

Master thesis : Optimization of a tomosynthesis trajectory on an existing CT system (including introduction to research methodology)

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Optimization of a tomosynthesis trajectory on an existing CT system



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Summary

This master thesis investigates the optimization of tomosynthesis acquisition trajectories for the Dx-Box C5 multi-axis X-ray inspection system developed by X-RIS. The Dx-Box C5 combines a C-arm with a five-axis motion stage, enabling flexible acquisition geometries for industrial non-destructive testing.

The objective of this work is to identify the acquisition trajectories that achieve the best image quality. Four simple trajectories and several combined configurations were evaluated through numerical simulations performed using the gVXR framework. Image quality was assessed using a composite score combining contrast and depth resolution metrics.

The results show that rotational trajectories provide better depth localization, while translation trajectories yield higher contrast. Among all configurations evaluated, the R1-R2 combination — combining C-arm rotation and turntable rotation — achieved the highest overall score and is identified as the most suitable trajectory for implementation on the Dx-Box C5 system.

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Chapter 1

Introduction

1.1 Industrial context

Non-destructive testing (NDT) plays a crucial role in modern manufacturing industries. These inspection techniques allow manufacturers to detect internal defects, verify dimensional accuracy, and ensure product quality without damaging the parts being examined [1]. Among the various NDT methods available today, X-ray imaging has become particularly valuable due to its ability to visualize the internal structure of dense materials such as metals, composites, and ceramics [2]. X-ray based inspection systems are now widely used across multiple industrial sectors. In the aerospace industry, for example, they help detect critical defects like porosities in turbine blades or cracks in structural components [3]. Similarly, the automotive sector relies on X-ray imaging to inspect cast aluminum parts, welds, and battery assemblies [4]. In the electronics industry, these systems verify solder joints and detect voids in circuit boards. Regarding the medical device sector, it can be used to ensure the quality of implants, prosthetics, and surgical instruments, supporting strict safety standards.

1.2 X-RIS: Company Overview and the Dx-Box C5 System

1.2.1 About X-RIS: X-Ray Imaging Solution

Founded in 2010, X-RIS is a Belgian company specializing in digital X-ray imaging solutions for industrial quality control and non-destructive testing. The company designs and manufactures a wide range of inspection systems that address the demanding requirements of modern manufacturing environments. Its expertise covers both compact cabinet systems for production workflows and customized platforms for research and development applications.

X-RIS places strong emphasis on image quality, mechanical robustness, and user-friendliness. Its systems comply with industrial and military standards and are designed for reliable operation in demanding environments. The company employs a multidisciplinary team of engineers and technical experts, and collaborates actively with universities and R&D centers to support innovation in digital radiography and computed tomography (CT).

The company has customers in diverse industrial sectors, including aerospace, automotive, electronics, and advanced manufacturing. Figure 1.1 illustrates some examples of X-ray inspection applications, highlighting the versatility of X-RIS's systems.

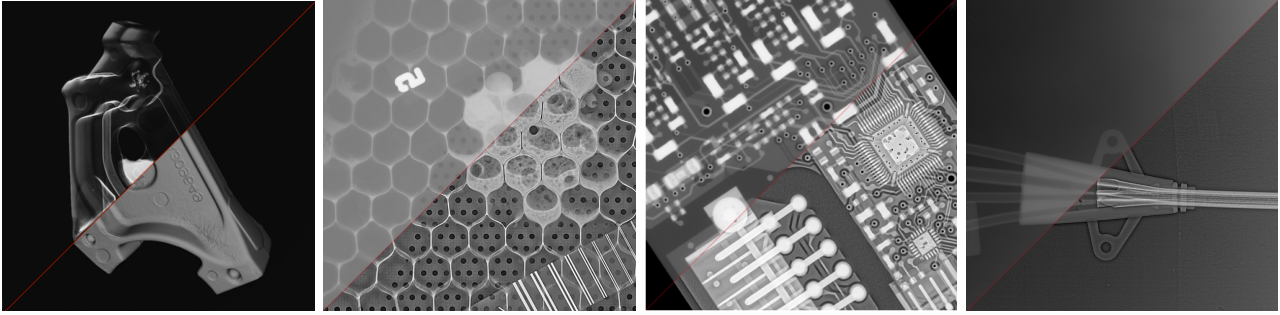


Figure 1.1: Examples of industrial X-ray inspection applications across different sectors: aerospace component inspection (left), composite and cellular structure inspection (center-left), electronic circuit board inspection (center-right), and medical device inspection (right). (adapted from [5])

X-RIS's product portfolio includes the Dx-Box series of cabinet-style X-ray systems (Figure 1.2), each tailored to specific inspection needs and part sizes. These systems integrate automated positioning stages, full safety enclosures, and a software platform, enabling streamlined radiography and CT workflows.



Figure 1.2: The family of cabinet-style X-ray systems used for industrial radiography and CT, illustrating the different system sizes and inspection capacities available within the Dx-Box product range. Approximate external dimensions are indicated below each system.(adapted from [5])

1.2.2 The Dx-Box C5 System

Among X-RIS's product line, the one representing the main subject of this work is the Dx-Box C5. It represents a versatile inspection platform that combines flexibility with performance. The system is built around a C-arm geometry, which is a configuration widely used in medical and industrial imaging due to its ability to provide variable viewing angles while maintaining a compact setup.

The C5 integrates a five-axis motion system that provides great flexibility in positioning both the inspected part and the X-ray source–detector pair, enabling different acquisition geometries. This feature is key to provide possibility of a full 3D reconstruction of the part.

Figure 1.3 shows an overview of the box, where the first two pictures represent an outer view of the system with the visible C-arm on the second picture, while the third displays a tray with sample parts that can be inspected.

In its current configuration, the Dx-Box C5 is primarily used for providing images of parts at relevant angles and for CT, producing high-fidelity 3D reconstructions of internal part geometry.

The entire system is controlled through X-RIS's Maestro software, which synchronizes the source, detector, and motion stages in a single interface. Maestro then allows operators to configure acquisition parameters, automate movement sequences, visualize results after reconstruction.

The detailed geometry and kinematic configuration of the Dx-Box C5, including its degrees of freedom and motion limits, will be described Section 3.1, as they are central to understanding the implementation of this project.



Figure 1.3: The X-RIS Dx-Box C5 inspection system with its C-arm.(adapted from [5])

1.3 Problem Statement

Despite its capabilities, CT scanning faces several practical limitations in industrial applications. The first major constraint is acquisition time. A complete CT scan typically requires hundreds to thousands of projections (typically 500–2000), resulting in acquisition times ranging from 30 minutes to 3 hours per scan, depending on the part size, material density, and desired resolution [6]. In high-volume production environments where inspection throughput is critical, this duration can create bottlenecks in the manufacturing workflow.

The second challenge concerns the geometry of the part and its material properties. Depending on the viewing direction, X-rays may be forced to traverse a long path through dense material. For instance, when examining a metallic plate edge-on, the attenuation becomes so strong that very few photons reach the detector, resulting in extremely noisy or even completely opaque projections.

Figure 1.4 illustrates this geometric constraint: the path length $A \rightarrow B$ (through the plate width) is significantly longer than the path $1 \rightarrow 2$ (through the plate thickness). As a result, X-rays experience much stronger attenuation for certain viewing directions, leading to insufficient transmission and unusable projections during conventional CT acquisition [2].

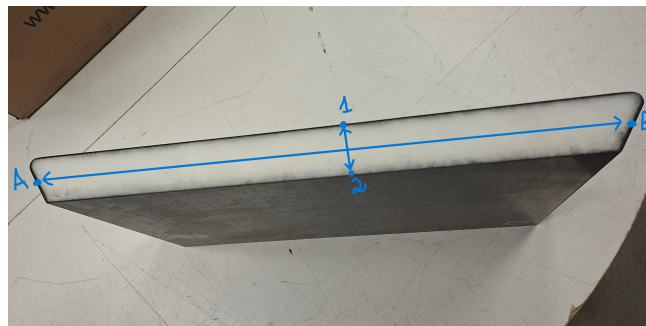


Figure 1.4: Illustration of geometric constraints in plates: view direction affects the X-ray path length through the material.

These limitations highlight the need for an alternative imaging approach that can provide useful 3D

information while avoiding the drawbacks of full CT scans.

1.4 Digital Tomosynthesis as a Solution

Digital tomosynthesis (DTS) offers a convincing alternative to traditional CT scanning. Unlike CT, which requires a complete 360° rotation, tomosynthesis acquires projections over a limited angular range, typically between 20° and 60° [8]. The projections acquired at different angles are then reconstructed into a stack of tomosynthesis slices, where each structure appears sharply focused only within the slice corresponding to its actual depth, usually spanning just a few adjacent slices.

Figure 1.5 illustrates the general principle of DTS: limited-angle projections are reconstructed into focal planes at different depths and structures in the focal plane appear sharp while out-of-plane structures are blurred.

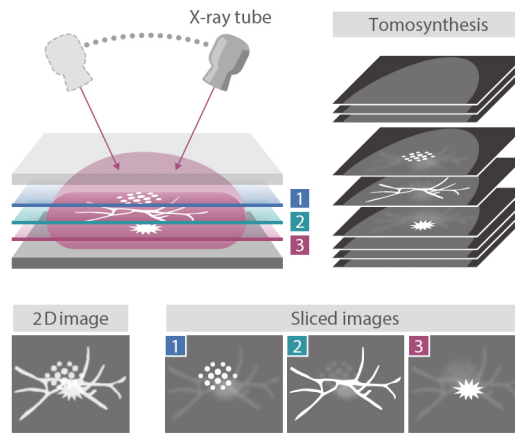


Figure 1.5: Illustration of the principle of tomosynthesis (adapted from [10])

This limited-angle acquisition strategy provides several practical advantages. First, tomosynthesis is significantly faster than CT. Since fewer projections are needed and the mechanical motion is limited, acquisition times can be reduced by a factor of 5 to 20 [9]. This speed improvement is particularly valuable in production environments where inspection throughput directly impacts manufacturing costs.

In addition, tomosynthesis is more forgiving regarding part geometry. Because the X-ray beam only needs to traverse the part from limited angles rather than from all directions, thick parts that would be problematic for CT can often be successfully imaged using tomosynthesis. The system can be configured to acquire projections only from angles where adequate X-ray transmission is achievable.

However, these advantages come with inherent trade-offs. Each reconstructed slice contains blurred contributions from out-of-focus structures at other depths, which can hinder analysis when overlapping features create confusing superpositions. Additionally, focal planes can only be reconstructed perpendicular to the imaging axis — the orientation is fixed by the acquisition geometry and cannot be changed retrospectively, unlike CT where arbitrary slice orientations are possible after reconstruction. [7]

1.5 Thesis Objectives

The primary goal of this master's thesis is to investigate and optimize tomosynthesis acquisition trajectories for the Dx-Box C5 system. More specifically, the objectives are:

- Characterize the geometry, degrees of freedom, and mechanical constraints of the Dx-Box C5 system.

- Design and compare different tomosynthesis acquisition trajectories based on the available system motions.
- Develop quantitative metrics and a composite scoring framework to evaluate trajectory performance in terms of reconstruction quality.
- Identify the most suitable trajectory among the configurations investigated through simulation and qualitative assessment.

1.6 Report Organization

The remainder of this thesis is organized as follows.

Chapter 2 presents the theoretical background on X-ray imaging, including the principles of conventional radiography, computed tomography, and digital tomosynthesis, as well as the reconstruction methods and image quality metrics used in this work.

Chapter 3 describes the Dx-Box C5 system, the simulation framework, the phantom design, and the acquisition trajectories investigated.

Chapter 4 presents and discusses the simulation results obtained for the different acquisition trajectories and evaluation methods considered in this study.

Finally, Chapter 5 summarizes the main findings of the work, identifies the recommended trajectory for implementation on the Dx-Box C5 system, and outlines the limitations, personal contributions, and perspectives for future research.

Chapter 2

State of the art

2.1 Elements of X-ray Physics

Understanding how X-rays interact with matter is essential for interpreting tomographic images and optimizing acquisition parameters. This section introduces the fundamental principles governing X-ray attenuation, image formation, and acquisition geometry.

2.1.1 Radiation–Matter Interaction

Beer–Lambert Law

When X-rays pass through matter, their intensity decreases exponentially due to absorption and scattering. For a monochromatic beam traversing a homogeneous material, this attenuation follows the Beer–Lambert law [2]:

$$I(x) = I_0 e^{-\mu x} \quad (2.1)$$

where I_0 is the incident X-ray intensity, $I(x)$ is the transmitted intensity after a thickness x , and μ is the linear attenuation coefficient (cm^{-1}). Figure 2.1 illustrates this exponential decay for three materials with different attenuation coefficients.

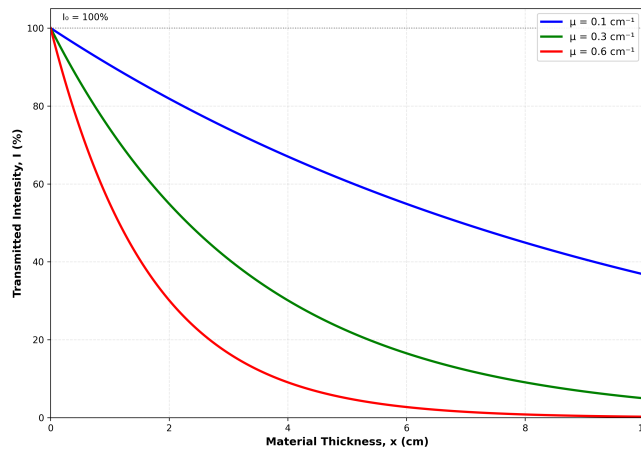


Figure 2.1: Exponential attenuation of X-ray intensity according to the Beer–Lambert law for three materials with increasing linear attenuation coefficients ($\mu = 0.1, 0.3, \text{ and } 0.6 \text{ cm}^{-1}$), illustrating the effect of attenuation coefficient on X-ray transmission.

For heterogeneous objects composed of multiple materials, the law generalizes to a line integral along

the ray path, where the attenuation coefficient μ varies spatially along the path L :

$$I(L) = I_0 \exp \left(- \int_0^L \mu(x) dx \right) \quad (2.2)$$

This relationship forms the mathematical foundation of tomographic reconstruction: by measuring transmitted intensities from many angles, algorithms can recover the spatial distribution $\mu(x, y, z)$ within the object [11].

Linear Attenuation Coefficient

The linear attenuation coefficient μ quantifies how strongly a material attenuates X-rays [12]. It depends on both the material properties and the photon energy and can be expressed as:

$$\mu(E) = \left(\frac{\mu}{\rho} \right) (E) \cdot \rho \quad (2.3)$$

where $(\mu/\rho)(E)$ is the mass attenuation coefficient (cm^2/g) at photon energy E , and ρ is the material density (g/cm^3). The mass attenuation coefficient is a fundamental material property that has been tabulated for all elements.

The physical meaning of μ relates to the mean free path: $1/\mu$ represents the average distance an X-ray photon travels before undergoing an interaction.

2.1.2 Energy and Material Dependence

X-ray attenuation varies significantly with both photon energy and atomic composition. In the 50–300 keV range typical of industrial CT, four main interaction mechanisms occur [2]:

- *Photoelectric absorption* dominates at low energies, particularly for high- Z materials. The cross-section scales approximately as Z^4/E^3 , making this process highly sensitive to both atomic number (Z) and photon energy (E).
- *Compton scattering* dominates at intermediate energies (100–300 keV), depending mainly on electron density.
- *Rayleigh/Thomson (coherent) scattering* occurs at low energies, involving elastic deflection of photons by atomic electrons without energy transfer or ionization. Its contribution is minor in typical industrial CT energy ranges but remains a secondary interaction mechanism.
- *Pair production* Pair production occurs only above 1.02 MeV and is negligible in most conventional industrial CT systems, though it may become relevant in high-energy systems using linear accelerators operating up to 6 MeV.

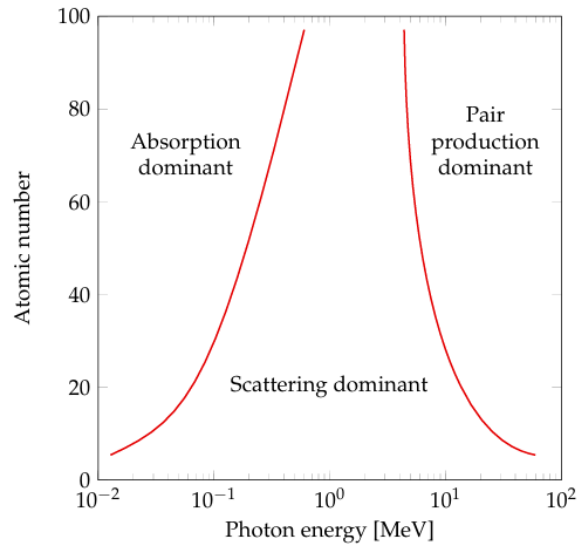


Figure 2.2: Dominant attenuation mechanisms as a function of photon energy and atomic number. (adapted from [8])

Industrial X-ray sources, unlike monochromatic synchrotron beams, emit polychromatic radiation consisting of a continuous spectrum of photon energies rather than a single value. The shape of this spectrum depends on the tube voltage and the filtration applied, which removes low-energy photons and modifies the energy distribution of the beam. [13]

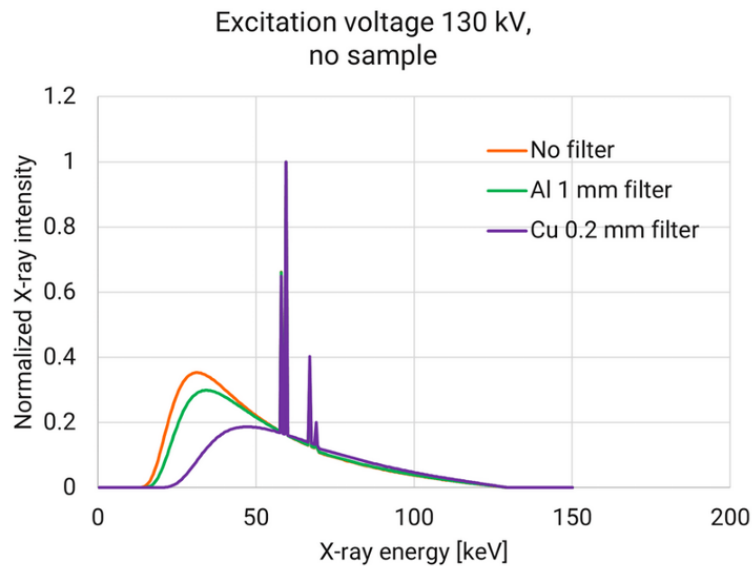


Figure 2.3: Polychromatic X-ray spectra at 130 kV excitation voltage showing the effect of aluminum and copper filtration on the spectral distribution. (adapted from [13])

Figure 2.3 illustrates a typical polychromatic X-ray spectrum at 130 kV excitation voltage. The characteristic peaks correspond to the anode material (tungsten), while the continuous background results from bremsstrahlung emission. Filtration with aluminum or copper sheets removes low-energy photons that contribute little to image formation but increase radiation dose and worsen beam hardening artifacts.

2.1.3 Cone-Beam CT Image Formation

Cone-Beam Projection

In cone-beam CT systems such as the Dx-Box C5, X-rays diverge from a compact focal spot and form a conical beam that illuminates the object and projects onto the detector simultaneously. Each detector pixel measures the transmitted X-ray intensity corresponding to a specific ray path from the source to the detector. The attenuation information is then obtained by comparing this intensity to the incident intensity measured without the object. [2, 14]

For a polychromatic X-ray source, as discussed in Section 2.1.2, the detector signal S is governed by the energy-dependent attenuation: [2, 15]

$$S \propto I = \int_0^{E_{\max}} I_0(E) \exp \left(- \int_{\text{ray}} \mu(E, s) ds \right) dE \quad (2.4)$$

where $I_0(E)$ is the incident spectrum (Figure 2.3), $\mu(E, s)$ is the energy- and position-dependent linear attenuation coefficient, and the integration is performed both over the ray path and across the energy spectrum.

In tomographic reconstruction, it is common practice to work with the line integral of attenuation, obtained through logarithmic transformation of the detector signal. For practical implementation, reconstruction algorithms typically employ a monochromatic approximation:

$$p = -\ln \left(\frac{I}{I_0} \right) \approx \int_{\text{ray}} \mu_{\text{eff}}(s) ds \quad (2.5)$$

where p represents the projection data and μ_{eff} is an effective attenuation coefficient averaged over the source spectrum. This simplification is valid when beam hardening effects are minimal or when appropriate preprocessing corrections are applied [12, 15].

The cone-beam geometry provides efficient data acquisition by capturing a large volume in each projection, but introduces geometric complexities that reconstruction algorithms must account for. Figure 2.4 illustrates the principle of cone-beam imaging, where the X-ray source and detector rotate around the object to capture projections from different angles.

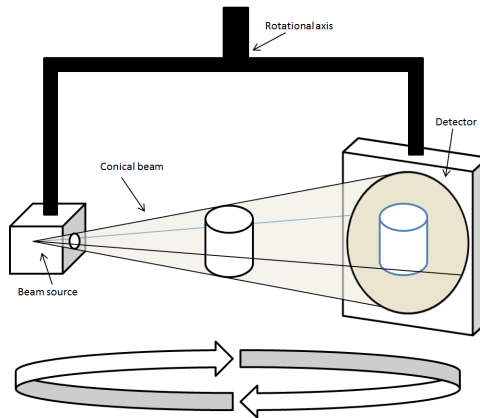


Figure 2.4: Schematic of cone-beam CT X-ray imaging geometry, showing the source, object, and flat-panel detector arrangement. [16]

This section reviewed the fundamental physics of X-ray imaging. The Beer–Lambert law describes exponential attenuation, with the linear attenuation coefficient depending on both material properties and photon energy. Energy-dependent interaction mechanisms determine how X-rays penetrate different materials. Radiographic image formation involves cone-beam projection geometry, with transmitted intensity related exponentially to path-integrated attenuation. These principles establish the physical foundation for understanding both the capabilities and fundamental limitations of tomographic imaging systems.

2.2 Conventional Radiography

Conventional radiography is the simplest and most widely used X-ray imaging technique. An X-ray beam passes through the object and the transmitted intensity is recorded by a detector, producing a two-dimensional (2D) projection image, as illustrated in Figure 2.5. The detected signal depends on the attenuation of the X-rays along each propagation path through the object according to the Beer–Lambert law (Section 2.1.1).

Because all structures located along the same X-ray path contribute to the detected intensity, information from different depths is superimposed in the resulting image. As a result, the exact depth position of internal features cannot be directly determined, and overlapping dense structures may partially or completely obscure smaller details or defects. This limitation motivated the development of tomographic imaging techniques, where multiple projections acquired from different viewing angles are combined to recover three-dimensional information.

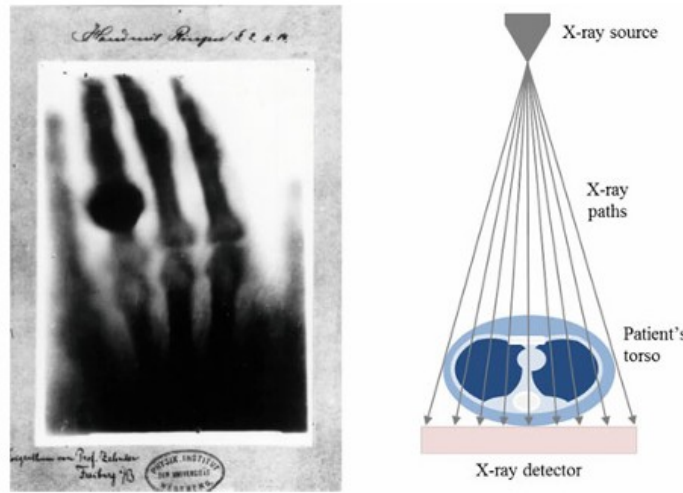


Figure 2.5: Left: one of the first X-ray radiographs, showing the hand of Wilhelm Röntgen's wife (1895), illustrating the projection nature of conventional radiography. Right: simplified X-ray acquisition setup showing the source, object, and detector arrangement. Adapted from [2].

Computed tomography extends this principle by acquiring multiple projections from different viewing angles to reconstruct the internal attenuation distribution of the object.

2.3 Computed Tomography: Acquisition and Mathematical Foundation

In conventional CT, projections are acquired as the X-ray source and detector rotate around the object. In the two-dimensional case, projections acquired over 180° are theoretically sufficient to reconstruct a slice. In practical three-dimensional cone-beam CT systems, a complete 360° rotation is commonly used for volumetric reconstruction. Each projection represents a set of line integrals through the object's attenuation distribution. Mathematically, the projection $p_\theta(t)$ acquired at angle θ corresponds to the integral of the linear attenuation coefficient $\mu(\mathbf{r})$ along rays [2]:

$$p_\theta(t) = \int_{\text{ray}} \mu(\mathbf{r}) ds \quad (2.6)$$

where the integration is performed along the ray path at position t and angle θ .

2.3.1 Radon Transform and Projection Sampling

The set of projections acquired at different angles can be interpreted as the Radon transform of the object, a concept introduced by the Austrian mathematician Johann Radon in 1917 [2, 17].

The 2D Radon transform

The Radon transform is formally defined in a 2D parallel-beam geometry, where X-rays are cast perpendicularly onto a line detector. The reconstruction problem then consists in recovering the spatial distribution of the attenuation coefficient $\mu(x, y)$ from a series of projections acquired at different angles. For a given projection angle γ , the measured signal at detector position ξ corresponds to the line integral of $\mu(x, y)$ along the corresponding projection line. As introduced in Section 2.1.3, taking the logarithm of the Beer–Lambert law yields the projection integral [14, 28]:

$$p_\gamma(\xi) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) \delta(x \cos \gamma + y \sin \gamma - \xi) dx dy \quad (2.7)$$

where γ is the projection angle and ξ is the detector coordinate defining the projection line, and $\delta(\cdot)$ is the Dirac delta function selecting the ray path. Figure 2.6 illustrates this geometry. This expression, denoted $\mathcal{R}_2\{\mu(x, y)\}$, is called the two-dimensional Radon transform of $\mu(x, y)$. Arranging the projection values $p_\gamma(\xi)$ in a (ξ, γ) diagram yields the sinogram representation of the object in Radon space.

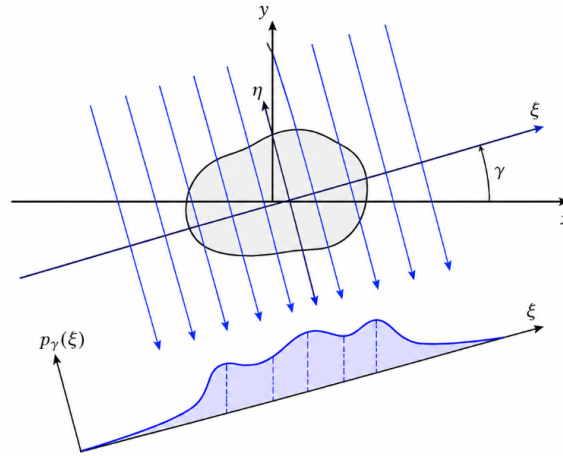


Figure 2.6: Illustration of the 2D parallel-beam acquisition geometry. X-rays are cast along the direction defined by angle γ and collected on a line detector along axis ξ . The resulting projection $p_\gamma(\xi)$ represents the line integrals of the attenuation coefficient $\mu(x, y)$ through the object. Adapted from [14].

It is important to note that this definition assumes parallel-beam geometry — a simplification that is mathematically convenient but not directly applicable to the divergent fan-beam or cone-beam geometry used in most modern CT systems.

The Fourier slice theorem

The reconstruction problem consists in recovering $\mu(x, y)$ from its projections $p_\gamma(\xi)$ — this is the inverse Radon transform. The mathematical foundation for this inversion is the Fourier slice theorem, also called the central slice theorem [2, 11].

Let $P_\gamma(q)$ denote the 1D Fourier transform of $p_\gamma(\xi)$ with respect to ξ :

$$P_\gamma(q) = \int_{-\infty}^{+\infty} p_\gamma(\xi) e^{-2\pi i q \xi} d\xi \quad (2.8)$$

Substituting Equation 2.7 into Equation 2.8 and rearranging:

$$P_\gamma(q) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mu(x, y) e^{-2\pi i q(x \cos \gamma + y \sin \gamma)} dx dy \quad (2.9)$$

The right-hand side is exactly the 2D Fourier transform $F(u, v)$ of $\mu(x, y)$, evaluated at the polar coordinates $(u, v) = (q \cos \gamma, q \sin \gamma)$:

$$P_\gamma(q) = F(q \cos \gamma, q \sin \gamma) \quad (2.10)$$

This is the Fourier slice theorem: *the 1D Fourier transform of a projection described by the Radon transform at angle γ equals a radial slice through the 2D Fourier transform of the object at the same angle*. As illustrated in Figure 2.7, each projection fills a radial line in the 2D Fourier domain. Projections acquired over 180° provide complete coverage of the 2D Fourier domain

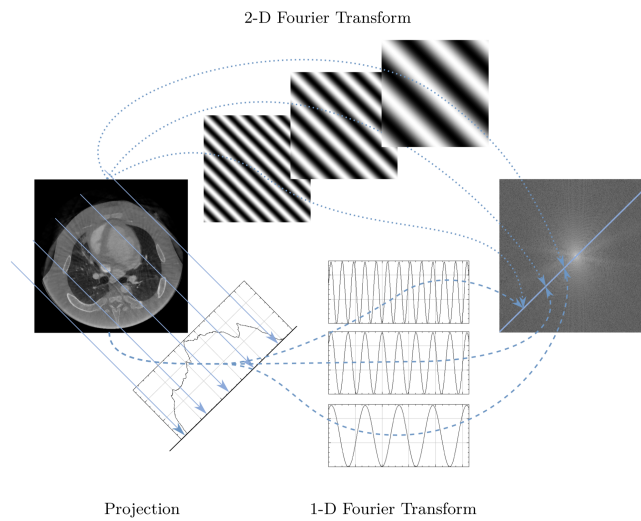


Figure 2.7: Illustration of the Fourier slice theorem. Each projection at angle γ contributes a radial line in the 2D Fourier domain of the object. Complete angular coverage fills the entire Fourier domain. Adapted from [21].

Filtered back-projection

While the Fourier slice theorem provides a theoretically exact reconstruction scheme, its direct implementation is impractical due to the non-uniform polar sampling of the Fourier domain — low frequencies are sampled much more densely than high frequencies, causing blur. The Filtered Back-Projection (FBP) algorithm offers a more practical alternative. Starting from the inverse 2D Fourier transform in polar coordinates and applying Equation 2.10, it can be shown [11] that the exact reconstruction is:

$$\mu(x, y) = \int_0^\pi \left[\int_{-\infty}^{+\infty} P_\gamma(q) |q| e^{2\pi i q \xi} dq \right]_{\xi=x \cos \gamma + y \sin \gamma} d\gamma \quad (2.11)$$

Reconstruction consists of two steps: first, each projection is filtered in the frequency domain by a ramp filter $|q|$, which compensates for the non-uniform sampling density; second, the filtered projections are back-projected along their acquisition direction and summed over all angles. This algorithm is mathematically exact in 2D parallel-beam geometry when projections cover a full 180° range.

Extension to 3D: the 3D Radon transform

In three dimensions, the Radon transform generalizes naturally. As illustrated in Figure 2.8, the key difference from the 2D case is the following: in 2D, one chooses a direction γ in the plane and integrates along the perpendicular line $\mathbf{L}$; in 3D, one chooses a direction in space defined by a unit normal vector $\mathbf{n}_\xi$ with spherical angles (γ, ϑ) , and integrates over the perpendicular plane $\mathbf{A}$. The 3D Radon value is therefore a plane integral [14]:

$$p_{n_\xi}(\xi) = \iint_{\mathbf{r} \cdot \mathbf{n}_\xi = \xi} \mu(\mathbf{r}) dA \quad (2.12)$$

where the plane is defined by $\xi = x \cos \gamma \sin \vartheta + y \sin \gamma \sin \vartheta + z \cos \vartheta$. The 3D Radon space is therefore parameterized by all possible planes through the object.

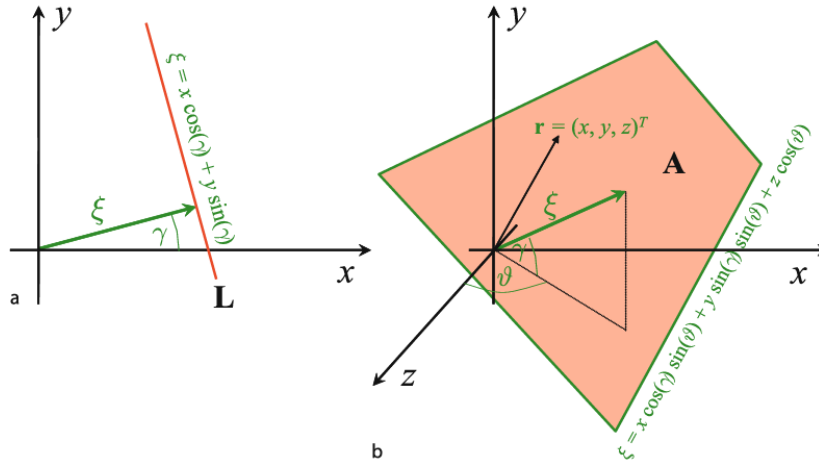


Figure 2.8: Geometrical analogy between the 2D and 3D Radon transforms. (a) In 2D, the Radon value is the line integral of $\mu(x, y)$ along a line $\mathbf{L}$ defined by angle γ and distance ξ . (b) In 3D, the Radon value is the surface integral of $\mu(x, y, z)$ over a plane $\mathbf{A}$ defined by its normal direction (γ, ϑ) and distance ξ from the origin. Adapted from [14].

The Fourier slice theorem can be extended to 3D: the 1D Fourier transform of $p_{n_\xi}(\xi)$ corresponds to a radial line in the 3D Fourier transform of $\mu(\mathbf{r})$ along $\mathbf{n}_\xi$. The corresponding 3D reconstruction framework is presented in Appendix A.

Extension to cone-beam geometry

The framework described above is valid for parallel-beam geometry. In cone-beam geometry — as used in the Dx-Box C5 system — the X-rays diverge from a point source, and the acquired projections are line integrals along diverging rays, not plane integrals of the 3D Radon transform. The central slice theorem is therefore no longer directly applicable [14].

An exact reconstruction formula for cone-beam geometry was derived by Grangeat [19], who established a relationship between the derivative of the cone-beam projections and the derivative of the 3D Radon transform, allowing an exact inversion in principle. In practice, the Feldkamp–Davis–Kress (FDK) algorithm [6] is the most widely used method. It reduces the 3D problem to a series of 2D sub-problems by treating each fan-shaped surface defined by the source and a detector row independently, applying a cosine weighting and a ramp filter before back-projecting along the cone-beam rays. While the FDK algorithm is an approximation, reconstruction accuracy decreases with increasing distance from the central plane and with increasing cone angle.

2.4 Digital Tomosynthesis

Digital tomosynthesis (DTS) is an imaging technique that occupies an intermediate position between conventional radiography and full computed tomography. Originally introduced in the medical field to improve depth discrimination while maintaining a low radiation dose [11], it has since been adapted for industrial non-destructive testing where rapid acquisition and mechanical flexibility are essential [22].

2.4.1 Principle

In this section, limited-angle tomosynthesis is first introduced, as it represents the most common configuration; other acquisition geometries are presented in the following sections. DTS relies on acquiring a limited number of projections over a restricted angular range, typically between 20° and 60° , instead of performing a full 360° rotation as in CT. The acquired projections are then combined to reconstruct a stack of slices parallel to the detector plane. Unlike CT, where a complete acquisition provides nearly complete sampling of the Fourier domain, (Section 2.3), tomosynthesis samples only a limited wedge-shaped region, resulting in incomplete frequency information. Consequently, structures located in the focal plane—the depth at which reconstruction is optimized—appear sharp, while features outside this plane become progressively blurred. This effect is a direct consequence of the incomplete angular sampling and limited Radon space coverage.

2.4.2 Visual comparison: CT vs DTS

The fundamental differences between CT and DTS reconstruction quality are illustrated in Figure 2.9. It presents gVXR [23] simulations of a complex 3D object (Welsh dragon phantom) reconstructed using both methods. In the CT reconstruction (panels a-c), each orthogonal slice exhibits sharp delineation of in-plane features with clean separation from out-of-plane structures, resulting in high-quality 3D representation. In contrast, the DTS reconstruction (panel d-f) demonstrates the characteristic behavior of limited-angle acquisition: while structures within the focal plane retain good in-plane resolution, significant out-of-plane blur is visible as ghosting artifacts from structures at different depths. Notably, slices parallel to the optical axis (panels e, f) contain virtually no usable geometric information due to severe depth blur, whereas perpendicular slices (panel d) remain representative of the object's structure. This visualization reflects the incomplete sampling of the object, leading to anisotropic image quality — good in-plane resolution but degraded depth resolution along the imaging axis.

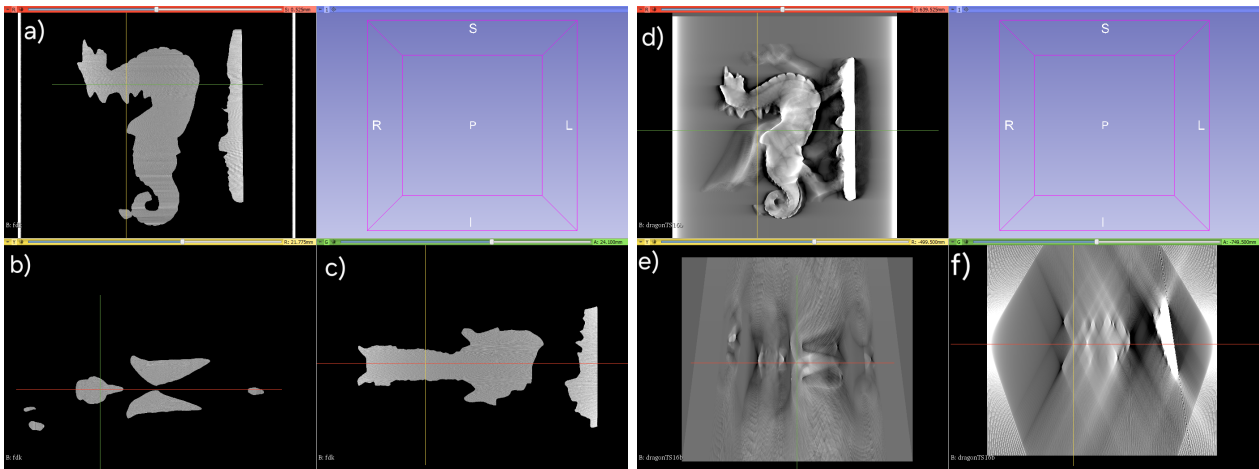


Figure 2.9: Comparative reconstruction of a 3D Welsh dragon phantom using CT (left) and DTS (right). CT provides isotropic resolution with clean slice separation, while DTS exhibits characteristic out-of-plane blur artifacts despite maintaining good in-plane resolution.

2.4.3 Advantages compared to CT

The reduced angular range leads to significantly shorter acquisition times, typically a factor of 5 to 20 compared to conventional CT. DTS is also more suitable for the inspection of thick or highly attenuating parts: by avoiding angles where transmission is too low, the acquisition can be restricted to orientations with adequate X-ray penetration [7], although this improvement comes at the expense of significantly reduced depth resolution, as demonstrated in Figure 2.9.

2.4.4 Limitations

The limited angular coverage inherently leads to missing information in Radon space, producing artifacts such as out-of-plane blur and streaks. As clearly visible in Figure 2.9, structures outside the focal plane appear as semi-transparent overlays, creating ambiguity in depth localization. Depth resolution is weaker than in CT and depends strongly on the total scan angle. As demonstrated by Cho et al. [9], scan angles below approximately 20° lead to severe blur artifacts, whereas angles above 60° yield more reliable reconstructions comparable to CT results (Figure 2.10).

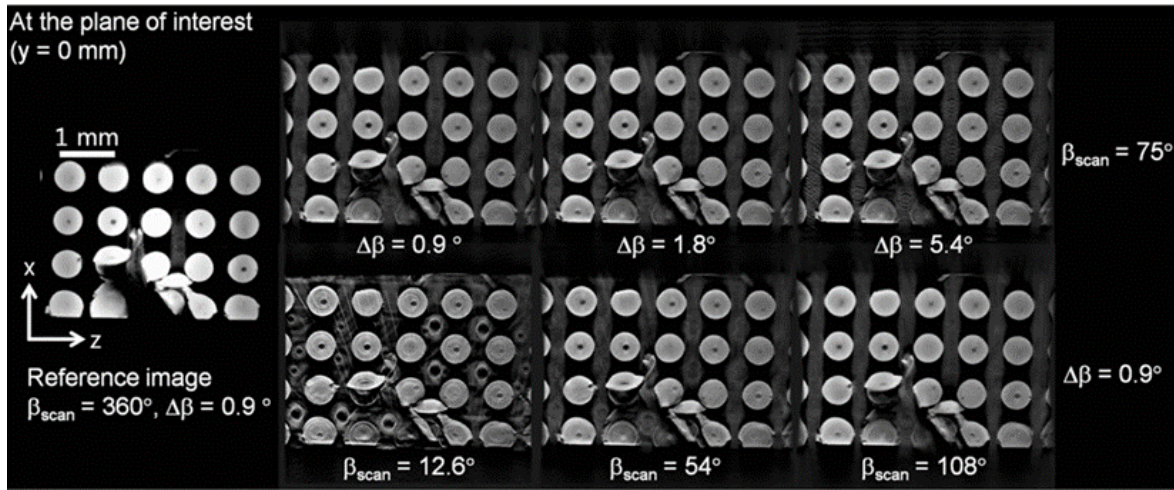


Figure 2.10: Impact of total scan angle on DTS reconstruction quality. The reference CT image (top left, $\beta_{\text{scan}} = 360^\circ$, $\Delta\beta = 0.9^\circ$) shows optimal quality. Narrow total scan angles ($\beta_{\text{scan}} < 20^\circ$) combined with small angular increments ($\Delta\beta < 1^\circ$) result in severe blur and streak artifacts, while wider angles ($\Delta\beta \approx 5^\circ$) and larger total scan ranges ($\beta_{\text{scan}} > 60^\circ$) approach CT-like quality. Adapted from [9].

2.4.5 Existing acquisition geometries of tomosynthesis

The trajectory followed by the X-ray source (and optionally the detector) during acquisition significantly impacts both the achievable image quality and the mechanical complexity of the system. Figure 2.11 illustrates the principal tomosynthesis acquisition geometries (a)–(e), together with the main parameters used to characterize them (f).

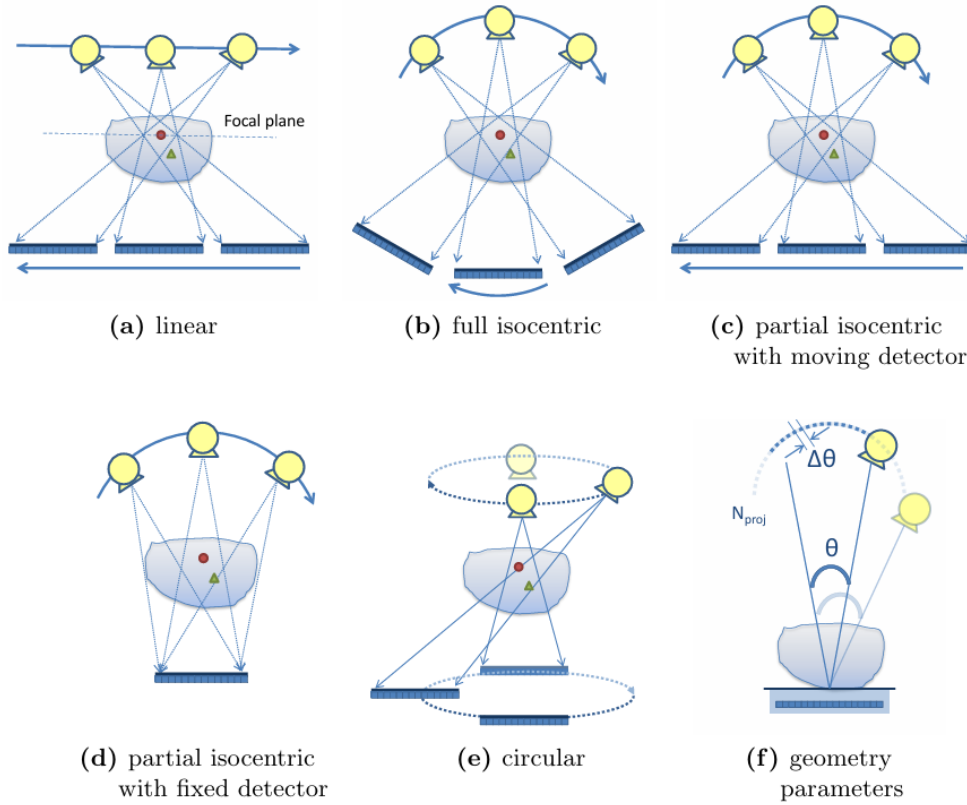


Figure 2.11: Tomosynthesis acquisition geometries: (a) linear, (b) full isocentric, (c)–(d) partial isocentric with moving/fixed detector, (e) circular, (f) key parameters (θ , N_{proj} , $\Delta\theta$). Adapted from [24].

Linear trajectory (Figure 2.11(a))

In this configuration, the X-ray tube translates along a straight line parallel to the detector plane, which remains stationary. This geometry is mechanically simple and corresponds to the earliest implementations of tomosynthesis [25].

Isocentric geometries (Figure 2.11(b)–(d))

Isocentric motion involves rotation around a fixed center point, analogous to conventional CT geometry. Three variants exist:

- **Full isocentric** (Figure 2.11(b)): Both the X-ray tube and detector rotate synchronously around a common axis. If extended to 360° , this becomes equivalent to cone-beam CT.
- **Partial isocentric with moving detector** (Figure 2.11(c)): The X-ray tube rotates along an arc while the detector follows to maintain geometric alignment.
- **Partial isocentric with fixed detector** (Figure 2.11(d)): Only the X-ray tube moves along an arc trajectory, while the detector remains stationary. This geometry is used in breast tomosynthesis systems where detector motion is mechanically constrained. However, it introduces magnification variations across projections that must be corrected during reconstruction. [24]

The choice between these isocentric variants depends on mechanical constraints, with moving-detector configurations generally providing more uniform image quality at the cost of increased mechanical complexity. In isocentric configurations, the region of interest can be positioned close to the isocenter, where the sampling is generally more uniform and depth localization is improved.

Circular trajectory (Figure 2.11(e))

In circular geometry, the tube and detector move along parallel circular paths without rotating about a common center. This configuration has been explored in early tomosynthesis literature but is less commonly used in modern systems due to its complex magnification behavior and non-uniform angular sampling. However, it is often used in certain industrial applications such as the inspection of flat parts like electronic circuit boards. This geometry is then especially well suited when the region of interest extends mainly parallel to the detector plane [25].

Geometry parameters (Figure 2.11(f))

Regardless of the trajectory type, any tomosynthesis acquisition is governed by three fundamental parameters:

- Angular range (θ or β_{scan}): The total angular span covered by the X-ray tube, corresponding to the arc described between the first and last projection positions around the rotation center. In tomosynthesis, this typically ranges from 20–60°.
- Number of projections (N_{proj}): The total number of X-ray images acquired over the angular range, typically 10–40 for clinical applications and 50–200 for industrial use depending on quality requirements and time constraints.
- Angular step ($\Delta\theta = \theta/N_{\text{proj}}$): The angular increment between successive projections, corresponding to the angular displacement between two consecutive tube positions. Smaller angular steps provide denser sampling but require more projections for a given angular range, directly impacting acquisition time.

2.4.6 Concept of focal plane

In digital tomosynthesis, image reconstruction produces a stack of slices parallel to the detector plane, each focused at a different depth within the object. The sharpness of reconstructed features depends on whether they lie within the selected depth, a concept governed by the *focal plane*.

During acquisition, the X-ray source moves along a predefined trajectory while projections are recorded at multiple angles. For a given reconstruction slice, contributions from different projections are combined in such a way that structures located at a specific depth—the focal plane—are coherently aligned across projections. This effect is achieved during reconstruction by shifting and combining projections so that a selected plane is brought into focus. As a result, features lying in this plane are reinforced and appear sharp in the reconstructed image. In contrast, structures located outside the focal plane are misaligned between projections and are therefore averaged out, producing characteristic out-of-plane blur.

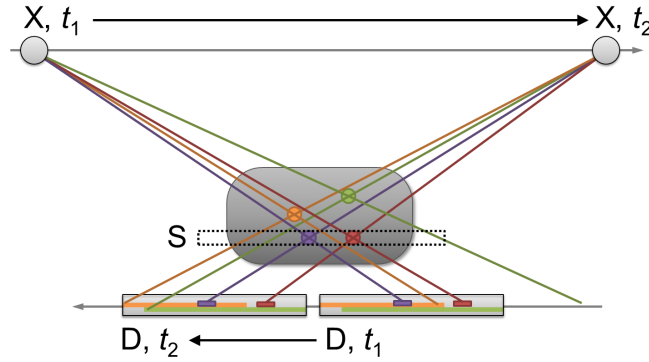


Figure 2.12: Principle of focal plane formation in tomosynthesis. The figure illustrates the acquisition geometry: structures located in the focal plane (S) are aligned across projections, while features at other depths are misaligned. During reconstruction, this alignment leads to sharp representation of the focal plane, whereas structures from other depths appear blurred. Adapted from [26]

The thickness of the focal plane depends on acquisition parameters such as the total angular range and the trajectory geometry [27]. In practice, the focal plane corresponds to a slice of finite thickness rather than an ideal plane. As illustrated in Figure 2.13, this thickness decreases as the sweep angle increases, thereby improving depth discrimination. Conversely, narrow angular ranges lead to thicker focal planes and stronger depth ambiguity.

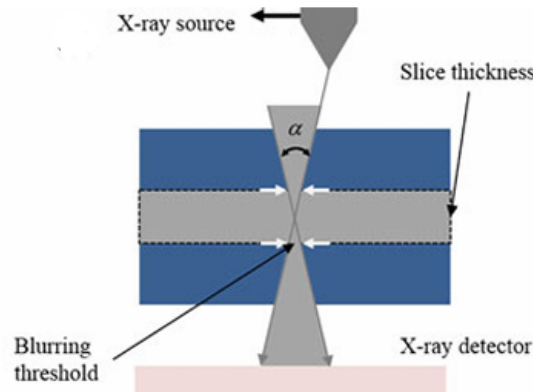


Figure 2.13: Slice thickness as a function of the sweep angle α in tomosynthesis. Increasing the angular range reduces slice thickness and improves depth discrimination. Adapted from [2]

This behavior explains why tomosynthesis is often described as a *2.5D* imaging technique, combining high in-plane resolution with limited depth discrimination, as demonstrated in Figure 2.9.

2.4.7 Reconstruction methods in DTS

Several reconstruction algorithms have been proposed for digital tomosynthesis to compensate for its limited angular coverage and the resulting incomplete sampling of Fourier space. These methods differ in computational complexity, robustness to limited-angle artifacts, and achievable image quality. Three representative approaches are reviewed here: Shift-And-Add (SAA), Filtered Back-Projection (FBP), and iterative algebraic methods.

Shift-And-Add (SAA)

Shift-And-Add is one of the earliest reconstruction methods used in tomosynthesis [8, 28]. The principle consists in shifting each projection by an amount proportional to the expected lateral displacement of structures at a selected depth, then summing the shifted projections, as illustrated in Figure 2.14. Features located at the target depth are coherently reinforced, while out-of-plane structures are spread across the image, producing characteristic blur.

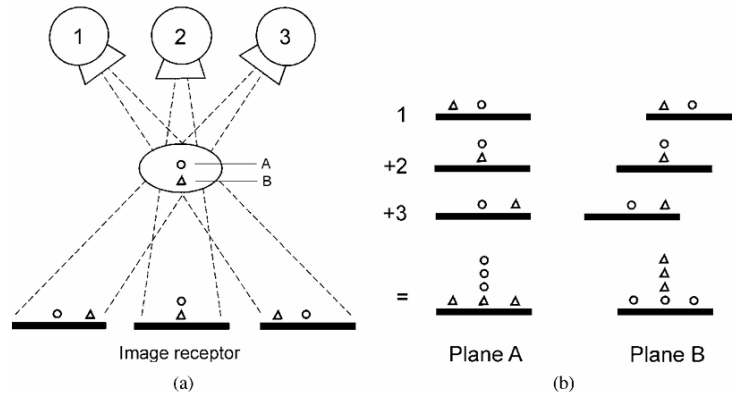


Figure 2.14: Principle of shift-and-add tomosynthesis. Projections acquired from different X-ray tube positions are shifted to align structures in a selected focal plane and then summed. Out-of-plane features are misaligned and produce blur. Adapted from [8]

SAA is computationally efficient and straightforward to implement, making it suitable for rapid qualitative visualization. However, SAA is most accurate for acquisition geometries where the X-ray source translates parallel to the detector plane at a fixed height [33]. A generalized formulation, Simple Back-Projection (SBP) [33, 34], extends SAA to arbitrary tube motions by introducing a depth-dependent scale factor, though at the cost of stronger out-of-plane streak artifacts [8].

Filtered Back-Projection (FBP)

Filtered Back-Projection is the standard analytical reconstruction algorithm in CT and has been adapted to tomosynthesis [33, 34]. As already explained in the Section 2.3.1, the method applies a ramp filter to each projection in the frequency domain — motivated by the Fourier slice theorem — before backprojecting the filtered data into the reconstruction volume. The filtering step compensates for the uneven density of radial samples in Fourier space and suppresses the low-frequency components responsible for image blurring that affects unfiltered backprojection, as illustrated in Figure 2.15

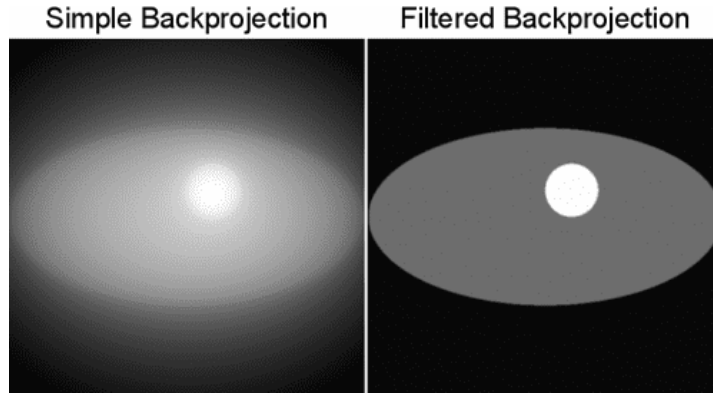


Figure 2.15: Illustration of the difference between simple backprojection and filtered backprojection. Simple backprojection produces a blurred reconstruction due to the accumulation of low-frequency components, whereas filtered backprojection restores sharper object boundaries by applying a ramp filter before reconstruction. Adapted from [35]

FBP produces high-quality reconstructions when projections sufficiently cover the angular range and data are consistent, as in conventional CT. In tomosynthesis, however, the limited movement range leaves a large portion of Radon space unsampled (Section 2.4), which the ramp filter cannot recover. This results in residual out-of-plane blur and streak artifacts, particularly for high-contrast features. [8, 34]

Algebraic Reconstruction Techniques

Iterative algebraic methods treat reconstruction as the solution of a large system of linear equations $\mathbf{p} = \mathbf{A}\mathbf{f}$, where $\mathbf{f}$ is the unknown voxel volume, $\mathbf{p}$ the measured projection data, and $\mathbf{A}$ the system matrix encoding the acquisition geometry [8].

The Algebraic Reconstruction Technique (ART) [36], updates the linear attenuation coefficients in a ray-by-ray manner: the difference between the measured and estimated projection values corrects each voxel along the ray being processed. ART converges quickly, but because it updates one ray at a time, the reconstruction tends to be noisy, especially in difficult cases like limited-angle tomosynthesis [8].

The Simultaneous Iterative Reconstruction Technique (SIRT) [37], considers all projection equations simultaneously before updating the volume estimate. Compared to ART, SIRT exhibits better noise properties but converges more slowly [29].

The Simultaneous Algebraic Reconstruction Technique (SART), combines the advantages of both approaches by processing one full projection at a time rather than a single ray or all projections simultaneously. The update rule for the j -th voxel is:

$$f_j^{(n+1)} \leftarrow f_j^{(n)} + \frac{1}{\sum_{i \in I_\theta} a_{ij}} \sum_{i \in I_\theta} a_{ij} \frac{p_i - \hat{p}_i(\mathbf{f}^{(n)})}{\sum_j a_{ij}} \quad (2.13)$$

where I_θ denotes the set of rays in a single projection at angle θ , a_{ij} is the contribution of voxel j to ray i , and $\hat{p}_i$ is the forward-projected estimate. This projection-by-projection update provides a favorable balance between convergence speed and noise properties [29]. SART can thus be seen as a compromise between ART and SIRT, combining fast convergence with improved noise stability.

A practical extension is Ordered Subsets SART (OS-SART) [38], which partitions the projections into ordered subsets and applies one SART update per subset per iteration, significantly accelerating convergence. In tomosynthesis, early stopping before full convergence acts as an implicit regularization, limiting noise amplification while preserving contrast.

2.4.8 Image quality metrics

Quantitative image quality metrics are essential for objectively evaluating reconstruction performance in DTS and for comparing different acquisition trajectories and reconstruction methods.

Contrast-to-Noise Ratio (CNR)

The contrast-to-noise ratio (CNR) is one of the most widely used metrics to quantify defect detectability in NDT radiology in general, including tomographic and tomosynthesis imaging. Unlike simple contrast measures, CNR explicitly accounts for image noise and therefore provides a more reliable indicator of the visibility of low-contrast structures.

For a region of interest (ROI) corresponding to a defect and a surrounding background region, CNR is classically defined as [9, 28]:

$$\text{CNR} = \frac{|\mu_{\text{defect}} - \mu_{\text{background}}|}{\sqrt{\sigma_{\text{defect}}^2 + \sigma_{\text{background}}^2}} \quad (2.14)$$

where μ_{defect} and $\mu_{\text{background}}$ denote the mean intensity values within the defect and background regions, respectively, and σ_{defect}^2 and $\sigma_{\text{background}}^2$ are the corresponding variances.

In industrial practice, particularly in NDT radiography, noise is often dominated by the background region. For this reason, CNR is commonly defined according to the ASTM E2698 standard [39] as:

$$\text{CNR} = \frac{|\mu_{\text{defect}} - \mu_{\text{background}}|}{\sigma_{\text{background}}} \quad (2.15)$$

A higher CNR indicates improved defect visibility and thus better image quality [28]. In the context of digital tomosynthesis, CNR is influenced by acquisition parameters such as angular range, number of projections, and reconstruction method, making it a key metric for trajectory optimization and quantitative image quality assessment.

Spatial Resolution

Spatial resolution quantifies the ability of an imaging system to distinguish fine details and separate closely spaced features within a reconstructed image. In tomographic imaging, it is commonly characterized by the Full Width at Half Maximum (FWHM) of the system's point spread function (PSF) — the response of the imaging system to a point object [40, 41]. The FWHM corresponds to the width of the intensity profile at half its maximum value, as illustrated in Figure 2.16. A smaller FWHM indicates a sharper response and therefore better spatial resolution.

In three-dimensional imaging, spatial resolution can therefore be characterized separately in the two in-plane directions and in the depth direction [22]:

- *In-plane resolution*, corresponding to the detector plane, is mainly governed by detector pixel size, magnification, and reconstruction filtering.
- *Depth resolution*, perpendicular to the detector plane, depends primarily on the total angular range or displacement range and the acquisition geometry.

FWHM is then measured along each axis — in-plane (FWHM_x , FWHM_y) and in depth (FWHM_z) — allowing the characterization of potential anisotropies in the reconstructed volume. In conventional CT, the complete 360° acquisition provides isotropic resolution ($\text{FWHM}_x \approx \text{FWHM}_y \approx \text{FWHM}_z$), whereas the limited angular coverage of tomosynthesis introduces significant anisotropy in the depth direction, as discussed in Section 2.4.6.

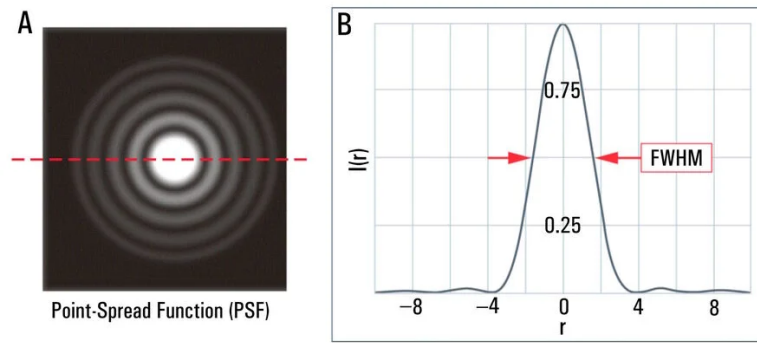


Figure 2.16: Illustration of the Point Spread Function (PSF) and the Full Width at Half Maximum (FWHM) as measures of spatial resolution. (A) The PSF represents the response of the imaging system to a point object. (B) Corresponding normalized intensity profile, where the FWHM is measured at half of the maximum intensity. The values 0.25 and 0.75 correspond to normalized intensity levels on the vertical axis. Adapted from [41].

Chapter 3

Materials and methods

This chapter describes the experimental framework used to develop and evaluate optimized tomosynthesis trajectories for the Dx-Box C5 system. It first presents the system architecture and its five degrees of freedom (Section 3.1), followed by the mechanical constraints that limit trajectory design (Section 3.2). The gVXR simulation tool used for trajectory validation is then introduced (Section 3.3), along with the numerical phantom used throughout the study (Section 3.4). The acquisition trajectories investigated — both simple and combined — are described in Section 3.5. Finally, the reconstruction method, image quality metrics, and Radon space coverage analysis used to compare the trajectories are detailed in Section 3.6.

3.1 Dx-Box C5 system description

The X-RIS Dx-Box C5 is a cabinet-style cone-beam X-ray inspection system designed for industrial non-destructive testing and computed tomography (CT).

3.1.1 General architecture and degrees of freedom

As introduced in Section 1.1, the system combines a C-arm geometry with a multi-axis positioning stage, providing flexibility for both conventional radiography inspection and tomographic acquisitions.

Figure 3.1 shows the internal configuration of the system. The X-ray source and flat-panel detector are mounted on a motorized C-arm capable of rotation, while the inspected part is positioned on a turntable mounted on a multi-axis translation stage.

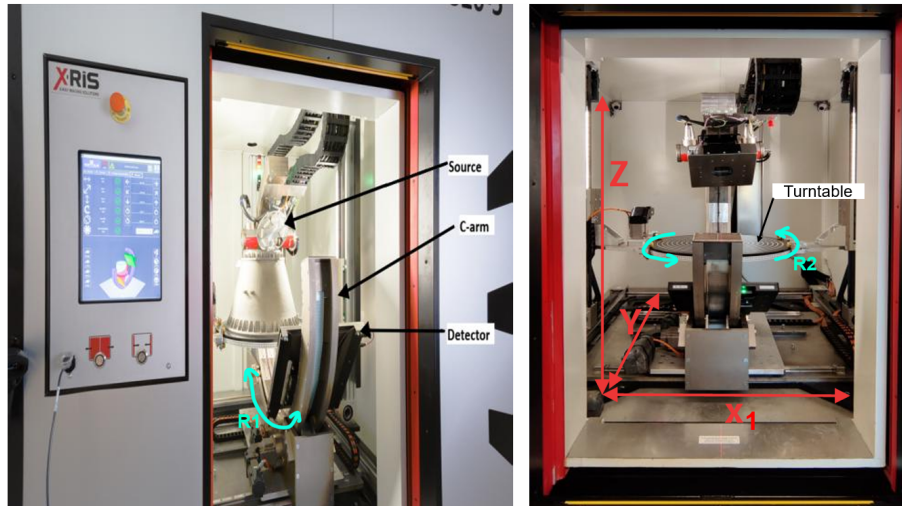


Figure 3.1: Interior view of the Dx-Box C5 system. Left: C-arm geometry showing the X-ray source, detector, and rotational axis. Right: Multi-axis translation stage with annotated coordinate system (X, Y, Z axes).

The system provides five independent degrees of freedom (DOF) that allow flexible positioning of the X-ray source, detector, and inspected part (See the annotations in Figure 3.1 for visual illustrations). These axes are summarized in Table 3.1 and are described below:

- **Z**: Vertical translation of the part through synchronized motion of the left and right supports, with a range of -450 to $+450$ mm relative to the rotation center.
- **X1**: Horizontal translation parallel to the cabinet door, with a range of -220 to $+220$ mm, allowing lateral displacement of the C-arm assembly.
- **Y**: Horizontal translation perpendicular to the cabinet door (forward/backward), with a range of -700 to $+220$ mm, allowing the distance between the X-ray source and the object to be adjusted.
- **R1**: Rotation of the C-arm about its horizontal rotation axis, with a range of 0° to $+180^\circ$, providing angular coverage for tomographic acquisitions.
- **R2**: Rotation of the turntable about the vertical axis, allowing rotation of the inspected part. The turntable has a diameter of 700 mm.

Table 3.1: Manipulator axes of the DX-Box C5 system (from the system documentation).

Axis	Function	Range	Zero position
Z	Vertical translation of the part	-450 to $+450$ mm	C-arm rotation center
X1	C-arm horizontal translation	-220 to $+220$ mm	Door center
Y	Horizontal translation	-700 to $+220$ mm	C-arm rotation center
R1	C-arm rotation	0° to $+180^\circ$	X-ray tube at top
R2	Turntable rotation	-720° to $+720^\circ$	No zero position

This kinematic configuration enables a wide range of acquisition geometries beyond conventional circular CT scans. By combining C-arm rotation (R1), C-arm translation (X1), object translations (Y and Z), and turntable rotation (R2), the system can support linear, arc, and composite acquisition trajectories. These trajectories are described in detail in Section 3.5.

3.1.2 Imaging geometry and detector characteristics

The geometric configuration of the Dx-Box C5 follows a cone-beam projection model, characterized by two key distances:

- **Source-to-Detector Distance (SDD):** the distance between the X-ray source and the flat-panel detector, fixed at 1150 mm.
- **Source-to-Isocenter Distance (SID):** the distance between the X-ray source and the rotation center of the system, fixed at 575 mm.

The geometric magnification factor M is defined as:

$$M = \frac{SDD}{SID} = \frac{1150}{575} = 2 \quad (3.1)$$

This magnification factor governs the relationship between physical object dimensions and their projection on the detector.

The flat-panel detector has a square active area of 430×430 mm, with a pixel pitch of 0.139 mm, resulting in a detector resolution of approximately 3072×3072 pixels. Figure 3.2 illustrates the overall cone-beam geometry of the system.

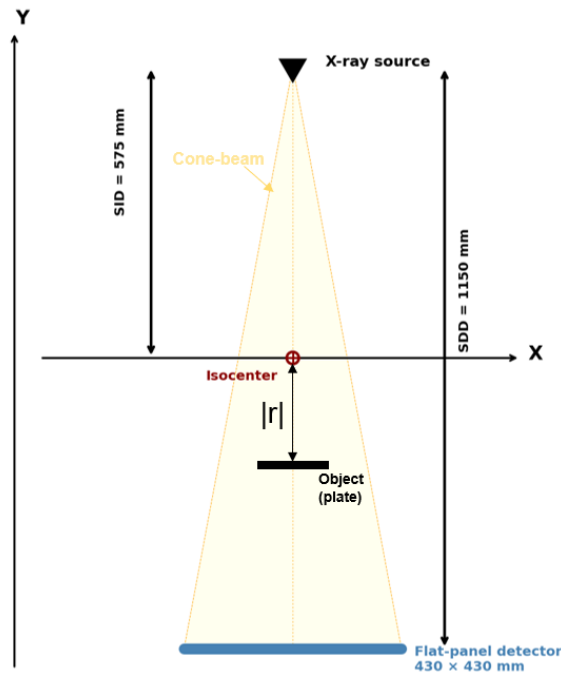


Figure 3.2: Simplified cone-beam imaging geometry of the Dx-Box C5 system, showing the source-to-isocenter distance (SID), source-to-detector distance (SDD), detector and object offset $|r|$ from the isocenter. The Z-axis (vertical direction) is perpendicular to the XY plane shown here.

Coordinate system

A fixed right-handed coordinate system is used throughout this work, as illustrated in Figure 3.1. The X-axis is defined parallel to the cabinet door, the Y-axis perpendicular to the door (forward/backward direction), and the Z-axis corresponds to the vertical direction.

For rotational motion, the zero position of the C-arm rotation (R1) corresponds to the X-ray tube located at the top of the system. The turntable rotation (R2) is defined around the vertical axis passing

through the rotation center or isocenter. These conventions are used consistently throughout this work for trajectory design and simulations.

3.2 Mechanical constraints and operational limits

The design of tomosynthesis trajectories on the Dx-Box C5 is subject to geometric constraints imposed by the finite size of the flat-panel detector. For any acquisition geometry, the projection of the inspected part must remain entirely within the detector field of view at every source position. Exceeding this limit results in truncated projections, which can introduce artifacts in the reconstructed volume.

In this work, a plate of width $W = 100$ mm is positioned at a distance $|r|$ from the isocenter, as illustrated in Figure 3.2. The parameter $|r|$ controls the position of the object relative to the isocenter and directly influences the detector field coverage and admissible trajectory range.

Angular range constraint (R1 and R2)

For rotational trajectories, the maximum admissible angular range $\theta_{\max}$ is determined by requiring that all points of the object remain within the detector field of view at every projection angle. For each value of $|r|$, $\theta_{\max}$ is determined numerically as the largest angle satisfying this condition. As shown in Figure 3.3, $\theta_{\max}$ decreases as $|r|$ increases, since objects further from the isocenter subtend a larger angle on the detector. For example, at $|r| = 300$ mm, $\theta_{\max} = \pm 21.45^\circ$, corresponding to a total angular range of 42.90° .

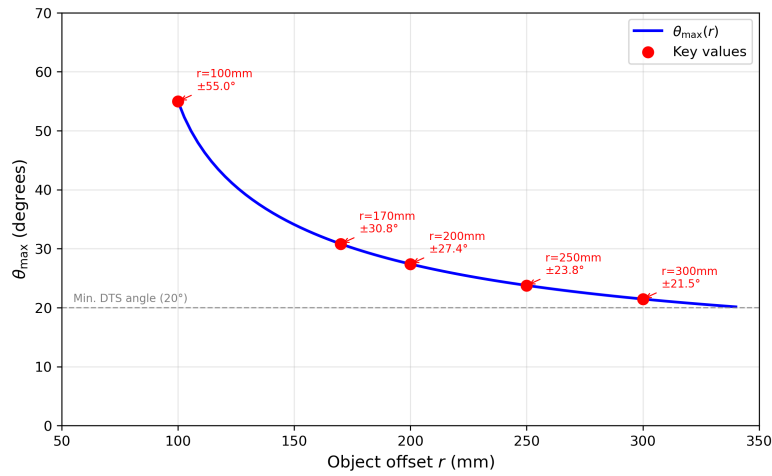


Figure 3.3: Maximum admissible angular range $\theta_{\max}$ as a function of object offset $|r|$, for a part of width $W = 100$ mm. The dashed line indicates the minimum angular range of 20° recommended for tomosynthesis.

Translation range constraint (X1 and Z)

For linear translation trajectories, the generator and detector move together along the X or Z axis while the object remains fixed. The maximum admissible translation $x_{\max}$ is derived analytically from the cone-beam projection geometry (see Appendix B for details):

$$x_{\max}(|r|) = \frac{D/2}{|t|} - \frac{W}{2} \quad \text{with} \quad |t| = \frac{SDD}{|SID - |r||} \quad (3.2)$$

As shown in Figure 3.4, $x_{\max}$ decreases linearly as $|r|$ increases. For $|r| = 170$ mm, $x_{\max} \approx 89$ mm, corresponding to a total translation range of approximately 180 mm. Since the detector is square ($430 \times$

430 mm), the same constraint applies identically to the vertical translation (Z axis).

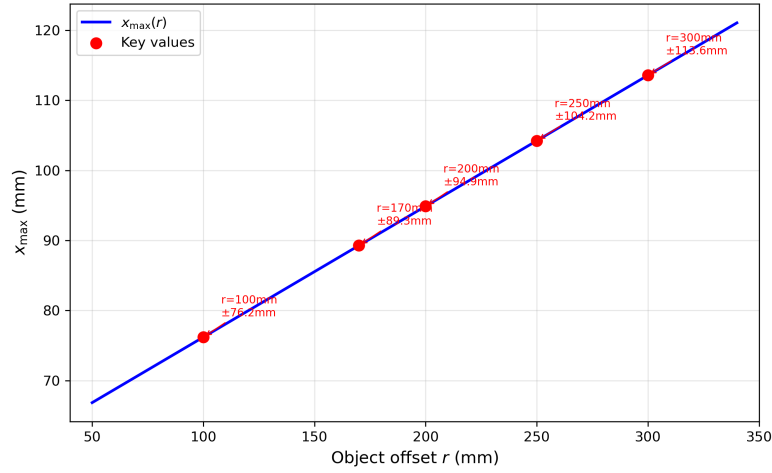


Figure 3.4: Maximum admissible translation range $x_{\max}$ as a function of object offset $|r|$, for a part of width $W = 100$ mm.

These geometric constraints define the admissible parameter space for the trajectories investigated in this work and are systematically applied throughout the parametric study.

3.3 Simulation framework: gVXR

All acquisition trajectories investigated in this work were validated through numerical simulation using gVXR (GPU-accelerated X-Ray simulation framework) [23], a Python-based tool for simulating cone-beam X-ray projection images from 3D mesh objects. gVXR computes X-ray projections by ray-casting through triangulated surface meshes while accounting for material attenuation along each ray path. Figure 3.5 shows a typical simulation scene used in this work.

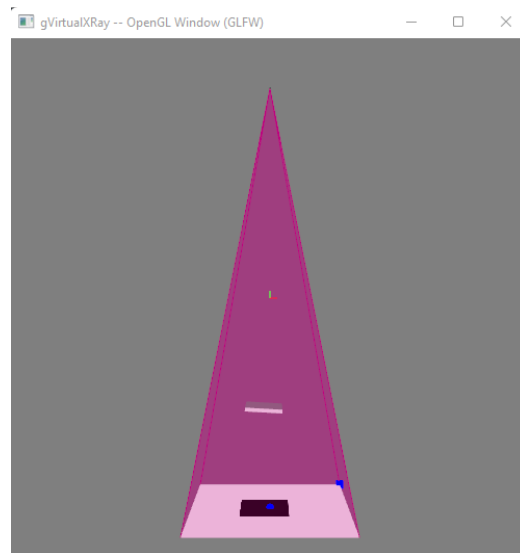


Figure 3.5: Example of a gVXR simulation scene showing the cone-beam acquisition geometry with the inspected part positioned between the X-ray source (top) and the flat-panel detector (bottom).

Simulation parameters

The simulation reproduces the geometric configuration of the Dx-Box C5 system described in Section 3.1.2. The source is modeled as an ideal point source emitting a monochromatic beam at 320 keV, with 10^5 photons per projection. The detector is configured with 3072×3072 pixels of 0.139 mm pitch, matching the physical detector specifications. Table 3.2 summarizes the simulation parameters used throughout this work.

Table 3.2: gVXR simulation parameters.

Parameter	Value	Unit
Source type	Point source, monochromatic	—
Beam energy	320	keV
Number of photons	10^5	per projection
SID	575	mm
SDD	1150	mm
Detector size	430×430	mm
Pixel pitch	0.139	mm
Detector pixels	3072×3072	—
Object material	Iron (Fe)	—

For each trajectory, projections are acquired sequentially by updating the relevant axis positions at each step. Each projection is computed as a floating-point image and subsequently normalized and exported as a 16-bit TIFF file, with intensity values mapped to the range [1000, 50000] to avoid saturation during export and reconstruction.

3.4 Phantom design

The numerical phantom used in this work consists of a set of six iron (Fe) plates, each measuring $100 \times 100 \times 10$ mm, modeled as 3D STL objects. Each plate contains a single spherical air-filled porosity located at its geometric center, with diameters ranging from 0.2 mm to 2 mm, as summarized in Table 3.3.

Table 3.3: Phantom plates and corresponding porosity diameters.

Plate	Porosity diameter D (mm)
1	0.2
2	0.5
3	0.8
4	1.2
5	1.6
6	2.0

This range of porosity sizes was chosen to cover both difficult-to-detect and more visible defects, representative of internal defects typically encountered in industrial parts.

The iron material was selected for its high X-ray attenuation, representative of dense industrial parts typically encountered in NDT applications. The plate geometry — thin and flat — represents a challenging inspection scenario where tomosynthesis is particularly relevant, as discussed in Section 1.1.

Figure 3.6 illustrates the 3D model of a phantom plate with its central spherical porosity.

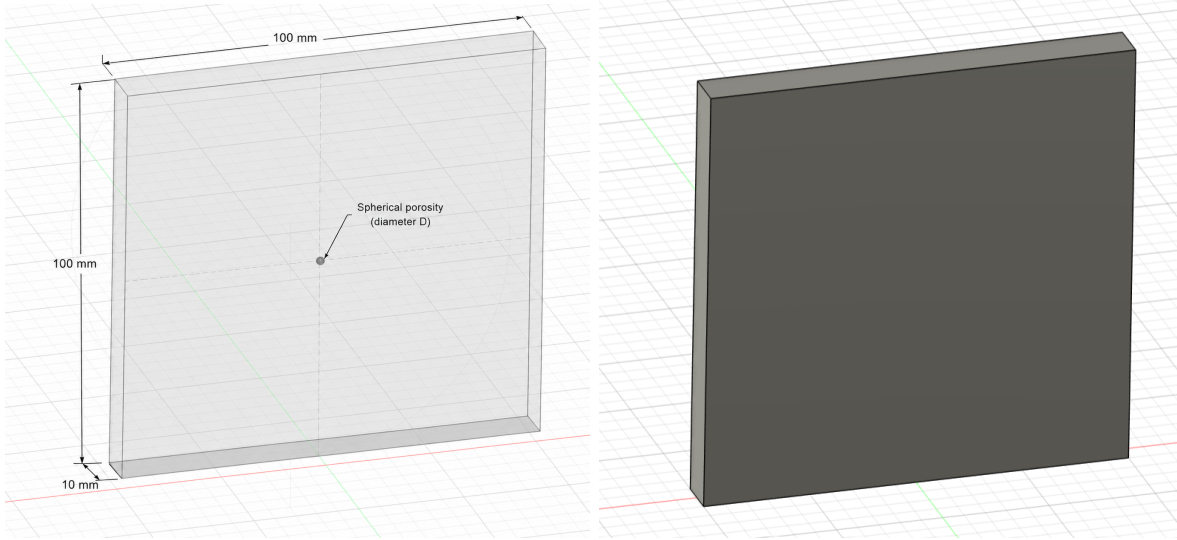


Figure 3.6: 3D model of the iron phantom plate ($100 \times 100 \times 10$ mm). Left: semi-transparent view showing the spherical porosity located at the plate center. Right: solid view of the plate.

3.5 Acquisition trajectory design

3.5.1 Simple trajectories

Four simple acquisition trajectories are investigated in this work, each corresponding to a distinct degree of freedom of the Dx-Box C5 system (Section 3.1): horizontal translation of the C-arm (X1), vertical translation of the object (Z), C-arm rotation (R1), and turntable rotation (R2). Together, they represent the main single-axis motions available on the system and form a representative basis for evaluating the influence of motion type and moving element on reconstruction quality, before investigating combined trajectories. These trajectories differ in the moving element (source–detector assembly or object) and in the nature of the motion (rotation or translation). Table 3.4 summarizes the key characteristics of each trajectory.

Table 3.4: Summary of the four simple acquisition trajectories.

Trajectory	Moving element	Motion type	Axis / Plane	Fixed element
X1	C-arm (source + detector)	Translation	X axis	Object
Z	Object (turntable)	Translation	Z axis	Source + detector
R1	C-arm (source + detector)	Rotation	YZ plane	Object
R2	Object (turntable)	Rotation	Z axis	Source + detector

X1 - Horizontal C-arm translation

In the X1 trajectory, the C-arm translates along the X-axis, moving the source–detector assembly while the object remains fixed. This corresponds to a linear tomosynthesis geometry, as introduced in Section 2.4.

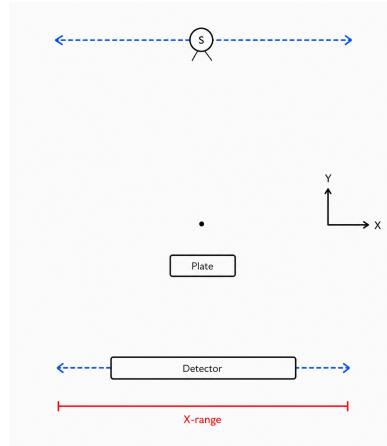


Figure 3.7: X1 trajectory showing the horizontal translation of the C-arm along the X-axis while the object remains fixed.

Z - Vertical object translation

In the Z trajectory, the source and detector remain fixed while the turntable translates vertically along the Z-axis. This geometry also corresponds to a linear tomosynthesis acquisition in the vertical direction, providing a complementary sampling geometry compared to the X1 trajectory.

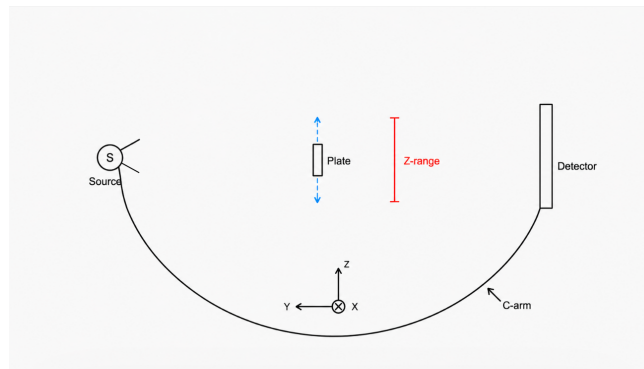


Figure 3.8: Z trajectory showing the vertical translation of the object along the Z-axis while the source and detector remain fixed.

R1 - C-arm rotation

In the R1 trajectory, the C-arm rotates in the YZ plane about the isocenter, moving the source and detector simultaneously while the object remains fixed at position $|r|$ along the Y-axis. The source follows a circular arc of radius $SID = 575$ mm about the isocenter, and the detector follows the symmetric arc at the same radius on the opposite side. In the simulation, detector up-vector is updated at each step to remain perpendicular to the source–detector axis.

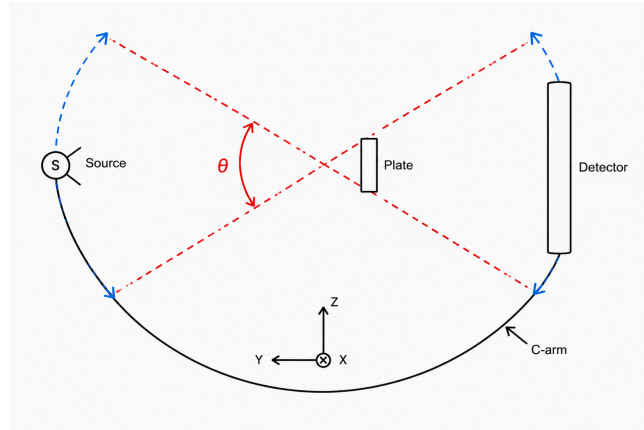


Figure 3.9: R1 trajectory showing the rotation of the C-arm in the YZ plane around the isocenter.

R2 - Turntable rotation

In the R2 trajectory, the source and detector remain fixed while the object rotates about the vertical Z-axis on the turntable. At each projection angle θ , the object is rotated around the Z-axis while the source and detector remain fixed. This geometry corresponds to a limited-angle acquisition with a fixed detector configuration, as described in Section 2.4.

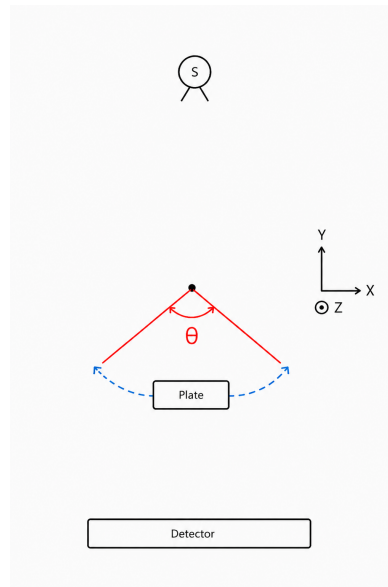


Figure 3.10: R2 trajectory showing the rotation of the object around the Z-axis while the source and detector remain fixed.

Acquisition parameters

For each simple trajectory, a parametric study was first conducted to evaluate the influence of the object offset $|r|$ and the angular or translation range on image quality. The admissible parameter ranges were determined from the geometric constraints derived in Section 3.2.

For the final comparison of these simple trajectories, a common set of acquisition parameters was used. For all trajectories, 41 projections were acquired with an object offset $|r| = 170$ mm. The translation range was set to ± 85 mm for the X1 and Z trajectories, while the angular range was set to $\pm 21.45^\circ$ for

the R1 and R2 trajectories. These values were selected to ensure a fair comparison between trajectories under equivalent acquisition conditions.

3.5.2 Combined trajectories

In addition to the four simple trajectories described in Section 3.5.1, combinations of two axes were also investigated. Most combined trajectories were implemented as sequential acquisitions, where different projection sets were concatenated before reconstruction.

X1 and Z combinations

Two types of combined X1–Z configurations were investigated. The first is a simultaneous configuration, referred to as *Diagonal XZ*, in which both X and Z translations are applied simultaneously at each projection step, producing a diagonal sampling trajectory across the detector field, as illustrated in Figure 3.11. This configuration was evaluated using 41 projections to allow direct comparison with the simple trajectory results.

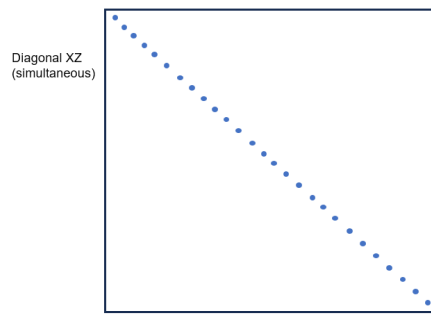


Figure 3.11: Diagonal XZ projection sampling trajectory obtained by simultaneous X and Z translations.

The second type investigates the effect of vertical projection position on image quality, using the X1 trajectory sampled at three vertical offsets: $Z = 0$ mm (detector center), $Z = 40$ mm, and $Z = 85$ mm, all at 160 projections and a symmetric translation range of ± 85 mm. The number of projections was increased from 41 to 160 in order to obtain a sufficiently dense two-dimensional sampling of the detector plane.

Several two-dimensional detector sampling patterns were also investigated at 160 projections each, as illustrated in Figure 3.12. These patterns were designed to evaluate the influence of projection distribution across the detector plane by comparing sampling concentrated near the detector center, sampling extending towards the detector edges, one-dimensional sampling paths, and fully two-dimensional distributions:

- **Full detector:** projections distributed over the full detector area in a regular grid pattern, with ± 85 mm in both X and Z.
- **Square (4 edges):** projections distributed along the four edges (40 projections per edges) of the detector field, with ± 85 mm in both X and Z.
- **Cross:** 80 projections along a horizontal line at $Z = 0$ and 80 projections along a vertical line at $X = 0$, with ± 85 mm in both directions.
- **Full centered:** projections distributed in a compact rectangular grid confined to the central detector region, with ± 40 mm in X and ± 20 mm in Z.
- **Cross centered:** same cross structure as above but restricted to ± 40 mm, keeping projections close to the detector center.

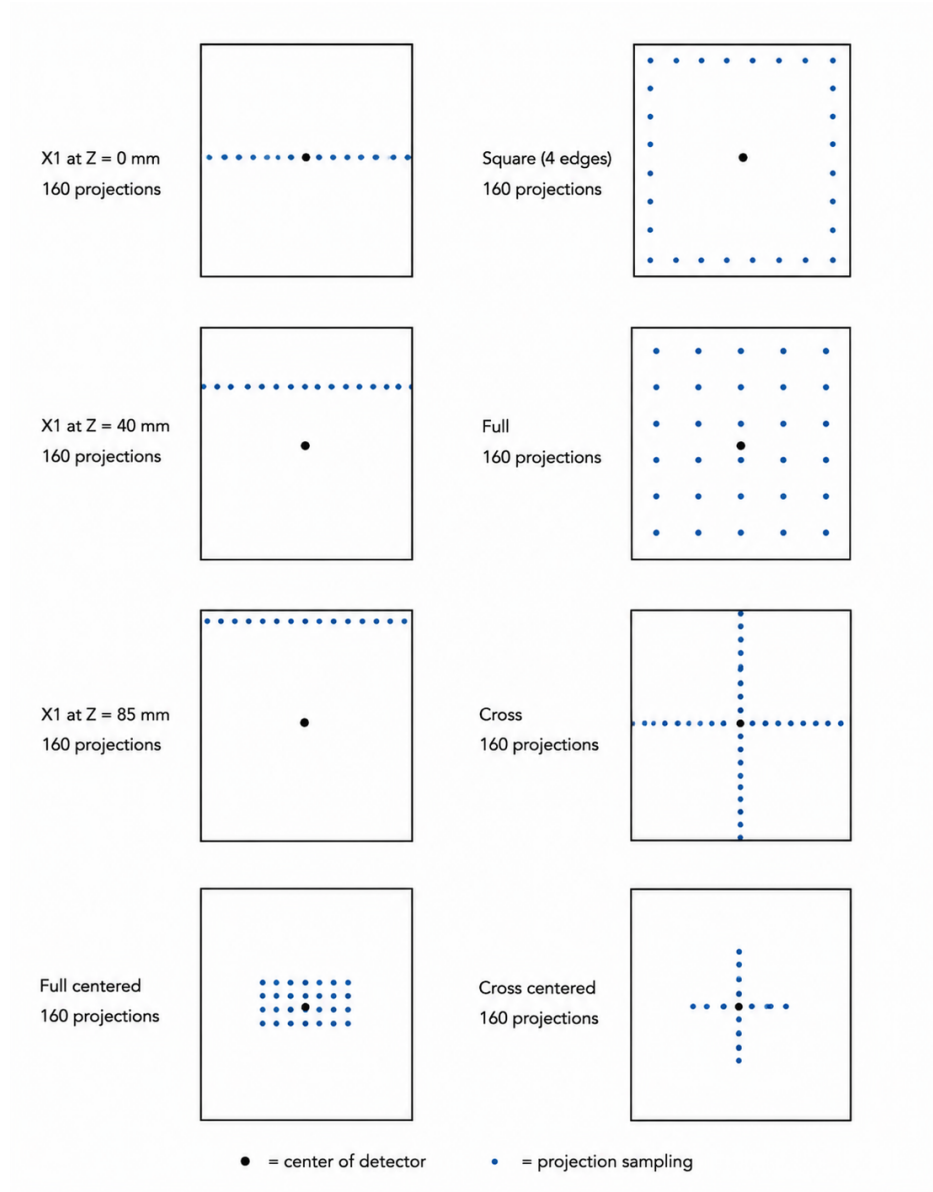


Figure 3.12: Detector-plane projection sampling patterns investigated for the X1 and X1+Z trajectories (160 projections each). Blue dots indicate the projection sampling positions.

Other combinations

Sequential combinations of the remaining axis pairs were also investigated: X1+R1, X1+R2, Z+R1, Z+R2, and R1+R2. Unlike the X1+Z study presented above, which aimed to evaluate the influence of projection distribution across the detector plane, these combinations were designed to assess whether adding a rotational component to a translation trajectory can improve overall image quality.

The translation range was set to ± 40 mm and the angular range to $\pm 15^\circ$ in order to keep the projection sampling concentrated around the detector center. A total of 200 projections were used for each configuration. For translation–rotation combinations, a larger fraction of the projections was assigned to the rotational component (120 versus 80) in order to increase the angular sampling density of the acquisition. For the R1+R2 combination, projections were split equally (100 each).

3.6 Reconstruction and evaluation protocol

3.6.1 Reconstruction method

All projections are reconstructed using Filtered Back-Projection (FBP), as described in Section 2.4.7, implemented through the Reconstruction Toolkit (RTK) [42]. The pipeline consists of two steps: a ramp filter applied to each projection using `rtkramp`, followed by cone-beam backprojection using `rtkbackprojection` with GPU acceleration (`CudaBackProjection`). The acquisition geometry is provided through an RTK XML file generated from the simulation parameters. The reconstructed volume has dimensions of $800 \times 800 \times 200$ voxels with an isotropic voxel spacing of 0.139 mm, giving a physical field of approximately $111 \times 111 \times 28$ mm.

3.6.2 Image quality metrics

The image quality metrics used in this work are defined in Section 2.4.8. This section describes their practical implementation.

Contrast and CNR

Since the porosity is located at the geometric center of each phantom plate, the central axial slice of the reconstructed volume corresponds to the focal plane of the defect and is selected for analysis. Two circular regions of interest (ROIs) are defined on this slice, as illustrated in Figure 3.13:

- **Signal ROI:** a circular region inside the porosity, with radius $r_{\text{sig}} = D/2 - \varepsilon$, where D is the porosity diameter and $\varepsilon = 0.05$ mm is a margin to limit partial volume effects.
- **Background ROI:** a concentric annular region surrounding the porosity, with inner radius $r_{\text{bg},\text{in}} = D/2 + \delta_{\text{gap}}$ and outer radius $r_{\text{bg},\text{out}} = \max(r_{\text{bg},\text{in}} + 2\Delta x, D \cdot f_{\text{bg}})$, where $\delta_{\text{gap}} = 0.5$ mm excludes the transition zone, $\Delta x = 0.139$ mm is the voxel spacing, and $f_{\text{bg}} = 2.5$ is a scaling factor.

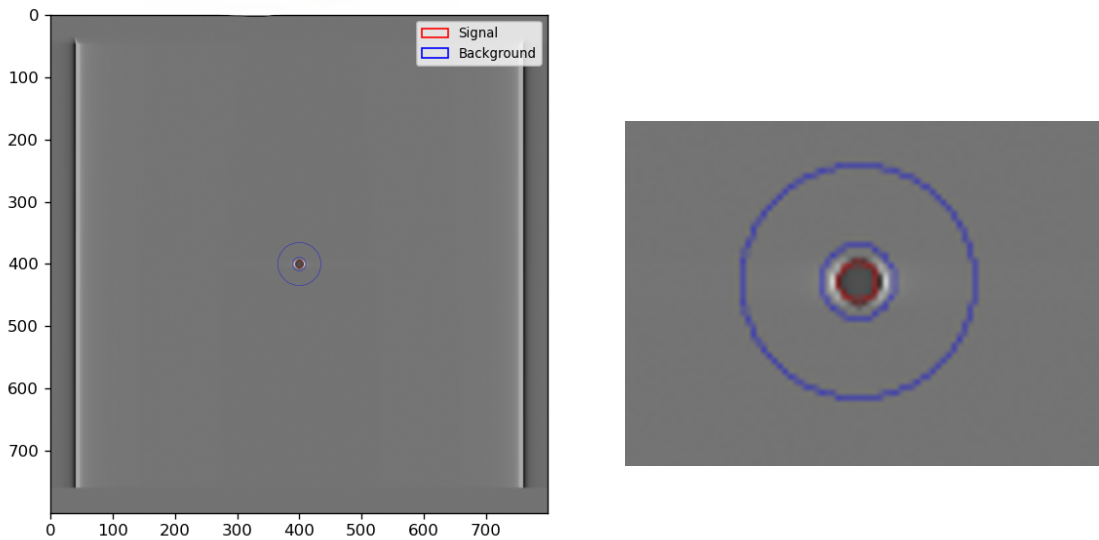


Figure 3.13: Signal ROI (red circle) and background ROI (blue annulus) defined on the central axial slice of a reconstructed volume, shown here for a 2 mm porosity. Left: full slice view. Right: zoomed view around the porosity. The gap between the two regions excludes the intensity transition zone around the porosity boundary.

Contrast and CNR are then computed according to the ASTM E2698 standard, as defined in Section 2.4.8. The absolute contrast $|\mu_{\text{sig}} - \mu_{\text{bg}}|$ is also recorded separately, as it serves as the primary visual quality metric in the trajectory comparison, independently of noise level.

Depth spatial resolution

Depth resolution is assessed along the Z-axis using the intensity profile extracted through the porosity center. The FWHM is computed around the porosity center using linear interpolation at the half-level crossings. The depth resolution ratio:

$$\text{Ratio}_Z = \frac{\text{FWHM}_Z}{d_{\text{real}}} \quad (3.3)$$

quantifies the depth blur relative to the true defect size. A value close to unity indicates that the reconstructed depth extent of the porosity matches its real size, while larger values reflect increasing out-of-plane blur.

The sharpness of the depth profile is further characterized by a dimensionless slope, defined as the mean of the 95th percentile of the normalized gradient on each side of the porosity:

$$\text{Slope} = \frac{1}{2} \left(P_{95} \left(\left| \frac{dp_{\text{norm}}}{dz} \right|_{z < 0} \right) + P_{95} \left(\left| \frac{dp_{\text{norm}}}{dz} \right|_{z > 0} \right) \right) \times d_{\text{real}} \quad (3.4)$$

where p_{norm} is the intensity profile normalized between its local minimum and maximum within the analysis window, and P_{95} denotes the 95th percentile operator. Using the 95th percentile rather than the maximum value reduces sensitivity to isolated noise peaks while still capturing the dominant gradient magnitude at the porosity boundaries. While Ratio_Z quantifies the overall extent of depth blur, the slope metric captures the sharpness of the intensity transitions, providing complementary information about axial localization quality.

Composite score

To rank trajectories according to both contrast and depth resolution simultaneously, a composite score is defined as:

$$\text{Score} = \text{Contrast} \times \frac{\text{Slope}}{\text{Ratio}_Z} \quad (3.5)$$

This score rewards trajectories that combine high contrast with sharp and well-localized depth transitions. It is computed for each porosity diameter independently and averaged over the full set of phantoms to provide a global ranking of the tested trajectories.

Radon space coverage and reachability

The image-based metrics defined above — contrast, depth resolution, and composite score — are tied to the specific phantom used in this work. To complement them with a more geometry-driven perspective, a theoretical criterion based on Radon space coverage is introduced here, referred to as the *reachability* criterion.

Reachability vs. sampling. Two distinct concepts must be carefully distinguished. The first is *reachability*: for a given source trajectory Γ , which points of Radon space are theoretically accessible? A point of Radon space is reachable if there exists at least one source position along Γ for which the corresponding sphere S_s — defined with the source position s and the object center O as diameter endpoints — passes through that point. The union of all such spheres along the trajectory defines the reachable region:

$$\mathcal{V}(\Gamma) = \bigcup_{s \in \Gamma} S_s \quad (3.6)$$

Figure 3.14 illustrates this principle schematically. Each source position generates a characteristic sphere in Radon space, and the union of all such spheres forms the reachable region $\mathcal{V}(\Gamma)$. The smaller sphere $\mathcal{V}(\Omega)$ represents the Radon region associated with the object support.

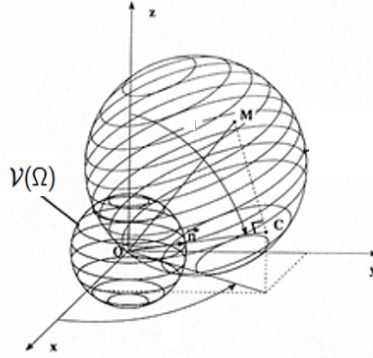


Figure 3.14: Illustration of the characteristic spheres associated with a source trajectory in Radon space. Each source position defines a sphere, and the union of all such spheres forms the reachable region $\mathcal{V}(\Gamma)$. The smaller sphere $\mathcal{V}(\Omega)$ represents the Radon region associated with the object support. Adapted from [19].

The second concept is *sampling*: among the reachable points, which ones are actually visited during the acquisition? This depends on how many projections are acquired and how they are distributed along Γ . Reachability defines what is theoretically accessible; sampling determines how well that region is actually covered.

In tomosynthesis, reachability is inherently limited since the source does not complete a full rotation — some portions of Radon space remain inaccessible because the source trajectory does not provide complete angular coverage. The optimization problem in this work can therefore be viewed from two complementary angles: combining multiple axes increases reachability, while optimizing the projection distribution for a given trajectory improves sampling within the already reachable region.

The reachability criterion. For the object to be fully reconstructable, every point of Radon space associated with the object must be reachable. Let $\mathcal{V}(\Omega)$ denote the region of Radon space associated with the object support Ω — the region that needs to be covered to reconstruct the object. The ideal condition is:

$$\mathcal{V}(\Omega) \subseteq \mathcal{V}(\Gamma) \quad (3.7)$$

In practice, this condition is never fully satisfied in tomosynthesis. The reachability is therefore quantified by the intersection volume:

$$V_{\text{cov}} = V(\mathcal{V}(\Omega) \cap \mathcal{V}(\Gamma)) \quad (3.8)$$

A larger V_{cov} indicates that the trajectory reaches a greater portion of the Radon data required for reconstruction, enabling a comparison between trajectories that is less dependent on the specific phantom configuration than image-based metrics.

Definition of $\mathcal{V}(\Omega)$. In this work, $\mathcal{V}(\Omega)$ is approximated by the region of Radon space associated with the circumscribed sphere of the phantom plate ($100 \times 100 \times 10$ mm), with radius:

$$r = \frac{1}{2} \sqrt{100^2 + 100^2 + 10^2} \approx 70.9 \text{ mm} \quad (3.9)$$

This sphere represents the space around the object that remains visible on the detector throughout the acquisition and that needs to be covered for a complete reconstruction. It is therefore a practical approximation of the detector-visible space around the object, rather than the object itself.

Reachable regions for X1 and R1. The reachability is evaluated for the X1 (linear translation) and R1 (arc rotation) trajectories, as these represent different sampling geometries in Radon space.

For a complete circular trajectory, $\mathcal{V}(\Gamma)$ forms a full torus in Radon space [20], [19]. However, this torus does not fully cover $\mathcal{V}(\Omega)$ — some portions of the object's Radon region fall outside the torus, forming a shadow zone of missing data, as illustrated in Figure 3.15. This is precisely the geometric expression of Tuy's incompleteness condition for circular cone-beam trajectories [2].

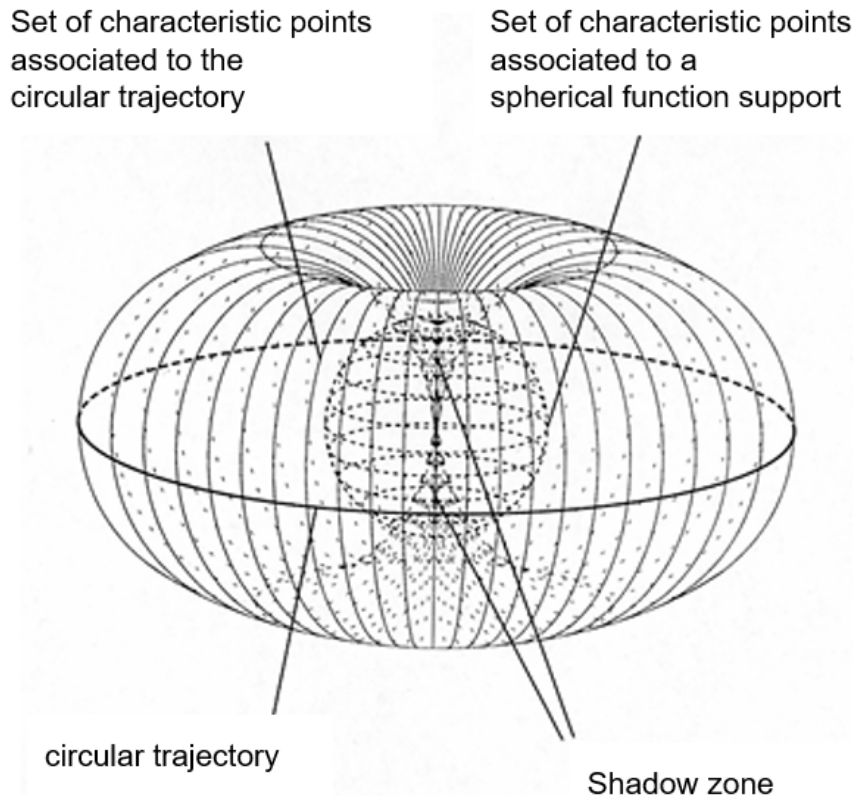


Figure 3.15: Shadow zone associated with a circular cone-beam trajectory in Radon space. The toroidal reachable region generated by the trajectory does not fully cover the region $\mathcal{V}(\Omega)$ associated with the object support, resulting in incomplete Radon data. Adapted from [19].

For a partial arc of angular range $[-\theta_{\max}, +\theta_{\max}]$ as in R1, the reachable region forms a partial torus subtended by that arc, as illustrated in Figure 3.16. The intersection volume $V(\mathcal{V}(\Omega) \cap \mathcal{V}(\Gamma_{R1}))$ is computed numerically using mesh-based methods, since no closed-form expression is available for the partial torus case.

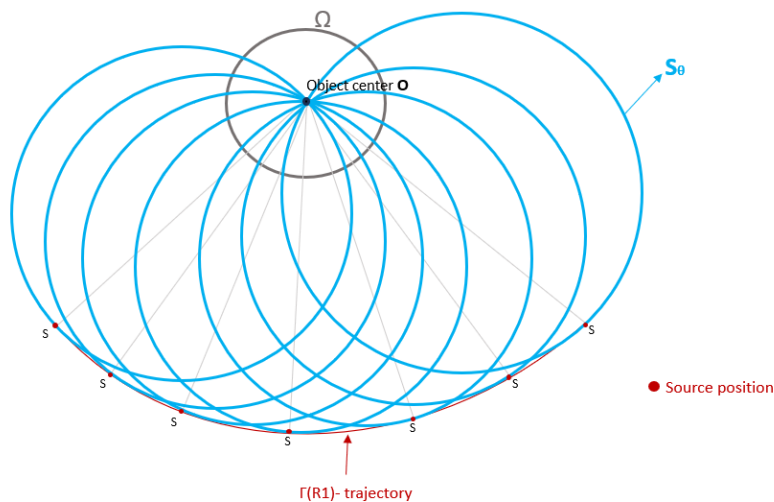


Figure 3.16: Radon space coverage for the R1 trajectory. The source follows a circular arc Γ_{R1} , and the union of the corresponding spheres S_θ forms a partial torus $\mathcal{V}(\Gamma_{R1})$.

For the linear translation trajectory X1, it can be shown (see Appendix C) that the two extreme spheres $S_{x_{\min}}$ and $S_{x_{\max}}$ cover all intermediate spheres along the trajectory, so that $\mathcal{V}(\Gamma_{X1}) = S_{x_{\min}} \cup S_{x_{\max}}$.

The intersection volume is therefore computed from the geometry of these two boundary spheres and the circumscribed sphere Ω , as illustrated in Figure 3.17. The maximum translation range was chosen as ± 85 mm so that the object remained fully visible on the detector for all source positions along the trajectory.

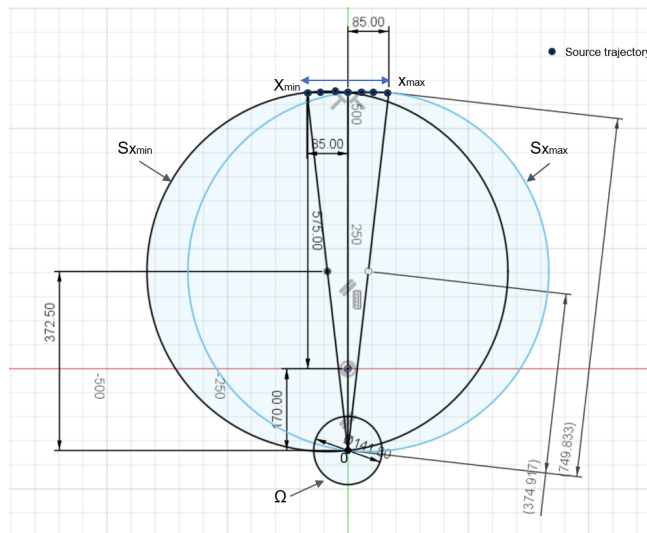


Figure 3.17: Geometric configuration used for the reachability computation of the X1 trajectory. The intersection volume is computed from the circumscribed sphere Ω and the two extreme spheres $S_{x_{\min}}$ and $S_{x_{\max}}$.

The purpose of this analysis is to verify whether a higher reachability translates into a better composite score, and to assess whether the reachability criterion can serve as a geometry-based predictor of trajectory quality, independently of the simulation parameters.

Chapter 4

Results and discussion

This chapter presents the simulation results obtained for the acquisition trajectories described in Chapter 3. The analysis is organized into two main parts: simple trajectories (Section 4.1) and combined trajectories (Section 4.2).

4.1 Simple trajectories

4.1.1 Porosities visualization

Figure 4.1 provides a visual overview of the six phantom plates used throughout this work. The top row shows cropped axial slices centered on the porosity at the plate mid-plane. The middle row shows the corresponding ROI definitions used for CNR analysis, and the bottom row provides a more zoomed view around the defect center.

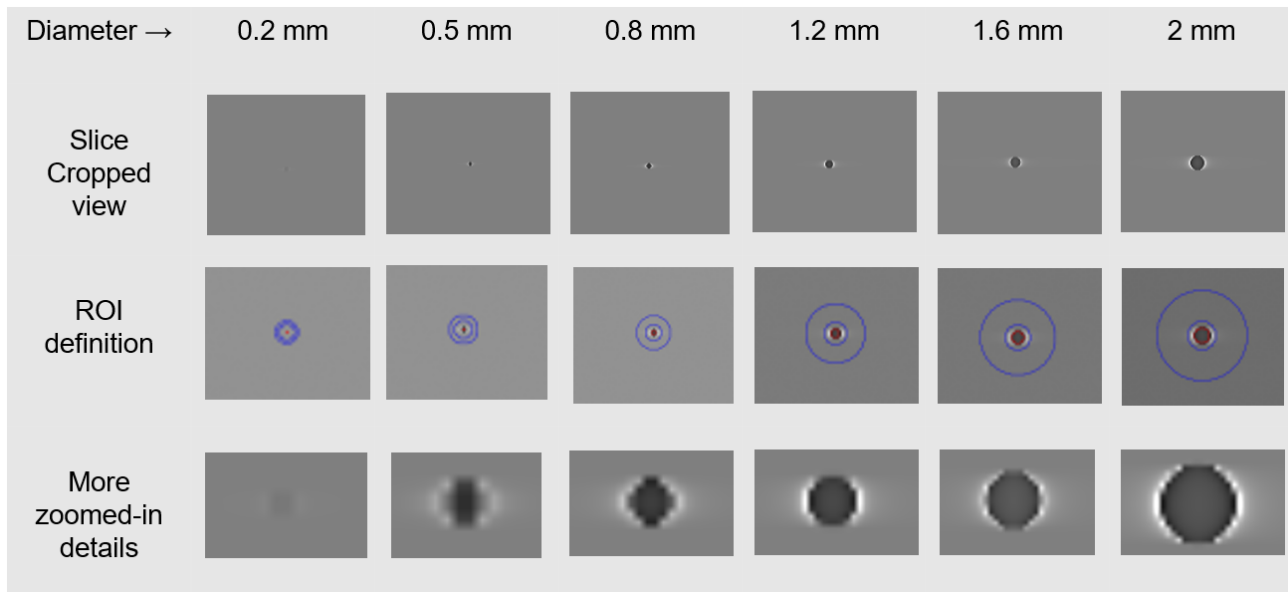


Figure 4.1: Visualization of reconstructed porosities for the six phantom plates (diameters: 0.2, 0.5, 0.8, 1.2, 1.6, and 2.0 mm, from left to right). Top row: local reconstructed slice. Middle row: ROI overlay showing the signal ROI (red) and background ROI (blue). Bottom row: zoomed detail around the porosity center.

As expected, the visibility of the porosity increases with its diameter. The smallest defect (0.2 mm) is barely distinguishable from the background, while the 2 mm porosity produces a clearly visible dark region. It is also worth noting that the two smallest porosities (0.2 and 0.5 mm) correspond to only approximately

1.4 and 3.6 voxels in diameter, respectively (based on the voxel size of 0.139 mm), which limits the reliability of quantitative measurements for these defect sizes, as discussed in the following section.

4.1.2 Contrast and CNR analysis

Figure 4.2 presents the contrast, background noise, and CNR as a function of porosity diameter, obtained for the X1 trajectory at $|r| = 170$ mm, $x = \pm 85$ mm, and 41 projections. These results are used to characterize the behavior of the image quality metrics before comparing trajectories.

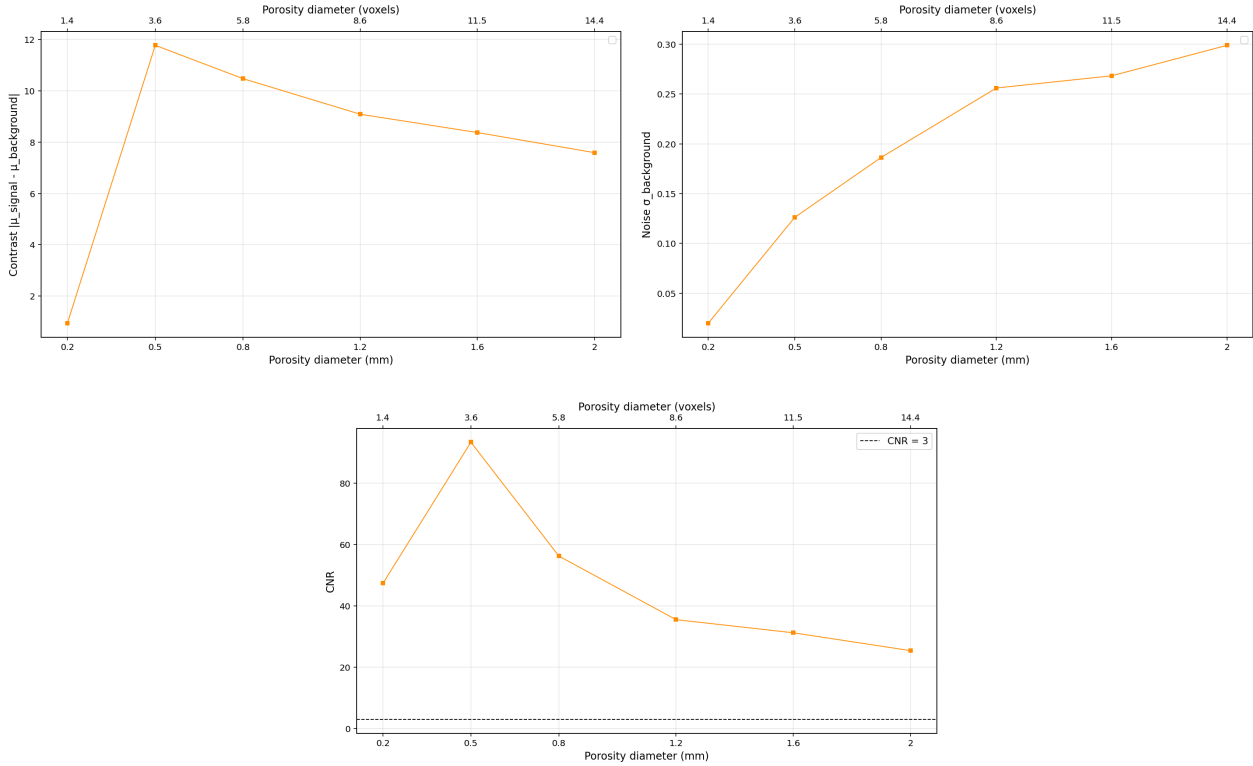


Figure 4.2: Contrast (top left), background noise (top right), and CNR (bottom) as a function of porosity diameter for the X1 trajectory ($|r| = 170$ mm from the isocenter, $x = \pm 85$ mm, 41 projections)

Apart from the smallest porosity (0.2 mm, ≈ 1.4 voxels), which is barely measurable due to its sub-voxel scale, contrast decreases monotonically with porosity diameter. Background noise increases with defect size: as the background ROI scales proportionally with the porosity diameter, it samples a larger area around the defect, capturing more out-of-plane blur and reconstruction artifacts, which raises the measured intensity variance. The CNR consequently exhibits a local maximum around intermediate porosity sizes, reflecting the combined effect of decreasing contrast and increasing background noise.

It should be noted that this ROI scaling introduces a size-dependent bias: larger defects are evaluated against a larger background region, making absolute CNR values not directly comparable across porosity sizes. A fixed-size ROI could provide a more uniform comparison basis but would risk sampling background regions not locally representative of the noise around smaller defects. The proportional design was therefore preferred.

At 0.5 mm (≈ 3.6 voxels), the signal ROI contains very few pixels, making the mean intensity estimate sensitive to discretization effects. The same limitation applies to the smallest porosity at 0.2 mm (≈ 1.4 voxels), where the ROI is barely larger than a single voxel. Quantitative results for these two porosity sizes should therefore be interpreted with caution.

These observations suggest that contrast is a more stable and informative metric than CNR in this context, as it directly reflects defect visibility without being strongly affected by background noise variations.

This choice is further supported by the trajectory comparison presented in Section 4.1.3.

4.1.3 Simple trajectories comparison

Figure 4.3 compares the contrast, background noise, and CNR curves for the four simple trajectories under equivalent acquisition conditions (41 projections, $|r| = 170$ mm).

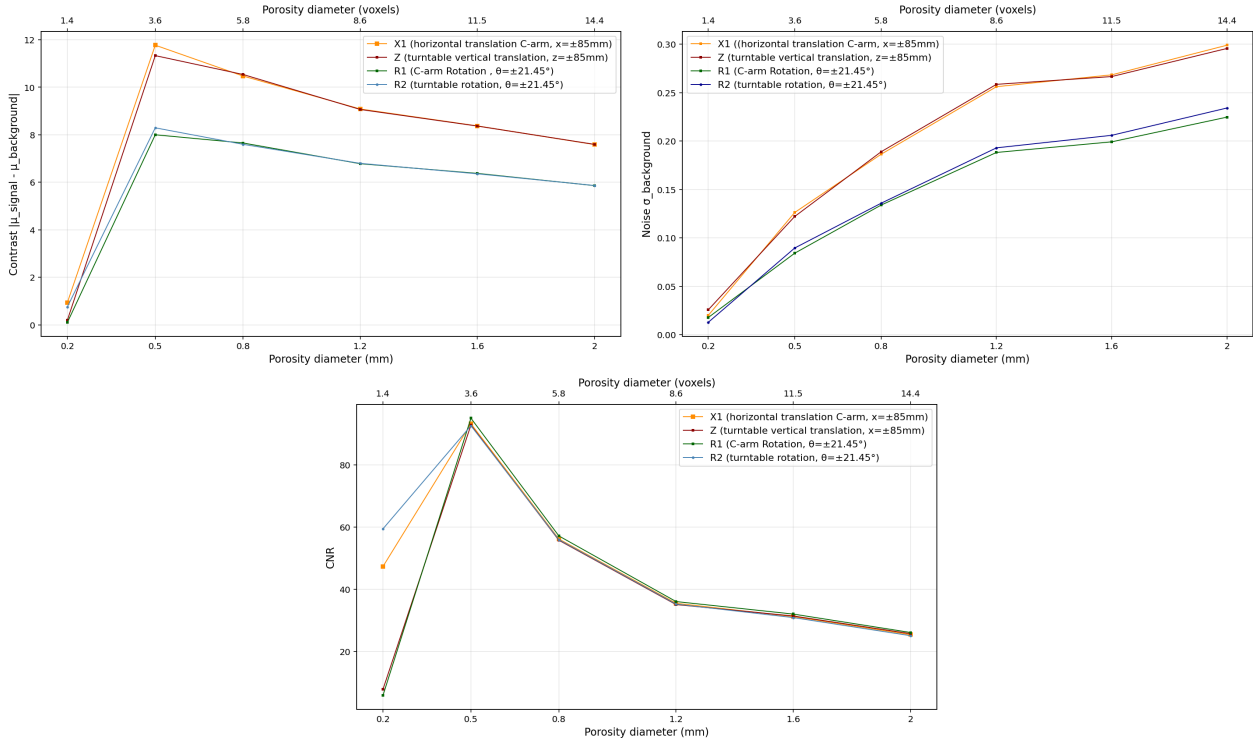


Figure 4.3: Contrast (top left), background noise (top right), and CNR (bottom) as a function of porosity diameter for the four simple trajectories under equivalent acquisition conditions (41 projections, $|r| = 170$ mm, translation range ± 85 mm, angular range $\pm 21.45^\circ$).

Two groups of trajectories can be distinguished. The translation trajectories (X1 and Z) yield consistently higher contrast than the rotation trajectories (R1 and R2) across all porosity sizes, suggesting stronger in-plane signal reinforcement with linear translation geometries. The background noise follows the inverse trend: rotation trajectories produce lower noise levels, which partially compensates for their lower contrast. As a result, the four CNR curves are nearly superimposed — when ranked by CNR, the four trajectories appear essentially equivalent. This illustrates that the choice of metric directly conditions the ranking: contrast and CNR lead to different conclusions, which motivated the selection of contrast as the primary metric in this work.

The X1 and Z curves are nearly identical, as are R1 and R2, suggesting that the direction of motion has little influence on image quality and that the nature of the motion — translation versus rotation — is the dominant factor. Based on these observations, the parametric study in the following section focuses on X1 as a representative of translation trajectories and R2 as a representative of rotation trajectories.

4.1.4 Parametric study

Effect of object offset $|r|$

The object offset $|r|$ controls the distance between the object and the isocenter, which directly affects both the projection magnification and the maximum admissible trajectory range, as derived in Section 3.2.

The normalized offset $\rho = |r|/R$, where $R = 350$ mm is the turntable radius, is introduced to compare results across configurations, with values $\rho \in \{0.29, 0.49, 0.57, 0.71, 0.86\}$ corresponding to $|r| \in \{100, 170, 200, 250, 300\}$ mm.

Since the acquisition geometry is symmetric with respect to the X and Z directions and X1 and Z produced nearly identical contrast curves in Section 4.1.3, only X1 is studied here as a representative of translation trajectories. Similarly, only R2 is studied for rotation trajectories, as R1 and R2 are geometrically symmetric and were shown to behave equivalently in Section 4.1.3. Results for the remaining trajectories are provided in Appendix D.

Figure 4.4 shows the effect of ρ on contrast and CNR for the X1 trajectory.

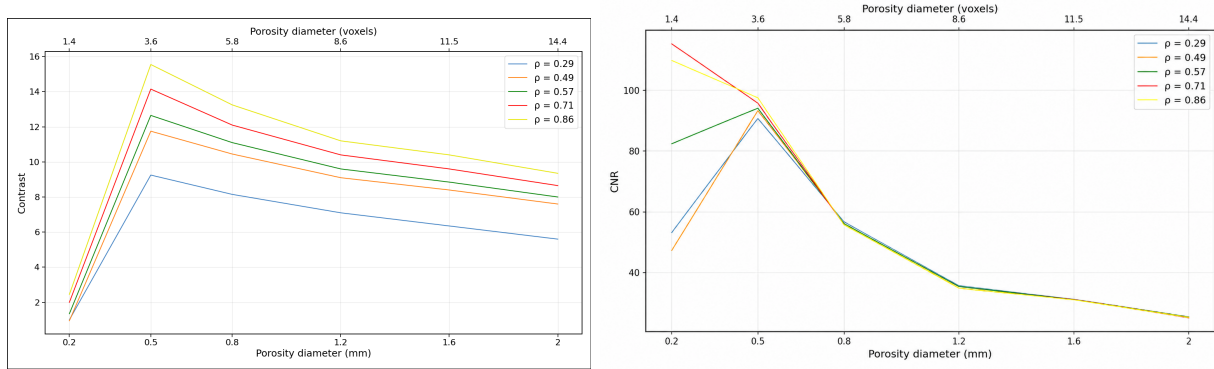


Figure 4.4: Effect of normalized object offset $\rho = |r|/R$ on contrast (left) and CNR (right) for the X1 trajectory (translation range ± 85 mm, 41 projections).

Contrast increases monotonically with ρ . This can be attributed to the increase in projection magnification as the object moves further from the isocenter: the porosity signal is spread over more detector pixels, which reduces partial volume effects and improves defect visibility. The CNR curves follow the same general trend and are not discussed further, as contrast is used as the primary metric throughout this study (Section 4.1.2).

Figure 4.5 shows the same analysis for the R2 trajectory.

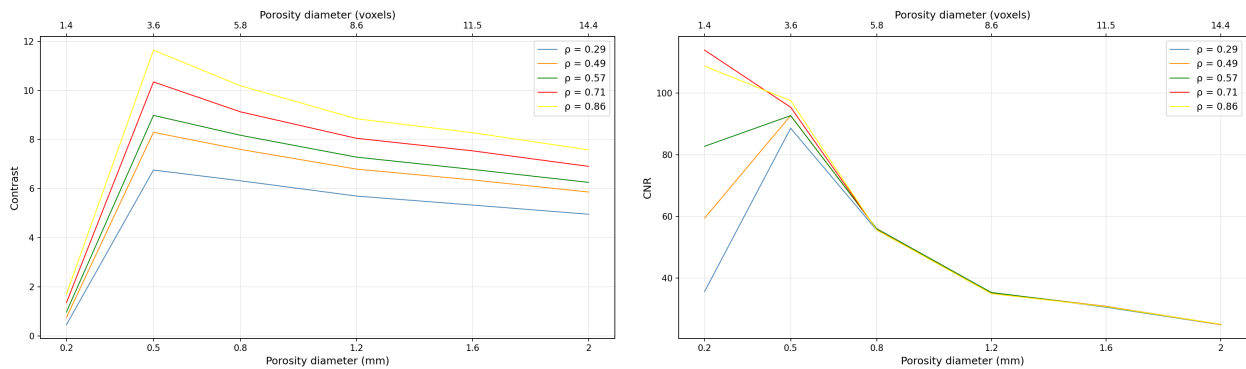


Figure 4.5: Effect of normalized object offset $\rho = |r|/R$ on contrast (left) and CNR (right) for the R2 trajectory (angular range $\pm 21.45^\circ$, 41 projections).

The same trend is observed for R2: contrast increases monotonically with ρ , consistently with the behavior observed for X1. This suggests that the influence of object offset is not strongly dependent on the trajectory type. However, increasing ρ reduces the maximum admissible trajectory range (Section 3.2): this gain in contrast therefore comes at the expense of a reduced angular or translation coverage, which must be considered when selecting acquisition parameters.

Effect of angular range

Figure 4.6 shows the influence of the total angular range on contrast and CNR for the R2 trajectory at fixed $|r| = 170$ mm.

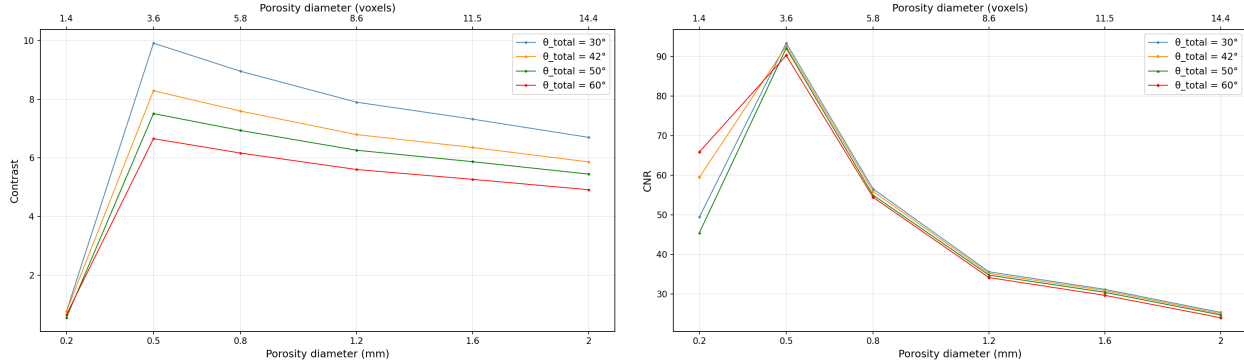


Figure 4.6: Effect of total angular range on contrast (left) and CNR (right) for the R2 trajectory ($|r| = 170$ mm, 41 projections).

Contrast decreases as the total angular range increases. A wider angular range distributes the reconstruction energy over a larger volume, reducing the intensity reinforcement at the focal plane and therefore lowering in-plane contrast.

Effect of translation range

Figure 4.7 shows the influence of the total translation range on contrast and CNR for the X1 trajectory at fixed $|r| = 170$ mm.

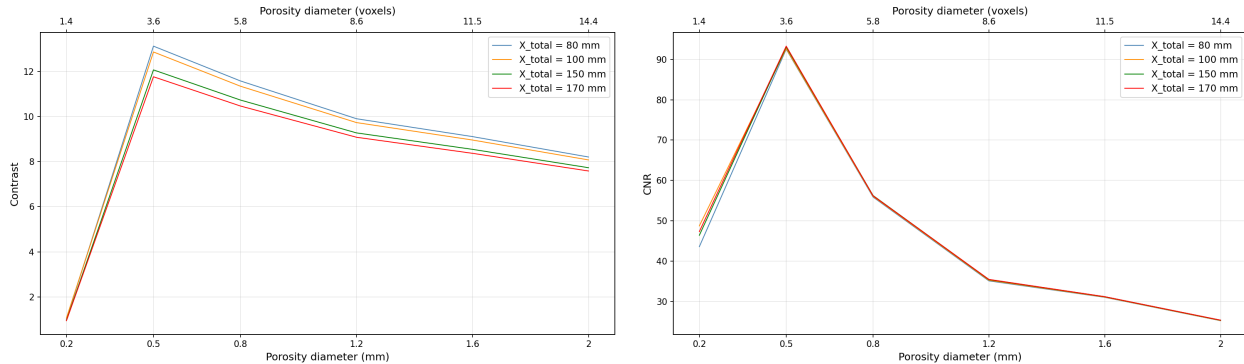


Figure 4.7: Effect of total translation range on contrast (left) and CNR (right) for the X1 trajectory ($|r| = 170$ mm, 41 projections).

Contrast decreases slightly as the total translation range increases, with shorter travel ranges yielding higher contrast values. This effect remains modest across all porosity sizes, suggesting that the translation range has a limited influence on defect visibility for the X1 trajectory.

4.1.5 Spatial resolution and depth blur

The depth resolution analysis is performed on the 2 mm porosity phantom, which provides sufficient signal for reliable FWHM estimation. Figure 4.8 shows the intensity profiles along the Z-axis through the porosity center for the four simple trajectories.

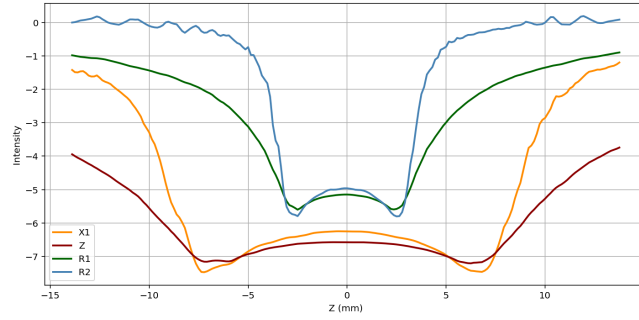


Figure 4.8: Intensity profiles along the Z-axis through the porosity center for the four simple trajectories (2 mm porosity, $|r| = 170$ mm, 41 projections). The central plateau corresponds to the in-focus porosity signal, while the extended wings on either side reflect out-of-plane blur. A narrower and deeper profile indicates better depth localization.

The profiles reveal a clear difference between translation and rotation trajectories. X1 and Z exhibit broad, shallow profiles with out-of-plane blur extending over approximately ± 10 mm, while R1 and R2 produce narrower and deeper responses, indicating better depth localization. Among the four trajectories, R2 produces the narrowest and most localized response.

Table 4.1 summarizes the quantitative resolution metrics for each trajectory.

Table 4.1: Spatial resolution metrics and composite score for the four simple trajectories (2 mm porosity, 41 projections).

Trajectory	Ratio _X	Ratio _Y	Ratio _Z	Slope	Contrast	Score
X1	0.97	0.98	6.12	1.047	7.588	1.299
Z	0.97	0.98	6.14	0.810	7.596	1.002
R1	0.97	0.99	2.42	0.794	5.859	1.923
R2	0.97	0.99	2.46	1.060	5.864	2.527

The in-plane resolution is nearly identical across all trajectories ($\text{Ratio}_X \approx \text{Ratio}_Y \approx 0.97\text{--}0.99$), confirming that in-plane resolution is well preserved in tomosynthesis and is not significantly affected by the trajectory type. The depth resolution, quantified by Ratio_Z , differs significantly: Ratio_Z is approximately 6 for translation trajectories and 2.4 for rotation trajectories. Given the 2 mm physical diameter of the porosity, these values correspond to an effective reconstructed depth extent of about 12 mm for translation trajectories and about 5 mm for rotation trajectories, illustrating the limited depth resolution inherent to limited-angle tomosynthesis.

The slope metric further differentiates the two rotation trajectories: despite similar Ratio_Z values, R2 produces sharper intensity transitions at the porosity boundaries (Slope = 1.060 vs. 0.794 for R1), consistent with the narrower and steeper profile visible in Figure 4.8. Combined with its in-plane contrast value, R2 achieves the highest composite score (2.527), making it the highest-scoring simple trajectory according to the proposed metric.

Although the acquisition geometry is expected to be symmetric with respect to the X and Z directions, small but systematic differences are observed in the profile shapes of the paired trajectories (X1 versus Z and R2 versus R1), as visible in Figure 4.8. These differences mainly affect the transition regions at the porosity boundaries — highlighted by the different transition slopes observed in the profile wings — and are therefore reflected in the Slope metric and composite score, while the contrast and Ratio_Z values remain nearly identical within each pair.

Several hypotheses were investigated to identify the origin of this difference, including the projection orientation, the moving component (object versus C-arm), and discretization effects related to detector pixel size (see Appendix E for more details). However, none of these investigations eliminated the ob-

served difference. The origin of this residual difference therefore remains unclear and may warrant further investigation. It is reported here for completeness, as it influences the composite score ranking despite the trajectories being theoretically equivalent under the symmetry of the system.

Figure 4.9 summarizes the composite scores for the four simple trajectories.

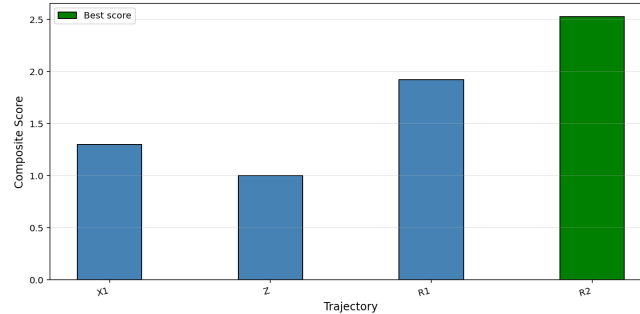


Figure 4.9: Composite score for the four simple trajectories (41 projections). R2 achieves the highest score, reflecting its superior depth localization combined with adequate contrast.

Figure 4.10 provides a visual confirmation of these results through depth views of the reconstructed volumes for the four trajectories.

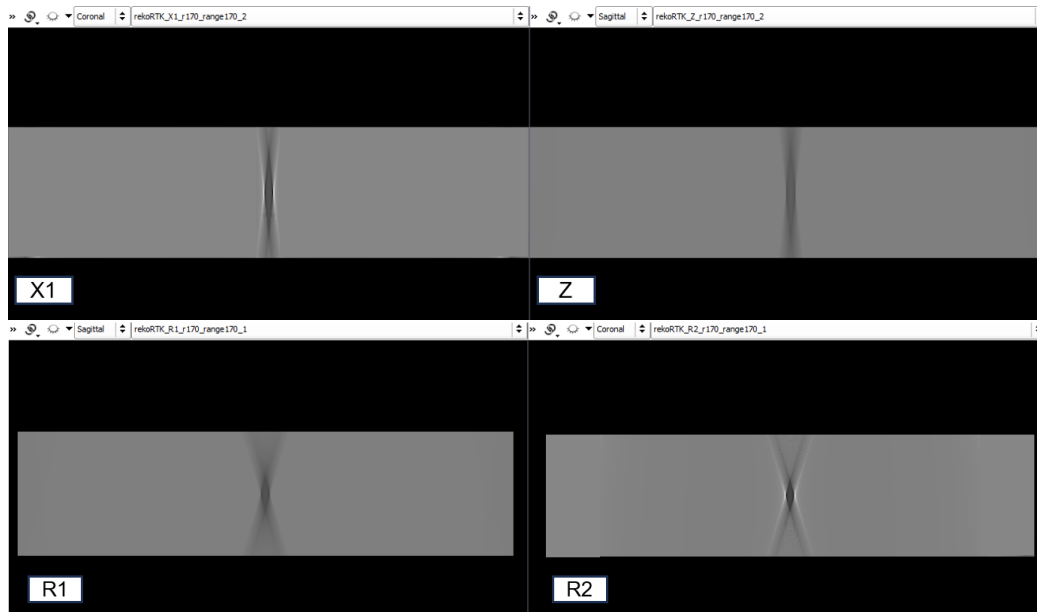


Figure 4.10: Depth views of the reconstructed volumes for the four simple trajectories (2 mm porosity). For each trajectory, the view plane was selected to reveal the most representative depth blur pattern associated with the acquisition motion. Coronal views are shown for X1 and R2, while sagittal views are shown for Z and R1; these correspond to geometrically equivalent cross-sectional planes through the plate thickness.

The depth views confirm that translation trajectories produce strongly directional blur patterns, while rotation trajectories generate a more localized and symmetric response. They are also consistent with the profile analysis, showing similar overall blur patterns within each trajectory pair (X1/Z and R1/R2) while revealing subtle differences near the porosity boundaries that are reflected in the Slope metric.

4.1.6 Radon space reachability

Table 4.2 summarizes the intersection volumes V_{cov} computed for the X1 and R1 trajectories, using the circumscribed sphere of the phantom plate as $\mathcal{V}(\Omega)$ (radius $r \approx 70.9$ mm, volume $V(\Omega) = 1\,492\,556$ mm³).

Table 4.2: Radon space reachability for the X1 and R1 trajectories. $V(\Omega)$ is the volume of the object support sphere, V_{cov} is the intersection volume, and the coverage ratio is $V_{\text{cov}}/V(\Omega)$.

Trajectory	$V(\Omega)$ (mm ³)	V_{cov} (mm ³)	Coverage (%)
X1 (± 85 mm)	1 492 556	746 565	$\approx 50.0\%$
R1 ($\pm 21.45^\circ$)	1 492 556	825 296	$\approx 55.3\%$

For the X1 trajectory, the two extreme spheres $S_{x_{\min}}$ and $S_{x_{\max}}$ each intersect the object support for a volume of 692 728 mm³, but their union covers 746 565 mm³ — less than the sum of the two individual intersections, due to the overlap between $S_{x_{\min}}$ and $S_{x_{\max}}$.

For the R1 trajectory, the partial torus $\mathcal{V}(\Gamma_{R1})$ intersects the object support for 825 296 mm³, corresponding to a coverage of 55.3%.

R1 achieves a higher coverage than X1 (55.3% vs 50.0%), indicating that the arc rotation trajectory reaches a greater portion of the Radon data associated with the object.

Consistency with image-based results

This ordering is consistent with the image-based results presented in Section 4.1.5: R1 produced better depth resolution than X1, as reflected by its lower Ratio_Z value (2.42 vs. 6.12). The trajectory with the higher reachability (R1) also achieves better depth localization in the reconstructed volume, which suggests that the reachability criterion captures relevant geometric information about trajectory quality.

This agreement supports the use of reachability as a geometry-based first-order indicator for trajectory comparison. However, as discussed in Section 4.1.6, the criterion has limitations that prevent it from being used as a standalone predictor of image quality.

Discussion

The agreement between reachability and the composite score is encouraging, but several aspects of the criterion deserve closer examination.

- *Reachability and sampling are two different problems.*

The reachability criterion characterizes what is theoretically accessible in Radon space for a given trajectory. However, the actual image quality also depends on how densely the reachable region is sampled. Two trajectories with the same reachability could produce very different images if one samples the reachable region uniformly while the other concentrates projections in a small portion of it. Reachability sets an upper bound on what can be reconstructed; sampling determines how well that bound is exploited in practice.

This distinction also helps frame the optimization problem addressed in this work: combining multiple axes increases reachability, while optimizing the projection distribution improves sampling within the reachable region. These are two complementary but independent levers.

- *Dependence on the visible region around the object.*

The criterion is computed with respect to $\mathcal{V}(\Omega)$, approximated here by the circumscribed sphere of the phantom plate. This sphere represents the space around the object that remains visible on the detector, rather than the object itself. It introduces an implicit dependence on both the object size and the detector

field of view. One could also ask what happens when the object is not fully visible at every projection angle — in that case, parts of $\mathcal{V}(\Omega)$ become unreachable regardless of the trajectory, which is not captured by the current formulation.

- *The reachability criterion is geometry-driven but not fully trajectory-independent.*

At first sight, the criterion appears to be purely geometric and independent of the simulation parameters. This is its main advantage over image-based metrics, which are tied to the specific phantom and would require re-simulation to generalize. However, $\mathcal{V}(\Omega)$ depends on the visible space around the object, which depends on the object geometry. Claiming that the criterion is purely theoretical would therefore be an overstatement.

- *Representativeness of the circumscribed sphere.*

The phantom plate ($100 \times 100 \times 10$ mm) is anisotropic — much thinner in one direction than in the other two. Its circumscribed sphere therefore contains a large volume of space that does not correspond to any part of the plate. Part of the computed intersection volume may correspond to Radon data that would not contribute to the reconstruction of the plate itself, which could make the criterion overly optimistic for flat objects.

The reachability criterion provides a useful, geometry-based first-order indicator for comparing acquisition trajectories. Its agreement with the image-based composite score in this work suggests that it captures relevant information about trajectory quality. However, it should be interpreted with caution: it does not account for sampling density, it depends implicitly on the visible space around the object, and the approximation by a circumscribed sphere may introduce inaccuracies for highly anisotropic objects. Used alongside image-based metrics, it offers a complementary perspective that is less tied to the specific simulation parameters.

Overall, the results obtained for the simple trajectories highlight a tension between in-plane contrast and depth localization performance. Translation trajectories (X1 and Z) provide higher contrast but suffer from strong directional depth blur, whereas rotation trajectories (R1 and R2) achieve better depth localization and more symmetric blur patterns at the expense of lower contrast. Among the four simple trajectories, R2 achieved the highest composite score, suggesting the best balance between contrast and depth resolution. These observations motivate the investigation of combined trajectories in Section 4.2, with the objective of exploring whether combining multiple acquisition motions can improve the compromise between contrast and depth localization.

4.2 Combined trajectories

This section presents the results obtained for the combined trajectories described in Section 3.5.2. The analysis is organized into two parts. First, combinations of the X1 and Z translations are investigated to evaluate the effect of two-dimensional detector sampling patterns on image quality (Section 4.2.1). Second, sequential combinations involving rotational axes are studied to assess whether adding a rotational component to a translation trajectory can improve the compromise between contrast and depth localization identified in Section 4.1.5

4.2.1 X1 and Z combinations

Simultaneous X1-Z translation: Diagonal XZ

The diagonal XZ configuration, described in Section 3.5.2, was evaluated in two opposite orientations to verify the symmetry of the acquisition geometry. Figure 4.11 compares the contrast curves obtained for both orientations, alongside the simple X1 and Z trajectories for reference.

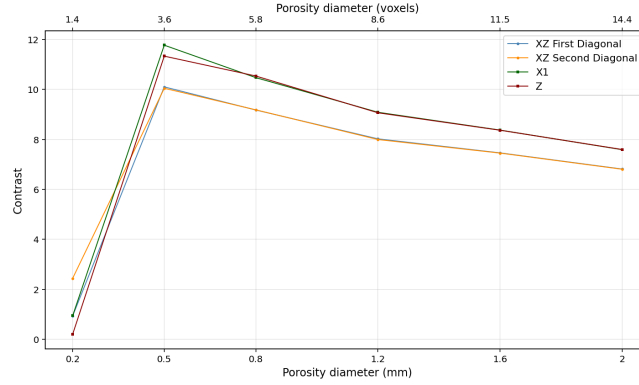


Figure 4.11: Contrast as a function of porosity diameter for the two diagonal XZ trajectories compared to the simple X1 and Z trajectories (41 projections, translation range ± 85 mm).

The two diagonal configurations produce nearly identical contrast curves across all porosity sizes, confirming the expected symmetry of the acquisition geometry. Both diagonal trajectories remain below the contrast levels obtained for the simple X1 and Z trajectories, suggesting that simultaneously combining X and Z motions distributes the projection sampling over two directions, reducing the intensity reinforcement along a single axis and therefore lowering the in-plane contrast under these acquisition conditions.

Effect of vertical offset

The projection sampling patterns investigated in this subsection are illustrated in Figure 3.12. The effect of vertical offset is first assessed using the X1 trajectory at three positions: $Z = 0$ mm, $Z = 40$ mm, and $Z = 85$ mm (160 projections, translation range ± 85 mm). Figure 4.12 shows the contrast curves and depth profiles along the Z-axis obtained for each configuration.

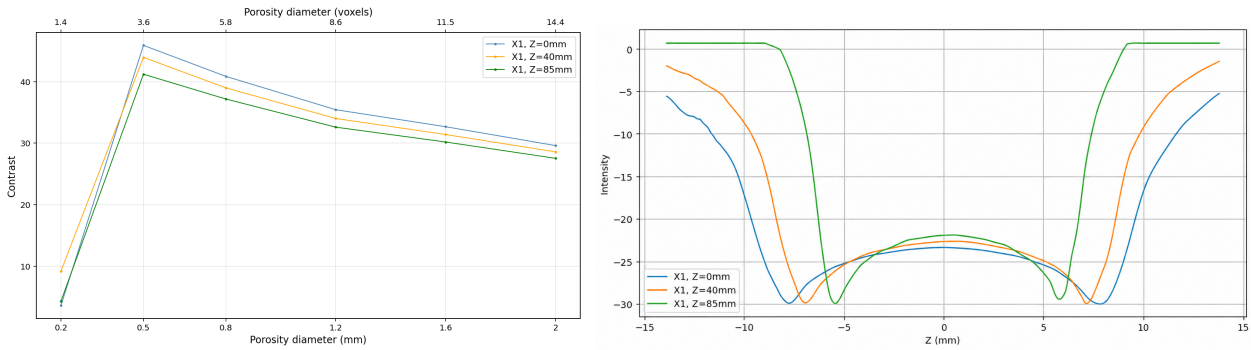


Figure 4.12: Effect of vertical offset on contrast (left) and depth profile along the Z-axis (right) for the X1 trajectory at $Z = 0, 40, \text{ and } 85$ mm (160 projections, translation range ± 85 mm, 2 mm porosity).

The contrast curves remain relatively close across the three vertical offsets, although a slight decrease in contrast is observed as the offset increases. The depth profiles, however, reveal a clear trend: as the vertical offset increases, the profile narrows progressively, as reflected by the decreasing Ratio_Z values (6.10, 5.76, and 4.54 for $Z = 0, 40, \text{ and } 85$ mm respectively). Projections concentrated at the detector center ($Z = 0$ mm) produce a broader depth response, while projections distributed towards the detector periphery ($Z = 85$ mm) result in a more localized depth response.

Figure 4.13 provides a visual confirmation of this observation.

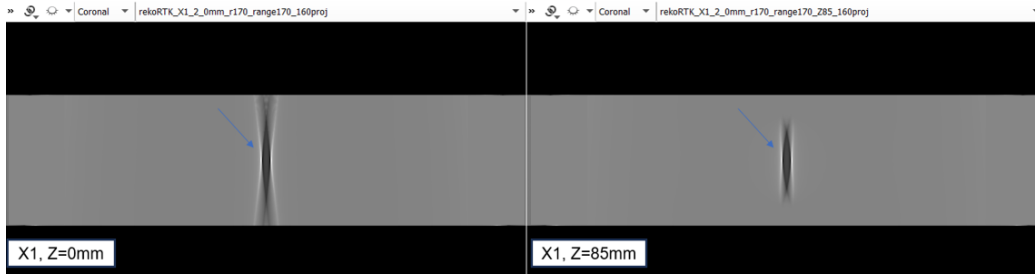


Figure 4.13: Coronal depth views for $Z = 0$ mm (left) and $Z = 85$ mm (right). The depth blur extent is visibly reduced at $Z = 85$ mm, leading to a more localized porosity response.

These results suggest that distributing projections away from the detector center improves depth localization, at the expense of a reduced contrast. This observation motivates the investigation of two dimensional sampling patterns, where projections are distributed across different regions of the detector plane.

Two-dimensional sampling patterns

The five two-dimensional sampling patterns described in Section 3.5.2 and illustrated in Figure 3.12 are compared here. Figure 4.14 presents the contrast curves and depth profiles along the Z -axis for each configuration, alongside the simple $X1$ trajectory as reference.

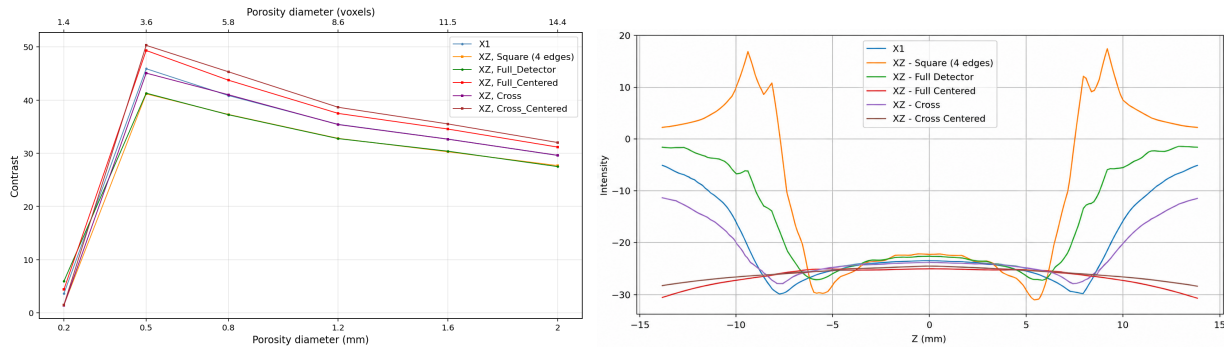


Figure 4.14: Contrast as a function of porosity diameter (left) and depth profiles along the Z -axis (right) for the five two-dimensional sampling patterns and the simple $X1$ trajectory (160 projections, 2 mm porosity).

A clear opposition emerges between centered and edge-distributed configurations. Patterns concentrated near the detector center (Full Centered and Cross Centered) yield the highest contrast values but produce broad depth profiles, with Ratio_Z values close to those of the simple $X1$ trajectory (5.57–5.68). In contrast, patterns extending towards the detector edges (Full Detector and Square (4 edges)) reduce contrast but significantly improve depth localization, with Ratio_Z values dropping to approximately 4.27–4.29. The Cross pattern occupies an intermediate position, with depth resolution close to the simple $X1$ trajectory despite combining both horizontal and vertical sampling directions.

Notably, the Square (4 edges) pattern produces strongly non-monotonic depth profiles with pronounced intensity peaks at the porosity boundaries, as visible in Figure 4.14. This behavior is likely associated with the concentration of projections at the detector periphery, which appears to introduce reconstruction artifacts at the porosity edges.

Figure 4.15 summarizes the composite scores for all configurations.

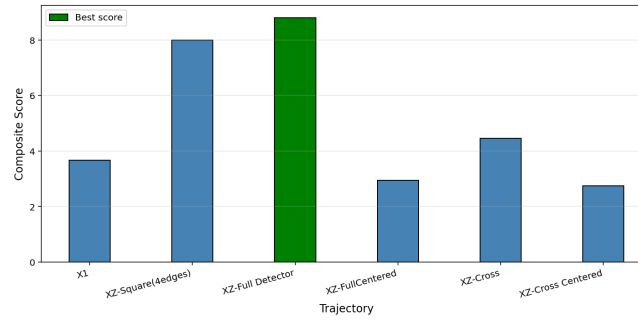


Figure 4.15: Composite scores for the five two-dimensional sampling patterns and the simple X1 trajectory (160 projections). XZ-Full Detector achieves the highest score, reflecting the best balance between contrast and depth localization.

The XZ Full Detector configuration achieves the highest composite score (8.811), followed by Square (4 edges) (7.991). The relatively small difference between these two configurations suggests that projections acquired near the detector boundaries contribute substantially to the observed improvement in depth localization. Although its contrast is slightly lower than that of the centered configurations, the XZ Full Detector pattern benefits from good depth localization and sharp profile transitions, which together produce the best overall score. The centered configurations (Full Centered and Cross Centered), despite their higher contrast, are penalized by their poor depth resolution, resulting in the lowest composite scores.

Figure 4.16 provides a visual confirmation of these results through coronal depth views of the reconstructed volumes.

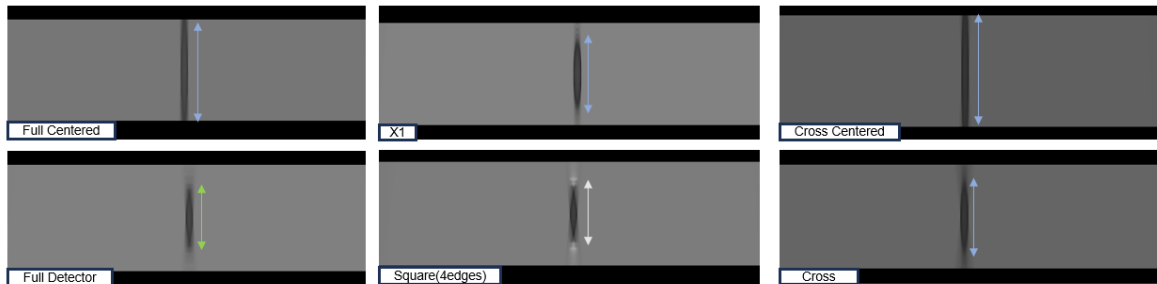


Figure 4.16: Coronal depth views of the reconstructed volumes for the five two-dimensional sampling patterns and the simple X1 trajectory (2 mm porosity). Top row, from left to right: Full Centered, X1, Cross Centered. Bottom row: Full Detector, Square (4 edges), Cross. The arrows indicate the depth blur extent for each configuration. Configurations with edge-distributed projections (Full Detector, Square (4 edges)) show a more localized depth response (shorter arrows), while center-concentrated patterns (Full Centered, Cross Centered) show a more extended depth blur (longer arrows).

These results confirm that the spatial distribution of projections across the detector plane has a significant influence on the balance between contrast and depth localization. Distributing projections towards the detector edges improves depth resolution at the expense of contrast, while concentrating them near the center has the opposite effect. Among the configurations investigated, XZ Full Detector achieves the best overall performance according to the composite score and is therefore retained as the best XZ combination.

4.2.2 Sequential translation–rotation combinations

The five sequential combinations involving rotational trajectories described in Section 3.5.2 are evaluated here. These combinations were evaluated using the acquisition parameters described in Section 3.5.2,

with reduced translation and angular ranges to concentrate the projection sampling around the detector center, consistent with the findings of Section 4.2.1. Figure 4.17 presents the contrast curves and depth profiles along the Z-axis for each configuration.

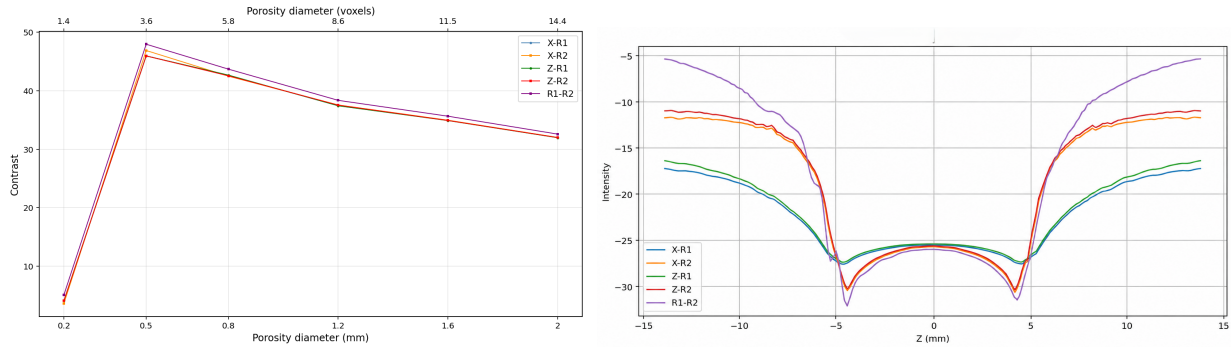


Figure 4.17: Contrast as a function of porosity diameter (left) and depth profiles along the Z-axis (right) for the five sequential combinations involving rotational trajectories (200 projections, 2 mm porosity).

All five combinations yield contrast values close to those of the simple translation trajectories, confirming that adding a rotational component does not significantly reduce in-plane defect visibility. The depth profiles, however, show a marked improvement compared to pure translation trajectories: Ratio_Z values drop to approximately 3.5 for all combinations, compared to 6.1 for the simple X1 trajectory, indicating a substantial gain in depth localization.

An interesting feature is observed in the depth profiles of X-R2 and Z-R2: the profiles exhibit two distinct components — a narrow central minimum and more extended surrounding wings. This observation suggests that the rotational component contributes primarily to the narrow central response, whereas the translational component contributes to the more extended depth distribution. This two-component structure is not observed for R1-R2, whose profile is more uniform and symmetric, consistent with both components being rotational.

Figure 4.18 summarizes the composite scores for all configurations.

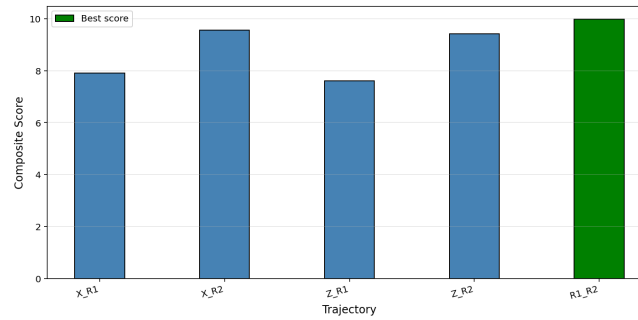


Figure 4.18: Composite scores for the five sequential combinations involving rotational trajectories (200 projections). R1-R2 achieves the highest score, reflecting its superior depth localization combined with adequate contrast.

R1-R2 achieves the highest composite score (9.976), followed closely by X-R2 (9.559) and Z-R2 (9.416). The combinations involving R2 consistently are better than those involving R1 alone, reflecting the higher Slope values associated with R2 as observed in Section 4.1.5. X-R1 and Z-R1 produce lower scores despite similar Ratio_Z values, confirming that depth profile sharpness — captured by the Slope metric — plays a decisive role in the composite score ranking.

Figure 4.19 provides a visual confirmation of these results through coronal depth views of the reconstructed volumes.

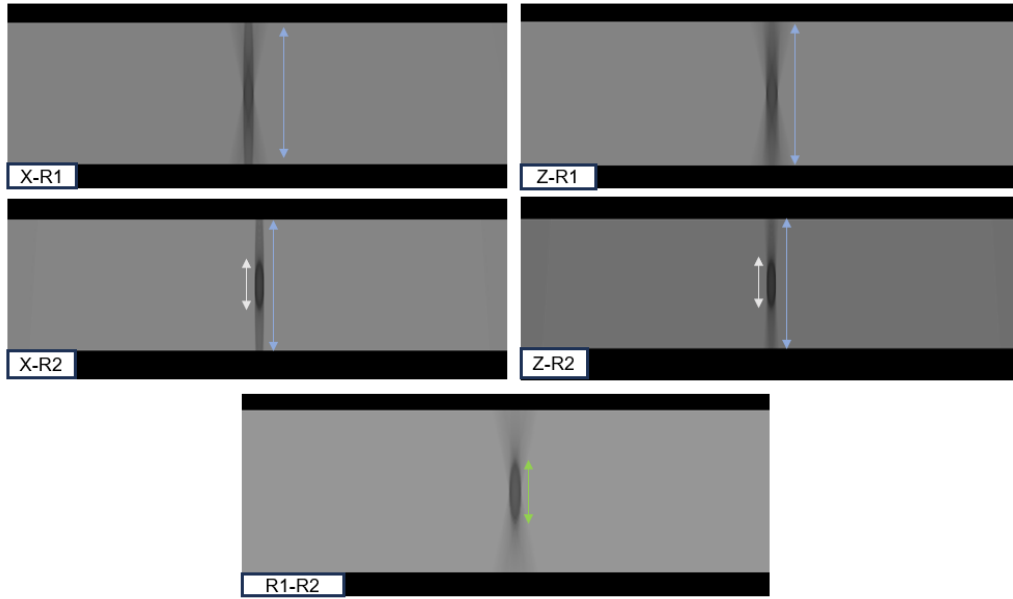


Figure 4.19: Coronal depth views of the reconstructed volumes for the five sequential combinations involving rotational trajectories (2 mm porosity). The arrows indicate the depth blur extent for each configuration. For X-R2 and Z-R2, two arrow lengths are shown: the shorter arrow indicates the narrow central component that may be associated with the rotational acquisition, while the longer arrow reflects the broader depth distribution likely originating from the translational component. R1-R2 exhibits the most localized and symmetric response overall.

These results confirm that combining translational and rotational trajectories significantly improves depth localization compared to simple translation trajectories, while maintaining comparable contrast levels. Among the configurations investigated, R1-R2 achieves the best overall composite score and is therefore retained as the best sequential combination in this group.

4.2.3 Application to a complex geometry

The reconstructed volumes obtained for the porosity phantom were qualitatively reviewed blindly by a level 2 radiography expert at X-RIS, who identified the R1-R2 configuration as providing the most favorable visual compromise between contrast and depth localization. Following the quantitative evaluation performed on the porosity phantom and the associated qualitative review, the R1-R2 trajectory was further evaluated on a more complex object—a dragon-shaped phantom—alongside X1 and R2 as reference trajectories. Figure 4.20 shows the object used, with dimensions of 68.5 mm $\times$ 155.1 mm $\times$ 124.4 mm.



Figure 4.20: Three-dimensional model of the dragon-shaped object used for qualitative evaluation.

Figure 4.21 presents the central axial slices extracted from the reconstructed volumes for the three trajectories (200 projections each).

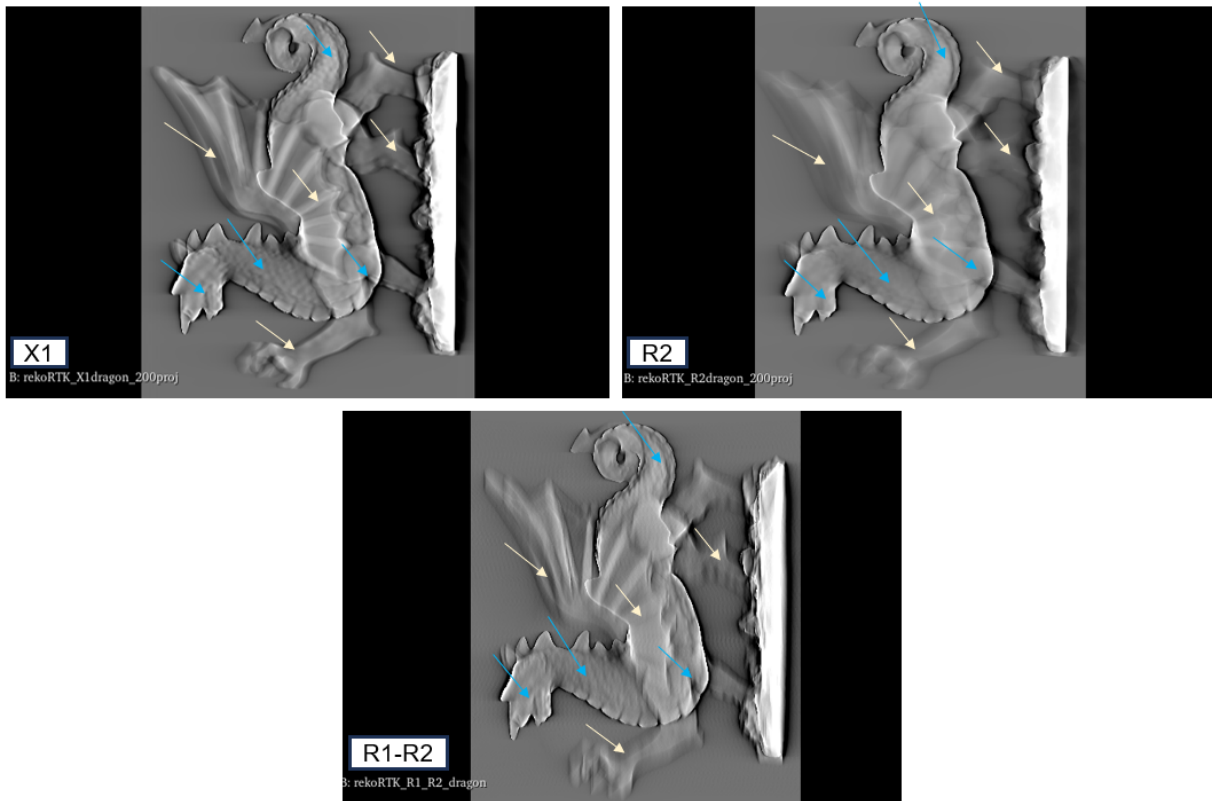


Figure 4.21: Central axial slices of the reconstructed dragon-shaped object for the X1 (top left), R2 (top right), and R1-R2 (bottom) trajectories (200 projections). Blue arrows indicate regions of good in-plane contrast and fine detail visibility, while yellow arrows highlight out-of-plane blur contributions from structures outside the focal plane.

Consistent with the phantom-based results, the X1 trajectory provides higher local contrast and reveals fine surface details such as scales and edge features, as indicated by the blue arrows. However, structures located outside the focal plane remain clearly visible, as shown by the yellow arrows, resulting in stronger out-of-plane blur. The R2 trajectory reduces the visibility of out-of-plane structures, improving depth localization at the expense of reduced contrast. The R1-R2 combination provides an intermediate behaviour, preserving most structural details while limiting the contribution of out-of-plane features.

It should be noted that the most suitable trajectory depends on the inspection objective. When maximizing in-plane contrast and fine detail visibility is the primary goal, translation trajectories such as X1 may be preferable. When reducing out-of-plane blur and improving depth localization is more critical, combined trajectories such as R1-R2 offer a more favorable response.

Although this evaluation was not intended as a systematic study, it confirms the trends observed on the porosity phantom and suggests that the R1-R2 trajectory provides a favorable compromise between contrast and depth localization for more complex geometries.

4.3 General discussion

The results presented in this chapter highlight several consistent trends across all trajectory types investigated.

Among the four simple trajectories (Section 4.1), translation trajectories (X1 and Z) consistently yield higher in-plane contrast than rotation trajectories (R1 and R2), while rotation trajectories provide significantly better depth localization. R2 achieves the highest composite score among simple trajectories, representing the best balance between contrast and depth resolution in this group.

The investigation of two-dimensional XZ sampling patterns (Section 4.2.1) reveals that the spatial distribution of projections across the detector plane has a significant influence on image quality. Projections distributed towards the detector edges improve depth localization at the expense of contrast, while centered configurations show the opposite behavior. Among the XZ configurations, XZ Full achieves the best composite score and is retained as the best combination in this group.

The sequential combinations involving rotational trajectories (Section 4.2.2) demonstrate that adding a rotational component to a translation trajectory substantially improves depth localization while maintaining contrast levels comparable to simple translation trajectories. R1-R2 achieves the highest composite score in this group, benefiting from the depth localization properties of both rotational components.

Overall, these results indicate that the inclusion of a rotational component generally improves depth localization while preserving adequate contrast. Although optimized two-dimensional translation patterns such as XZ Full can also provide substantial improvements, the highest composite scores are obtained for combinations involving rotational trajectories, particularly R1-R2. These results highlight the importance of angular diversity for improving depth localization in limited-angle tomosynthesis.

These findings were further supported by a qualitative review conducted by a level 2 radiography expert at X-RIS, who identified the R1-R2 configuration as providing the most favorable visual compromise between contrast and depth localization among the porosity phantom reconstructions reviewed. A complementary evaluation on a complex dragon-shaped object (Section 4.2.3) confirmed that the trends observed on the porosity phantom remain consistent for more realistic geometries.

Chapter 5

Conclusions and perspectives

5.1 Summary

This work investigated the optimization of tomosynthesis acquisition trajectories for the Dx-Box C5 system through numerical simulation using the gVXR framework. Different trajectory types — simple and combined — were evaluated on a numerical phantom consisting of iron plates with spherical porosities of varying sizes.

Two evaluation criteria were developed as part of this work. The first is a composite score combining in-plane contrast and depth resolution metrics (Ratio_Z and Slope), enabling a quantitative ranking of acquisition trajectories. The second is a geometry-based criterion derived from the coverage of Radon space, which characterizes the theoretical accessibility of reconstruction data for a given trajectory from a geometric perspective. Despite its limitations — notably its dependence on the visible region around the object, the approximation by a circumscribed sphere, and the absence of sampling-density information — this criterion provided results consistent with the image-based composite score for the simple trajectories investigated, supporting its use as a first-order indicator for trajectory comparison.

The results showed that translation trajectories consistently provided higher in-plane contrast, whereas rotation trajectories offered better depth localization, a trend that is consistent with the expected influence of angular diversity on tomosynthesis reconstruction. Among all configurations evaluated, sequential combinations involving rotational components — particularly R1-R2 — achieved the best composite scores, combining adequate contrast with significantly improved depth localization compared to simple translation trajectories.

Based on the quantitative results obtained in this work and the qualitative review conducted by an imaging specialist at X-RIS, the R1-R2 trajectory is identified as the most suitable configuration among the trajectories investigated in this work for implementation on the Dx-Box C5 system. It provides the best composite score among all evaluated configurations, combining adequate in-plane contrast with good depth localization and a symmetric blur response.

It should be noted, however, that the most suitable trajectory depends on the inspection objective. When maximizing in-plane contrast and fine detail visibility is the primary goal, translation trajectories such as X1 may be preferable. When reducing out-of-plane blur and improving depth localization is more critical, combined trajectories such as R1-R2 offer a more favorable result.

It is also important to acknowledge that the XZ Full and R1-R2 configurations were evaluated under different acquisition conditions — 160 and 200 projections respectively, with different translation and angular ranges. A direct quantitative comparison between these two groups is therefore not straightforward, and the recommendation of R1-R2 is therefore based on the consistency of its performance across the different evaluation metrics, together with its qualitative validation by the X-RIS radiography expert.

5.2 Limitations

Several limitations of this work should be acknowledged.

First, all results were obtained through numerical simulation using gVXR, with a monochromatic point source and an ideal detector. Real acquisition conditions involve polychromatic radiation, focal spot blur, detector noise, and mechanical positioning uncertainties, which may affect the quantitative results.

Second, the phantom used in this study — a flat iron plate with spherical porosities — represents a simplified inspection scenario. The qualitative evaluation on the dragon-shaped object (Section 4.2.3) suggests that the observed trends generalize to more complex geometries, but a systematic quantitative validation on realistic industrial parts remains to be performed.

Furthermore, only a single defect type and position were considered. The influence of defect shape, orientation, and location within the object was not investigated and could affect the relative performance of the evaluated trajectories.

Finally, a residual difference in depth profile shape was observed between geometrically equivalent trajectory pairs (X1 versus Z and R1 versus R2), whose origin could not be identified despite several investigations (Appendix E) and then remains unclear.

5.3 Personal contributions

The main personal contributions of this work include the derivation of the maximum admissible translation and angular acquisition ranges as a function of object dimensions and offset from the isocenter for the Dx-Box C5 system, ensuring that the inspected part remains within the detector field of view throughout the acquisition.

A composite score combining contrast and depth localization metrics was developed to enable a quantitative comparison and ranking of acquisition trajectories.

A geometry-based criterion derived from Radon space coverage was also implemented and evaluated for the simple trajectories, providing a complementary perspective on trajectory quality.

Finally, a systematic investigation of the influence of two-dimensional projection sampling patterns on image quality was conducted, revealing a consistent trade-off between contrast and depth localization depending on the projection distribution across the detector plane.

5.4 Perspectives

Several directions for future work can be identified.

The most immediate next step is the experimental validation of the R1-R2 trajectory on the physical Dx-Box C5 system, using real industrial parts. The Dx-Box C5 system was not available during the duration of this internship, which limited the study to numerical simulations. Such experimental validation would allow verification of the simulation-based results under realistic acquisition conditions.

Testing defects of different shapes, orientations, and positions within the object would provide a more complete characterization of trajectory performance and strengthen the generalizability of the conclusions.

Future work could also investigate iterative reconstruction methods such as SART or OS-SART, which may further improve depth localization and artifact reduction in limited-angle tomosynthesis compared to the FBP algorithm used in this work.

The unexplained asymmetry between geometrically symmetric trajectory pairs (X1 versus Z and R1 versus R2) deserves further investigation, as it may reflect a fundamental property of the reconstruction algorithm or the acquisition geometry that has not yet been identified.

Finally, the reachability criterion was evaluated only for the simple trajectories X1 and R1. Extending this analysis to combined trajectories such as R1-R2 and XZ Full would allow a more rigorous geometry-based comparison between trajectory families.

Appendix A

3D Fourier slice theorem and filtered back-projection

This appendix briefly presents the 3D extension of the Fourier slice theorem and the corresponding reconstruction framework. The derivation follows the same logic as the 2D proof in Section 2.3, and is presented here without repeating all intermediate steps.

3D Fourier slice theorem. Let $\mathbf{n}_\xi = (\cos \gamma \sin \vartheta, \sin \gamma \sin \vartheta, \cos \vartheta)^T$ be the unit normal defining a plane orientation in 3D space. The 1D Fourier transform of $p_{\mathbf{n}_\xi}(\xi)$ with respect to ξ , after substituting the definition of the 3D Radon transform and transforming to the fixed coordinate system, gives in full analogy with Equation 2.10 [18]:

$$P_{\mathbf{n}_\xi}(q) = F(q \cos \gamma \sin \vartheta, q \sin \gamma \sin \vartheta, q \cos \vartheta) \quad (\text{A.1})$$

where $F(u, v, w)$ is the 3D Fourier transform of $\mu(x, y, z)$. Each plane integral contributes a radial line in the 3D Fourier domain along direction $\mathbf{n}_\xi$.

3D filtered back-projection. Expressing the inverse 3D Fourier transform in spherical coordinates and applying Equation A.1, the 3D filtered back-projection formula is [18]:

$$\mu(x, y, z) = -\frac{1}{4\pi^2} \int_0^{2\pi} \int_0^\pi \left. \frac{\partial^2 p_{\mathbf{n}_\xi}(\xi)}{\partial \xi^2} \right|_{\xi=\mathbf{r} \cdot \mathbf{n}_\xi} \sin \vartheta \, d\vartheta \, d\gamma \quad (\text{A.2})$$

This formula requires integrating the second derivative of the plane integrals over all plane orientations on the unit sphere. As noted in Section 2.3, it applies only to parallel-beam geometry. For cone-beam acquisitions, Grangeat's approach [19] or the FDK approximation [6] must be used instead.

Grangeat's cone-beam reconstruction framework

Figure A.1 summarizes the reconstruction framework proposed by Grangeat for exact cone-beam tomography. Unlike the parallel-beam framework presented previously, the measured cone-beam projections do not directly correspond to the 3D Radon transform. Grangeat established a relationship between the derivative of the cone-beam projections and the derivative of the 3D Radon transform, allowing an exact inversion of the cone-beam acquisition geometry.

A complete derivation is beyond the scope of this work. However, this framework provides the theoretical basis for exact cone-beam reconstruction methods and highlights the fundamental difference between parallel-beam and cone-beam geometries.

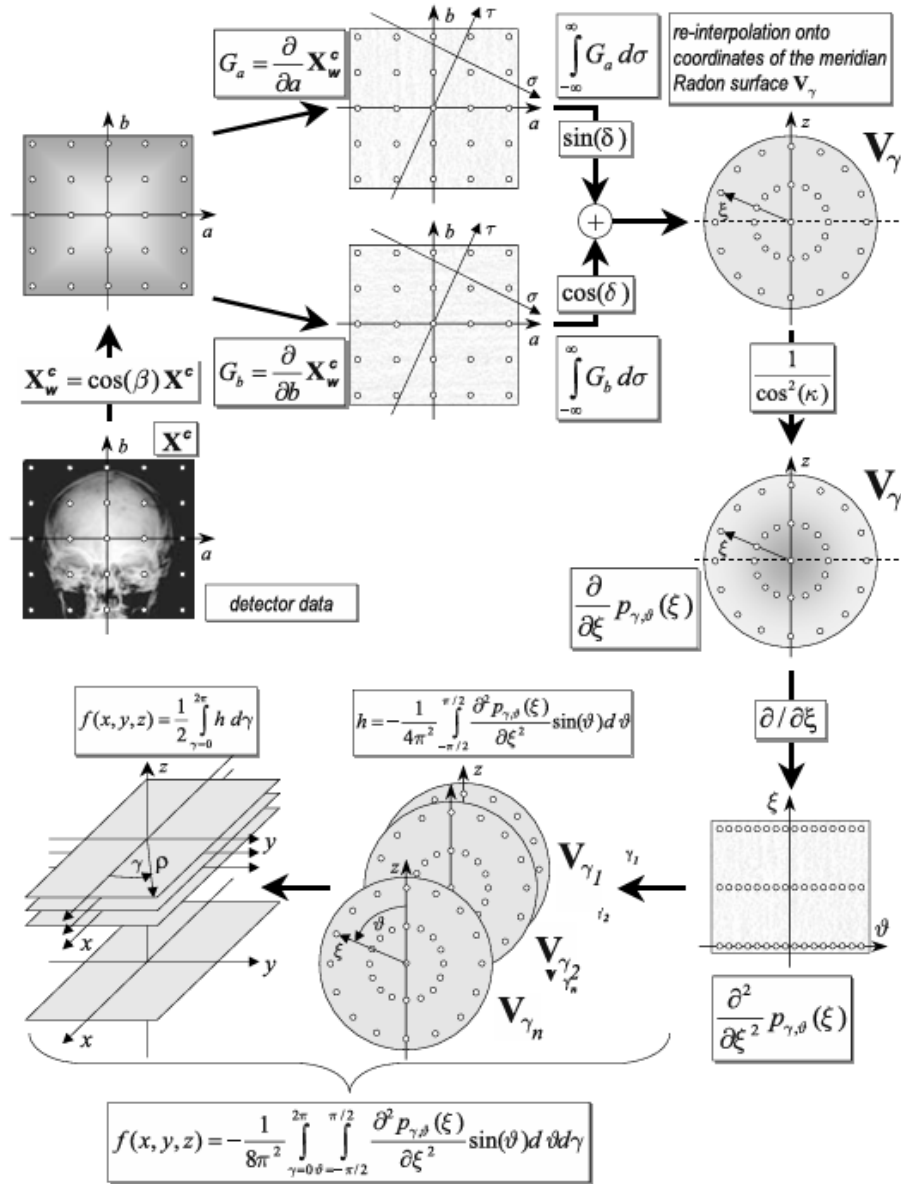


Figure A.1: Summary of Grangeat's exact cone-beam reconstruction framework. Adapted from [18].

Appendix B

Derivation of the maximum translation range x_{max}

This appendix derives the analytical expression for the maximum admissible translation range x_{max} for linear translation trajectories (X1 and Z) on the Dx-Box C5 system.

Geometric setup

In the X1 translation trajectory, the C-arm (source and detector) translates together along the X axis by a displacement x_{pos} , while the object remains fixed. The positions of the key elements are:

- **X-ray source:** $(x_{pos}, +SID, 0)$
- **Detector center:** $(x_{pos}, -(SDD - SID), 0)$
- **Object point:** $(x_p, |r|, 0)$, where $x_p \in [-W/2, W/2]$

Projection onto the detector

A ray from the source through the object point $(x_p, |r|, 0)$ intersects the detector plane at $y_{det} = -(SDD - SID)$. The parametric equation of the ray gives:

$$t = \frac{y_{det} - SID}{|r| - SID} = \frac{-(SDD - SID) - SID}{|r| - SID} \quad (B.1)$$

The position of the projection on the detector, expressed relative to the detector center, is:

$$u_{rel} = (x_p - x_{pos}) \cdot t \quad (B.2)$$

Field of view constraint

The constraint requires that $|u_{rel}| \leq D/2$ for all object points $x_p \in [-W/2, W/2]$. The most critical case occurs at the object edge producing the largest detector displacement:

$$(x_{pos} + W/2) \cdot |t| \leq D/2 \quad (B.3)$$

Maximum translation range

Solving Equation B.3 for equality yields the maximum admissible translation:

$$x_{\max}(|r|) = \frac{D/2}{|t|} - \frac{W}{2} \quad \text{with} \quad |t| = \left| \frac{-(SDD - SID) - SID}{|r| - SID} \right| \quad (\text{B.4})$$

For the Dx-Box C5 parameters ($SDD = 1150$ mm, $SID = 575$ mm, $D = 430$ mm) and a plate of width $W = 100$ mm positioned at $|r| = 170$ mm, this gives $x_{\max} \approx 89$ mm, corresponding to a total translation range of approximately 180 mm. This result is consistent with the experimental observations reported in Section 3.2.

```
1  # -*- coding: utf-8 -*-
2  """
3  Created on Mon May 11 10:33:41 2026
4
5  @author: Liliane
6  """
7
8  import numpy as np
9
10 # Paramètres
11 SDD = 1150
12 SID = 575
13 D = 430 #detector width
14 W = 100 #objet width
15
16 def compute_x_max(r):
17     """
18     Calculate the maximum X-axis displacement of the C-arm so that
19     the object remains within the detector's field of view
20
21     Source      : (x_pos, +SID, 0)
22     Detector    : centré en (x_pos, -(SDD-SID), 0)
23     Object      : points (xp, r, 0), r négatif
24     """
25     det_y = -(SDD - SID)
26     t = abs((det_y - SID) / (r - SID))
27     x_max = D/2 / t - W/2
28     return x_max
29
30 # Test for differents values of r
31 #r_values = [-100, -170, -200, -250, -300]
32 r_values = [-170] #position of the object from isocenter
33
34 print(f"{'r (mm)':<12} {'x_max (mm)':<15} {'Range total (mm)'}")
35 print("-" * 45)
36 for r in r_values:
37     x_max = compute_x_max(r)
38     print(f"{'r':<12} {'x_max':<15.2f} {'2*x_max:~.1f}'")
39
```

Figure B.1: Python script to compute the **maximum translation range**, for the object to remain on the detector field of view, knowing the detector size, the detector object and the object offset from the isocenter.

```
1  # -*- coding: utf-8 -*-
2  """
3  Created on Thu Feb 12 14:06:59 2026
4
5  @author: Liliane
6  """
7  import numpy as np
8  from scipy.optimize import brentq
9
10 # Parameters
11 SDD = 1150
12 SID = 575
13 D = 430 #detector width
14 W = 100 #objet width
15 r = 300 #position of the object from isocenter (absolute value)
16 def max_abs_u(theta):
17     """maximal Position in the detector for an angle theta"""
18     xps = np.linspace(-W/2, W/2, 2001)
19     xc = r * np.sin(theta)
20     yc = r * np.cos(theta)
21     x = xc + xps * np.cos(theta)
22     y = yc - xps * np.sin(theta)
23     u = x * SDD / (y + SID)
24     return np.max(np.abs(u))
25
26 def condition(theta):
27     return max_abs_u(theta) - D/2
28
29 # Calcul theta_max
30 theta_max_rad = brentq(condition, np.deg2rad(0.1), np.deg2rad(89))
31 theta_max_deg = np.rad2deg(theta_max_rad)
32
33 print(f"θ_max = ±{theta_max_deg:.2f}°")
34 print(f"Plage total = {2*theta_max_deg:.1f}°")
35
36
```

Figure B.2: Python script to compute the **maximum angular range**, for the object to remain on the detector field of view, knowing the detector size, the detector object and the object offset from the isocenter.

Appendix C

Radon space reachable regions for X1 and R1 trajectories

This appendix details the geometry of the reachable Radon regions $\mathcal{V}(\Gamma)$ for the linear translation trajectory X1 and the arc rotation trajectory R1, as introduced in Section 3.6.2.

X1 trajectory: the two extreme spheres cover all intermediate spheres

For the X1 trajectory, the source moves along the X-axis from $x_{\min}$ to $x_{\max}$ while the object center remains fixed at $O = (0, h, 0)$. At each source position $s_x = (x, 0, 0)$, a sphere S_x is defined with s_x and O as diameter endpoints, so that its center is $C_x = (s_x + O)/2$ and its radius is $R_x = \|s_x - O\|/2$. The diameter of S_x is defined by the source-to-object distance:

$$\text{SOD}(x) = \|s_x - O\| = \sqrt{x^2 + \text{SOD}_0^2} \quad (\text{C.1})$$

where $\text{SOD}_0 = h$ is the source-to-object distance at $x = 0$.

Proof of containment. We show that for any $x \in [x_{\min}, x_{\max}]$, every point belonging to S_x also belongs to $S_{x_{\min}}$ or $S_{x_{\max}}$. Consider a point $P = (X, Y, Z)$. P belongs to S_x if and only if:

$$\|P - C_x\|^2 \leq R_x^2 \quad (\text{C.2})$$

After expansion, this condition reduces to:

$$X^2 + Y^2 + Z^2 - xX - hY \leq 0 \quad (\text{C.3})$$

For a fixed point P , the only term depending on x is $-xX$. If $X > 0$, the condition is most easily satisfied for the largest positive x , i.e., $x = x_{\max}$. If $X < 0$, it is most easily satisfied for $x = x_{\min}$. Consequently, any point satisfying Equation C.3 for some intermediate x also satisfies it for $x_{\min}$ or $x_{\max}$. Therefore:

$$\mathcal{V}(\Gamma_{X1}) = S_{x_{\min}} \cup S_{x_{\max}} \quad (\text{C.4})$$

This property simplifies the coverage computation for X1 to the intersection of $\mathcal{V}(\Omega)$ with only the two boundary spheres, as illustrated in Figure C.1.

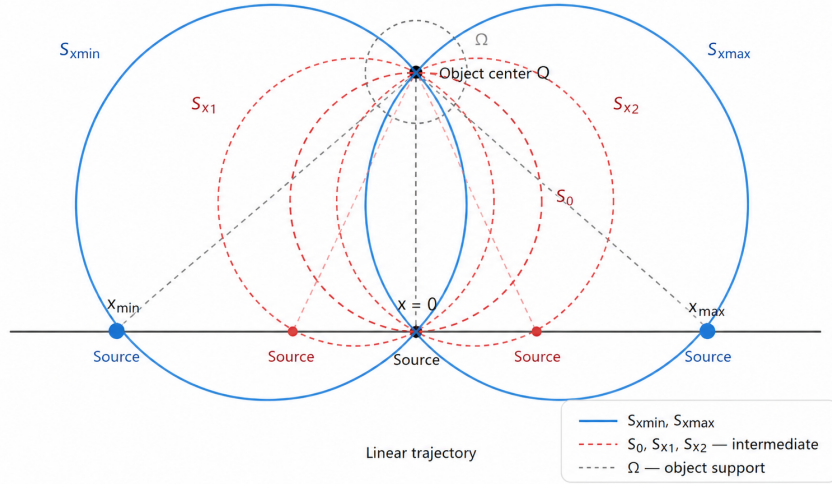


Figure C.1: Radon space coverage for the X1 trajectory. At each source position, a sphere S_x is constructed with diameter equal to SOD. The extreme spheres $S_{x_{\min}}$ and $S_{x_{\max}}$ (blue) cover all intermediate spheres (red), so $\mathcal{V}(\Gamma_{X1}) = S_{x_{\min}} \cup S_{x_{\max}}$.

R1 trajectory: partial Radon torus

For the R1 trajectory, the source follows a circular arc of angular range $[-\theta_{\max}, +\theta_{\max}]$ around the isocenter. At each angle θ , a sphere S_θ is constructed with the same principle — centered at the midpoint between the source position and O , with diameter equal to the source-to-object distance at that angle.

For a complete circular trajectory (360°), it can be shown [20] that the union of all such spheres forms a torus-shaped region in Radon space — the Radon torus. This result is well established in the literature and corresponds to the classical torus illustrated in Figure 3.15 of Section 3.6.2.

For a partial arc of angular range $[-\theta_{\max}, +\theta_{\max}]$, the union of spheres naturally forms a partial torus subtended by that arc, as illustrated in Figure C.2. Unlike the X1 case, no simplification analogous to the two-extreme-spheres property exists for the rotational trajectory: all spheres along the arc must be considered, and the intersection volume $\mathcal{V}(\mathcal{V}(\Omega) \cap \mathcal{V}(\Gamma_{R1}))$ is computed numerically using mesh-based methods.

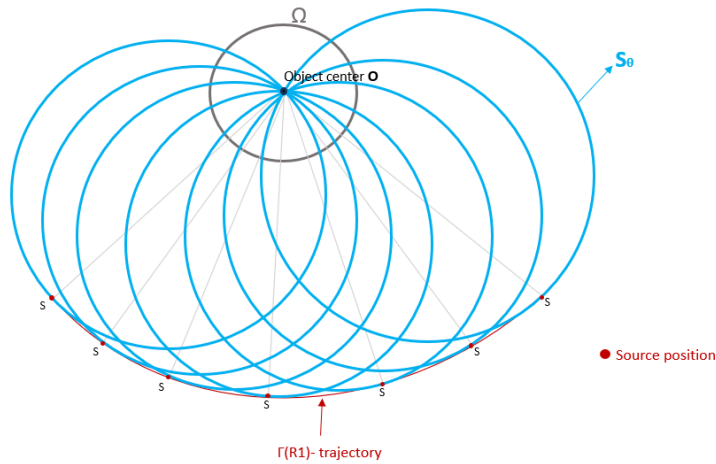


Figure C.2: Radon space coverage for the R1 trajectory. The source follows a circular arc Γ_{R1} , and the union of the corresponding spheres S_θ forms a partial Radon torus $\mathcal{V}(\Gamma_{R1})$. For a complete 360° rotation, this union reduces to the classical Radon torus [20].

Appendix D

Parametric study: additional trajectories

This appendix presents the effect of object offset $\rho = |r|/R$ on contrast and CNR for the Z and R1 trajectories. These results complement those presented in Section 4.1.4 for X1 and R2, and confirm that the observed trends are consistent across all four simple trajectories.

- Effect of object offset $|r|$ Z trajectory

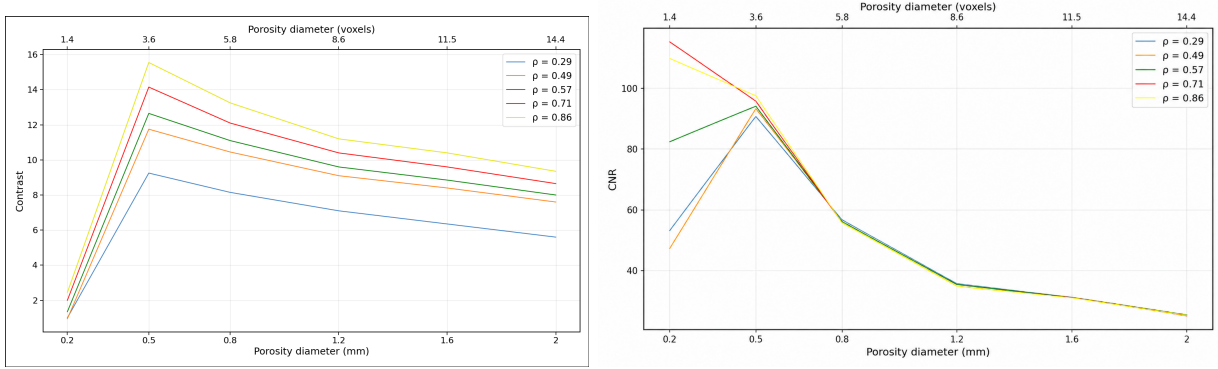


Figure D.1: Effect of normalized object offset $\rho = |r|/R$ on contrast (left) and CNR (right) for the Z trajectory (translation range ± 85 mm, 41 projections).

- Effect of object offset $|r|$ R1 trajectory

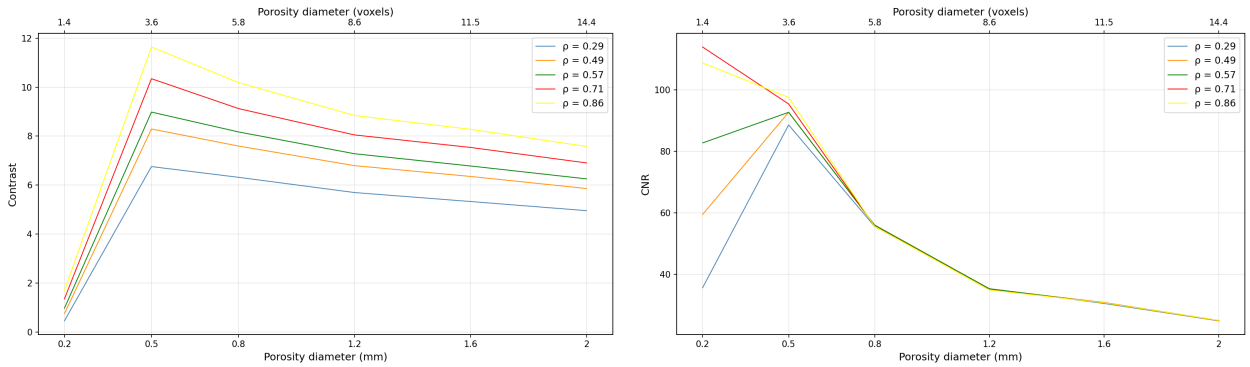


Figure D.2: Effect of normalized object offset $\rho = |r|/R$ on contrast (left) and CNR (right) for the R1 trajectory (angular range $\pm 21.45^\circ$, 41 projections).

- Effect of angular range on R1 trajectory

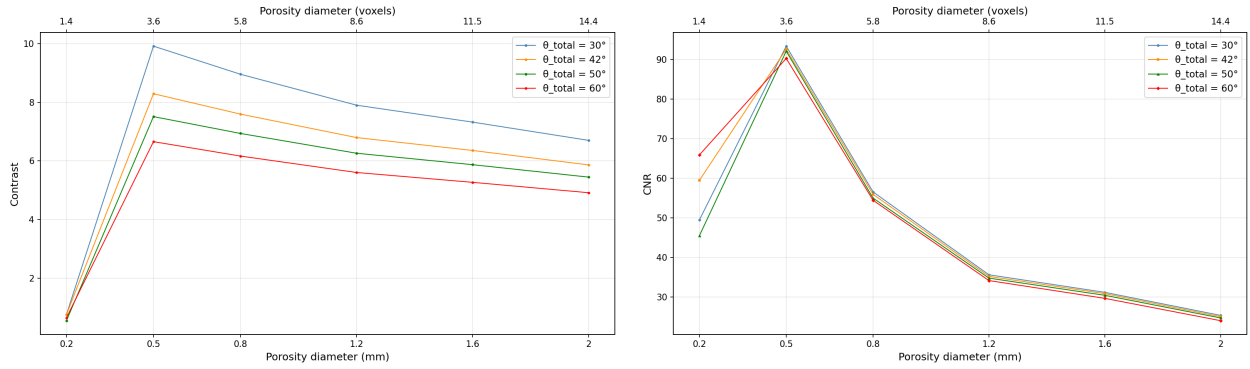


Figure D.3: Effect of total angular range on contrast (left) and CNR (right) for the R2 trajectory ($|r| = 170$ mm, 41 projections).

- Effect of translation range on Z trajectory

Figure D.4 shows the influence of the total translation range on contrast and CNR for the Z trajectory at fixed $|r| = 170$ mm.

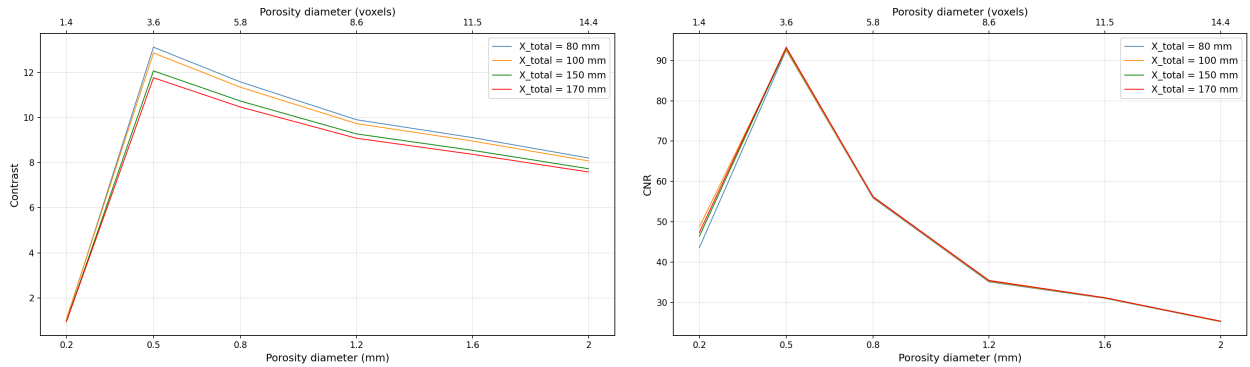


Figure D.4: Effect of total translation range on contrast (left) and CNR (right) for the X1 trajectory ($|r| = 170$ mm, 41 projections).

Appendix E

Investigation of the asymmetry between symmetric trajectory pairs

As noted in Section 4.1.5, a systematic difference in depth profile shape was observed between geometrically equivalent trajectory pairs (X1 versus Z and R1 versus R2), despite the theoretical symmetry of the acquisition geometry. Several hypotheses were investigated to identify the origin of this difference.

Effect of projection orientation. The projections generated for the X1 trajectory were rotated by 90° before reconstruction, producing a configuration equivalent to a Z acquisition. Figure E.1 illustrates this transformation. The resulting depth profiles remained unchanged, indicating that the orientation of the projections relative to the detector is not responsible for the observed asymmetry.

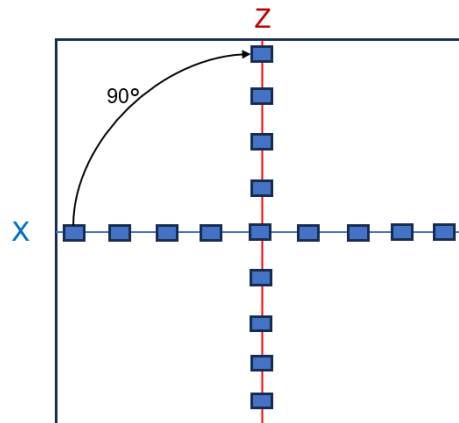


Figure E.1: Schematic illustration of the projection orientation test. The projections acquired for the X1 trajectory were rotated by 90° before reconstruction, producing a configuration equivalent to a Z acquisition while preserving the original acquisition geometry.

Effect of the moving component. To evaluate the influence of which element moves during acquisition, the turntable was displaced along the X axis in simulation — a configuration that is not physically available on the Dx-Box C5 but is geometrically equivalent to the X1 trajectory. The resulting depth profiles were again identical to those of Figure 4.8, ruling out the moving component as the primary cause.

Effect of detector pixel size. The pixel size was reduced from 0.139 mm to 0.05 mm. Figures E.2 and E.3 present the corresponding results. Figures E.2 shows that the relative difference between paired

trajectories (X1 versus Z and R2 versus R1) remains consistent with the results obtained at 0.139 mm (Figure 4.8), confirming that discretization effects are not the primary source of the observed asymmetry.

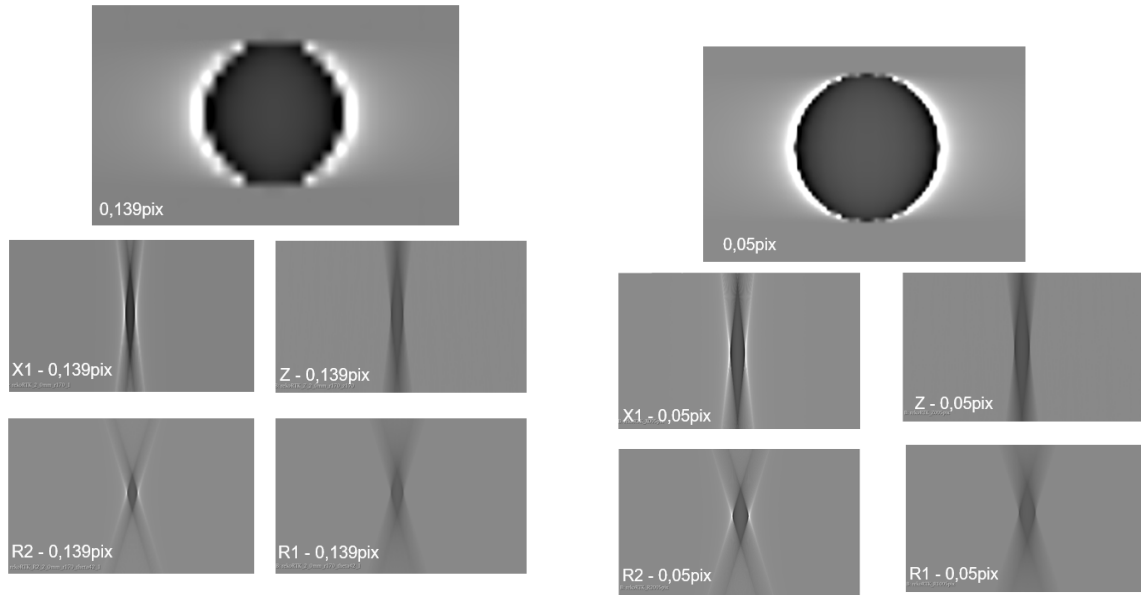


Figure E.2: Reconstructed volumes for the four simple trajectories at pixel sizes of 0.139 mm (left) and 0.05 mm (right). Top: axial slice of the 2 mm porosity. Middle and bottom: coronal and sagittal depth views. While the overall image sharpness improves with the smaller pixel size, the relative difference between paired trajectories (X1 versus Z and R2 versus R1) remains unchanged.

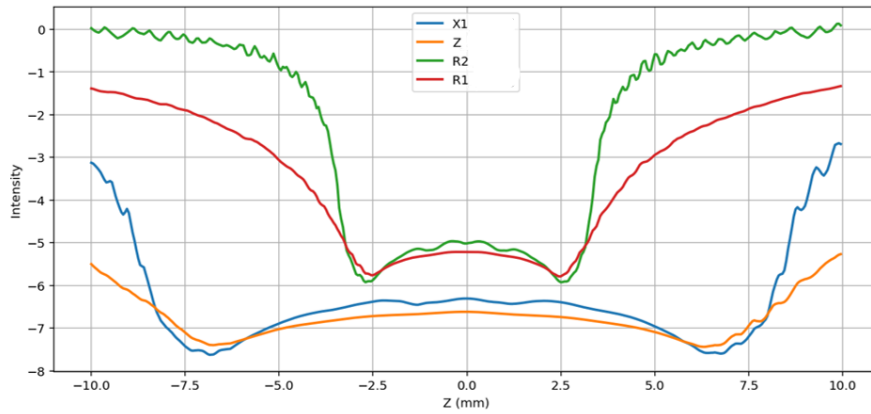


Figure E.3: Depth profiles along the Z-axis for the four simple trajectories reconstructed with a pixel size of 0.05 mm.

As expected, the overall image sharpness improved with the smaller pixel size. However, the relative difference between paired trajectories remained unchanged, indicating that discretization is not the primary factor.

These investigations were unable to identify the origin of the residual difference. Since the effect persists when changing the projection orientation, the moving component, and the detector discretization, it may be related to subtle implementation details of the reconstruction pipeline, interpolation procedures, or numerical approximations within the FBP algorithm. Further investigation would be required to identify its exact origin.

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