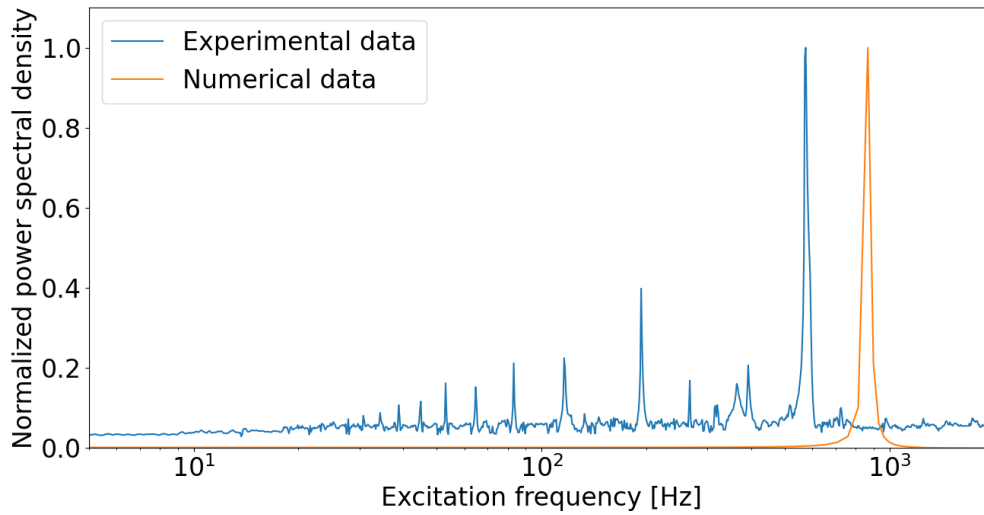


Figure 2.15: Power spectral density comparison with the experimental and numerical responses to a excitation along the X-axis

Figure 2.15 shows similar behaviour between the experimental test and the numerical model. The differences in the natural frequencies at the peaks can be explained by material uncertainties, differences in boundary conditions, damping effects, or structural non-linearities at the fixations, for example. These non-linearities may also explain the lower peaks observed before the main resonance peak. Non-linearities can generate harmonics preceding the natural frequency, these are referred to as subharmonics [25]. Their frequency corresponds to the resonance frequency divided by an odd integer. As the model assumes a linear response, these peaks cannot be reproduced by the software. Peaks that do not correspond to a fractional division of the resonance frequency may be interpreted as subharmonics of higher-frequency modes (above 2000 Hz) or as experimental noise. The experimental and numerical frequencies are compared in Table 2.7.

Experimental [Hz]	Numerical [Hz]	Effective mass [%]	Relative error [%]
574.45	866.64	85.67	50.86

Table 2.7: Frequency comparison shaker and model results X-axis

The error is not negligible, but the purpose of this study is to predict the first mode and its corresponding

frequency, the third mode shows a larger relative error than the first one. Qualitatively, the dynamic behaviour is dominated by a single mode influencing the acceleration along the X-axis.

Acceleration along Y-Axis

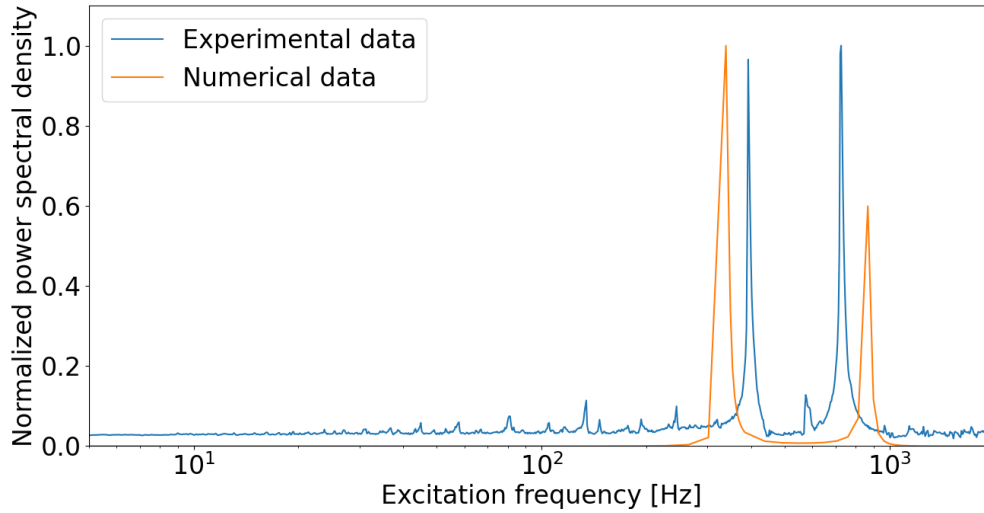


Figure 2.16: Power spectral density comparison with the experimental and numerical responses to an excitation along the Y-axis

The same analysis can be performed for Figure 2.16, where the numerical model appears to represent the dynamics of the structure adequately. The two peaks in the shaker test at 393 and 726 [Hz] are compared with the four peaks at 338, 383, 866, and 1159 [Hz] obtained from the numerical simulation. Once again, nonlinear effects and subharmonics can be observed in the shaker data. It can also be noted that, as the mode number increases, the agreement becomes less clear.

Experimental [Hz]	Numerical [Hz]	Effective mass [%]	Relative error [%]
392.57	338.73	0.00004	13.71
392.57	383.19	84.66	2.39
726.10	866.64	0.00000	19.36
726.10	1159.48	3.84	59.69

Table 2.8: Frequency comparison shaker and model results Y-axis

Acceleration along Z-Axis

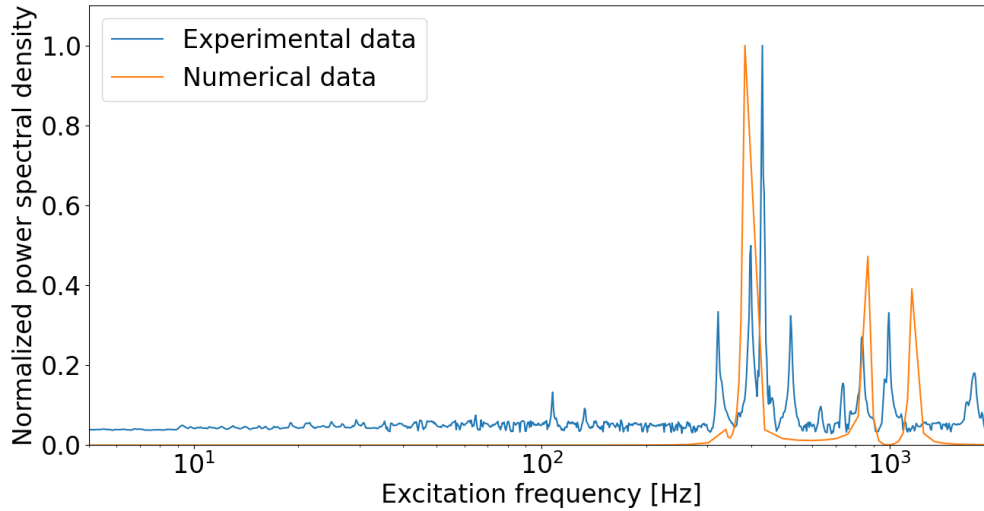


Figure 2.17: Power spectral density comparison with the experimental and numerical responses to a excitation along the Y-axis

The numerical model produces results that are similar to the experimental data. Two peaks can be observed in both graphs in Figure 2.17. The frequencies of the experimental peaks are 392 and 726 [Hz], and they are compared to the numerical results of 338 and 866 [Hz]. Based on this comparison, the two modes at 386 and 1159 [Hz] identified in the Y-direction are not supported and are likely misidentified. The peak frequency values are summarized in Table 2.9.

Experimental [Hz]	Numerical [Hz]	Effective mass [%]	Relative error [%]
392.57	338.73	73.96	13.71
726.10	866.64	0.0076	19.36

Table 2.9: Frequency comparison shaker and model results 2-axis

The error on the first mode is close to 14 [%]. This error is considered acceptable, as the purpose of this thesis is to estimate the first natural frequency for preliminary design. High accuracy is not the primary objective. The error may be influenced by the assumptions made in the bracket model. However, the brackets will not be modeled in the remainder of this study. The error on the first mode induced by the support model is therefore less than 14 [%] when only the blade support is considered.

2.2 Methodology

The remainder of the study conducted with *NX Siemens* requires multiple modal simulations. A Python script was developed to automate the simulation process and reduce the workload for the rest of this thesis. This was made possible thanks to the *NXOpen API in Python*. The script is able to generate the parameter list for each analysis, apply these parameters to the model, and run the simulations. The results are stored in *Excel* files and analyzed using a separate *Python* script to generate plots and meaningful results.

2.3 Parameter's influence

This section aims to analyze the effect of each parameter on the modal response of the structure. The variation is performed by varying each parameter independently in order to isolate its influence. The baseline case is defined using the same blade geometry as the EUI instrument, while the assembly geometry and detector mass are chosen within a realistic range. The baseline case parameters are summarized in Table 2.10. These values are the same as those used for the mode shape representation in Table 2.2. When varying a given parameter, all others are kept constant and equal to the values listed in the table mentioned above.

Length	Width	Thickness	Triangle's base	Triangle's height	Detector's mass
25 [mm]	7 [mm]	0.4 [mm]	40 [mm]	40 [mm]	0.1 [kg]

Table 2.10: Standard value for the parameter's variation

This study is divided into two parts: first, without any consideration of the thermal constraint, then, a thermal limitation is introduced to ensure the required thermal decoupling.

2.3.1 Excluding thermal limitation

The influence of the first three parameters is presented in Section 2.1.3. However, one difference is the addition of the detector mass at the end of the support. The variation of the last three parameters is studied to visualize their impact on the first natural frequency. In this section, thermal conduction is neglected as a design constraint, and the dimensions are free to take any value. The first analysis concerns the variance of the first three parameters (length, width, thickness). The second study focuses on the shape of the assembly triangle, followed by an investigation of the influence of its size. Finally, the influence of the detector mass is studied.

Blade's geometry variation

In this section, the variation of the blade geometry is analyzed. Similar results are presented in Figures 2.10, 2.11, and 2.12, but in this section the mass of the detector is also considered. The variation of the three blade parameters is studied and presented in Figure 2.18. The parameter ranges are the same as those used in Section 2.1.3.

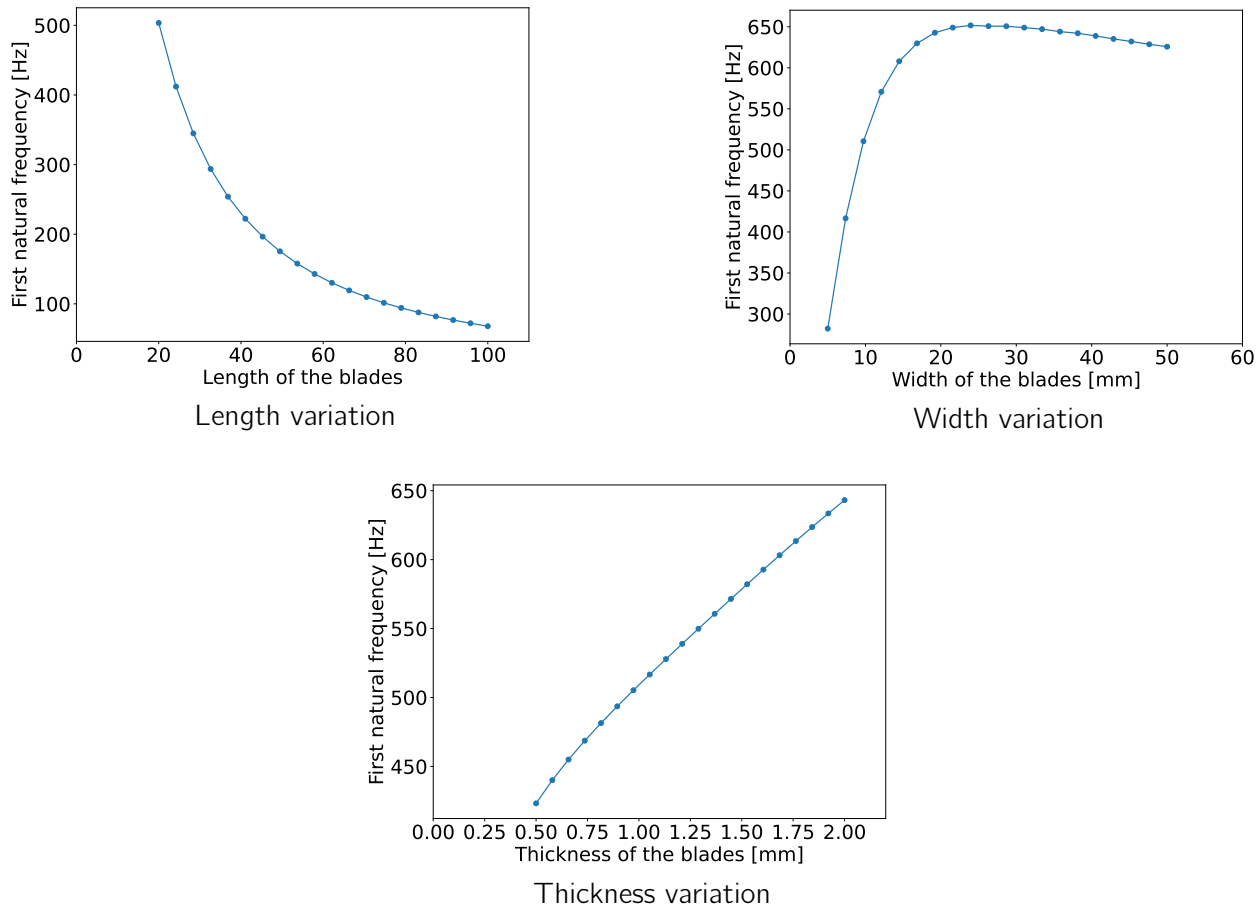


Figure 2.18: Blades' parameters variation

The variation of the length with mass follows a similar trend to that shown in Figure 2.10. The difference lies in the order of magnitude of the resonance frequency. The effect of the applied mass is studied in Section 2.3.1. The presence of mass leads to a decrease in the natural frequencies, while the trend induced by the variation of length remains unchanged. The first natural frequency decreases exponentially with increasing length. The total variation of the frequency ranges from 68 to 503 [Hz] and represents 87 [%] of the maximum frequency. The length therefore has a major impact on the frequency.

The width has a different impact on the modal analysis with and without the detector mass. We expect a similar behaviour to that observed in Section 2.1.3, but in this case the frequency appears to decrease at a slower rate as the width increases. The main discrepancy is that the detector mass is taken into account in these simulations, whereas it is not included in the comparison with the analytical solution. The influence of the added mass is reduced because the ratio between the structural mass and the detector mass is small.

Quantitatively, the variation of the blade width affects the first natural frequency, which ranges from 282 [Hz] for the smallest width to 625 [Hz] for the largest width, passing through a maximum of 651 [Hz] at a width of 24 [mm]. This variation represents 56 [%] of the maximum frequency. This difference corresponds to the behaviour before the maximum, after the peak, the variation is only 4 [%], which means that the influence of width becomes nearly negligible at high values. In other words, it is preferable to have a width greater than the optimal value rather than a lower width.

The thickness variation results are similar with and without consideration of the detector mass. The curves show an approximately linear increase in frequency as a function of blade thickness. As in the

simulations above, the added mass reduces the first natural frequency. The variation is bounded between 423 and 643 [Hz]. This range represents 34 [%] of the maximum frequency.

Table 2.11 summarizes the variation of the frequency induced by changes in the parameters.

Parameter	Parameter's range	Frequency range [Hz]	Frequency variation [%]
Length [mm]	20-100	67.89-503.45	86.50
Width [mm]	5-50	282.18-651.71	56.70
Thickness [mm]	0.5-2	423.23-643.17	34.20

Table 2.11: Blades' parameters variation results

These results can be used to classify the parameters according to their importance and their impact on the first natural frequency, based on the frequency variation. It appears that the order of importance is length > width > thickness. This analysis is further investigated in the next section using the ANalysis Of VAriance (ANOVA) method.

ANalysis Of VAriance (ANOVA) on the three first parameters

The ANOVA method is commonly used in parametric statistical studies. The method relies on the fundamental identity given in Equation 2.3. This formula is applicable to the variation of one parameter with a levels and n observations. Multiple observations are useful in real-life experiments where measurement errors may occur. In a deterministic analysis such as a prediction of a FEM modal response, a single observation is sufficient.

$$\sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y})^2 = n \sum_{i=1}^a (\bar{y}_i - \bar{y})^2 + \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_i)^2 \quad (2.3)$$

where:

- a is the number of level of the parameter.
- n is the number of observation for each level.
- y_{ij} is the j th observation for the i th level.
- $\bar{y}$ is the mean of all the observations.
- $\bar{y}_i$ is the mean for the i th level.

The equation above can be rewritten as Equation 2.4 considering the total sum of squares SS_T , the sum of square due to the parameter $SS_{parameter}$ and the sum of square of the error SS_E [26].

$$SS_T = SS_{parameter} + SS_E \quad (2.4)$$

These equations can be adapted to multiple parameters, in this case, three: the length, the width, and the thickness of the blades. Equation 2.3 then becomes:

$$\sum_{i=1}^N \sum_{j=1}^a (y_{ij} - \bar{y})^2 = \sum_{i=1}^N N^2 \sum_{j=1}^a (\bar{y}_j - \bar{y})^2 + \sum_{i=1}^N \sum_{j=1}^a (y_{ij} - \bar{y}_i)^2 \quad (2.5)$$

$$SS_T = SS_L + SS_W + SS_T + SS_E \quad (2.6)$$

Where N represents the number of parameters. Since several parameters are considered, their interactions can be included in the error term in Equation 2.5. There is no error if all interactions are taken into account, in that case, the sum of squares associated with the error can be decomposed as follows:

$$SS_E = SS_{LW} + SS_{LT} + SS_{WT} + SS_{LWT} \quad (2.7)$$

$$SS_{LW} = N \sum_{i=1}^a \sum_{j=1}^a (y_{ij} - \bar{y}_{L,i} - \bar{y}_{W,j} + \bar{y})^2 \quad (2.8)$$

$$SS_{LT} = N \sum_{i=1}^a \sum_{j=1}^a (y_{ij} - \bar{y}_{L,i} - \bar{y}_{T,j} + \bar{y})^2 \quad (2.9)$$

$$SS_{WT} = N \sum_{i=1}^a \sum_{j=1}^a (y_{ij} - \bar{y}_{W,i} - \bar{y}_{T,j} + \bar{y})^2 \quad (2.10)$$

$$SS_{LWT} = \sum_{i=1}^a \sum_{j=1}^a \sum_{k=1}^a (y_{ijk} - \bar{y}_{L,i} - \bar{y}_{W,j} - \bar{y}_{T,k} + \bar{y}_{LW,ij} + \bar{y}_{LT,ik} + \bar{y}_{WT,jk} - \bar{y})^2 \quad (2.11)$$

$$(2.12)$$

where:

- $\bar{y}_{Parameter,i}$ is the mean of the observation with the i -th value of the parameter.

The ratio of each sum of squares to the total sum of squares gives the importance of each term. Comparing these ratios provides the ranking of the parameters and their interactions in terms of influence.

In this study, this method is applied with five levels for each parameter. The parameter values are summarized in Table 2.12. A full factorial design is used with three factors at five levels, covering all possible combinations. The total number of simulations is 5^3 , i.e. 125

Parameter	Value 1	Value 2	Value 3	Value 4	Value 5
Length [mm]	20	27.5	35	42.5	50
Width [mm]	5	7.5	10	12.5	15
Thickness [mm]	0.5	0.875	1.25	1.625	2

Table 2.12: Parameter's values for the ANOVA method

The results are computed and summarized in Table 2.13. The table shows the ratio of each term to the total sum of squares. The influence of each term is quantified for the first six modes. Based on these results, the physical behaviour of the model can be better understood.

Term	Formula	Mode 1 [%]	Mode 2 [%]	Mode 3 [%]	Mode 4 [%]	Mode 5 [%]	Mode 6 [%]
L	SS_L/SS_T	59.21	59.46	45.76	81.63	69.12	70.38
W	SS_W/SS_T	30.58	30.86	11.66	2.37	$9.27 \cdot 10^{-2}$	$4.64 \cdot 10^{-2}$
T	SS_T/SS_T	9.17	8.65	38.52	9.92	29.88	28.29
L-W	SS_{LW}/SS_T	$4.19 \cdot 10^{-3}$	$4.25 \cdot 10^{-3}$	$5.56 \cdot 10^{-3}$	$6.98 \cdot 10^{-3}$	$8.77 \cdot 10^{-4}$	$1.75 \cdot 10^{-3}$
L-T	SS_{LT}/SS_T	$6.66 \cdot 10^{-5}$	$7.23 \cdot 10^{-5}$	$2.32 \cdot 10^{-2}$	$4.93 \cdot 10^{-2}$	$6.69 \cdot 10^{-3}$	$7.27 \cdot 10^{-3}$
W-T	SS_{WT}/SS_T	$5.94 \cdot 10^{-3}$	$5.80 \cdot 10^{-3}$	$4.35 \cdot 10^{-3}$	$6.40 \cdot 10^{-4}$	$1.11 \cdot 10^{-3}$	$2.75 \cdot 10^{-3}$
L-W-T	SS_{LWT}/SS_T	1.04	1.03	4.06	6.08	$9.11 \cdot 10^{-1}$	1.29

Table 2.13: ANOVA method results

A quick look at Table 2.13 shows that second-order interactions are negligible. All interaction terms between two parameters are below 0.1 [%] and can be considered physically negligible. The influence of

each factor depends on the mode considered.

For the first two modes, which correspond to bending behaviour, the length has a dominant impact of nearly 60 [%]. The influence of the width accounts for 30 [%] of the total effect on the natural frequency. The remaining 10 [%] is distributed between the thickness 9 [%] and higher-order interactions 1 [%]. The ranking defined in Section 2.3.1 is therefore confirmed for the first two modes.

The third mode corresponds to a compression of all three blades. The length and thickness are important because the response is governed by bending in each blade. Their influences are respectively 46 [%] and 39 [%]. The influence of the width represents 11.66 [%] of the total variation. The remaining 4 [%] is attributed to third-order interactions. Its impact is negligible with respect to the primary factors. For the third mode, the ranking changes, with thickness having a greater influence than width.

The fourth mode corresponds to a torsional deformation of the global structure. The variation in frequency is mainly driven by the blade length, with an influence of 82 [%]. The width, thickness, and third-order interactions account for 2 [%], 10 [%], and 6 [%], respectively. This mode exhibits the largest contribution from third-order interactions among the six first modes. Although this result may appear counterintuitive, it is consistent with the variation range of the fourth natural frequency with respect to the three parameters. The frequency range associated with length variation is 216–4016 [Hz], corresponding to a 95 [%] variation relative to the maximum value. The frequency variations due to width and thickness changes are 24 [%] and 17 [%], respectively, of the maximum frequency obtained in these simulations. The results are presented in Appendix B.

The fifth and sixth modes are similar and show similar results in their factors of influence. The length is the main parameter, accounting for around 70 [%], while the width is negligible, with an influence on the order of magnitude of 10^{-2} . The thickness represents approximately 29 [%] of the frequency variation, and the remaining 1 [%] is due to third-order interactions.

Equilateral ratio variation

The equilateral ratio categorizes the shape of the triangle. It is dimensionless and defined as:

$$Eq_{rat} = \frac{2height}{base\sqrt{3}} \quad (2.13)$$

$$Eq_{rat} = \frac{2P_5}{P_4\sqrt{3}} \quad (2.14)$$

A ratio of less than one means that the triangle is wider than it is tall, while a ratio greater than one indicates the opposite. If the ratio is equal to one, the triangle is equilateral. The variation is performed within the following boundaries:

$$Eq_{rat} \in 0.2 - 2[-] \quad (2.15)$$

The shapes at the bounds are represented in Figure 2.19. The triangle base remains constant at 40 [mm], while the height varies from 7 mm to 70 [mm]. Beyond these boundaries, the triangle becomes ill-shaped and no longer corresponds to past or potential future applications. Such ill-shaped triangles would exhibit poor performance in terms of modal response.

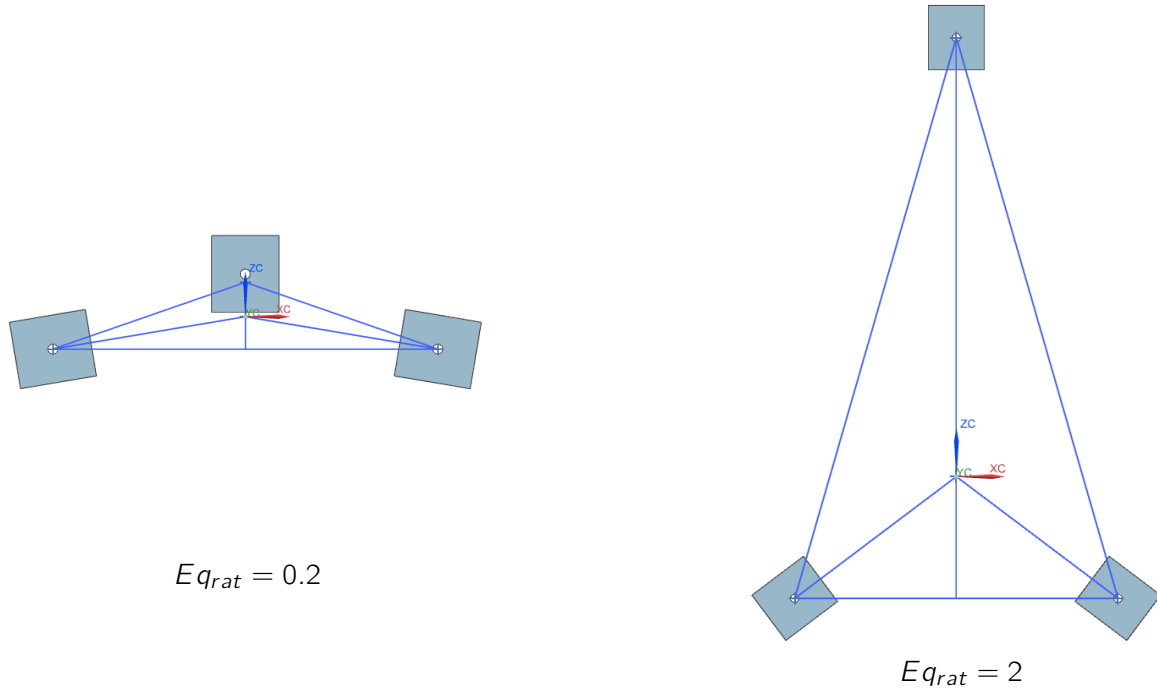


Figure 2.19: Representation of the boundaries of the equilateral ratio

The variation is performed over 20 iterations. It is linear, with a constant step between two successive iterations. The first natural frequency is represented in Figure 2.20. From a physical standpoint, the first natural frequency is expected to be maximized for an equilateral triangle, as this configuration ensures a more uniform stress distribution.

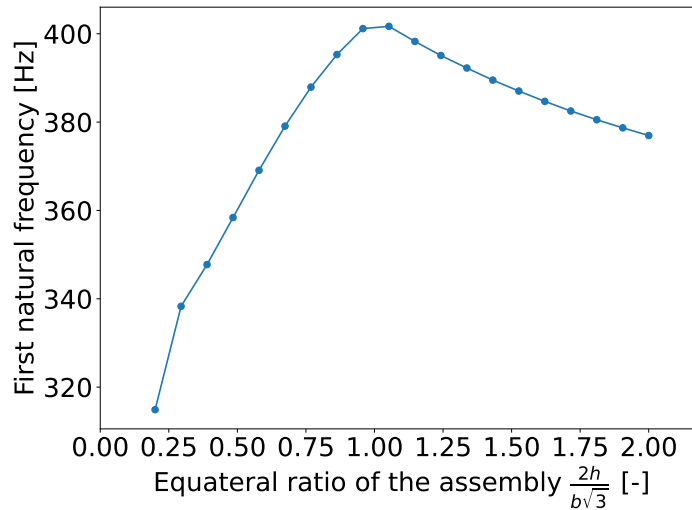


Figure 2.20: Variation of the equilateral ratio

The maximum is obtained when the equilateral ratio is equal to one. This result was expected, as an equilateral triangle is the most regular and symmetric geometry possible for a triangle. The curve decreases on both sides of this maximum. The minimum value is obtained for the simulation with the lowest equilateral ratio. The minimum frequency is 312 [Hz] and the maximum is 397 [Hz]. This variation corresponds to 21 [%] of the maximum frequency. This parameter therefore has a major influence on the design of an FPA support.

An interesting observation in Figure 2.20 is that the slope for ratios lower than one is steeper than for ratios greater than one. As a result, for a given shape, a three-point fixation yields a higher first natural frequency when the height of the assembly triangle is greater than its base. In other words, in Figure 2.21, for supporting the same rectangular shape, the green fixation configuration is stiffer than the red fixation configuration.

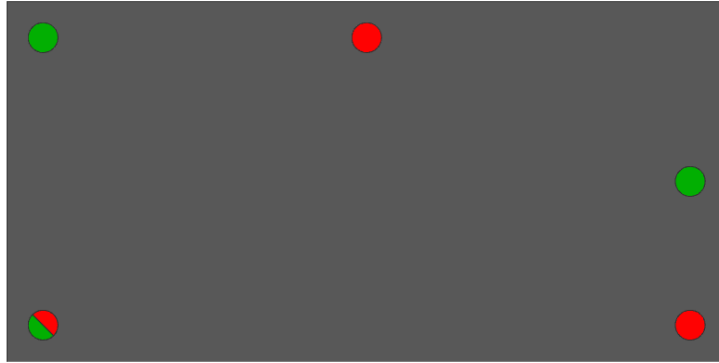


Figure 2.21: Fixation optimization representation

For future applications, if an equilateral triangle is not possible, it is preferable to mount the detector using a triangle with an equilateral ratio greater than one. This results in a triangle that is taller than it is wide in its own reference frame.

Assembly's triangle size variation

The next variation concerns the size of the assembly triangle. The equilateral ratio is kept constant while the side length of the triangle is varied. The equilateral ratio is set to the same value as in the standard parameters in Table 2.10. This ratio is equal to 1.15, as in the base case. The side length of the triangle varies within the following range.

$$P_3, P_4 \in 20 - 200[mm] \quad (2.16)$$

Although the considered parameter range exceeds typical application values, it is deliberately chosen to encompass a wide range of potential configurations and to provide a comprehensive understanding of the system behaviour. The variation is performed using 20 simulations, with a linear sampling of the parameter range. The results of these simulations are presented in Figure 2.22. This graph shows the first natural frequency as a function of the size of the assembly triangle.

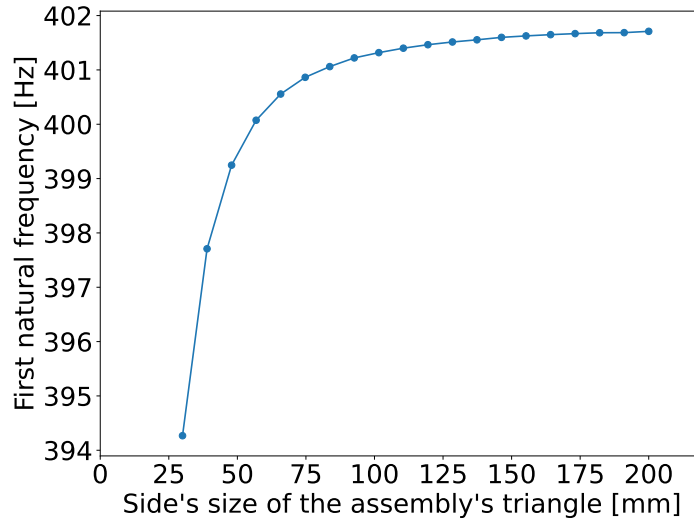


Figure 2.22: Variation of the assembly triangle's size

The results of the variation show a logarithmic increase in the first natural frequency. The minimum frequency is obtained for the smallest triangle side, while the maximum is obtained for the largest value in the range. The slope of the curve decreases as the triangle side increases. Quantitatively, the frequency lies in the interval 390-397 [Hz]. This range represents 2 [%] of the maximum value. This parameter therefore has a limited impact on the modal response of the structure.

As stated above, the parameter range is broad, and in a real application a 20 [mm] triangle cannot be integrated into the instrument due to the inability to access the screw fixation holes. The variation in frequency is even lower than 1.82 [%]. As long as the mounting triangle can be integrated, the influence of the triangle side length on the first natural frequency of the structure can be considered negligible.

Detector's mass variation

In this section, the mass of the detector, modelled as a lumped mass at a node, is varied. This is important in order to account for cases where the detector changes during the design phase of its support. To mount a mirror, a similar design can be applied, and mirrors are usually heavier than a detector. The mass is expected to have a major impact on the modal response. For a system with a lumped mass at a point, the natural frequency is defined as:

$$f = \sqrt{\frac{k}{m}} \quad (2.17)$$

This implies that the frequency is inversely proportional to the square root of the mass. The frequency decreases as the mass increases. The effective stiffness k_{eff} of the baseline case can be computed from the known mass and the first resonance frequency of the structure.

$$k_{eff} = mf^2 \quad (2.18)$$

$$k_{eff} = 16680.41[kg/m] \quad (2.19)$$

The natural frequency of the structure can be estimated for different masses using the effective stiffness of the structure in Equation 2.17. This computation is compared with the numerical results in Figure 2.23. The numerical results are computed for a mass varying from 0.05 [kg] to 0.5 [kg]. This range is wide,

but it includes applications involving heavier mirrors. The study is performed using 20 simulations, with a linear discretization of the mass range.

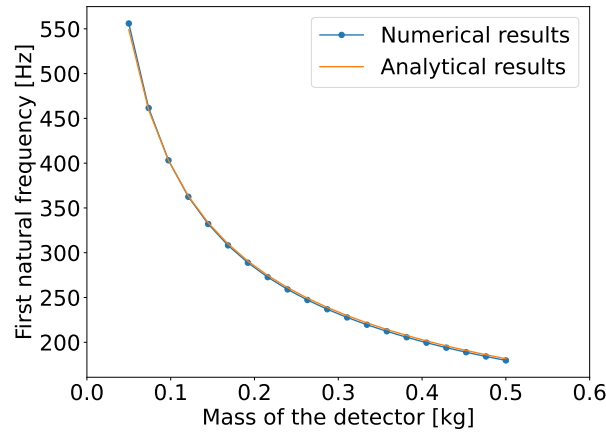


Figure 2.23: Variation of the detector's mass

As predicted, the decrease in frequency is inversely proportional to the square root of the mass. The impact of the same variation in mass is greater at low mass values than at higher mass values. A larger mass generally requires a stiffer, and therefore heavier, structure. For this mass range, the frequency varies from 180 Hz to 556 [Hz]. This variation represents 67 [%] of the maximum value.

The results are very close to each other. This is due to the fact that the detector is modeled as a node, increasing the mass does not modify the stiffness of the structure, and the effective stiffness of the initial design can be used to compute the new natural frequency. In a real structure, this would not be the case. A variation in mass would also affect the detector's inertia, and the response would not remain as close to the analytical prediction. This effect arises from the assumption that the detector is much stiffer than the support and can be represented as a lumped mass at a node. However, the differences are negligible because this hypothesis is close to reality.

The results show that the mass of the supported object is critical for first natural frequency optimization. A decrease by a factor of three can be observed if the mass is multiplied by ten. Even if such a change in order of magnitude would not occur in practical applications, the mass should be minimized to ensure favorable modal behaviour. These results emphasize the importance of optimizing the detector or mirror mass.

Conclusion

Those five variations presented above can be used to establish a guideline to maximize, or at least increase, the first natural frequency of a blade support. These conclusions are drawn without considering thermal conduction limitations through the blades. The table below summarizes which parameters should be maximized, minimized, or have no significant influence on the result.

Parameter	Impact	Optimization process
Length	High	Minimize
Width	Moderate	Maximize
Thickness	Little	Maximize
Equilateral ratio	Moderate	Greater than or equal to 1
Triangle's size	Negligible	/
Detector's mass	High	Minimize

Table 2.14: Optimization process summary for a support without thermal consideration

The most important parameters are the blade length and the detector mass. Both parameters should be minimized, and the evolution of the first natural frequency with increasing parameter values shows an exponential decay. Next in order of importance are the width and the equilateral ratio. The width should be maximized, while the equilateral ratio is optimal at one and becomes nearly negligible for values greater than one compared to the optimum. If the equilateral ratio is greater than or equal to one, the design can be considered acceptable. The thickness has a smaller influence on the natural frequency, but increasing thickness still improves the response. Finally, the size of the triangle has a negligible impact on the frequency, realistic sizes can therefore be considered optimal.

2.3.2 Including thermal limitation

This section aims to consider thermal conduction through the support. Thermal decoupling of the detector or other optical components is a requirement for many space instruments. The maximum allowable thermal conductance is known, therefore, it will be kept constant in future computations for all parameter variations. The interactions between the three parameters are studied:

- Length-Width
- Width-Thickness
- Length-thickness

The parameter tables for the simulations are generated using a linear variation of the smallest parameter, with the second parameter computed using Equation 1.3. If the second parameter falls outside the range defined in Table 2.1, the boundary value of the range is used, and the first parameter is then computed using the thermal conduction equation.

The following simulations are performed using a common approach. Since thermal conduction is included, only the three parameters related to the blade geometry are varied. Then, for each interaction between two parameters, three simulations are performed, varying the thermal conductance, the third parameter, and the detector mass.

Width variation with a length adaptation

The first interaction studied is the relationship between the length and the width. The width is varied linearly from 5 to 50 [mm]. The blade length is then computed using Equation 1.3. As mentioned above, several values of thermal conductance, thickness, and mass are fixed in order to explore the evolution of the first natural frequency. The variation for different thermal conductivities is shown in Figure 2.24.

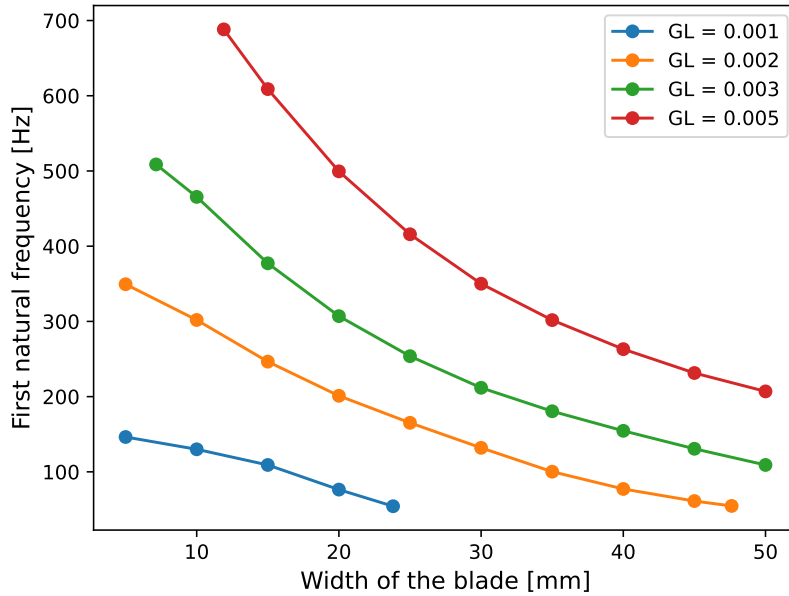


Figure 2.24: Width-length interaction with different thermal conductance

The general behavior is that the frequency decreases as the width increases. This is due to the fact that a larger width, for a fixed thermal conductance, implies a shorter length, which in turn reduces the frequency. As discussed in Section 2.3.1, variations in length have a greater impact on the frequency than variations in width. It can also be observed that higher thermal conductance increases the feasible range of parameters.

Another observation concerns the second derivative of the curves, i.e., their concavity. For high thermal conductance, the curve is strictly concave, whereas for lower thermal conductance there is an inflection point where the concavity changes. This phenomenon is due to the fact that the frequency decays exponentially with the length of the support. For large lengths, the rate of decrease slows down, which explains the concave shape of the curves.

The inflection point does not occur at the same width because the length computed for a given width depends on the thermal conductance. For example, with a conductance of 0.001 [W/K], a width of 15 [mm] corresponds to a length of 126 [mm], whereas for the same width and a conductance of 0.002 [W/K], the corresponding length is 63 [mm]. The exponential decay of frequency with respect to length is therefore shifted depending on thermal conductance.

It is also possible that a maximum could appear for very low thermal conductance, but this was not observed within the chosen parameter range. If smaller widths were considered, an optimum might emerge for low conductance cases. However, in general, smaller widths lead to higher first natural frequencies within the explored parameter space.

The same variation is performed for several thickness values. The results are presented in Figure 2.25. The thermal conductance is fixed at 0.002 [W/K].

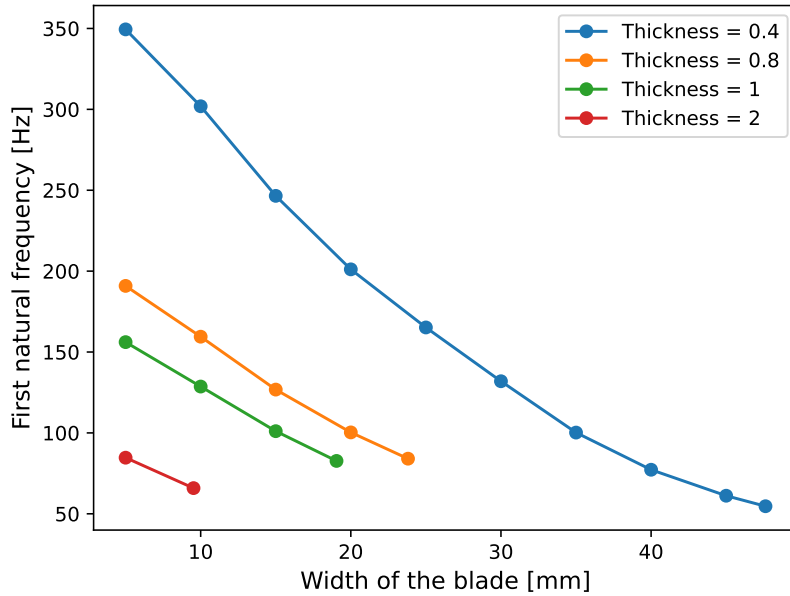


Figure 2.25: Width-length interaction with different blade's thickness

The behavior of the curves is consistent, showing a decreasing frequency as the width increases. The same phenomenon appears with an inflection point, where the rate of decrease in frequency due to increasing length begins to slow down. The effect of thickness variation is to limit the feasible range of the other two parameters. A large thickness prevents the width from taking high values as well as low values. Minimizing the thickness is critical to obtaining favorable modal behavior. However, there is a limit to this minimization: since the support is structural, it must withstand static loads without failure. These loads and constraints are not considered in this thesis. As long as the blades satisfy the static load requirements, the thickness should be minimized.

The mass of the detector is varied in Figure 2.26 to assess its impact.

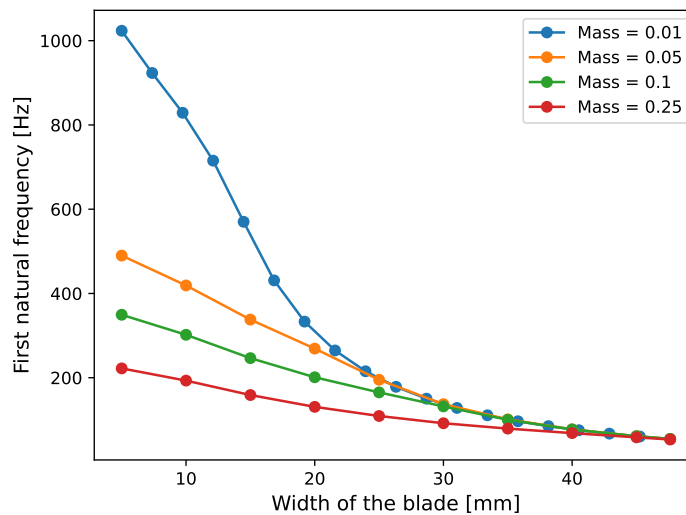


Figure 2.26: Width-length interaction with different detector's mass

The curve still shows a decreasing trend in the first natural frequency, with an inflection point where the curvature changes from convex to concave. The effect of the mass is to shift the curves upward as the mass decreases. However, for large widths, which imply large lengths, the effect of reducing the mass becomes less significant. This is a consequence of the support mass increasing and becoming dominant compared to the detector mass. At maximum width and length, the support mass is 50 [g], which is non-negligible compared to the total mass. This is why the detector mass has less influence for larger supports.

Thickness variation with a width adaptation

In this section, a linear variation is applied to the blade thickness. An adjustment is made to the width to ensure the required thermal conduction. The same methodology is applied to the parameters: if the computed width is outside the bounds defined in Table 2.1, the boundary value is used, and the thickness is then computed using Equation 1.3. The results of three variations are shown below for different thermal conductivities, blade lengths, and detector masses. The first analysis is performed with varying thermal conductance and is shown in Figure 2.27. The simulations presented use the standard values for blade length and detector mass, these values are summarized in Table 2.10.

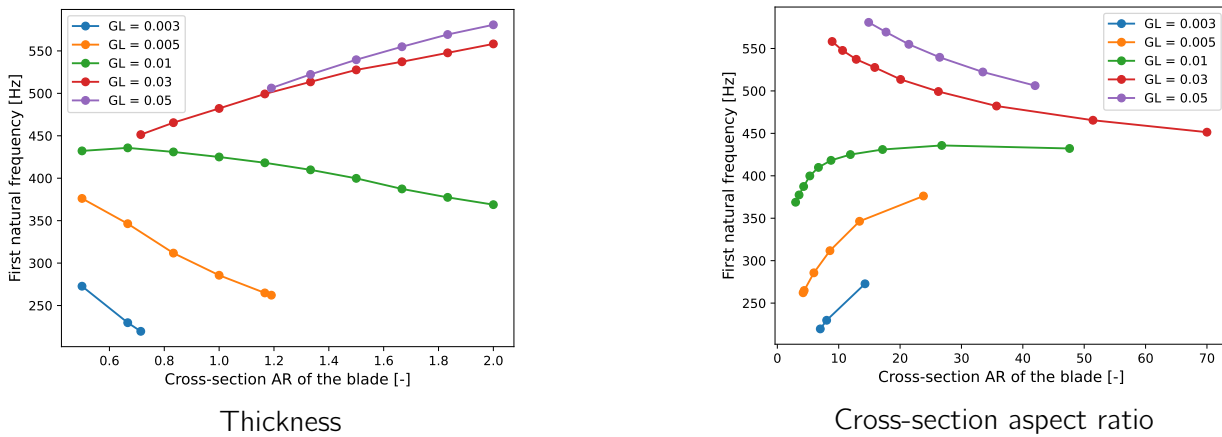


Figure 2.27: Thickness-width interaction with different thermal conduction

Figure 2.27 shows the evolution of the first natural frequency as a function of thickness (left) and cross-sectional Aspect-Ratio (AR) (right). The formula for the cross-sectional AR is given as follows:

$$AR_{cross-section} = \frac{thickness}{width} \quad (2.20)$$

The second representation is necessary because it is easier to analyze and identify trends. The following analyses are performed on the second graph.

The curves show different behavior depending on the allowed thermal conduction. For high conductance, the curves are strictly decreasing with respect to the AR, whereas for low conductance they increase with the AR. For some intermediate values of GL , the curves exhibit a maximum, which defines an optimal value of the cross-sectional AR. The optimization process therefore depends on the conductance.

As in the simulation presented in Figure 2.24, low thermal conductance restricts the number of feasible configurations. The minimum GL can be computed using Equation 1.3, based on the smallest width and thickness. For this thermal conductance, only one case is possible.

Overall, increasing the allowed thermal conductance increases design freedom and leads to higher first natural frequencies. This is due to the width and thickness taking larger values, thereby stiffening the

structure.

The next variation studied involves several blade lengths. It is represented in Figure 2.28. The simulations are performed with a fixed thermal conductance of 0.003 [W/K].

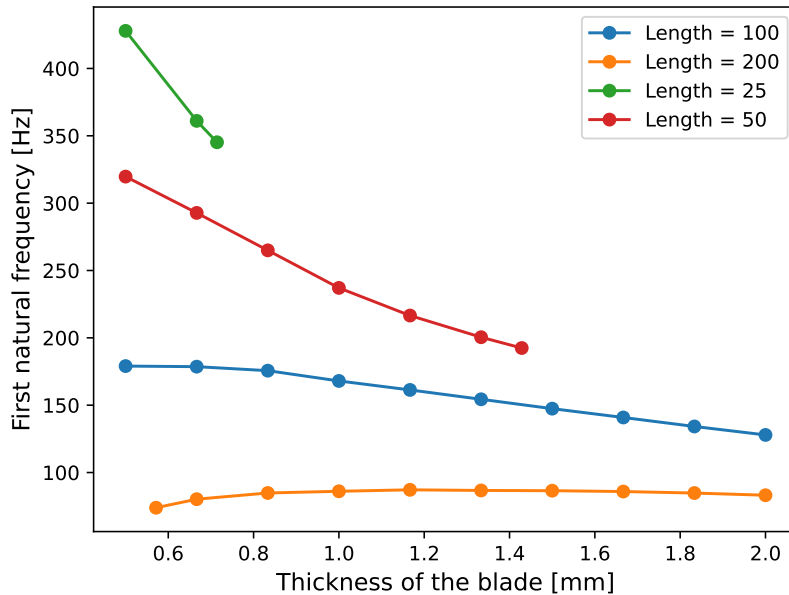


Figure 2.28: Thickness-width interaction with different length

The graph displayed in Figure 2.28 shows that varying the thickness while adapting the width to ensure a given thermal conductance leads to a decreasing natural frequency, with an inflection point that changes the concavity of the curves. At low thickness, the curves are concave and may therefore exhibit a maximum for a given thickness. However, this maximum lies outside the range considered in this study. For larger lengths, the maximum shifts toward higher thickness values. This explains why a maximum natural frequency is observed for a length of 200 [mm]. The decreasing trend comes from the fact that increasing thickness at constant thermal conductance implies a reduction in width. As discussed in Section 2.3.1, the width has a greater influence on the first natural frequency. The reduction in width decreases the frequency even as thickness increases.

As expected, shorter lengths lead to higher frequencies. However, reducing the length also limits the feasible combinations of width and thickness. It can also be observed that at low length, the slope of the frequency curves is steeper. Changing the thickness by the same amount has a greater effect on the frequency at shorter lengths than at larger lengths.

The last variation studied involves changing the thickness while adapting the width, and is performed for several detector masses. The resulting curves are displayed in Figure 2.29. The simulations are performed with a thermal conductance of 0.005 [W/K].

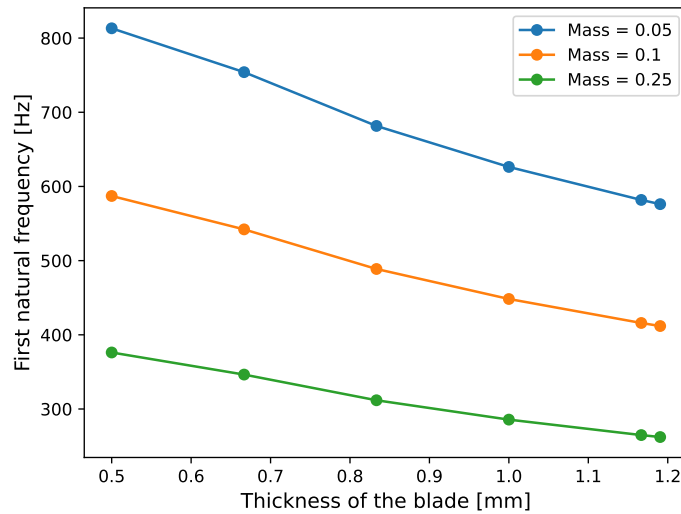


Figure 2.29: Thickness-width interaction with different detector's mass

The frequency shows a decreasing trend, as explained above, but the influence of mass shifts the curves upward or downward. The curves corresponding to different masses are almost parallel to each other. This can be explained by the geometry, which does not change, only the mass at a single node is modified in the FEM model. The same method as in Section 2.3.1 is used, relying on the effective stiffness to recompute the frequency without requiring additional FEM simulations, thereby reducing computation time.

Thickness variation with a length adaptation

The last interaction studied involves varying the thickness while adapting the length to match the thermal conductance. The thickness variation is the same as before, with a linear range from 0.5 to 2 [mm]. The evolution of the first natural frequency is represented in three graphs for several thermal conductivities, widths, and detector masses, respectively in Figures 2.30, 2.31, and 2.32.

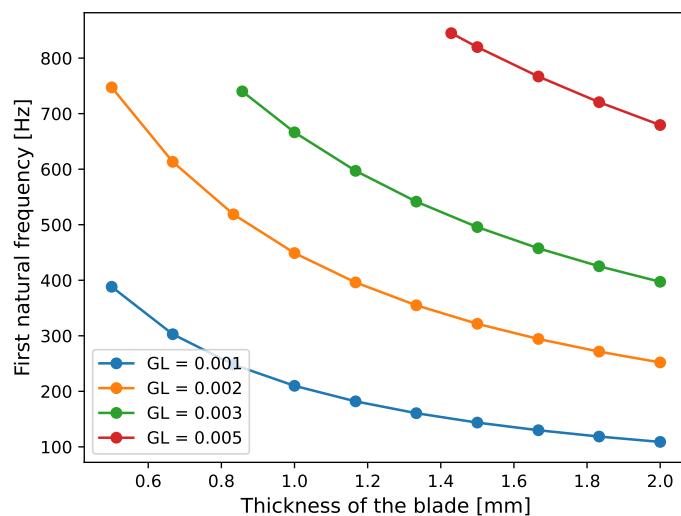


Figure 2.30: Thickness-length interaction with different thermal conductance

The evolution of the natural frequency is mainly driven by the increasing length resulting from the increase in thickness. The curves show an exponential decay with increasing thickness. The ANOVA analysis indicates that length has a greater impact on the frequency than thickness, as shown in Figure 2.30. The variation in thermal conductance influences the range of feasible parameters: higher thermal conductance allows a wider range of thickness and length values. The general optimization strategy is to reduce the thickness in order to shorten the length and increase the frequency.

The next variation concerns the third blade dimension, namely the width. Using four width values, the evolution of the natural frequency is represented in Figure 2.31. The thermal conductance fixed for these simulations is 0.006 [W/K].

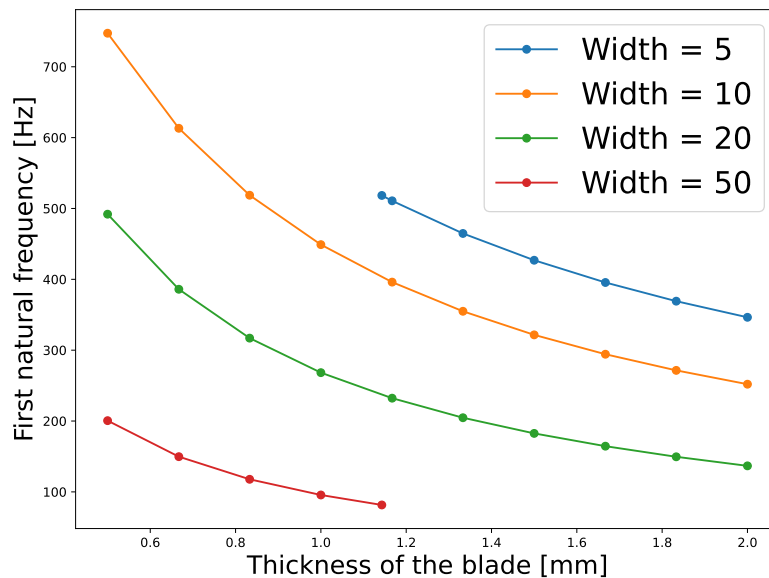


Figure 2.31: Thickness-length interaction with different width

The evolution of the frequency shows an exponential decay influenced by the increasing length of the blades. The lower the width, the higher the first natural frequency. An interesting phenomenon observed in Figure 2.31 is that for a width of 5 [mm], the frequency would be higher with lower thickness, but the resulting length becomes too short and falls outside the feasible range. The issue with a short length is that the assembly process of the support cannot be performed properly. The blade length must be large enough to allow insertion of the fixation screws connecting the blades to the detector and the bracket.

The last variation involves implementing several detector masses. This is represented in Figure 2.32. The thermal conductance is set to 0.006 [W/K].

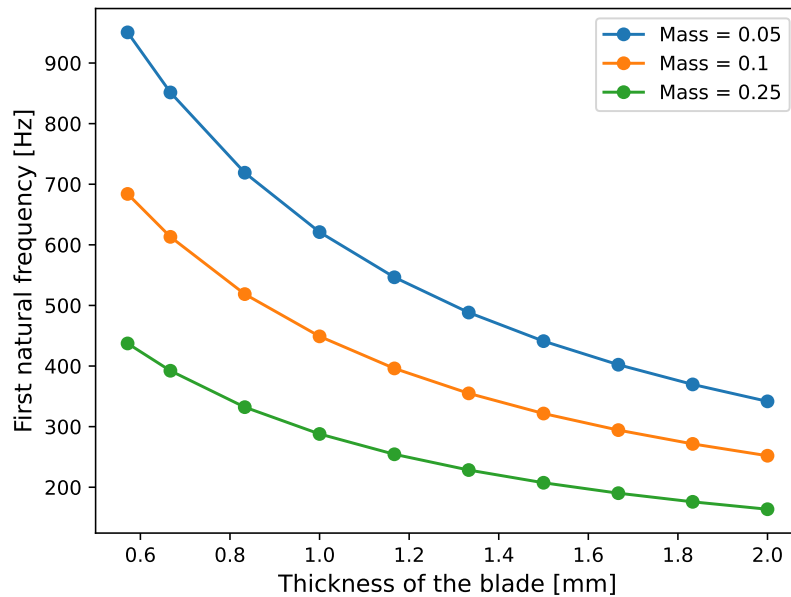


Figure 2.32: Thickness-length interaction with different detector's mass

The effect of mass on the frequency, for the variation of thickness while adapting the length, is similar to that observed in the previous interaction (see Figure 2.29). The mass shifts the curves vertically: increasing the mass decreases the frequency, and vice versa. The frequency can be predicted using Equation 2.17 by using the effective stiffness and updating the mass.

Conclusion

This section summarizes the choices required to optimize the first natural frequency of a support, including a thermal conduction constraint. The process is summarized in Table 2.15 for each blade dimension.

Parameter's name	Optimization choice
Length	Minimize
Width	Minimize
Thickness	Minimize

Table 2.15: Optimization process on the blades' dimensions including thermal limitation

Table 2.15 shows that all dimensions should be minimized. This is due to the strong influence of length on the first natural frequency. Decreasing the length increases the first natural frequency, as shown in Section 2.3.1. This can seem counterintuitive but the objectives of this study is to maximize the frequency not minimize the thermal conductance. The two remaining dimensions must also be minimized to allow the length to take lower values under the thermal constraint. As long as the support can be integrated into the instrument, the first natural frequency is maximized when the support dimensions are as small as possible.

2.4 Frequency prediction

The next step of this study is to predict the first natural frequency for a given parameter set. This can be achieved using regression based on a large number of simulated cases. The regression fits coefficients

to the results of modal simulations. The result is a set of coefficients multiplying the parameters in a polynomial form.

The first step in frequency prediction is generating data from modal simulations. Each parameter set is chosen randomly using the Latin Hypercube Sampling (LHS) method. The regression is then computed, and its degree is selected to ensure accurate results while maintaining a simple analytical expression. A convergence study is performed on the relative error between the regression and the numerical simulations. It is used to determine the appropriate number of training samples for the regression, ensuring accuracy while reducing computation time. The regression can then be improved by neglecting less influential coefficients and adjusting the parameter ranges. The results of both predictions are compared and analyzed.

2.4.1 Latin Hypercube Sampling (LHS)

The LHS method is chosen over pure random sampling because it covers the domain more uniformly. The LHS method consists of dividing each parameter range into equal intervals. If m points are required, the range is divided into m subintervals. Then, one random point is selected within each interval. This method avoids repetitive patterns in the sampling and improves the distribution of points. It is therefore well suited for covering large domains with a limited number of samples. For the regression performed in this study, the sampled domain is six-dimensional because there are six parameters. The comparison between the LHS method and pure random sampling is shown in Figure 2.33.

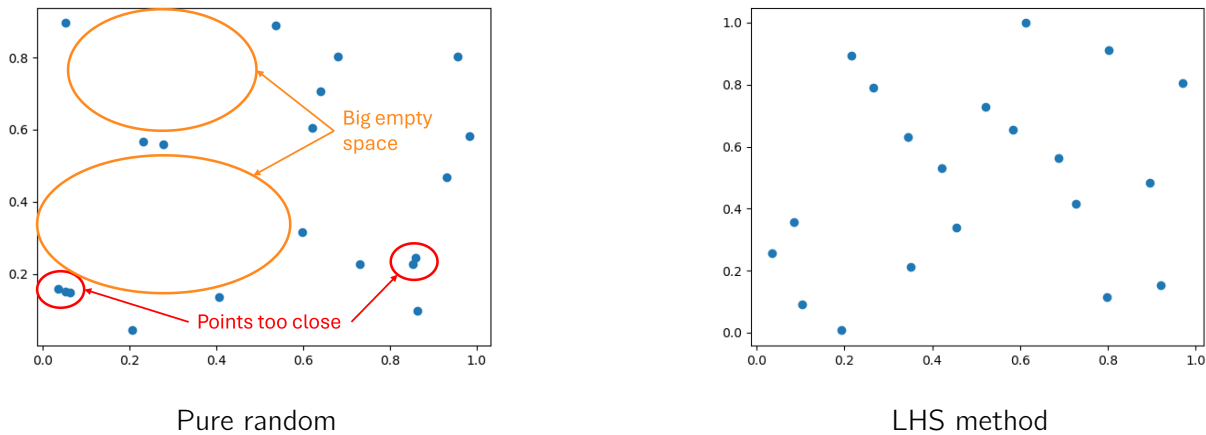


Figure 2.33: Comparison of two different sampling method in a 2D domain

The figure above shows that pure random sampling can produce samples that are clustered close to each other, introducing unnecessary redundancy, while leaving other regions unsampled. The LHS method distributes samples more evenly, this is why this thesis adopts this sampling method [27].

2.4.2 Frequency regression

In this section, the regression is explained in order to understand the obtained results and the possible improvements. The goal of the regression is to derive a formula capable of predicting the first natural frequency of the support. This formula depends only on the structural parameters. The polynomial degree of the regression is four, and first-order nonlinear interactions between the parameters are included. The resulting formula is defined by the equation below:

$$f(\mathbf{P}) = a_0 + \sum_{i=1}^6 b_i P_i + \sum_{i=1}^6 c_i P_i^2 + \sum_{i=1}^6 \sum_{j=i+1}^6 d_{ij} P_i P_j + \sum_{i=1}^6 e_i P_i^3 + \sum_{i=1}^6 f_i P_i^4 \quad (2.21)$$

The number of coefficients is 40: one corresponding to the constant term a_0 , six linear coefficients b_i , six quadratic coefficients c_i , six cubic coefficients e_i , and six quartic coefficients f_i . The remaining 15 coefficients correspond to parameter interaction terms d_{ij} . The regression is fitted on 1000 frequency samples randomly generated using the LHS method described above.

The regression is performed using a *Python* script with the *linalg.lstsq* function from the *Numpy* library. The regression finds the coefficient vector $\mathbf{x}$ that solves the following linear equation:

$$\mathbf{P}\mathbf{x} = \mathbf{b} \quad (2.22)$$

Where $\mathbf{x}$ is the coefficient vector. It is a one-dimensional vector containing the regression coefficients. Its length corresponds to the number of coefficients in the regression, here 40. The matrix $\mathbf{P}$ contains the terms multiplying the coefficients in the analytical expression. It depends on the parameters, which have been normalized to the interval $[-1,1]$ using Equation 2.23. The dimensions of this matrix are the number of samples by the number of coefficients in the model. The vector $\mathbf{b}$ contains the frequencies computed for each sample, and its length equals the number of samples. The number of samples is chosen in Section 2.4.3, which determines the size of the matrix introduced above.

$$P_{normalized} = 2 \frac{P_{unnormalized} - P_{min}}{P_{max} - P_{min}} - 1 \quad (2.23)$$

Where:

- $P_{normalized}$ is the normalized parameter within $[-1,1]$
- $P_{unnormalized}$ is the unnormalized parameter
- P_{min} is the minimum boundary of the parameter's range
- P_{max} is the maximum boundary of the parameter's range

The function solves Equation 2.22, or, if an exact solution is not possible which, is usually the case for large datasets, it minimizes the Euclidean 2-norm: $\|\mathbf{b} - \mathbf{P}\mathbf{x}\|$. The resulting solution is the set of 40 coefficients contained in the vector $\mathbf{x}$, which best fits the frequencies obtained from the sampled numerical simulations.

2.4.3 Convergence study

This section aims to determine the appropriate number of training samples to achieve acceptable accuracy while maintaining low computational time. The error is defined as the difference between the frequency obtained from numerical computations with *NX Siemens* and the frequency predicted by the regression for the same parameters. The error is computed over 1000 simulations sampled using the LHS method across the parameter domain. The standard deviation of the error is then computed from these 1000 errors for each training dataset size. The standard deviation is chosen as a convergence indicator instead of the mean or median error, which oscillate around zero. As the number of samples increases, the error tends toward a normal distribution with a mean of zero because there is a similar number of overestimations and underestimations in the predictions. The standard deviation is always positive and characterizes the uncertainty of the prediction through a confidence interval. The confidence interval is discussed in Section 2.4.6. The evolution of the standard deviation of the error is shown in the figure below.

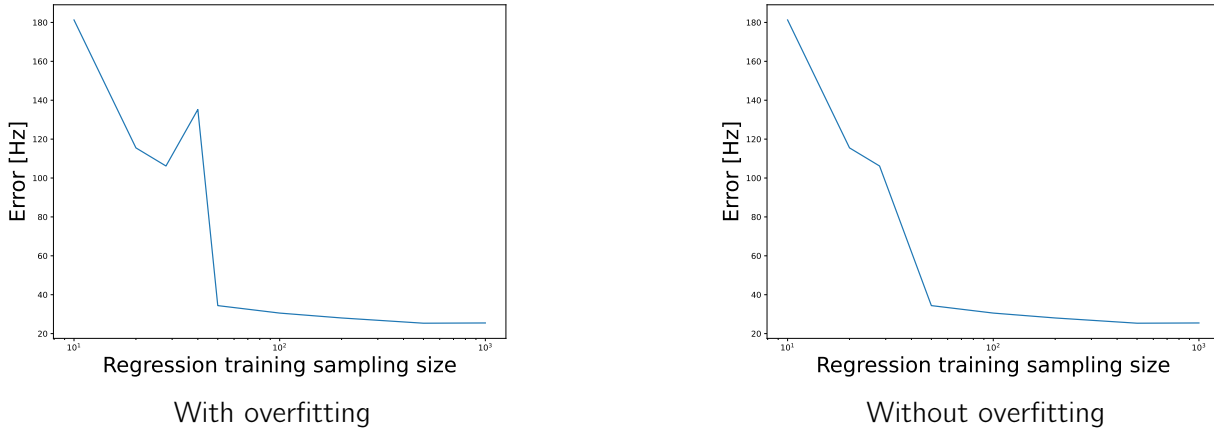


Figure 2.34: Convergence study of the error from the regression

Figure 2.34 shows that the number of training data required to achieve accurate prediction is 50 samples. The figure also reveals an interesting phenomenon, namely overfitting at 40 samples. This is due to the fact that the number of samples is equal to the number of coefficients in the regression. The least-squares method fits the coefficients exactly to the samples when there are 40 equations and 40 unknowns. The fitting error is therefore very low, but the coefficients do not generalize well to other samples, leading to a significant increase in the standard deviation of the error computed over 1000 test samples. To avoid such issues, increasing the number of training samples is generally recommended. The study by Yeonjong Shin and Dongbin Xiu [28] states that a general rule of thumb is to use a number of samples 1.5 to 3 times larger than the number of coefficients. For a stable regression, the number of samples (M) should therefore satisfy:

$$M = p \log(p) \quad (2.24)$$

Where p is the number of coefficients in the regression. In this case, the number of samples should be greater than 64. For the rest of the study, a regression with 100 samples is used to ensure good accuracy and stability.

2.4.4 Results

The regression provides coefficients to compute the first natural frequency of the system. These coefficients are summarized in the table below, including for each coefficient its symbol from Equation 2.21, the parameters it multiplies, and its numerical value.

a_0	1	205.81	c_2	W^2	-54.06	d_4	Lh	5.41	d_{12}	Tm	28.65	e_5	h^3	-1.90
b_1	L	-118.55	c_3	T^2	2.73	d_5	Lm	-0.41	d_{13}	bh	-11.84	e_6	m^3	-7.00
b_2	W	35.79	c_4	b^2	-9.11	d_6	WT	-38.54	d_{14}	bm	-34.82	f_1	L^4	0.55
b_3	T	27.97	c_5	h^2	-4.42	d_7	Wb	107.61	d_{15}	hm	-1.98	f_2	W^4	1.11
b_4	b	-4.48	c_6	m^2	40.54	d_8	Wh	-4.44	e_1	L^3	-0.62	f_3	T^4	-12.44
b_5	h	5.69	d_1	LW	-87.72	d_9	Wm	-10.07	e_2	W^3	78.57	f_4	b^4	5.96
b_6	m	-64.45	d_2	LT	28.32	d_{10}	Tb	-0.96	e_3	T^3	4.19	f_5	h^4	-3.49
c_1	L^2	39.89	d_3	Lb	-5.01	d_{11}	Th	0.29	e_4	b^3	0.98	f_6	m^4	-5.95

Table 2.16: Regression coefficients results

With the coefficients from the table 2.16, several observations can be made. The coefficient analysis is done based on their magnitude in absolute value and also their sign. The range of the absolute value

of the coefficients is from less than one to 206. A coefficient with a value less than 10 is considered to be negligible due to the two orders of magnitude between the coefficient and the maximum value of the coefficients. This simplification can be done because the parameters are normalized in the regression. Their value is in the $[-1,1]$ range. Multiplying the parameters together or applying a power keeps the multiplying terms in the same range.

The first observation is that the overall values of the coefficients are decreasing with the complexity of the multiplying factor. This is predictable because the higher the order of the polynomial terms, the lower the impact it has on the frequency. The quartic terms of the regression are related to coefficients f_i , which are lower than the coefficient for the linear terms b_i . In the set of coefficients relative to the linear terms b_i , only those related to the assembly geometry are negligible (b_4 and b_5). For the quadratic coefficients c_i , in addition to the two related to the assembly geometry (c_4 and c_5), the thickness coefficient can be neglected (c_3). Lastly, for the cubic and quartic terms only one coefficient is sufficient not to be neglected, respectively the width e_2 and the thickness f_3 . The rest of the coefficients can be neglected for these two orders of terms.

The sign of the coefficient is also important. A positive sign of the coefficient means that the frequency increases with the corresponding parameter, as is the case for the width and thickness and for the parameters b_2 and b_3 . On the other hand, a negative sign means that an increase in the corresponding parameter decreases the natural frequency, for example the length or the mass, associated with the coefficients b_1 and b_6 .

The linear terms associated with the coefficients b_i confirm what the ANOVA method states and the results obtained from the parameter variation in Section 2.3.1. The length and the mass have the greatest negative influence on the natural frequency. The width and the thickness have a moderate positive influence on the natural frequency. Lastly, the assembly triangle dimensions are negligible regarding their impact on the first natural frequency of the structure.

The magnitude of the coefficients relative to the interaction terms d_{ij} can be explained by the decoupling between the blade geometry and the assembly geometry. In other words, coefficients associated with interactions linking a parameter from the blade geometry (L , W or T) and a parameter from the assembly geometry (b or h) should be small. This is confirmed by the coefficients d_3 , d_4 , d_8 , d_{10} and d_{11} , which are lower than ten in absolute value. The exception is the coefficient d_7 , which is significantly larger than expected, with a value of 107. The coefficients associated with interactions within each geometry are not negligible, as can be expected, this applies to the coefficients d_1 , d_2 , d_6 and d_{13} .

The next step is to neglect the terms associated with coefficients lower than ten. A comparison with the original regression is performed in order to assess the impact of low-order coefficients.

2.4.5 Regression improvement

In this section, the objective is to improve the regression model. Two factors can be modified in order to increase either the accuracy or the simplicity of the regression. The first method consists of neglecting the terms associated with low coefficients. The number of coefficients becomes 18 instead of 40. The regression computation is faster, and the formula is shorter, with less than half of the original terms. The other method is to change the range of the parameters because the regression's precision depends on the domain. Areas where the regression has lower accuracy due to parameters taking extreme values are excluded.

Coefficient reduction

As stated above, the coefficients with an absolute value under ten are neglected to reduce the number of parameters from 40 to 18. The 22 neglected coefficients are summarized in Table 2.17.

b_4	b	-4.48	d_3	Lb	-5.01	d_{11}	Th	0.29	e_5	h^3	-1.90	f_5	h^4	-3.49
b_5	h	5.69	d_4	Lh	5.41	d_{15}	hm	-1.98	e_6	m^3	-7.00	f_6	m^4	-5.95
c_3	T^2	2.73	d_5	Lm	-0.41	e_1	L^3	-0.62	f_1	L^4	0.55			
c_4	b^2	-9.11	d_8	Wh	-4.44	e_3	T^3	4.19	f_2	W^4	1.11			
c_5	h^2	-4.42	d_{10}	Tb	-0.96	e_4	b^3	0.98	f_4	b^4	5.96			

Table 2.17: Neglected terms for the enhanced regression

The same convergence study can be performed on the regression by increasing the number of training samples. The regression is then compared with a testing set of 1000 samples, as in the previous section. The standard deviation of the error made on those 1000 predictions as a function of the training dataset size is shown in the figure below.

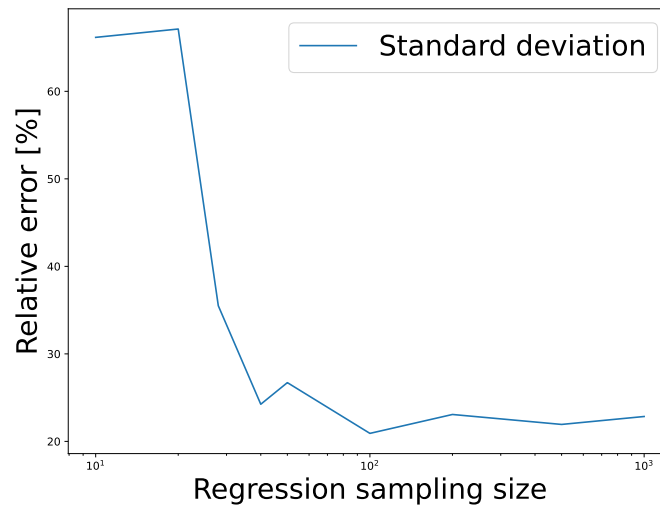


Figure 2.35: Convergence study of the enhanced regression with 28 coefficients

The number of coefficients required to achieve an accurate prediction is above 50. Above this threshold, the error converges, and increasing the number of training samples does not significantly affect the standard deviation of the error. In this case, there is no overfitting when the number of samples equals the number of coefficients computed by the regression. In the upcoming computations, the number of training samples used for the regression is 100. With this training dataset, the regression yields 18 coefficients, summarized in Table 2.18.

a_0	1	189.69	b_6	m	-93.31	d_1	LW	19.41	d_9	Wm	7.43	e_2	W^3	2.36
b_1	L	-181.85	c_1	L^2	138.19	d_2	LT	-3.89	d_{12}	Tm	-1.77	f_3	T^4	4.57
b_2	W	48.18	c_2	W^2	-59.40	d_6	WT	3.42	d_{13}	bh	-15.33			
b_3	T	18.57	c_6	m^2	44.78	d_7	Wb	-18.09	d_{13}	bm	-15.18			

Table 2.18: Enhanced regression coefficients results

Domain reduction

In this section, the full regression is considered, with 40 coefficients, but the parameter domain is modified to increase accuracy. With a large testing sample set, the mean of the parameters for which the error is highest can be used to identify the region where the regression does not fit the FEM computation. The

figure below shows the relative error from the regression using a training dataset of 100 samples and a testing dataset of 2000 samples.

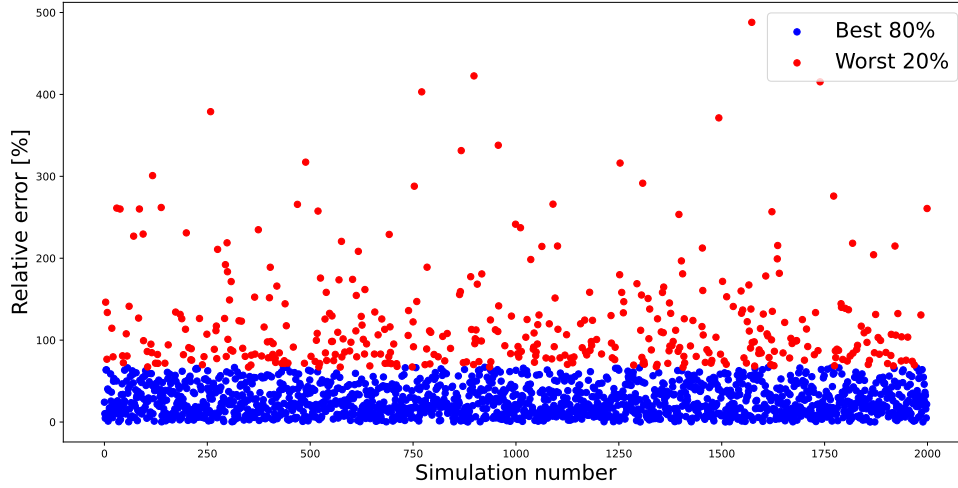


Figure 2.36: Relative error from a 2000 testing dataset with a regression of 100 training samples

In Figure 2.36, the errors from the frequency prediction are separated into two categories: the 80 [%] of points with the lowest error, shown in blue, and the remaining points with the highest error, shown in red. The mean values of the parameters used to compute the worst predictions are summarized in the table below. The average relative error on the worst 20 [%] of predictions is 120 [%], the regression does not give satisfactory results for these sets of parameters.

	Length [mm]	Width [mm]	Thickness [mm]	Base [mm]	Height [mm]	Detector's mass [kg]
Range	20-100	5-50	0.5-2	20-200	20-200	0.05-0.5
Range's mean	60	27.5	1.25	110	110	0.275
Worst 20% mean	76.63	30.62	1.09	109.49	112.51	0.27

Table 2.19: Mean parameter values for the worst 20% predictions compared to the overall ranges

Table 2.19 shows that the highest errors occur across the entire domain for most parameters, except for the length. The worst 20 [%] set of parameters has an average length of 77 [mm], which is significantly greater than the range average of 60 [mm]. The regression does not fit well for large lengths. For the enhanced regression, the length range is set to 20–50 [mm]. To understand the region where the regression does not fit the FEM results, a histogram of the number of simulations for each parameter is shown in the following figure.

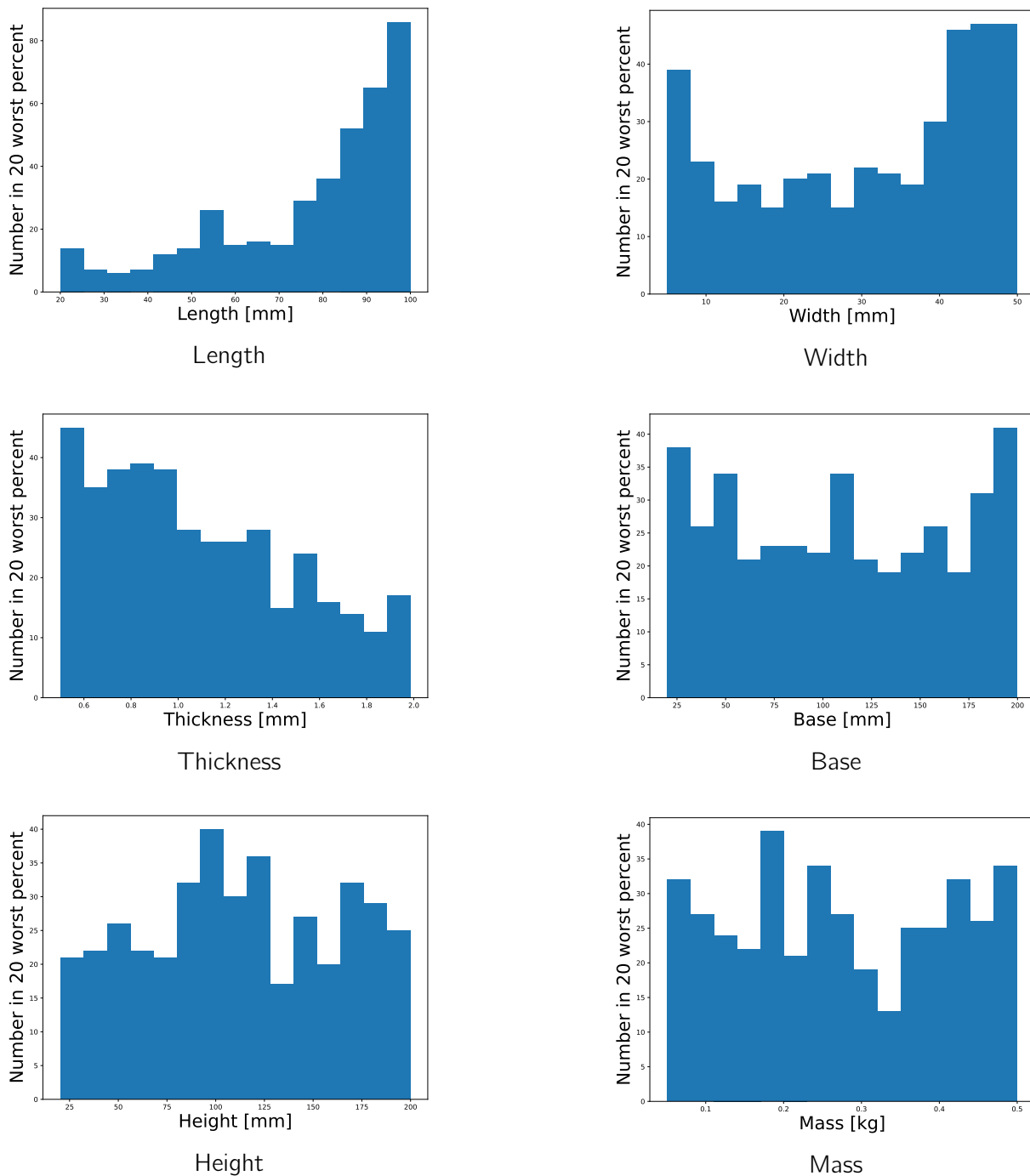


Figure 2.37: Histogram of the number of error in the 20 % worst case for each parameter

The figure 2.37 confirms what has been stated previously based on the average of the parameters: the probability of high error increases with length. Reducing its range is useful to decrease the error on the predicted first natural frequency. The second histogram shows that the probability of high error is higher for large widths. This is because the mode exhibits some flexion along the width of the blade for large widths. The probability of a non-negligible error also increases for small widths, but since the width must be minimized to increase the frequency, low widths will not be removed from the range. The new range for the width becomes 5-25 [mm]. The probability of a high error increases for low thicknesses. However, as with the width, the thickness must be reduced to ensure thermal insulation. The thickness range will therefore not be changed for the same reason. The last three parameters, the triangle base and

height, and the detector mass, show a nearly constant distribution of the probability of high error over their entire ranges. Their values do not affect the error in a significant way. The new parameter intervals are summarized in the table below.

Parameter	Old range [mm]	New range [mm]
Length	20-100	20-50
Width	5-50	5-25

Table 2.20: New parameter's range for the enhanced regression

The regression is computed using the same method as previously, with the new parameter range. The convergence of the error with respect to the training dataset size is shown in Figure 2.38.

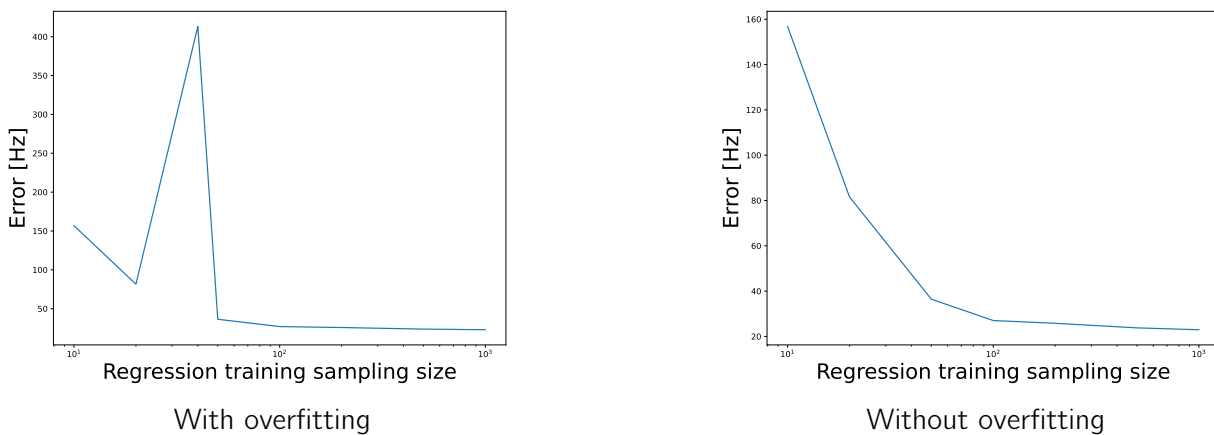


Figure 2.38: Convergence study of the regression with a reduced parameter domain

Figure 2.38 shows the error evolution, with and without overfitting, as a function of the training dataset size. The left graph shows the phenomenon of overfitting when the training dataset has 40 samples. On the right graph, the results are the same but without the results from the 40-sample training dataset. The error curves seem to stabilize for a training dataset size of 500 samples. This value is used for the comparison of the three regressions in the following section.

2.4.6 Comparison of the three regression

The comparison of these three regressions is based on their accuracy, ease of computing predictions, and computation time. The table below summarizes the regression characteristics and results.

	# Coefficients	# Samples	Mean[Hz]	Median [Hz]	Standard deviation [Hz]	Computation time [s]
Normal	40	100	4.64	2.02	30.55	0.05018
Coefficient reduction	18	100	0.39	-3.71	51.84	0.04205
Domain reduction	40	500	1.22	0.68	23.77	0.05969

Table 2.21: Comparison of the three regressions

Table 2.21 shows the number of coefficients computed by each regression and the number of samples used for each regression. It also summarizes the mean, median error, and standard deviation of the error computed from 1000 predictions over the entire domain. The last column shows the average computation time over 100 regression evaluations.

The first two columns are related to the ease of computation of the regression. The regression with the reduced number of coefficients is easier to implement, as it involves only 18 coefficients and not 40, see

Table 2.18. The formula is shorter than for the two other regressions. However, the coefficient selection is not straightforward, with some coefficients neglected for the quadratic terms and others retained for the cubic and quartic terms. Knowing which interactions are kept is also not intuitive. The second column shows the sample size of the training data required for the regression to converge. The last regression requires a larger training dataset than the other two. This is reflected in the computation time, which is faster for the second regression, as it has the lowest number of coefficients and a small number of samples needed for convergence. The slowest regression is the third one because it requires an increased number of samples to converge. This higher computational cost is balanced by its accuracy, which is discussed later in the study. However, the difference in computation time is not significant, as it is less than a tenth of a second.

The accuracy of the surrogate model is also very important, even more so than the ease of implementation. As stated previously, the mean and median converge toward zero because, for a sufficient sample size, the error distribution tends to a normal distribution centered at zero. The analysis that can be made from these results is that the full regression tends to overestimate the frequency, as both the mean and median are positive. Similarly, the regression with the reduced domain tends to overestimate the frequency, and the regression with 18 coefficients cannot be characterized by its mean or median error, as they have opposite signs. The standard deviation is an indicator of confidence in the results because it is used to compute the confidence intervals. It is computed by adding and subtracting the standard deviation from the mean. The table below summarizes the confidence interval of each regression based on their mean error and standard deviation.

Regression	68 % interval [Hz]	95 % interval [Hz]	99.7 % interval [Hz]
Normal	[-25.91;35.19]	[-56.46;65.74]	[-87.01;96.29]
Coefficient reduction	[-51.45;52.23]	[-103.29;104.07]	[-155.13;155.91]
Domain reduction	[-22.55;24.99]	[-46.32;48.76]	[-70.09;72.53]

Table 2.22: 68-95-99.7 % confidence interval for the regression

These confidence intervals characterize the probability that the true solution lies within each interval. For the 68 [%] confidence interval, the frequency computed by solving the FEM model has a 68 [%] chance of falling within this interval relative to the predicted frequency. This is valid under the assumption that the error distribution follows a normal distribution. The error distribution for each regression is shown in the figure below.

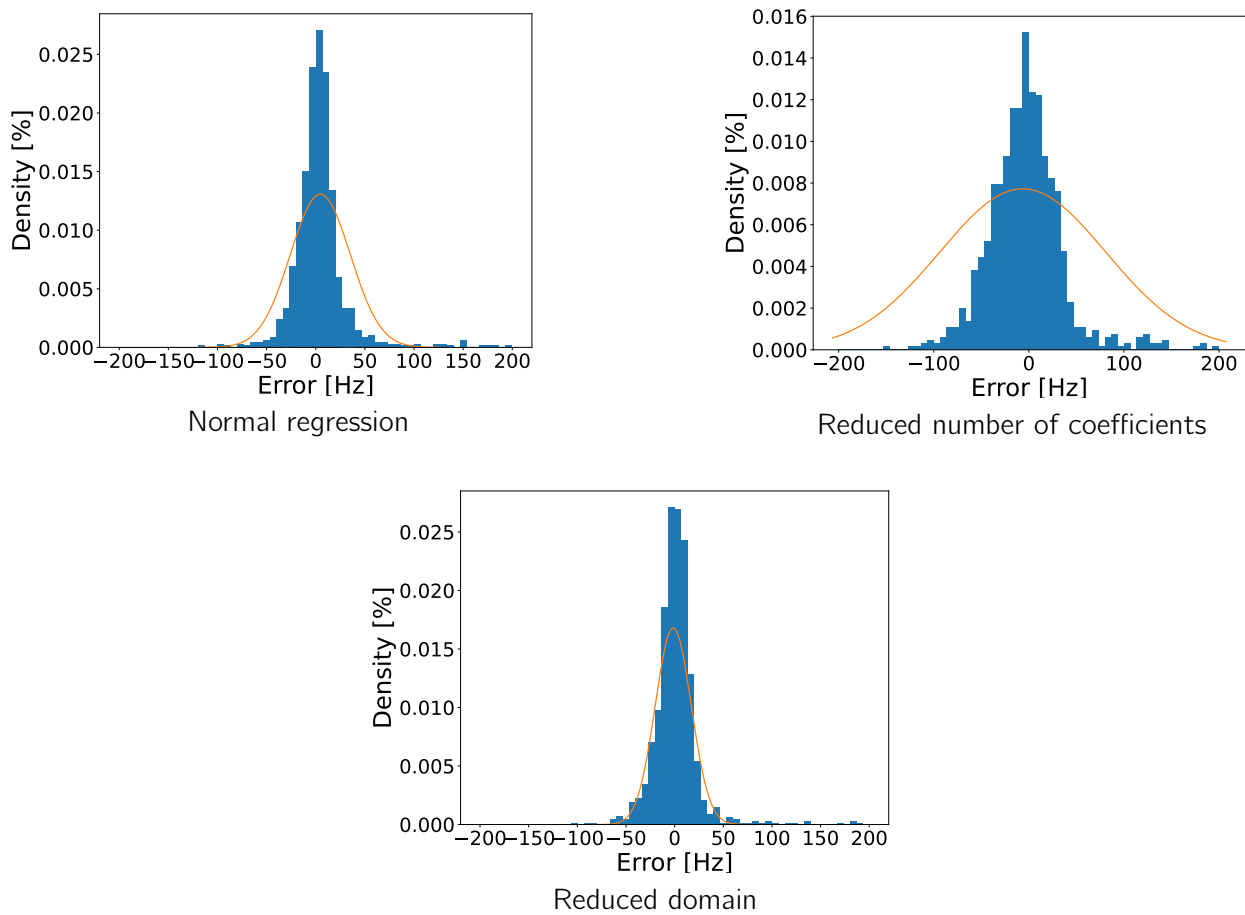


Figure 2.39: Regressions' histogram compared with a theoretical normal distribution

The error distribution shown in Figure 2.39 appears to follow a normal distribution, as expected. The normal curves in orange in each graph correspond to the normal distributions computed using the mean and standard deviation of the error. The theoretical curves differ from the observed distributions due to outliers in the error data, which increase the standard deviation. The curves are wider than the actual distributions and do not fit the data as intended. However, the error distributions from the predictions follow a Gaussian distribution with a lower standard deviation if outliers are not considered. The confidence intervals from Table 2.22 can be used to estimate the margin of error of the predictions. A comparison between the frequencies obtained from the predictions and from the numerical computations is shown in the figure below. The 95 [%] confidence interval is also represented in order to assess whether it includes the majority of the predictions.

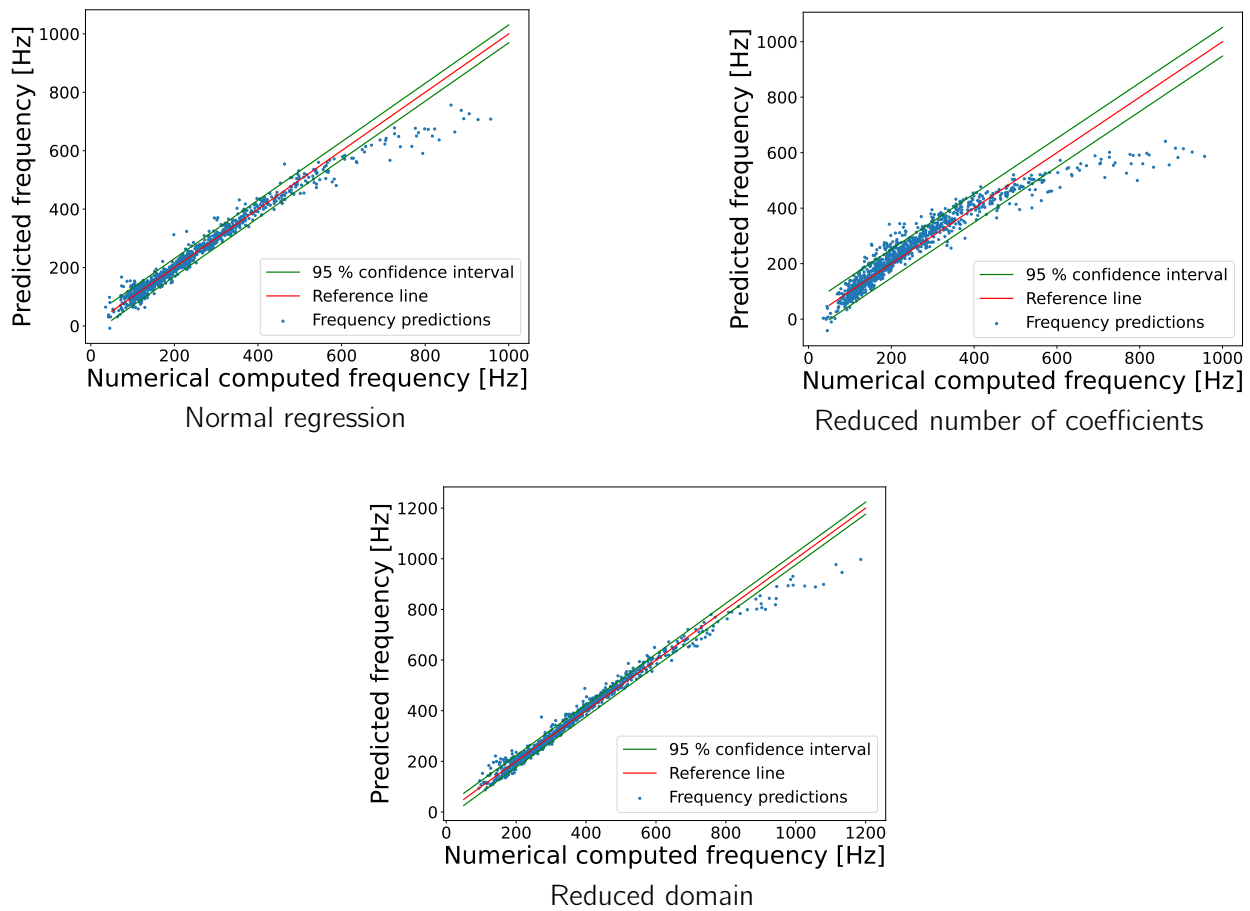


Figure 2.40: Comparison of the prediction with numerically computed frequency

Figure 2.40 represents the comparison between the predicted frequency and the frequency computed numerically with *NX Siemens*. The red line corresponds to the region where the prediction matches the numerical frequency perfectly. The green lines represent the lower and upper bounds of the 95 [%] confidence interval from Table 2.22. Overall, the predictions are included within the confidence interval. The predictions tend to underestimate high frequencies, with several data points falling below the confidence interval. However, this region is not of primary interest because the frequencies are typically much lower, around 300–500 [Hz], where the predictions lie within the interval. If high frequencies are required to fall within the 95 [%] interval, the standard deviation of the relative error could be used to compute the boundaries. The interval would then be wider for high frequencies and narrower for low frequencies. This standard deviation is not considered here because using a fixed interval is more meaningful in a real-world case.

The original regression has a 95 [%] confidence interval of -56 to +66 [Hz] relative to the true value of the frequency. This can be considered a wide range of inaccuracy, as it can lead to an error of 20 [%] for a predicted frequency of 300 [Hz]. The regression begins to underestimate frequencies above 420 [Hz], and for frequencies above 600 [Hz], the predictions are all below the confidence interval. An underestimation is not a major issue, as the true natural frequency is higher than the prediction. If the prediction meets the minimum frequency requirement, the true frequency would also satisfy the same requirement.

The regression with 18 coefficients shows the widest confidence interval of all the regressions. Its 95 [%] confidence range is -104 to +104 [Hz], which is not satisfactory. For a typical minimum frequency requirement of 300 [Hz], the relative error can reach up to 30 [%]. This regression was implemented to simplify the regression computation at the cost of accuracy. The reduction in coefficients makes the

regression faster, but the model becomes too inaccurate.

The regression with the reduced parameter domains has the narrowest confidence interval, indicating that it is the most accurate. The confidence interval ranges from -46 to +48 [Hz], which can be adequate for preliminary design considerations. This interval is most influenced by errors at high frequencies. This regression provides the best accuracy among the three cases considered.

2.5 Discussion

Several results have been obtained with this parametric simulation. First, the variation of each parameters have been studied and the optimization process are resumed in the section 2.3.1 if the thermal limitation is not considered and in the section 2.3.2 otherwise.

Then a surrogate model have been created using a regression to predict the frequency with an analytical formula. The regression have been subject to modification to decrease its computation time or increase its accuracy. The comparison and analysis of the results obtained from the surrogate model are resumed in the section 2.4.6.

These results are valid for a design with three blade in a isosceles assembly. This design have been used in several recent space mission as states in the section 1.3.1. The three blades setup provides satisfactory results, but its optimality has not been demonstrated. This is explored in the following section with topology tools.

Chapter 3

Topology optimization

The second part of this study is to explore different support designs applicable to similar requirements. This research is conducted using the topology optimization software *OOFELIE* developed by the *Open Engineering* company. The use of designs resulting from topology optimization is made possible by additive manufacturing. This technology is particularly attractive for aerospace applications, as it enables the creation of optimized designs for each part of a space instrument depending on its requirements [29]. This manufacturing method can reproduce the complexity of topology optimization results.

The first step is to translate the requirements into objectives and constraints that can be applied in a topology optimization problem. The main requirement is to achieve a high first natural frequency. Thermal decoupling between the fixation and the supported object can also be considered.

This study using topology optimization is divided into two parts. The first section excludes the thermal decoupling constraint, and the second section focuses on optimized designs considering the minimization of thermal conduction through the support. In each part, two different boundary conditions are studied: a parallel and a perpendicular fixation. These two boundary conditions are represented in the figure below.

In each configuration, two cases are considered: the first with fixed attachment points on the detector to simulate screw holes, for example. The second case does not impose any fixation points, the optimizer is free to create a support that connects to the detector anywhere on its back surface. The study and comparison of these two cases is interesting because, in topology optimization, increasing design freedom generally leads to more efficient optimal solutions. Imposing predefined attachment points restricts the design space and can prevent the optimizer from converging toward potentially better configurations that do not involve these regions.

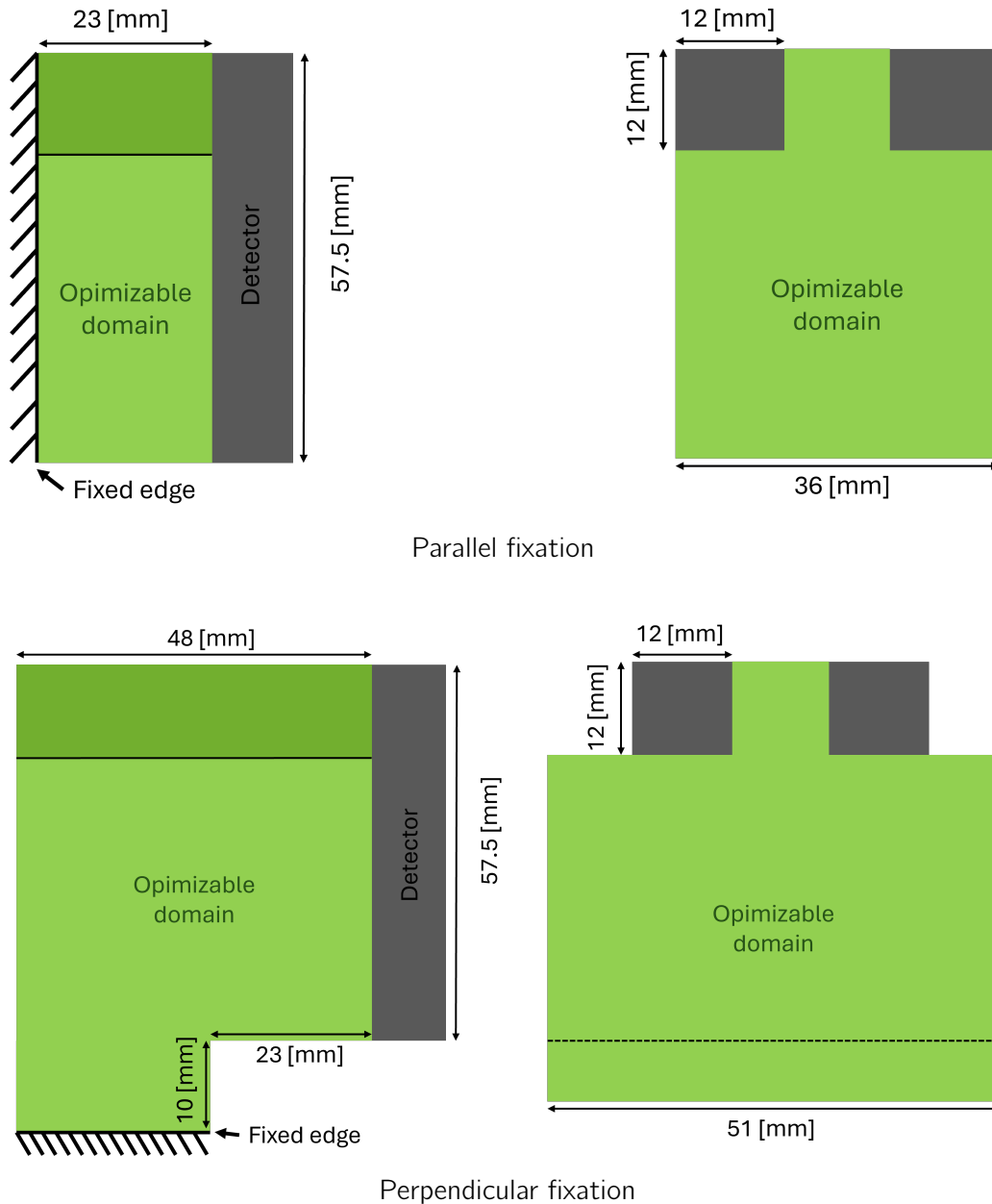


Figure 3.1: Side view (left) and rear view (right) of the two boundary conditions

In Figure 3.1, the two geometries are shown. The parallel fixation is applicable when a structure similar to the blades is used with a bracket that is not included in the optimization domain. This design is also valid if the detector is mounted on a vertical panel of the instrument, for example the rear panel of the instrument casing.

In the second geometry, the optimizer searches for a structure replacing both the blades and the bracket. The domain is larger than for the parallel fixation case. The boundary condition is a fixed surface perpendicular to the detector's face. The results obtained from this configuration can be applied to a fixation on a base plate, similar to what has been done in the EUI.

The dimensions of the optimizable domain are taken from the EUI instrument. The indentations on the top of the optimizable domain are due to copper braids linking the detector to a passive radiator. These links cool the detector to maintain its operational temperature.

3.1 Without thermal limitation

The purpose of this section is to find an optimized shape that satisfies the mechanical requirement: a minimum frequency. A topology optimization problem is usually formulated as follows:

$$\text{Minimize } f_i(\mathbf{x}) \quad (3.1)$$

$$\text{Subject to } g_j(\mathbf{x}) \leq 0 \quad (3.2)$$

Where $f_i(x)$ are called objective functions, $g_j(x)$ are inequality constraints, and $\mathbf{x}$ is the vector of design variables: the density of each element. Without any thermal consideration, the objective function in this section is the material volume. Minimizing the volume reduces the mass of the support, which when increasing can lead to higher launch energy requirements for the instrument and a decrease in natural frequency, as studied in Chapter 2. The objective function is stated as follows:

$$f_1(\mathbf{x}) = V(\mathbf{x}) \quad (3.3)$$

The constraint applied to the topology optimization problem is that the frequency must remain above certain limits: 300 [Hz] and 500 [Hz]. These two values are chosen because the first natural frequency of a sub-system in a space instrument is usually set to be approximately twice the requirement for the overall instrument. In addition, the results obtained with these two constraints can be compared to determine whether the optimizer finds two radically different topologies. The constraint formulation for the topology optimization problem is as follows:

$$g_1(\mathbf{x}) \geq 300[\text{Hz}] \quad (3.4)$$

$$g_1(\mathbf{x}) \geq 500[\text{Hz}] \quad (3.5)$$

The objective function and constraints describe the optimization problem to be solved. However, the optimizer does not converge to a feasible solution when only these two functions are considered. In fact, the optimization problem sets the density of every element to zero, effectively disconnecting the detector. The volume is then minimized. To avoid this issue, a static analysis is performed, including a downward acceleration equivalent to Earth's gravity. An objective function is introduced to minimize the displacement at the top of the detector. This second objective prevents forward tilting of the detector. This second function is stated as follows:

$$f_2(\mathbf{x}) = u_{node}(\mathbf{x}) \quad (3.6)$$

The optimization problem is formulated with two objective functions and one constraint. The two objective functions are combined into one using a weighted sum of their values. The weighted sum can be written as the following equation.

$$f_3(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x}) \quad (3.7)$$

$$f_3(\mathbf{x}) = w_1 V(\mathbf{x}) + w_2 u_{node}(\mathbf{x}) \quad (3.8)$$

$$f_3(\mathbf{x}) = 0.99V(\mathbf{x}) + 0.01u_{node}(\mathbf{x}) \quad (3.9)$$

The weights are defined to prioritize the minimization of the first objective function over the second, as the second is included to ensure that the optimizer connects the detector to the boundary condition. Setting the second weight to 1 [%] of the first allows the optimizer to focus on minimizing the volume.

The problem can be formulated as the equation below.

$$\text{Minimize } f_3(\mathbf{x}) = 0.99V(\mathbf{x}) + 0.01u_{node}(\mathbf{x}) \quad (3.10)$$

$$\text{Subject to } g_1(\mathbf{x}) \geq 300[Hz] \quad (3.11)$$

$$\text{Or subject to } g_1(\mathbf{x}) \geq 500[Hz] \quad (3.12)$$

3.1.1 Parallel fixation

In this section, the first configuration is considered, and the problem stated in Equation 3.10. This topology optimization problem is applied to the geometry represented in Figure 3.1 for the parallel fixation. The detector, shown in gray in the figure above, is meshed using 3D hexahedral elements with a side length of two millimeters. In the case of the parallel fixation, since the optimizable domain is smaller than for the perpendicular fixation, this region is meshed using 3D hexahedral elements with a side length of one millimeter. The mesh is perfectly symmetric with respect to the front-to-back vertical plane. This symmetry ensures that the computed solution is not biased toward one side. As the problem is symmetric, the solution should also be symmetric. A symmetric mesh ensures the symmetry of the solution.

The detector is considered a passive region, in other words, its mesh is not subject to a SIMP law. Its density is constant and equal to one. This region is not considered in the sensitivity analysis but is taken into account in the eigenfrequency computation. To ensure the imposed fixation points, three passive regions are modeled between the optimizable domain and the passive detector region. They are located at the screw holes used in the EUI instrument. The optimizer is free to connect to one or several of these links, but they are the only paths available to support the detector. In the case of the free fixation, the optimization domain is directly connected to the passive detector. These models are represented in Figures 3.2 and 3.3.

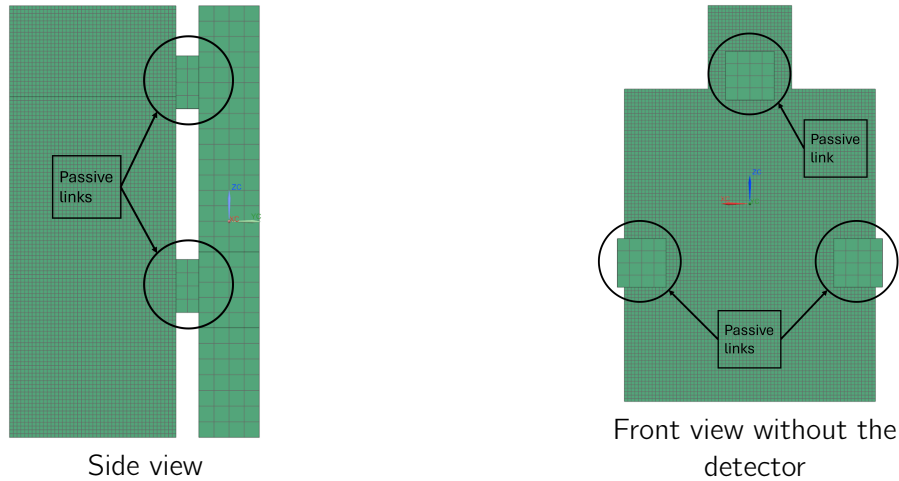


Figure 3.2: Meshed geometry with parallel imposed fixation points

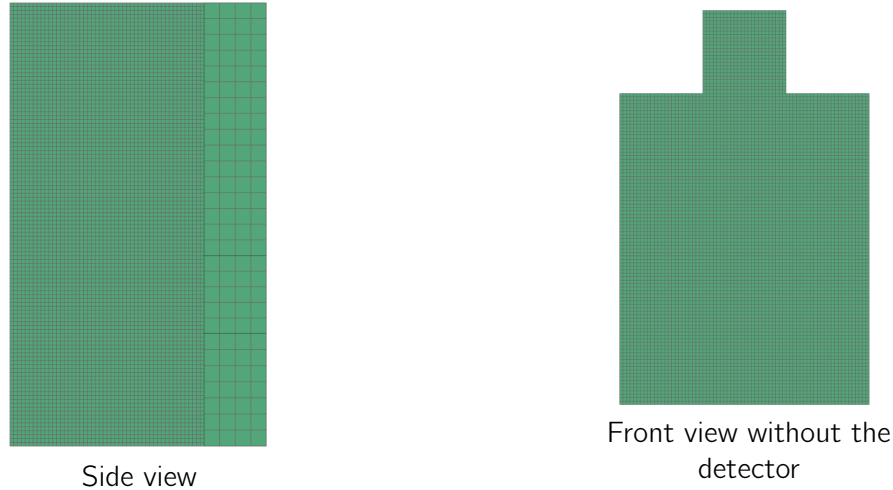


Figure 3.3: Meshed geometry with parallel free fixation

The result obtained with these simulation are analyzed in the following section.

With imposed fixation point

This section aims to discuss the results obtained from the topology optimization stated in Equation 3.10, with the geometry represented in Figure 3.2. The number of iterations was set to 100 to ensure convergence and a low NDR. A convergence criterion on the NDR was set to 0.5 [%]. The final topology obtained at the last iteration is displayed in Figure 3.4, with a 300 [Hz] minimum frequency constraint, and in Figure 3.5, for a 500 [Hz] minimum frequency constraint.

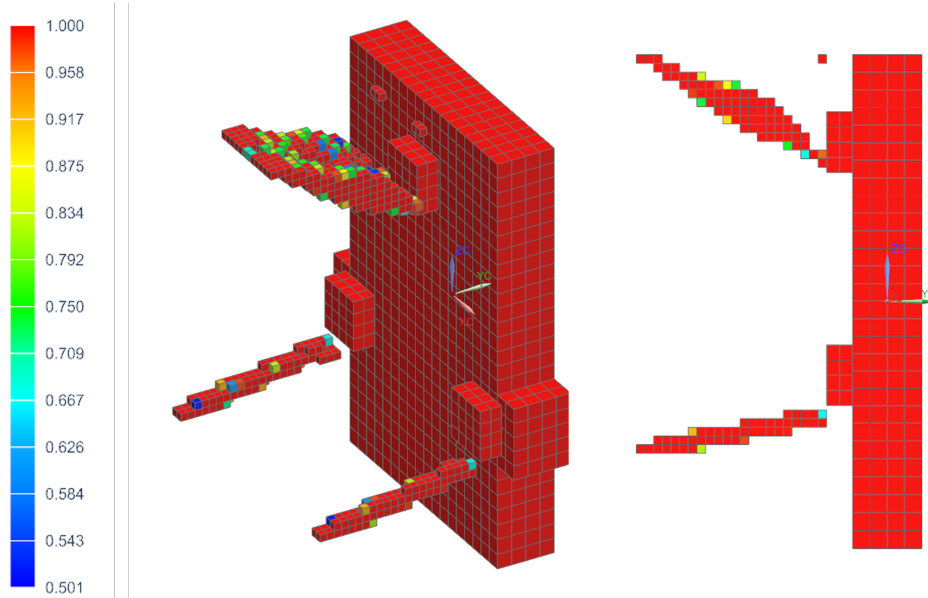


Figure 3.4: Mechanical topology optimization result for a parallel fixation with fixed attachment point with a minimum frequency constraint of 300 [Hz]

Figure 3.4 shows results similar to a three-blade design, with three diagonal elements connecting each fixation point. The top blade is the widest, with a separation in the middle forming a V-shape. The two bottom elements are more like beams with a constant cross-section. They differ from blades because their cross-section is not a thin rectangle but closer to a square.

The results also show disconnected elements near the top of the detector. These are numerical artifacts and should be neglected. The two bottom beams appear to be disconnected from the detector, but this is not the case due to the formulation in Equation 1.20. A mesh with a density of zero still has a small influence on the dynamics of the system. This influence is not negligible. It is discussed after the analysis of the results with a minimum frequency constraint of 500 [Hz].

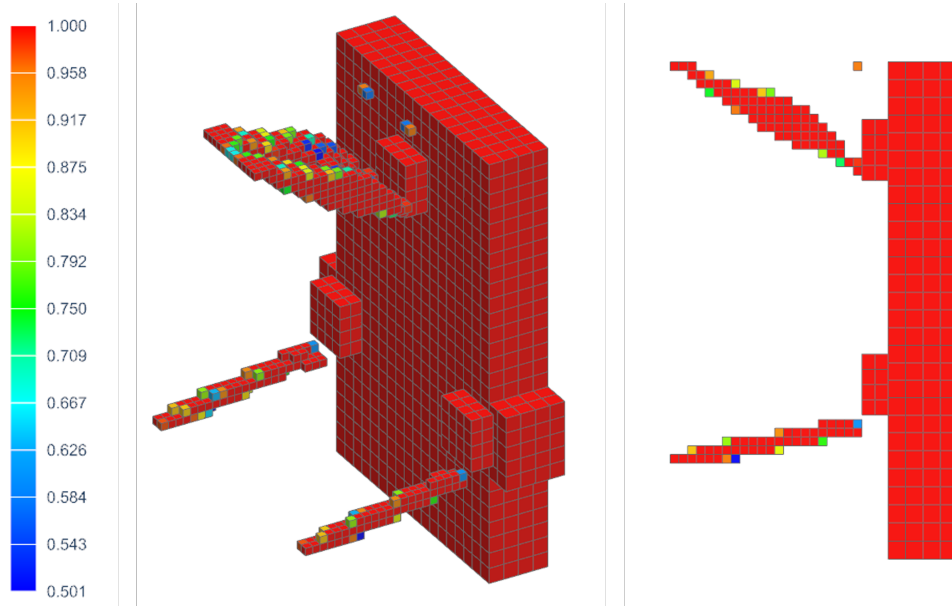


Figure 3.5: Mechanical topology optimization result for a parallel fixation with fixed attachment point with a minimum frequency constraint of 500 [Hz]

Figure 3.5 shows results similar to those obtained with the 300 [Hz] minimum frequency constraint. The similarity arises because the converged topology, obtained with a minimum frequency constraint, achieves a final frequency above the 500 [Hz] requirement. The frequency convergence is represented in Figure 3.6.

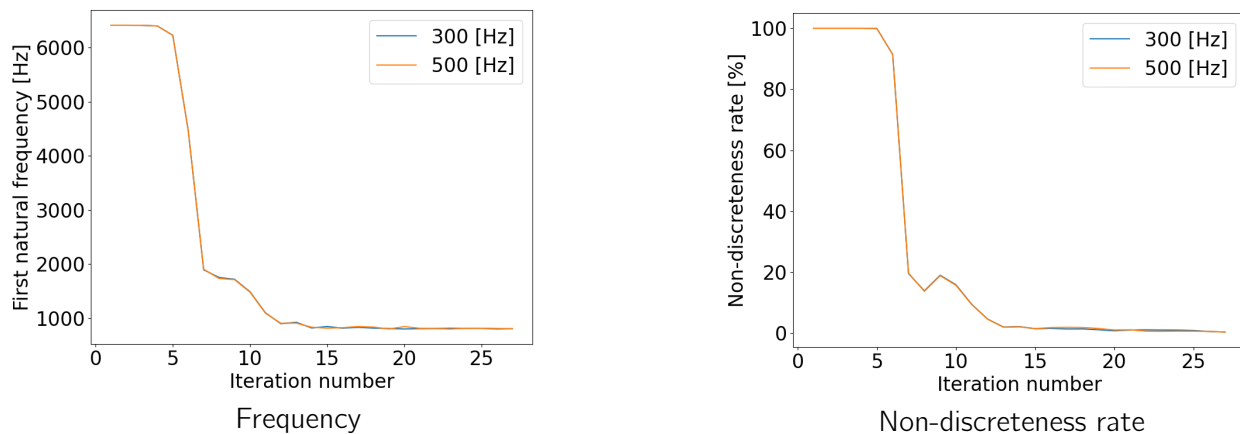


Figure 3.6: Convergence of the frequency and the NDR for the parallel imposed fixation

Figure 3.6 shows similar frequency and NDR evolution over the iterations. The two curves overlap almost perfectly. This explains the fact that the two different constraints produce the same result. The final frequency obtained with the 300 [Hz] constraint is above 800 [Hz]. There is no need to change the result for the 500 [Hz] constraint, as the first result already obtained with the 300 [Hz] constraint already

satisfies the second constraint.

However, the numerical treatment of material properties means that material is still present in regions where the density is close to zero. The material with minimum density properties still has an impact on the modal computation. The frequency obtained from the converged result may therefore be overestimated. In future studies, a model reproducing the optimal topology should be analyzed to confirm these results

With free fixation point

This section aims to find the optimal design for an optimizable parallel fixation without imposed fixation points. The design is free to connect to the detector anywhere. The same topology optimization problem from Equation 3.10 is considered. The first results shown in Figure 3.7 have a first natural frequency above the 300 [Hz] constraint.

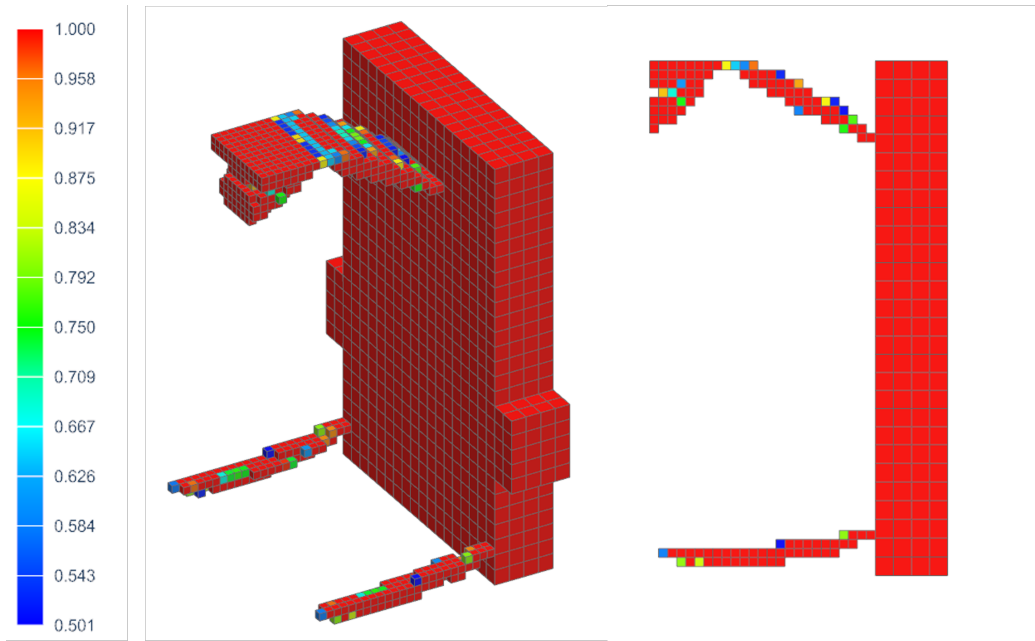


Figure 3.7: Mechanical topology optimization result for a parallel fixation with free attachment point with a minimum frequency constraint of 300 [Hz]

Figure 3.7 represents a design similar to the one obtained with imposed fixation points. The design is composed of three elements connecting the bracket to the detector. The top blade is wide and thin, while the bottom two are like beams with an approximately square cross-section. The blades seem to have a larger section at the connection with the bracket.

The difference between the two previous simulations is that the fixation points are not located in the same places. The top fixation is roughly in the same position as in the imposed fixation case. However, the bottom two are much lower than in the previous simulation.

The same topology optimization problem can be applied with a minimum frequency of 500 [Hz]. The result obtained is represented in Figure 3.8.

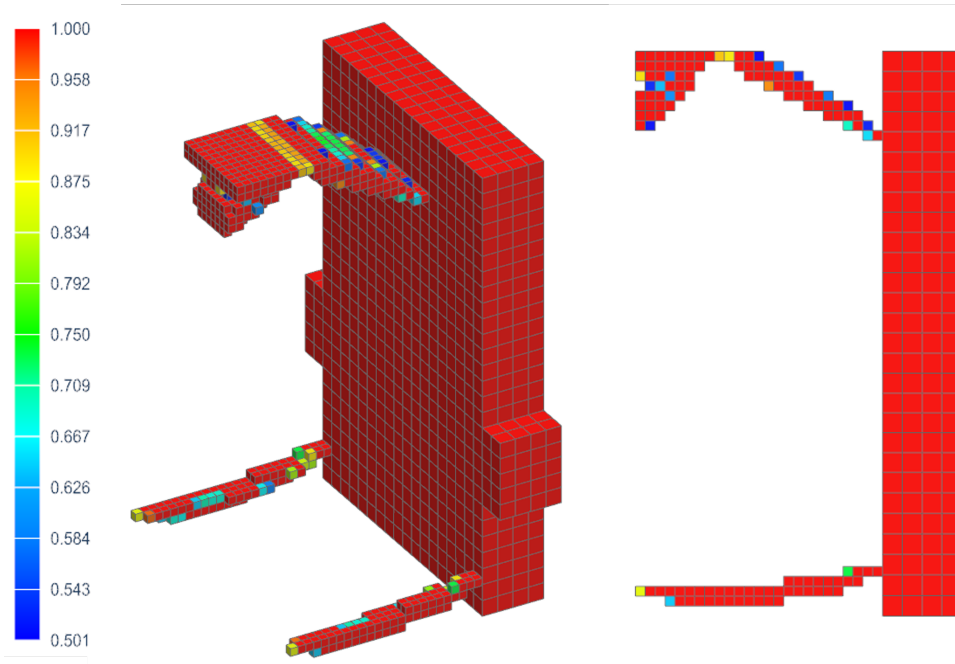


Figure 3.8: Mechanical topology optimization result for a parallel fixation with free attachment point with a minimum frequency constraint of 500 [Hz]

The result displayed in Figure 3.8 is identical to the result obtained with the 300 [Hz] constraint. The topology consists of one blade fixing the top of the detector and two beams connecting the bottom of the detector on each side. As for the previous simulations, the evolution of the frequency and NDR can be represented over the iterations. The curves are shown in the following graphs.

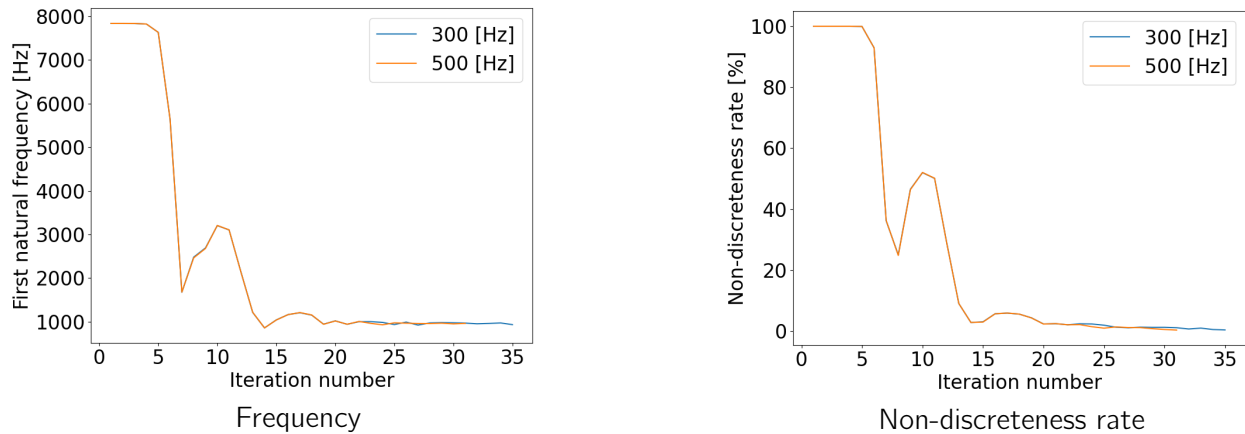


Figure 3.9: Convergence of the frequency and the NDR for the parallel free fixation

Figure 3.9 shows two overlapping curves for each minimum frequency constraint. The first topology optimization problem, with a minimum frequency of 300 [Hz], converges to a frequency of 931 [Hz]. The second topology optimization problem converges to the same topology, as the first already satisfies the 500 [Hz] minimum frequency constraint. This explains the similarity and the equivalent results for the two topology optimization problems.

3.1.2 Perpendicular fixation

After the parallel fixation case, the bracket is also optimized. The model is composed of a topology region subjected to a SIMP-alpha law and a detector modeled as a passive region. The model is represented in the scheme in Figure 3.1. For the same reasons as in the parallel imposed fixation case, three passive links are modeled in order to force the topology optimizer to connect to the fixation points.

The passive region modeling the detector is meshed with 2 [mm] hexahedral elements. Meanwhile, the optimizable region is meshed with 1.5 [mm] hexahedral elements. As for the parallel fixation case, the mesh is perfectly symmetric with respect to the vertical plane passing through the longitudinal axis of the structure. This symmetry avoids side-biased solutions that would not be consistent, since the problem is symmetric.

The mesh is represented in Figures 3.10 and 3.11.

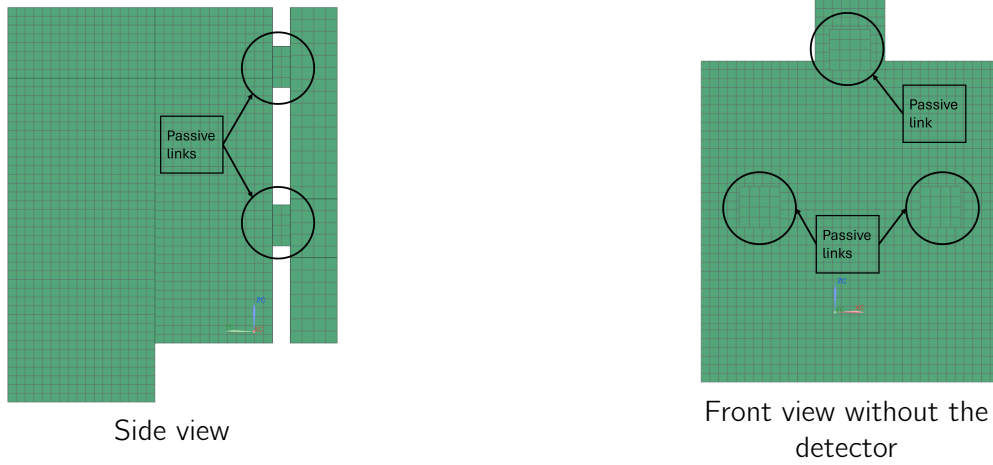


Figure 3.10: Meshed geometry with perpendicular imposed fixation points

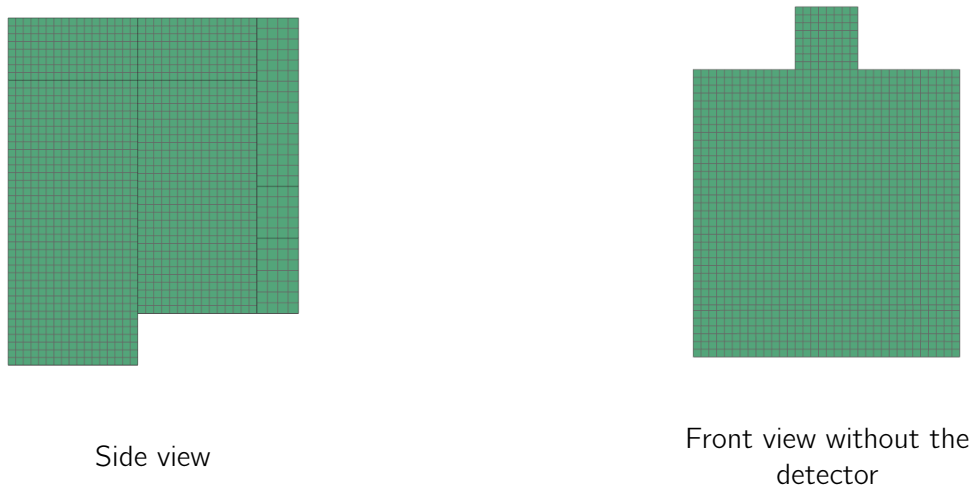


Figure 3.11: Meshed geometry with perpendicular free fixation

The problem from Equation 3.10 is applied to the described geometry, with a convergence criterion defined as the NDR becoming less than 0.5 [%]. The results obtained are separated into two categories: imposed and free fixation. Each category includes two topology optimization problems, corresponding to minimum frequency constraints of 300 and 500 [Hz].

With imposed fixation point

The results obtained by applying the topology optimization problem from Equation 3.10 to the numerical domain represented in Figure 3.10 are shown in Figures 3.12 and 3.13.

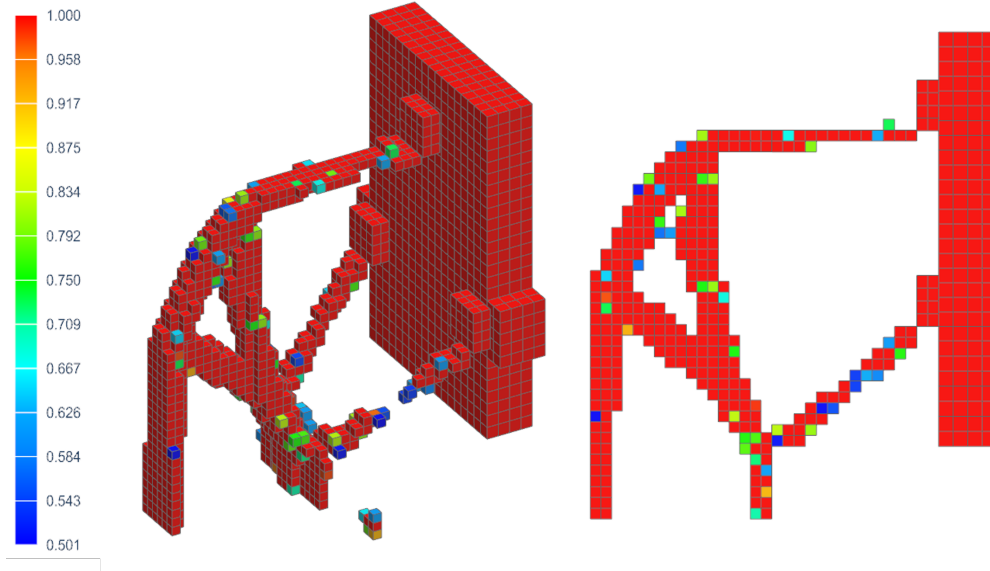


Figure 3.12: Mechanical topology optimization result for a perpendicular fixation with imposed attachment point with a minimum frequency constraint of 300 [Hz]

Figure 3.12 shows a truss structure that can be divided into two parts, similar to the bracket and the blades. The bracket consists of a planar truss connecting the optical bench to the blades, similar to the EUI design. The bracket can be reduced to four beams. The three blades behave more like beams with a square cross-section. The top beam is horizontal, continuing the curvature of the bracket. The bottom two beams connect the base of the bracket to the detector in a diagonal V-shaped configuration.

The next topology optimization is performed with the same problem but with a minimum frequency constraint of 500 [Hz]. The result is represented in Figure 3.13.

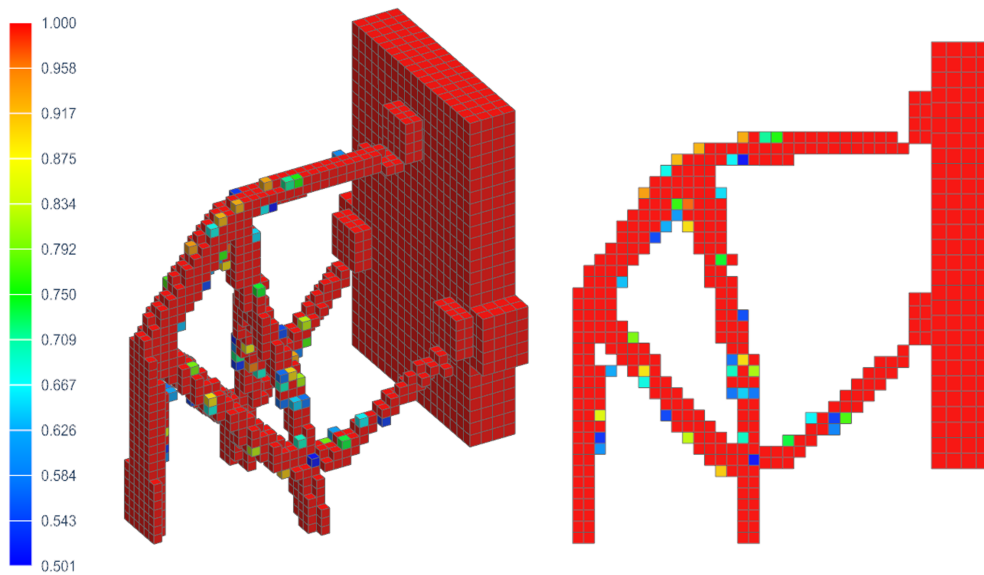


Figure 3.13: Mechanical topology optimization result for a perpendicular fixation with imposed attachment point with a minimum frequency constraint of 500 [Hz]

The results show a similar truss structure that can also be divided into two parts: the bracket and three beams. As the minimum frequency increases, the bracket is no longer a 2D truss but consists of six beams with two V-shaped assemblies. The three beams connecting the detector are similar to the previous result. The top beam is horizontal, and the bottom two connect the base of the bracket to the fixation points diagonally. They are more parallel than in the previous result in the horizontal plane.

The two results obtained with the two different constraints differ, unlike the case with the parallel fixation. The convergence plot of the frequency and NDR is represented in Figure 3.14.

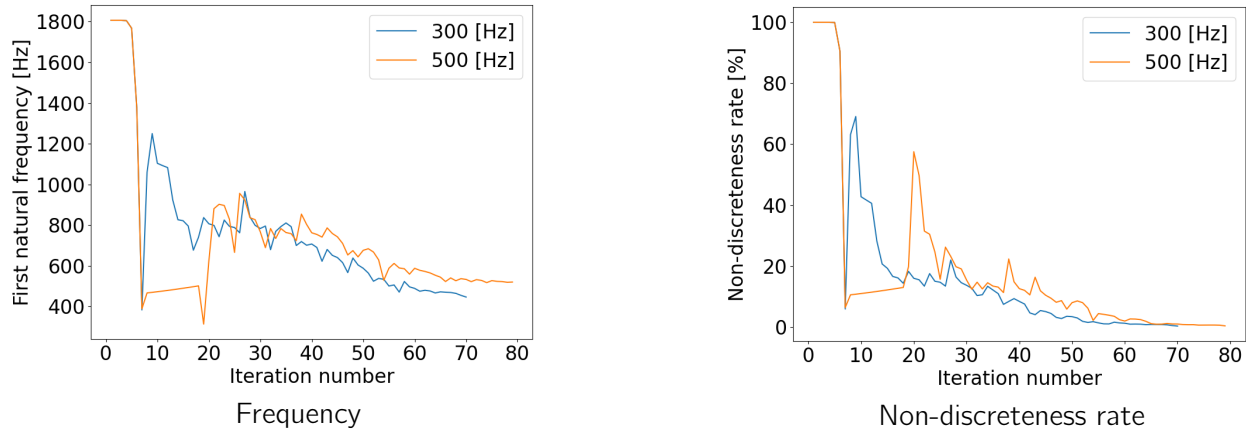


Figure 3.14: Convergence of the frequency and the NDR for the perpendicular imposed fixation

The two evolutions are clearly different, and the first optimization converges to a frequency of 446, while the second converges to a frequency of 520. The second optimization converges closer to the constraint and appears consistent. A post-processing analysis is still required to ensure that the frequency is correct and not too strongly influenced by the numerical treatment of elements with zero density.

As the problem is more complex, the optimization takes more time and iterations to converge. The first optimization converges with a NDR below 0.5 [%] after 70 iterations, and the second takes 79 iterations to meet the convergence criterion.

With free fixation point

In this section, the optimizer is free to choose the fixation points on the detector. In general, the freer the problem, the more optimal the solution. The problem from Equation 3.10 is applied to the domain shown in Figure 3.11. Two constraints are considered, with minimum frequencies of 300 [Hz] and 500 [Hz]. The converged topology with a 300 [Hz] minimum frequency is represented in Figure 3.15.

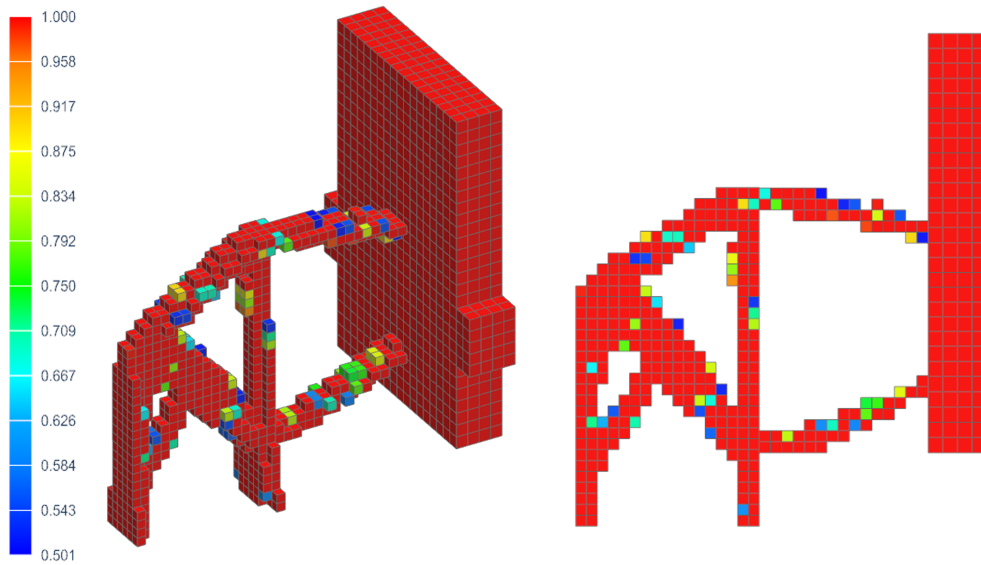


Figure 3.15: Mechanical topology optimization result for a perpendicular fixation with free attachment point with a minimum frequency constraint of 300 [Hz]

The topology obtained is a truss structure with two fixation points to the detector. The entire structure lies in a vertical plane. The structure can be decomposed into seven beams with a constant cross-section. This result may be over-influenced by the vertical downward acceleration used to ensure that the optimizer connects to the detector. In reality, such a structure has a high probability of not fulfilling the requirements and therefore cannot be used.

The next optimization is performed with a minimum frequency constraint of 500 [Hz]. The optimization did not converge within 100 iterations, so a new optimization with 200 iterations was performed in order to satisfy the convergence criterion. The obtained topology is shown in Figure 3.16.

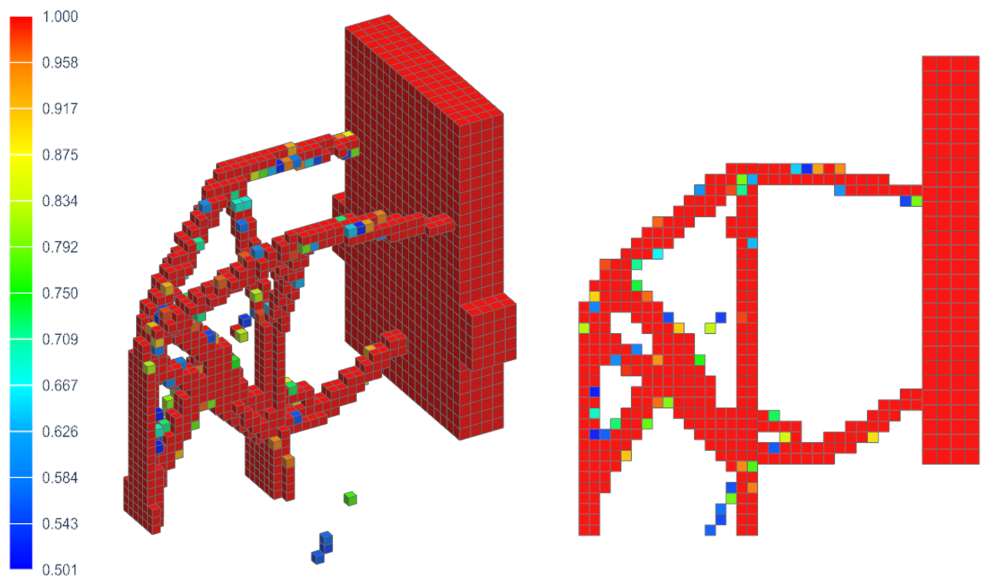


Figure 3.16: Mechanical topology optimization result for a perpendicular fixation with free attachment point with a minimum frequency constraint of 500 [Hz]

Unlike the 300 [Hz] frequency constraint case, Figure 3.16 shows a truss structure connecting the detector through three fixation points. This design appears to be more effective than the previous one

with an isostatic fixation. The fixation locations are: two at the top of the detector, on the left and right sides of the rear face, and one at the bottom center. This setup is the opposite of the imposed fixation configuration.

The truss structure is composed of eleven beams. Three are used to connect the bracket-equivalent structure to the detector, imitating the blades from the three-blade design. The bracket consists of the eight remaining beams with two V-shaped assemblies.

The frequency and NDR evolution can be represented over all iterations. This representation is shown in Figure 3.17.

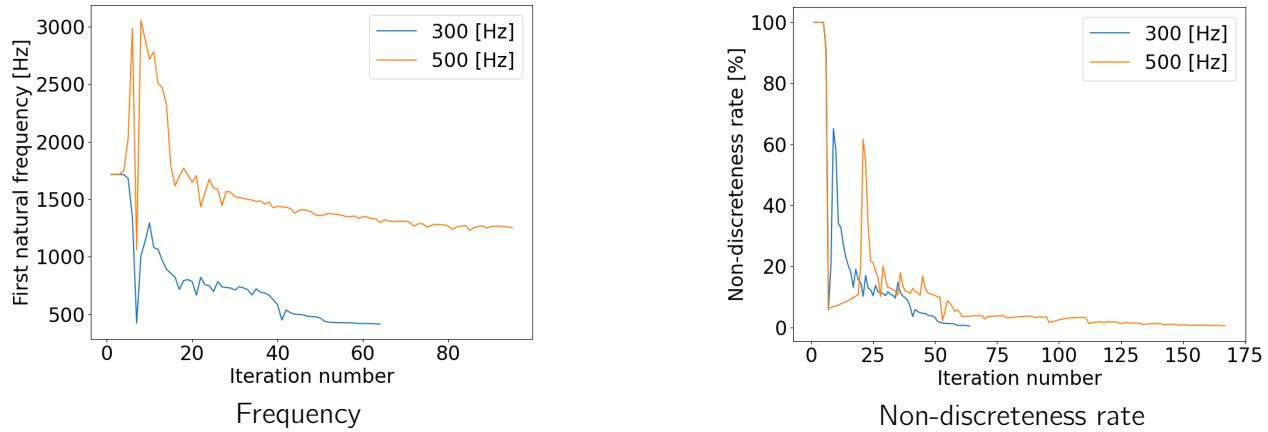


Figure 3.17: Convergence of the frequency and the NDR for the perpendicular free fixation

Figure 3.17 shows the frequency decreasing as the number of iterations increases. The evolutions exhibit sawtooth-shaped curves. This is due to the fact that the optimizer gradually converges toward the final solution through a trial-and-error process, which can lead to an initially too low frequency estimate. This can be particularly seen in the curves with the 500 [Hz] constraint. The frequency curve drops from 1346 [Hz] at the sixth iteration to 431 [Hz] at the next iteration. This frequency is below the minimum frequency constraint, so the optimizer must find a new topology that satisfies the constraint.

The NDR curves follow the same trend, with a sharp decrease when the optimizer finds a topology that reduces the objective function. If this topology does not meet the constraint requirements, the NDR starts to increase as the optimizer searches for a new topology. This process is repeated until the convergence criterion is met.

3.2 With thermal limitation

This section aims to consider the thermal properties of the materials and minimize heat conduction through the support. The topology optimization problem becomes a thermo-mechanical problem. The geometry of the problem is the same as for the mechanical optimization. The mesh includes a temperature field in addition to the displacement field. The topology optimization is formulated in the same way as in Equation 3.1, with objectives and constraints. The thermal properties of the material are summarized in Table 3.1 [30–32].

Material	Thermal conductivity [W/mK]	Coefficient of thermal expansion [W^{-1}]
Ti6Al4V	7	8.5e-6
AlN	150	5.6e-6

Table 3.1: Thermal properties of materials

The main objective is still the volume minimization, the formulation is the following:

$$f_1(\mathbf{x}) = V(\mathbf{x}) \quad (3.13)$$

The other objective is the minimization of heat flux conduction through the support. However, computing a flux crossing a surface is not yet possible in *OOFELIE*. The second objective must therefore be formulated to synthesize the same effect in the optimization. Using the thermal conduction equation, the thermal conductance of the support can be computed.

$$q = GL_{th}\Delta T \quad (3.14)$$

$$GL_{th} = \frac{q}{\Delta T} \quad (3.15)$$

The minimization of thermal conductance can be achieved either by minimizing heat conduction or by maximizing the temperature difference between the two ends of the support. In this study, the second option was chosen. The optimization of the temperature gradient is represented in Figure 3.18.

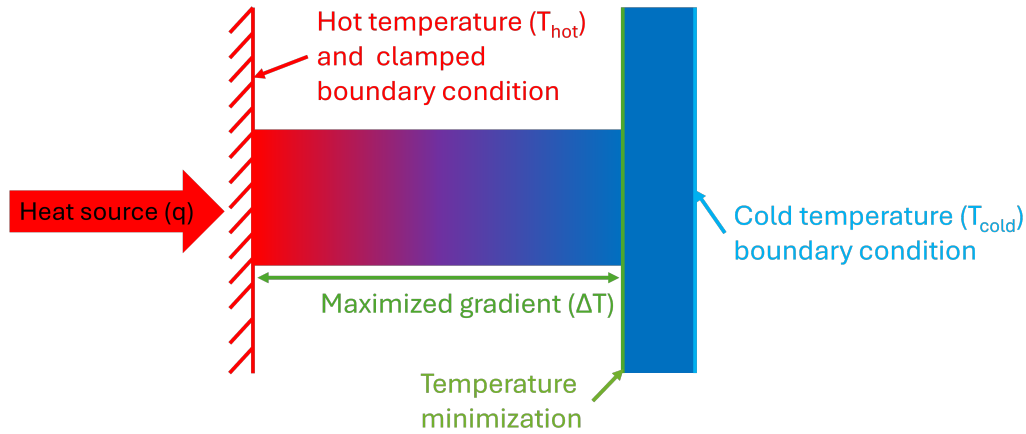


Figure 3.18: Thermo-mechanical optimization scheme

Figure 3.18 shows that minimizing the temperature at the connection between the support and the detector maximizes the temperature gradient. The thermal conductance is minimized for a given constant heat source. This objective is easy to implement with the imposed fixation case: the temperature of the middle node of the passive links is minimized.

However, with the free fixation case, the fixation point is unknown. Minimizing the temperature of each node on the rear surface of the detector would require the sensitivity analysis of each objective function and would significantly increase the computational cost. The approach adopted in this thesis is to maximize the thermal compliance [33]. The thermal compliance in a FEM model is computed using Equation 3.16.

$$C_{th} = \mathbf{q}^T \mathbf{T} \quad (3.16)$$

Where C_{th} is the thermal compliance, $\mathbf{q}^T$ is the heat flux vector, and $\mathbf{T}$ is the temperature field. Equation 3.16 shows that, for a given heat flux, maximizing the thermal compliance involves maximizing the temperature field. The only way to achieve this is by increasing the temperature and thus limiting heat conduction to the detector.

The second objective function can be formulated in two ways depending on whether the fixation is imposed or not. The objective functions are summarized in Equation 3.17.

$$f_{2,imposed}(\mathbf{x}) = T_{node}(\mathbf{x}) \quad (3.17)$$

$$f_{2,free}(\mathbf{x}) = C_{th}(\mathbf{x}) \quad (3.18)$$

$$(3.19)$$

As already explained above, these two objective functions lead the optimizer to completely disconnect the detector. A third objective function is added in order to force the optimizer to connect the clamped boundary condition to the detector. The same objective function from Equation 3.6 is added to the problem. This function consists of minimizing the displacement of the middle node on the top surface of the detector.

The two objective functions are combined into a single sum. This sum is not weighted due to instabilities in the optimizer that can lead to crashes. The combination of the objective functions is written in Equation 3.20.

$$f_4(\mathbf{x}) = f_1(\mathbf{x}) + f_3(\mathbf{x}) \quad (3.20)$$

$$f_4(\mathbf{x}) = V(\mathbf{x}) + u_{node}(\mathbf{x}) \quad (3.21)$$

The constraints are the same as in the purely mechanical optimization problem, with a minimum first natural frequency requirement. Since the topology does not change significantly between the 300 and 500 [Hz] minimum frequency cases, the optimization is performed only with a 500 [Hz] constraint. The formulation of the constraint is written in Equation 3.22.

$$g_1(\mathbf{x}) \geq 500[Hz] \quad (3.22)$$

The topology optimization problem can be written using the objectives and constraint described above. The formulation in Equation 3.23 was applied in all subsequent optimizations where the fixation location is imposed.

$$\text{Minimize } f_4(\mathbf{x}) = V(\mathbf{x}) + T_{node}(\mathbf{x}) + u_{node}(\mathbf{x}) \quad (3.23)$$

$$\text{Subject to } g_1(\mathbf{x}) \geq 500[Hz] \quad (3.24)$$

$$(3.25)$$

The other problem, formulated in Equation 3.26, was used when the fixation locations are undetermined.

$$\text{Minimize } f_4(\mathbf{x}) = V(\mathbf{x}) + u_{node}(\mathbf{x}) \quad (3.26)$$

$$\text{Maximize } f_2(\mathbf{x}) = C_{th}(\mathbf{x}) \quad (3.27)$$

$$\text{Subject to } g_1(\mathbf{x}) \geq 500[Hz] \quad (3.28)$$

$$(3.29)$$

3.2.1 Parallel fixation

In this section, the thermo-mechanical topology optimization problems formulated in Equation 3.23 and Equation 3.26 are applied to the domains represented in Figures 3.2 and 3.3, respectively. The mesh is the same as for the purely mechanical problem, using thermo-mechanical hexahedral elements with a one-millimeter edge length. As the temperature field is unknown in the model, each DoF has seven unknowns: three translations, three rotations, and the temperature.

The following optimizations are performed with 100 iterations and a convergence criterion defined by the NDR falling below 0.5 [%]. A density filter is used to avoid checkerboard patterns. Its radius is set equal to three times the average mesh element size.

With imposed fixation point

In this section, the fixation considered is parallel, and the fixation points are imposed at the same locations as in the EUI. The result obtained is represented in Figure 3.19. The Heaviside filter explained in Section 1.4.2 is applied to improve convergence, because without it the optimization would not converge to a coherent topology.

The result obtained from the topology optimization problem in Equation 3.23 is represented in Figure 3.19.

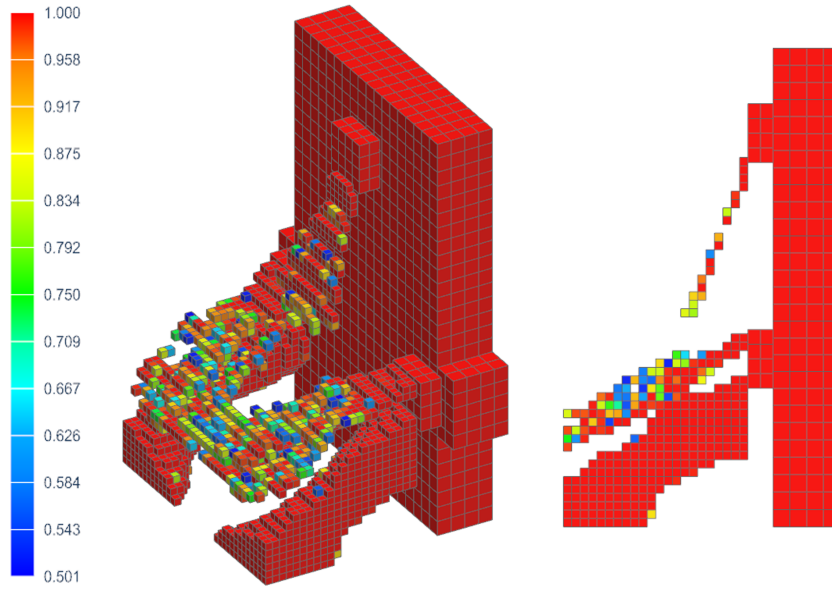


Figure 3.19: Thermo-mechanical topology optimization result for a parallel fixation with imposed attachment point with a minimum frequency constraint of 500 [Hz]

Despite the Heaviside filter, Figure 3.19 shows that the optimization has not converged after 100 iterations. If the number of iterations is increased, the filter would create holes in regions where the mesh has not converged. Another problem formulation must be found in order to obtain a converged topology.

However, the optimization seems to converge toward a structure with two supports connecting the two bottom fixation points. A thin organic-blade-like structure connecting the three upper fixation points above the two other structures can also be observed. This thin part has not converged, and the problem in Equation 3.23 cannot be solved by the optimizer.

The graphs of the frequency and NDR for each iteration are represented in Figure 3.20.

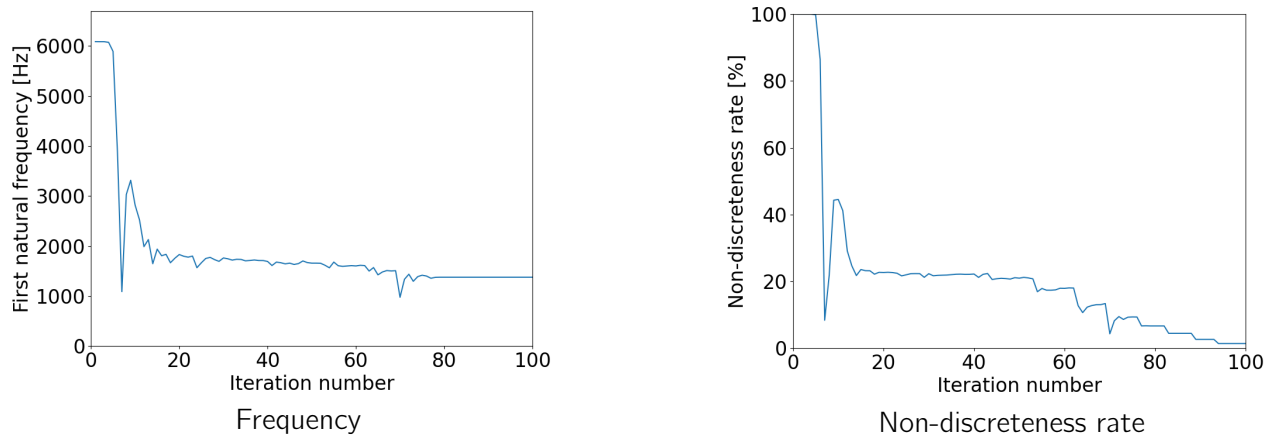


Figure 3.20: Convergence of the frequency and the NDR for the perpendicular free fixation

The graph of the frequency in Figure 3.20 shows a slow decrease in frequency from iteration 15 to iteration 79. After iteration 79, the frequency remains constant until the last iteration. This phenomenon occurs because the Heaviside filter modifies the density of mesh elements that are already disconnected and therefore do not influence the frequency.

The NDR evolution shows a slowly decreasing trend until iteration 53, when the Heaviside filter starts and increases the slope of the projection. Each step observed from this iteration corresponds to a change in the filter slope.

With free fixation point

In this section, the parallel fixation case with no imposed fixation locations is considered. The topology optimization problem from Equation 3.26 is applied to the domain in Figure 3.3. The Heaviside filter is also applied to improve the convergence of the optimizer. The results obtained by the software are represented in Figure 3.21.

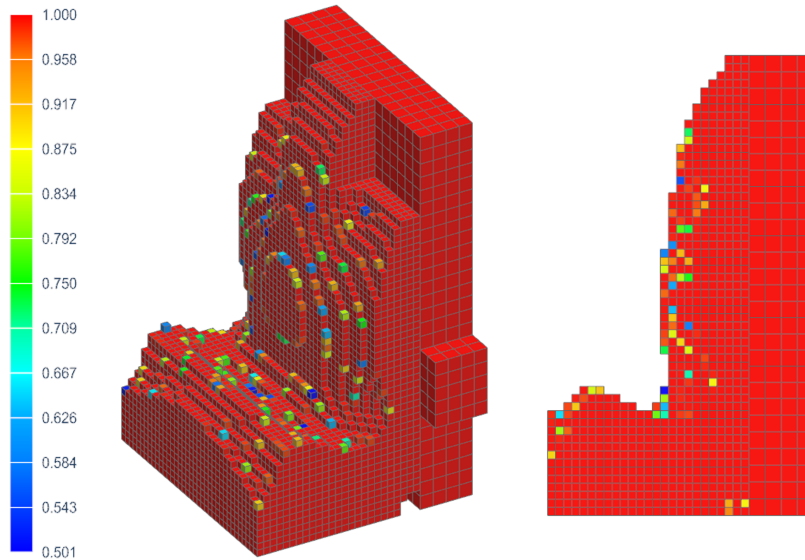


Figure 3.21: Thermo-mechanical topology optimization result for a parallel fixation with free attachment with a minimum frequency constraint of 500 [Hz]

In Figure 3.21, the obtained topology shows a clamped detector. The topology does not appear op-

timal: the material volume is too high, and the topology does not provide sufficient thermal decoupling between the detector and the bracket fixation. As mentioned previously, the software is still under development, the issue may arise either from the topology optimization problem formulation or from the software itself.

However, the evolution of the frequency and NDR can be analyzed using their representation in Figure 3.22.

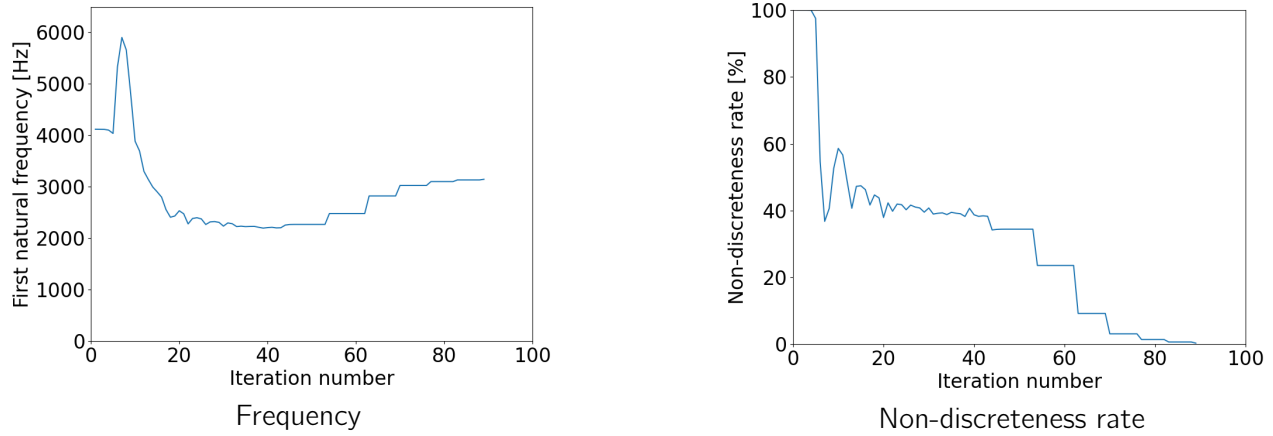


Figure 3.22: Convergence of the frequency and the NDR for the perpendicular free fixation

The frequency evolution shows a relatively constant plateau from iteration 20 to 53, when the Heaviside filter starts its projections. After this iteration, the frequency increases at each change in the filter slope until the convergence of the optimization.

The NDR shows a slowly decreasing trend over the iteration interval 20–53. After that, the Heaviside filter causes a sharp decrease in the NDR until convergence at iteration 89.

3.2.2 Perpendicular fixation

The purpose of this section is to apply the thermo-mechanical problems from Equation 3.23 and Equation 3.26 to a perpendicular fixation configuration. This configuration is represented in Figure 3.1, and the mesh is shown in Figures 3.10 and 3.11. The mesh is composed of thermo-mechanical hexahedral elements with a size of 1.5 [mm] in the optimizable domain and 2 [mm] in the passive regions.

The topology optimization is performed with a maximum of 100 iterations and a convergence criterion based on the NDR falling below 0.5 [%]. A density filter with a radius equal to three times the average mesh element size is applied to avoid checkerboard patterns.

With imposed fixation point

The first optimization problem considers the perpendicular imposed fixation, including thermal conduction through the support. The problem from Equation 3.23 is solved by the optimizer. The optimizer converges to a solution quickly enough that the Heaviside filter is not included in this topology optimization run. The result obtained is represented in Figure 3.23.

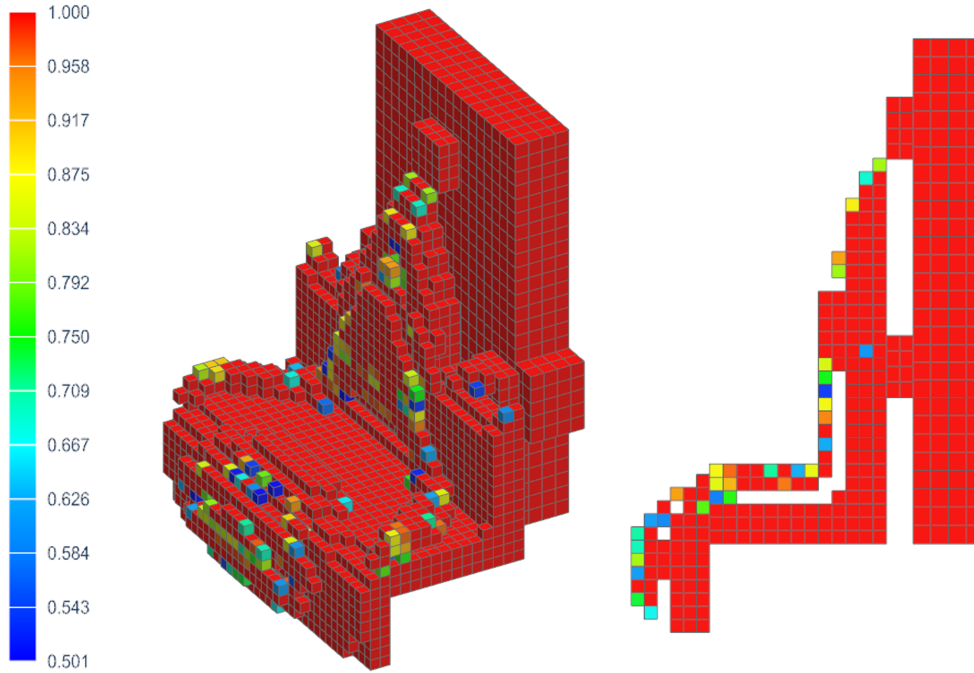


Figure 3.23: Thermo-mechanical topology optimization result for a perpendicular fixation with imposed attachment point with a minimum frequency constraint of 500 [Hz]

The topology obtained by the optimizer is a stair-like structure connecting the closest boundary conditions to the passive links. The final structure has a large volume and does not minimize thermal conduction through the support. The optimization may have converged to a local minimum that is not the optimal solution to the problem.

The first natural frequency and NDR evolution are represented in Figure 3.24.

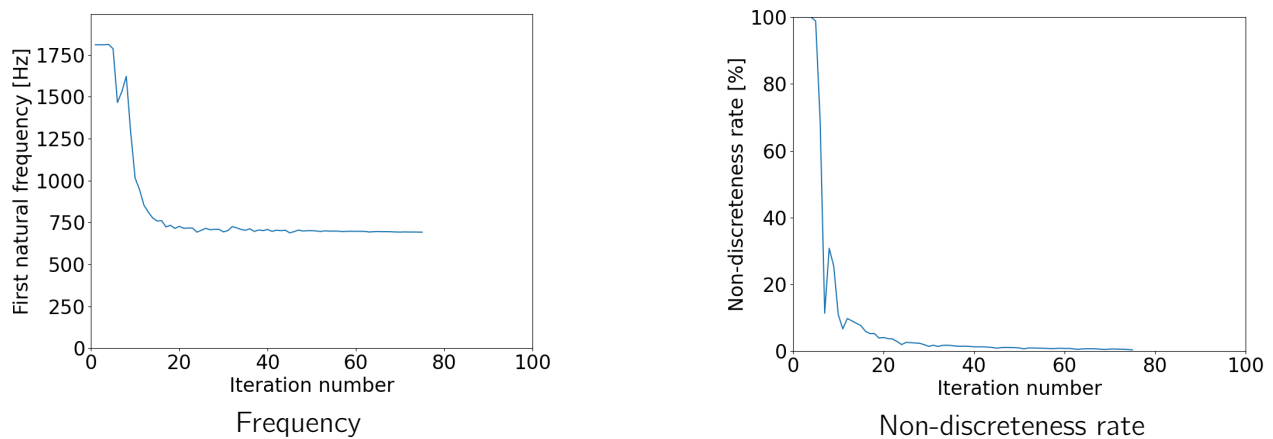


Figure 3.24: Convergence of the frequency and the NDR for the perpendicular imposed fixation

The curves seem to follow a typical optimization evolution, with a large decrease at the beginning up to iteration 15, followed by a slow decrease in both the frequency and the NDR until convergence at the 75th iteration. The use of the Heaviside filter to improve convergence is not necessary.

With free fixation point

In this section, the final topology optimization of this study is performed. The optimization problem from Equation 3.26 is applied to the domain represented in Figure 3.11. Both density and Heaviside filters are applied to improve the solution. The radius of the density filter is set to three times the average mesh element size. The maximum number of iterations is the same as in the previous thermo-mechanical optimizations, with a value of 100. The NDR convergence criterion is set to 0.5 [%]. The converged topology is shown in Figure 3.25.

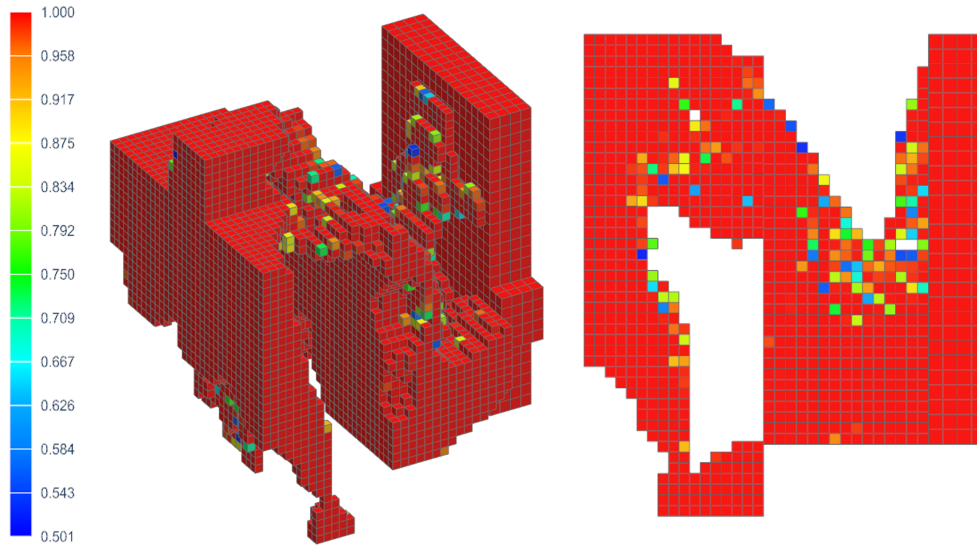


Figure 3.25: Thermo-mechanical topology optimization result for a perpendicular fixation with free attachment point with a minimum frequency constraint of 500 [Hz]

Figure 3.25 represents the optimal topology corresponding to the problem in Equation 3.26. It can be clearly seen that the obtained topology does not make sense and that the optimizer did not converge properly, while the Heaviside filter forces convergence. This phenomenon is visible in the frequency and NDR evolution graphs represented in Figure 3.26.

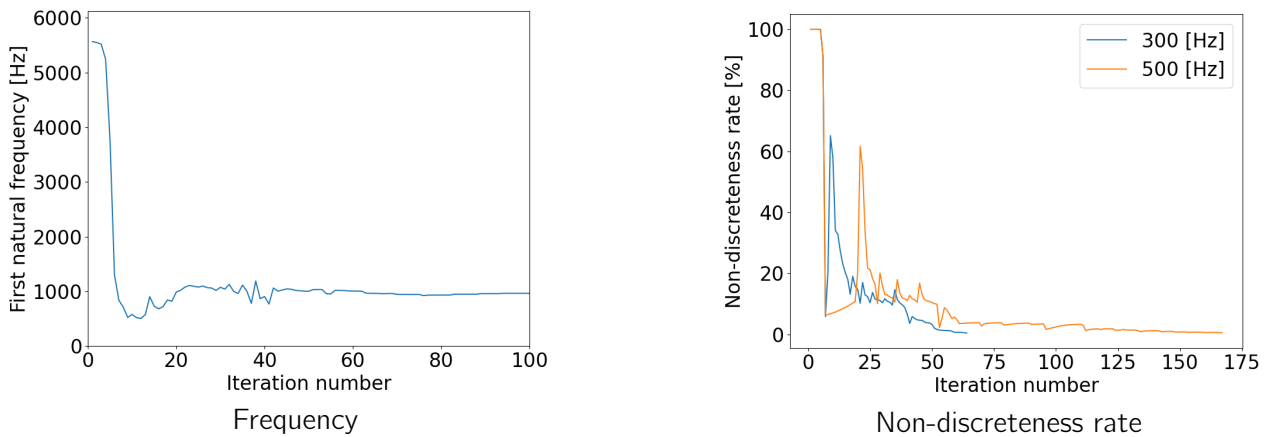


Figure 3.26: Convergence of the frequency and the NDR for the perpendicular free fixation

The curves in Figure 3.26 show that the frequency converges to a value of 965 [Hz] with normal behavior. The NDR curve shows an almost linear decrease with respect to the iterations. This is not a

typical trend compared to the curves obtained in previous optimizations. The stair-step pattern from the 44th iteration until the last iteration shows that the NDR decreases from 43 [%] to 1 [%] only due to the Heaviside filter. The optimizer did not find an optimal topology.

3.3 Conclusion

In conclusion, topology optimization proves to be a powerful tool for identifying designs that satisfy thermo-mechanical requirements. The results obtained for the purely mechanical topology optimization are consistent and demonstrate good physical relevance, suggesting their potential applicability to real engineering cases. Furthermore, the optimized topologies presented in Section 3.1 can be effectively interpreted and simplified into equivalent truss-like structures composed of a relatively small number of structural members. This highlights the ability of topology optimization to provide not only optimal material distributions but also meaningful conceptual designs that can serve as a basis for subsequent structural interpretation and engineering design.

However, the thermo-mechanical modal topology optimization cases presented in Section 3.2 were not handled satisfactorily by the *OOFELIE* software. The observed difficulties may originate from several sources, including limitations in the problem formulation itself or shortcomings in the current implementation of the topology optimization algorithm. In particular, the coupling between thermal, mechanical, and modal objectives or constraints introduces a high level of complexity, which can lead to convergence issues or non-physical optimization results. Such limitations are not unexpected, as thermo-mechanical topology optimization functionalities are still under active development within the software. Consequently, the results obtained in this section should be interpreted with caution and are primarily intended as an assessment of the current capabilities and limitations of the implemented framework.

Chapter 4

Conclusions and perspectives

4.1 Conclusions

The first section of this study aims to implement a parametrical simulation to explore the influence of several parameters on the first natural frequency. The first step with this parametrical simulation consisted in validating the numerical model of the parametrical simulation in order to ensure its reliability for subsequent parametric analyses and optimization tasks. Several complementary approaches were used to achieve this objective.

A first validation was performed through comparison with the analytical Euler–Bernoulli beam formulation, which showed a consistent agreement in terms of global structural behaviour and confirmed that the model correctly captures the expected bending response.

In addition, a mesh convergence study was carried out in order to assess the sensitivity of the results to spatial discretization. This study made it possible to determine the minimum number of finite elements required, depending on the characteristic dimensions of the geometry, to ensure that the numerical error remains below 0.5 [%], thus guaranteeing a sufficient level of accuracy for design purposes.

Finally, the model was further validated against experimental vibration measurements performed on the FPA of the EUI instrument. This comparison showed a discrepancy of approximately 14 [%] for the first vibration mode, which, while not negligible, remains acceptable within the context of a preliminary design phase and given the inherent uncertainties associated with experimental boundary conditions and modeling assumptions.

Once the model had been validated, a purely mechanical parametric simulation was conducted in order to identify and quantify optimization trends based on geometric parameters. The influence of each design variable was systematically evaluated, which made it possible to distinguish the most sensitive parameters governing the global stiffness and dynamic behaviour of the structure. This parametric study provided a clearer understanding of how each geometric feature affects the overall performance, and allowed the definition of relevant directions for optimization.

A similar methodology was then extended to a multiphysics framework by taking into account heat conduction through the support structure. In this context, the objective was to identify trade-offs between mechanical performance and thermal constraints. The resulting analysis led to the definition of optimization strategies aimed at increasing the first natural frequency while simultaneously respecting a constraint on maximum thermal conductance. This coupling highlighted the inherent compromise between thermal and mechanical requirements, and provided quantitative guidance for the design of an optimized structure.

The final part of this section focused on developing a predictive approach for estimating the resonance frequency of the model without relying on full numerical simulations. To this end, three regression models were constructed based on the sampling using the Latin Hypercube method. These regressions were compared with their accuracy, computational time and ease to implement. The best regression enables

the prediction of the resonance frequency with an accuracy of approximately ± 40 [Hz], corresponding to a 95 [%] confidence interval. Although simplified, this predictive tool provides sufficiently reliable estimates for early-stage design iterations and allows rapid evaluation of design changes without the need for computationally expensive simulations. Overall, this level of accuracy is considered fully acceptable for the preliminary design of a detector support structure, where rapid exploration of the design space is prioritized over high-fidelity prediction.

The second part of this work consisted in the use of topology optimization techniques in order to explore and potentially identify new optimal design concepts. For this purpose, the software *OOFELIE*, developed by *Open-Engineering*, was employed. The approach aimed at going beyond classical parametric studies by allowing the material distribution itself to evolve under the influence of some objective and constraint function, thereby enabling the discovery of non-intuitive structural layouts.

A first purely mechanical optimization was carried out, with the objective of minimizing the structural volume while imposing a constraint on a minimum required natural frequency. The results obtained were consistent and physically meaningful, leading to solutions that naturally converged towards truss-type structures. These optimal topologies also revealed a clear dependence on boundary conditions: configurations with parallel fixation tended to converge towards blades support structures which confirm the optimality of this design, while imposing a perpendicular freed the optimizer to found truss-structure that are totally consistent with the problem.

A similar procedure was then applied in a thermo-mechanical context, with the intention of incorporating both structural and thermal performance criteria into the optimization process. However, the results obtained in this case were not fully conclusive. This limitation is mainly attributed to the fact that the software is still under active development and may contain numerical instabilities or unresolved bugs affecting the robustness of coupled thermo-mechanical simulations. Consequently, the outcomes of this part can not be used or interpreted to design a detector support.

4.2 Perspectives

This section outlines several possible directions for extending the present work. It is divided into two main parts, focusing respectively on the continuation and improvement of the parametric simulation approach and on further developments in topology optimization. The objective is to identify potential enhancements of the results presented in this thesis.

4.2.1 Three blades design parametrical simulation

A first area for improvement concerns the validation of the numerical model of the parametrical simulation. Although the comparison with the experimental data available from the EUI instrument yielded satisfactory results, the observed discrepancies indicate that further refinement of the model may be possible. A more comprehensive validation campaign could therefore be conducted through dedicated vibration testing on representative support structures. Such experimental work would provide additional data for calibrating the numerical model, improving the representation of boundary conditions, and reducing uncertainties associated with material properties. Ultimately, this would increase confidence in the results of the simulation framework.

Then, the frequency prediction methodology could be further improved through the implementation of Kriging-based surrogate models. This approach was frequently encountered in the literature reviewed during this work, particularly in studies focusing on the prediction of structural dynamic behaviour without resorting to computationally expensive numerical simulations. Kriging has the potential to provide more accurate predictions than conventional regression models, especially when dealing with highly non-linear relationships between design parameters and system responses. However, this method was not pursued within the scope of the present work due to its greater implementation complexity and lower interpretability. In contrast, the analytical regression model developed here offers a straightforward mathematical formulation that can be easily understood and applied during the preliminary design phase.

Finally, the parametric study and frequency prediction model developed in this work can be regarded as the first building blocks towards the implementation of a complete shape optimization framework. Such a tool could automatically generate optimal geometries based on a set of user-defined objectives and constraints, including dynamic, thermal, mass. By combining predictive models with optimization algorithms, it would become possible to efficiently explore large design spaces and identify candidate solutions that best satisfy the system requirements. This would significantly reduce design iteration times and provide valuable support during the early stages of detector support structure development.

4.2.2 Thermo-mechanical topology optimization

First, the results obtained could be compared with those produced by alternative topology optimization formulations. Different objective functions, constraint definitions, and optimization strategies may lead to significantly different optimal topologies and performance levels. Such a comparative study would make it possible to identify the most suitable formulation for the design of detector support structures. In particular, alternative formulations may help overcome the limitations encountered in the thermo-mechanical optimization cases, where the results obtained in the present work were not entirely satisfactory. Consequently, the observed limitations may originate not only from software-related issues but also from the formulation of the optimization problem itself.

Another promising avenue would be to perform the optimization process using finer finite element meshes. The simulations conducted in this work were necessarily limited by computational resources, and mesh refinement could provide a more accurate representation of the material distribution and structural

behaviour. Such an investigation would also make it possible to evaluate the sensitivity of the resulting topologies to the mesh density and potentially reveal structural features or optimization trends that were not captured with the current discretization.

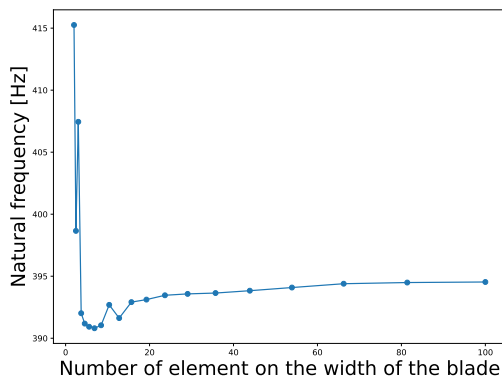
Furthermore, the optimized topologies should be validated by reconstructing and analyzing the final geometries after removing the elements considered void by the optimization algorithm. This verification step is important because the dynamic behaviour predicted during the optimization process may differ from that of the reconstructed geometry. Performing additional finite element analyses on the final designs would therefore allow the consistency and reliability of the optimization results to be assessed and would provide greater confidence in the practical applicability of the generated structures.

Finally, the static loading objective introduced through a vertical acceleration constraint to ensure structural connectivity may have influenced the resulting topologies by favoring stiffness in a specific direction. As a consequence, the optimized structures may exhibit anisotropic behaviour and perform less effectively under loads applied in other directions. To evaluate the robustness of the obtained designs, similar optimization studies could be performed using acceleration constraints along different axes. Such investigations would help determine whether the resulting topologies are genuinely optimal for the overall design requirements or whether they are biased by the specific loading conditions considered during the optimization process.

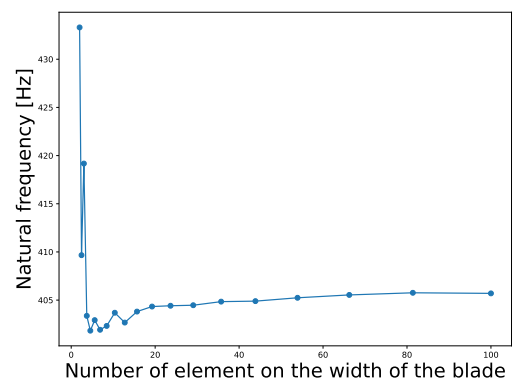
Overall, these future developments would contribute to a more comprehensive understanding of the topology optimization process and increase confidence in the applicability of the generated designs to real detector support structures. They would also provide valuable insights into the influence of modeling assumptions, optimization formulations, and numerical discretization on the final structural performance.

Appendix A

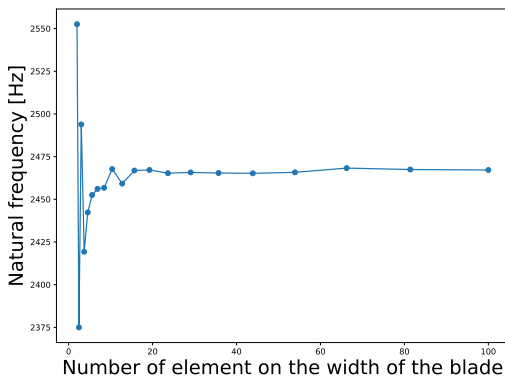
Convergence study of mode 1-6



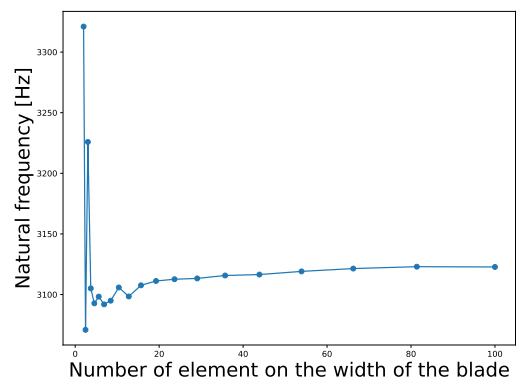
Frequency convergence of the fourth mode



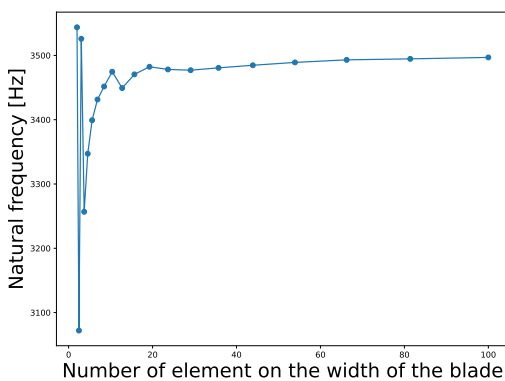
Relative error of the fourth mode



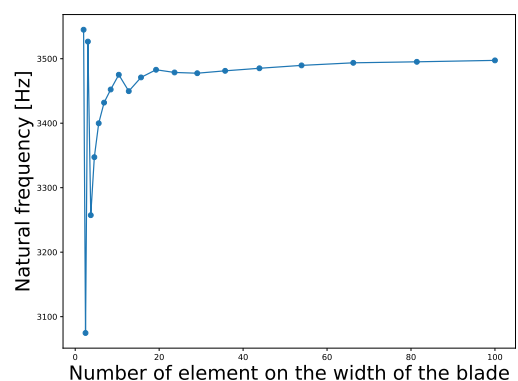
Frequency convergence of the fifth mode



Relative error of the fifth mode



Frequency convergence of the sixth mode



Relative error of the sixth mode

Appendix B

Fourth mode variation

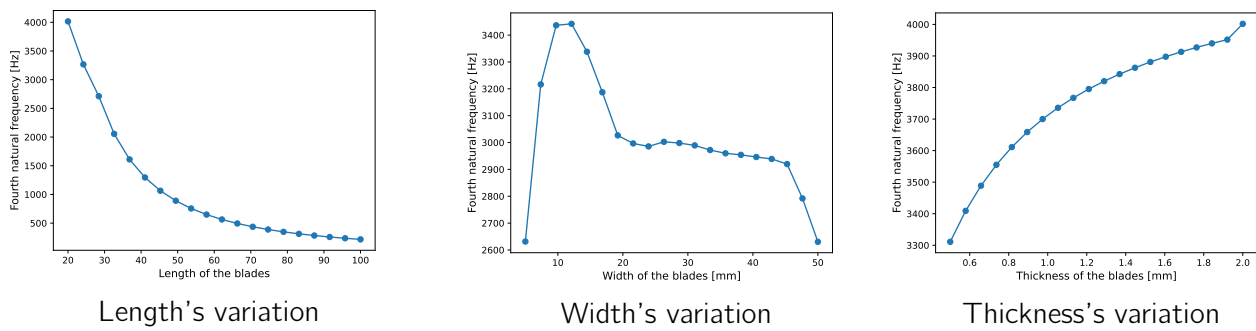


Figure B.1: Fourth natural frequency with blade geometry variation

Bibliography

- [1] Luntama Juha-Pekka. Vigil mission objectives and payload description. Technical Report ESA-S2P-LGR-MO-0002, European Space Agency, 2022.
- [2] European Space Agency (ESA). Airbus to build esa's vigil space weather forecasting mission. https://www.esa.int/Space_Safety/Vigil/Airbus_to_build_ESA_s_Vigil_space_weather_forecasting_mission. Viewed on the 09th of March, 2026.
- [3] National Aeronautics and Space Administration (NASA). Heliophysics space weather vigil focused mission of opportunity (vigil fmo) nasa space weather program (swxp) homepage. <https://soma.larc.nasa.gov/lws/vfmo/>. Viewed on the 09th of March, 2026.
- [4] Structural engineering. Technical Report ECSS-E-ST-32, European Cooperation for Space Standardization (ECSS), 2008.
- [5] Hassler-Don and the JEDI Team. Jedi and vigil mission at I5, 2025.
- [6] William T. Thomson and Marie Dillon Dahleh. *Theory of vibration with applications*. Prentice Hall, 1998.
- [7] Robert D. Blevins. *Formulas for natural frequency and mode shape*. 2001.
- [8] Kenneth Krane. *Modern Physics*. John Wiley Sons,INC, 2012.
- [9] Weibo Chen, Michael DiPirro, Ian McKinley, Chulhee Cho, and Howard Tseng. Active cryocooling needs for nasa space instruments and future technology development. *Cryogenics*, 141:103877, 2024.
- [10] National Aeronautics and Space Administration (NASA). Webb telescope's coldest instrument reaches operating temperature. <https://www.nasa.gov/solar-system/webb-telescopes-coldest-instrument-reaches-operating-temperature> Viewed on the 19th of March, 2026.
- [11] Ibsen Photonics. The effect of cooling ccd detectors for spectroscopy. <https://ibsen.com/resources/detector-resources/the-effect-of-cooling-ccd-detectors-for-spectroscopy> Viewed on the 19th of March, 2026.
- [12] Xavier Chauffleur et al. VNIR and SWIR FPA designs for 3MI instrument: thermal, mechanical and dimensional stability challenges. In Zoran Sodnik, Nikos Karafolas, and Bruno Cugny, editors, *International Conference on Space Optics — ICSO 2018*, volume 11180, page 111806S. International Society for Optics and Photonics, SPIE, 2019.
- [13] Euclid-definition study report. Technical Report ESA/SRE(2011)12, European Space Agency (ESA), 2011.
- [14] J Treis et al. Pixel detectors for x-ray imaging spectroscopy in space. *Journal of Instrumentation*, 4(03):P03012, mar 2009.

- [15] European Space Agency. Nirspec – the near-infrared spectrograph on jwst. <https://sci.esa.int/web/jwst/-/45694-nirspec-the-near-infrared-spectrograph>. Viewed on the 02th of June, 2026.
- [16] James D. et al. 2K X 2K molecular beam epitaxy HgCdTe detectors for the James Webb Space Telescope NIRCам instrument. In James D. Garnett and James W. Beletic, editors, *Optical and Infrared Detectors for Astronomy*, volume 5499, pages 35 – 46. International Society for Optics and Photonics, SPIE, 2004.
- [17] L. Rossi. loc design description. Technical Report MIRI-TN-00022-CSL, Centre spatial de Liege (CSL), 2006.
- [18] Heike Rauer et al. The plato mission, 2024.
- [19] L. Jacques. Eui fpa thermal analysis. Technical Report TN-CSL-SO-11020, Centre spatial de Liege (CSL), 2011.
- [20] AMOS Team. Solar orbiter eui focal plane assembly mechanical structural analyses. Technical Report 2166-01-02, AMOS, 2015.
- [21] O.A. Bauchau and J.I Craig. *Structural analysis and application to aerospace structure*. 2009.
- [22] Michaël Bruyneel and Pierre Duysinx. Note on topology optimization of continuum structures including self-weight. *Structural and Multidisciplinary Optimization*, 29(4):245–256, 2005.
- [23] David Liović, Marina Franulović, Ervin Kamenar, and Dražan Kozak. Nano-mechanical behavior of ti6al4v alloy manufactured using laser powder bed fusion. *Materials*, 16(12), 2023.
- [24] Hongan Xu et al. Modal analysis of general plate structures. 2014.
- [25] J.H. Yang, Miguel A.F. Sanjuán, and H.G. Liu. Vibrational subharmonic and superharmonic resonances. *Communications in Nonlinear Science and Numerical Simulation*, 30(1):362–372, 2016.
- [26] DOUGLAS C. MONTGOMERY. *Design and Analysis of Experiments*. John Wiley Sons, INC, 2017.
- [27] Shuwei Li, Shutian Fan, Jinan Gu, Xingjia Li, and Zedong Huang. Blind-kriging based natural frequency modeling of industrial robot. *Precision Engineering*, 74:126–139, 2022.
- [28] Yeonjong Shin and Dongbin Xiu. On a near optimal sampling strategy for least squares polynomial regression. *Journal of Computational Physics*, 326:931–946, 2016.
- [29] Jihong ZHU et al. A review of topology optimization for additive manufacturing: Status and challenges. *Chinese Journal of Aeronautics*, 34(1):91–110, 2021.
- [30] AZO Materials. Aluminium nitride / aluminum nitride (aln) - properties and applications. <https://www.azom.com/article.aspx?ArticleID=610&utm> Viewed on the 05th of May, 2026.
- [31] W. Michael Yim and Richard J. Paff. Thermal expansion of aln, sapphire, and silicon. *Journal of Applied Physics*, 45:1456–1457, 1974.
- [32] Karel Dvořák, Jana Dvořáková, Dusan Majtas, Radek evík, and Lucie Zárybnická. Wide-temperature characteristics of additively pbf-lb/m processed material ti6al4v. *Metals*, 2024.
- [33] Yongfeng Zheng, Zhen Luo, Yanzheng Wang, Zhengyang Li, Jinping Qu, and Chuanzeng Zhang. Optimized high thermal insulation by the topological design of hierarchical structures. *International Journal of Heat and Mass Transfer*, 186:122448, 2022.