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Neutrino Scattering and Oscillation in the Cosmic Neutrino Background: Probing the Gauged $L_\mu - L_\tau$ Boson

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Context

In 2011, astronomers got "fresh eyes", ones capable of seeing the unseen: astrophysical neutrinos of tremendous energies. Alongside IceCube, a new era in multi-messenger astrophysics has begun.

In parallel to this, theoretical physics advances, and Beyond the Standard Model (BSM) extensions pile up. As experimental tensions between theory and experiments come and go –think muon $g-2$ [1–3], for instance– the crux of the problem remains to probe the unimaginable scales of alleged theories, rather than making new ones.

To this end, neutrinos play an ever-increasing role. In the past decades, the possibility of using the tail end of the neutrino spectrum to probe secret neutrino self-interactions (ν SI) has arisen, while its flavor content has revealed itself to be a rich source of new physical signatures [4–6].

Previous work [7–20] has sought to probe one such BSM extension, the $U(1)_{L_\mu-L_\tau}$ symmetry's Z' boson, a possible "fifth force of nature", by leveraging IceCube's astrophysical neutrinos of very high energy (VHE, TeV–PeV). They derived the resonant absorption dips in the spectrum expected to arise from neutrinos scattering with the Cosmic Neutrino Background ($C\nu$ B) as a consequence of this new interaction.

This highlights two things. Firstly, the IceCube neutrino detector (and hopefully emerging next-generation infrastructures like KM3NeT or IceCube-Gen2) sets constraints on particle physics that laboratory experiments cannot currently access. Secondly, in the presence of such hypothetical interactions, the $C\nu$ B, a long sought after "Holy Grail" of cosmology –a relic of epochs far older than even recombination–, can become an opaque medium for neutrinos of VHE energies under appropriate conditions.

Abstract

This thesis aims at taking into account so-called "matter potentials" in the analysis: flavor-dependent potentials inevitably induced by the medium in the presence of the Z' . Because of their flavor dependence, these are *a priori* expected to alter neutrino oscillations, and thus the flavor composition at Earth, while keeping the total flux unscathed, an effect akin to the MSW solution to the solar neutrino problem.

I first reproduce, independently and numerically, the resonant-absorption effect by computing the Z' -induced $\nu - \nu$ cross-section in a QFT framework, before solving the Boltzmann integro-differential equation governing neutrino production, propagation and scattering through the $C\nu$ B. I then compute the effective potentials that such a Z' boson generates in the presence of the $C\nu$ B for two mediator-mass regimes: a light mediator (where absorption and matter effects overlap in energy), and a heavy mediator (where only matter effects fall within the observable spectrum).

In both regimes, an observable flavor signature requires $C\nu$ B overdensities that are either self-defeating –extinguishing the very part of the spectrum needed to see them– or physically implausible. This overdensity requirement scales with $M_{Z'}^2$, and is considerably more severe than in the analogous SM Z -boson case mentioned above [21], owing to the much smaller neutrino mass splittings invoked in their hypothetical scenario for generating neutrino masses (pseudo-Dirac particles).

The result is negative: within $C\nu$ B densities consistent with existing constraints (e.g. KATRIN), the $L_\mu - L_\tau$ symmetry's Z' mediator produces no observable matter-effect signatures in the VHE flavor ratios that would overlay with the resonant absorption features. It is noteworthy that KATRIN constrains only the $C\nu$ B overdensities in the vicinity of Earth.

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Introduction

"I have done a terrible thing: I have postulated a particle that cannot be detected."

— Wolfgang Pauli

Perhaps being permeable to billions of minuscule ghosts just seemed too ludicrous; perhaps postulating a particle that had never before been observed was too unconventional a solution. Still, in the midst of winter 1930, as Pauli first hypothesized their existence, neutrinos were but a "desperate remedy" that few would have imagined would allow us, nearly a century later, to explore virgin territories of particle physics and astrophysics.

Their neutrality and weak coupling, formidable obstacles to their detection, also make them one of the noblest narrators of the cosmic theater. Fierce travelers, propagating unbothered light-years upon light-years, neutrinos remain, at the dawn of multi-messenger astrophysics, but vanishing signatures of the remotest sectors of our Universe. Yet, despite this, the information they bring forth is preciously rare: the astrophysical distances and astrophysical energies probe sectors of physics usually "transparent" to typical lab-experiments.

Neutrinos oscillate from flavor to flavor as they propagate in empty space [22, 23]. Electronic neutrinos produced at the core of our Sun arrive at Earth in far fewer numbers than those at production, whereas atmospheric muon neutrinos are depleted in favor of their tau counterpart as they cross the Earth.

From creation to detection, neutrinos naturally give rise to three interconnected areas of queries: neutrino physics per se, their usage to understand the neutrino-genic environments where they are produced and the usage of neutrinos as a diagnostic tool of the medium they propagate through.

This thesis is embedded in the third endeavor, for astrophysical neutrinos in the very high energy (VHE, TeV–PeV) range detected by IceCube. These are produced with a flavor ratio of $\nu_e : \nu_\mu : \nu_\tau \approx 1 : 2 : 0$ [4], assuming classic pion production. Neutrino oscillations, given the best-fit parameters, are then expected to rotate the fluxes to a flavor ratio at Earth of $\nu_e : \nu_\mu : \nu_\tau \approx 1 : 1 : 1$. This result is derived in Chapter 1.

New physics along the propagation path can shift that expectation. One well-studied possibility [7–20] is a new gauge boson mediating neutrino self-interactions (ν 's scattering off ν 's): the Z' of a gauged $L_\mu - L_\tau$ symmetry. The core idea explored is that the rate of scattering of astrophysical neutrinos off the relic cosmological neutrino background (C ν B) would be enhanced by the existence of a new interaction, particularly at resonance, producing absorption features in the spectrum.

The $C\nu B$ is made of relic neutrinos, whose interaction rate fell below the expansion rate of the Universe at a temperature of roughly 1 MeV [24], effectively decoupling them. They have since been "redshifted" to extremely low energies, down to meV-scale. This makes their detection extremely difficult. Their estimated density is $n_\nu \sim 336 \text{ cm}^{-3}$ [25], a value adopted throughout this work.

The $C\nu B$ is the only background component considered throughout the Z' -IceCube literature as its estimated values –it has never been directly measured– are by far the greatest contribution to the neutrino background. This is also true in the vicinity of Earth, as is showcased through the Grand Unified Neutrino Spectrum (GUNS) in Fig. 1.

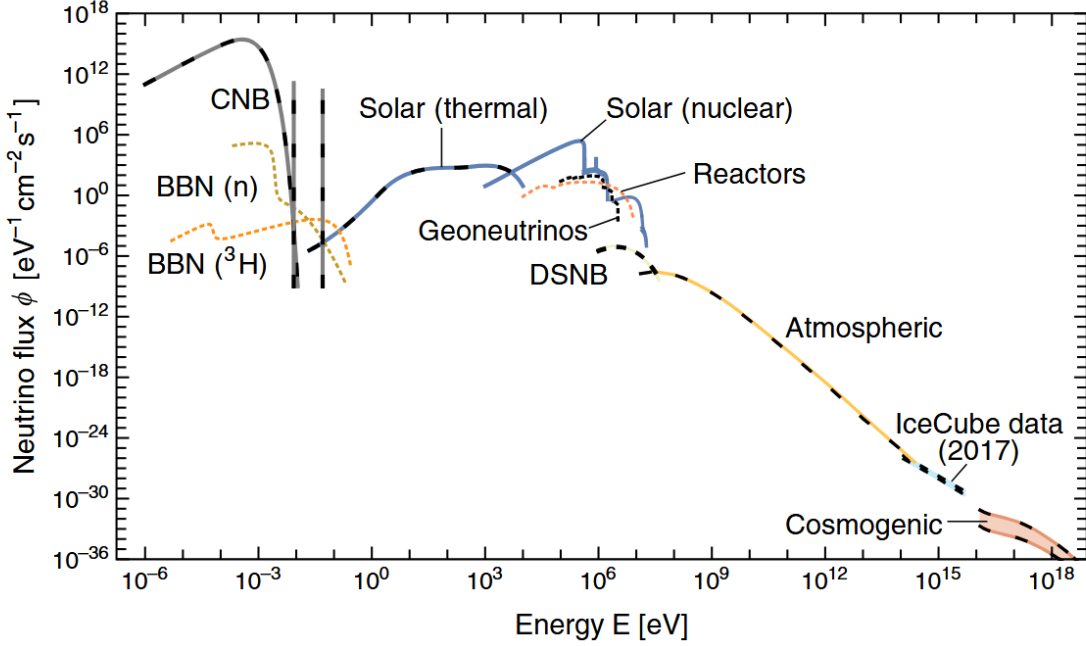


Figure 1: Grand Unified Neutrino Spectrum (summed over flavors) at Earth's surface. Figure by Vitagliano et al. (2020) [25]. Solar, atmospheric and reactor neutrinos contribute enormously to studying the oscillation parameters governing neutrino propagation. Accelerator neutrinos do not appear, unlike their reactor counterpart, as they do not dominate large portions of the spectrum over significant geographic regions [25]. The Big-Bang nucleosynthesis and geoneutrinos are of lesser importance in our understanding of neutrino physics, although the latter have been utilized to probe neutrino-genic reactions within the depths of our Earth [26]. The former have never been observed.

It is well known that the propagation medium alters the neutrino flux not only through overall scattering, but also through changes to the oscillations themselves. A prime example occurs when neutrinos leaving the Sun *forward scatter* with electrons. Tiny phase shifts add up coherently and change their oscillation pattern, without discarding any neutrinos from the Earth-bound flux [27]. Alongside the Sun's decreasing density layers, this process, the Mikheyev-Smirnov-Wolfenstein (MSW) effect, solves the solar neutrino problem.

Another way to grasp the phenomenon qualitatively is through an analogy with the change in the polarization content of light crossing a birefringent medium [27]: different polarizations gain different phase shifts, thus the polarization content changes ; different neutrino flavor gain different phase shifts, thus the oscillations/flavor content changes.

If the Z' boson does exist, then astrophysical neutrinos will undergo coherent forward scattering with the $C\nu B$ and experience a change to their oscillation pattern and final flavor content.

The main goals of this master's thesis are thus **(a)** to reproduce the scattering-only analysis independently ; **(b)** to determine under which conditions the matter potentials are strong enough to matter in the flavor ratio analysis ; **(c)** to exhibit the signatures of neutrino parameters, Z' physics and crucially $C\nu B$ density within the spectrum and flavor ratios ; **(d)** to give perspective on the role of current and upcoming neutrino detectors in unraveling such parameters and properties.

To achieve this, the underlying analysis on pure scattering with the $C\nu B$ is first derived independently. The cross-section associated with neutrino-neutrino scattering enabled by the new gauge boson is computed (Chapter 2) in the typical S-matrix formalism of quantum field theory (QFT). The resonance location, its dependence on the boson parameters and the mean-free-path it entails are discussed.

Solutions to the Boltzmann integro-differential propagation equation are numerically computed and presented in increasing levels of complexity of the physics (Chapter 3). The (necessary) effect of cosmological expansion, regeneration of the VHE neutrino beam and distributed sources throughout the Universe are progressively included in the problem statement. The convergence of the implementation's solution is verified where analytical solutions are tractable. Particular care is given to the stiffness and sparsity of the differential system at hand, and the semi-analytical bin-averaging method employed to address the extremely narrow resonance width.

Once this is done, the matter effects induced by the Z' on the flavor ratios are properly taken into account (Chapter 4). The approach is to compute the energy dependent, effective potentials each neutrino flavor will be subject to. This is done for two mediator-mass regimes: a light mediator scenario ($M_{Z'} \sim 10$ MeV) where the effective potentials distort the flavor content within the resonant absorption feature, and a heavy mediator scenario ($M_{Z'} \gg 10$ MeV) where only matter effects fall within the observable spectrum.

The modifications this entails to the oscillation parameters being too complex in the three-neutrino framework, numerical and analytical approximations used in typical matter environments are extrapolated to this work through a discussion on the charges of the $L_\mu - L_\tau$ -induced new force.

The effects on the flavor ratios are then presented (Chapter 5). The conditions for resonance with a monotonically decreasing density profile, i.e. the MSW effect adapted for the Z' in an expanding Universe permeated by the $C\nu B$, are discussed. This involves strong assumptions about $C\nu B$ overdensities and CP-asymmetry parameters, treated as free parameters for the sake of the analysis.

Finally, the main hypotheses and important choices made –to choose is to renounce– are critically assessed under the light of modern research (Chapter 6). Possible improvements and future perspectives beyond the scope of this work are also discussed before giving final conclusions.

Chapter 1

Prolegomena

1.1 Neutrino oscillations

With the Standard Model (SM) of particle physics comes the idea that leptons, those particles that do not couple to the strong force, which are either charged or neutral, come in three "flavors", or "families". The three charged leptons are the electron, the muon and the tau. The three neutral leptons are the electronic, muonic and tauc neutrinos. Particles, in this framework, are defined as *mass eigenstates*: quantum states of definite mass.

The caveat of this definition is that the charged leptons and the neutral leptons cannot be "particles", in this sense, at the same time. Indeed, if one defines weak interactions so that charged leptons mass eigenstates are their products (e.g. the electron), then the neutral leptons that take part in said processes (resp. the electronic neutrino) are superpositions of the neutral lepton mass eigenstates.

Conversely, if one defines the neutral leptons mass eigenstates to be the products of weak interactions, which we shall call ν_1 , ν_2 and ν_3 , then the charged leptons counterpart of these interactions will be superpositions of the traditional electron, muon and tau. These states will *not* have a definite mass, and cannot qualify as particles, in the traditional sense.

No prescription is intrinsically "truer" than the other. However, the comparative ease with which one can measure¹ an electronic state with respect to a neutrino mass eigenstate makes for the former convention to seem intrinsically superior. This universal convention is implicit throughout this thesis. To study neutrino oscillations, one typically describes the neutrino beam or particle as a direct analogy to the three-state quantum system.

1.1.1 Quantum mechanical setting

Richard Feynman, in his *Lectures on Physics*, devised quantum mechanics as the fundamental shift in paradigm with which one considers the *probability of a system to undergo a specific evolution*, given the knowledge of the intermediate steps of said evolution.

Indeed, in classical mechanics, the stochastic evolution of a given system which goes from state "i" to state "f" either through step "a", "b" or "c" will be governed by the following rule²:

$$P_{i \rightarrow f} = \sum_{\mu=a,b,c} P_{i \rightarrow \mu} \cdot P_{\mu \rightarrow f} \quad (1.1)$$

¹In the quantum mechanical spirit of the term.

²In direct analogy to the Chapman-Kolmogorov relation for Markov chains.

This is nothing but the straightforward formalization that, throughout its evolution and even if we only measure the system's state at the start and the end of the evolution, the system either goes through step "a", "b" or "c".

Quantum mechanics, then, arises from the imposition of a new rule replacing that previously stated. This composition law, which sums over all possible intermediate steps, is no longer adequate for probabilities of seeing the system go from "x" to "y". Instead, this composition law applies to an abstract complex number, $\phi_{x \rightarrow y}$, named the probability amplitude, related to the usual probability insofar as the latter is retrieved by taking the squared-norm of the former. We then have:

$$\phi_{i \rightarrow f} = \sum_{\mu=a,b,c} \phi_{i \rightarrow \mu} \cdot \phi_{\mu \rightarrow f} \quad (1.2)$$

$$P_{x \rightarrow y} = |\phi_{x \rightarrow y}|^2 \quad (1.3)$$

In stark opposition to classical mechanics, which typically describes a system's state by a point in its phase space, quantum mechanical systems are described by Hilbert spaces, conveniently chosen for their completeness and inner product, of which a state is represented by a given vector. This allows for the aforementioned probability amplitude $\phi_{x \rightarrow y}$ to be defined as the inner product $\langle y|x \rangle$, with $|x\rangle$ the state x of the Hilbert space and $\langle y|$ the linear form of the dual space associated with $|y\rangle$ ³.

Such systems typically extend throughout space-time and thus evolve within it. A system "evolving through space" is commonly said to have a "spatial dependency", while "time dependency" is simply referred to as "evolution". In quantum systems as in classical ones, those which respect the core idea that all space-time points are born equal -*i.e.* space-time symmetry- harbor, according to Noether's theorem, a set of conserved quantities: *momentum and energy*.

In the Hilbert space formalism pertaining to operator-based theories, such as quantum mechanics, because the normalization of a state is primordial in its relationship with the probability amplitude described above, space-time translations/evolutions are characterized by *unitary* operators acting upon the Hilbert space. They are typically denoted such that:

$$|\psi(t_f)\rangle = \hat{U}(t_f, t_i) |\psi(t_i)\rangle \quad (1.4)$$

$$|\psi(\mathbf{x} + \mathbf{r})\rangle = \hat{T}(\mathbf{r}) |\psi(\mathbf{x})\rangle \quad (1.5)$$

For infinitesimal space-time steps, these operators may only differ from the identity by a first-order term, which can be directly related to the respective space-time derivative. Taking the time dimension as an example, in direct analogy to space translations, one has:

$$|\psi(t + dt)\rangle = (\hat{1} + \hat{\alpha} \cdot dt) |\psi(t)\rangle \quad (1.6)$$

$$\frac{d}{dt} |\psi\rangle = \lim_{dt \rightarrow 0} \frac{|\psi(t + dt)\rangle - |\psi(t)\rangle}{dt} = \hat{\alpha} |\psi\rangle := \frac{\hat{H}}{i\hbar} |\psi\rangle \quad (1.7)$$

Where the last step embodies the fact that in QM, it is energy-momentum that generates time-space dependencies and evolutions. The factor i allows for the energy-momentum operators to have real eigenvalues by being hermitian, while $\hbar$ is a constant that has the units of two conjugate quantities multiplied together, namely *action*, and guarantees the dimensional coherence of the equation.

In a system where $\hbar$ is set to $1.05... \times 10^{-34} [J \cdot s = m \cdot \frac{kg \cdot m}{s}]$, space-time and energy-momentum are prescribed in SI units.

³Note how the hermitian nature of the Hilbert space's inner product yields the interpretation that $\phi_{x \rightarrow y}^* = \phi_{y \rightarrow x}$.

In a system where $\hbar$ is set to 1 [–], conjugate quantities have inverse units and time can be expressed in J^{-1} . If, in addition, one sets the speed of light c to one, time and space hold the same dimension and all physical quantities can be described in powers of the energy. This is the so-called *natural* system of units.

A quantum system's dynamics through space-time can then be expressed by defining a dispersion relation, relating its energy to its momentum, and applying the statement that they generate space-time translations, namely $\frac{d}{dt} |\psi\rangle = \frac{\hat{H}}{i\hbar} |\psi\rangle$ and $-\frac{d}{dx} |\psi\rangle = \frac{\hat{P}}{i\hbar} |\psi\rangle$ ⁴. For a non-relativistic particle whose energy is $E = \frac{p^2}{2m} + V$, this yields the famous Schrodinger equation: $i\hbar \frac{d}{dt} |\psi\rangle = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} |\psi\rangle + V(x) |\psi\rangle$.

1.1.2 Expression of the oscillation probability

Neutrino flavor-content oscillations in vacuum are typically described as a three-state quantum system. Neutrinos are produced as "flavor states" of the electronic, muonic and tauic kind, denoted $|\nu_\alpha\rangle$ with $\alpha = \{e, \mu, \tau\}$. The Hamiltonian is diagonalized in a basis of three other states called "mass eigenstates" $|\nu_1\rangle, |\nu_2\rangle$ and $|\nu_3\rangle$.

These two orthonormal bases are related by a unitary transformation U , the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix.⁵

$$|\nu_\alpha\rangle = U_{\alpha i} |\nu_i\rangle \quad (1.8)$$

$$(U^{-1})_{i\alpha} = (U_{\alpha i})^* \quad (1.9)$$

As flavor-basis neutrinos are produced with energy uncertainties larger than the mass differences between eigenstates, pure mass eigenstates $|\nu_i\rangle$ are never measured. Hence, following their creation in a particle accelerator, the core of the Sun, or a million parsecs away, neutrinos that we detect have undergone oscillations, being superpositions of three eigenstates whose respective phases evolve at different frequencies $\frac{E_i}{\hbar}$.

The probability that a neutrino of flavor α at position 0 is measured after a time t as a β -flavored neutrino is given by the rule on the composition of probability amplitudes 1.2, with the possible intermediate states being the three propagation mass eigenstates. The probability amplitude of transition is:

$$\phi_{\nu_\alpha \rightarrow \nu_\beta} = \langle \nu_\beta | \hat{U}(t, 0) | \nu_\alpha \rangle = \left(\sum_{j=1}^3 U_{\beta j}^* \langle \nu_j | \right) \left(\sum_{i=1}^3 U_{\alpha i} \cdot e^{-\frac{iE_i}{\hbar} t} \cdot |\nu_i\rangle \right) = \sum_{i=1}^3 U_{\alpha i} U_{\beta i}^* \cdot e^{-\frac{iE_i}{\hbar} t} \quad (1.10)$$

Such that the probability of transition, $P_{\nu_\alpha \rightarrow \nu_\beta} = |\phi_{\nu_\alpha \rightarrow \nu_\beta}|^2$, can be expressed as:

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \left(\sum_{i=1}^3 U_{\alpha i} U_{\beta i}^* \cdot e^{-\frac{iE_i}{\hbar} t} \right) \cdot \left(\sum_{j=1}^3 U_{\alpha j} U_{\beta j}^* \cdot e^{-\frac{iE_j}{\hbar} t} \right)^* = \sum_{i,j} \underbrace{U_{\alpha i} U_{\alpha j}^* U_{\beta j} U_{\beta i}^*}_{Q_{ij}^{\alpha\beta}} e^{-\frac{i(E_i - E_j)}{\hbar} t} \quad (1.11)$$

One can note that by virtue of the unitarity of U , $P_{\nu_\alpha \rightarrow \nu_\beta}(0) = \delta_{\alpha\beta} \cdot \delta_{\alpha\beta} = \delta_{\alpha\beta}$. In addition, the probability of having no transition is given by:

⁴The minus sign comes from the fact that space-time harbors a non-euclidean, Minkowski metric.

⁵Where the summation over the repeated indices of a term is made implicit.

$$P_{\nu_\alpha \rightarrow \nu_\alpha}(t) = \sum_{i,j=1}^3 |U_{\alpha i}|^2 |U_{\alpha j}|^2 \cdot e^{-\frac{i(E_i - E_j)}{\hbar} t} \quad (1.12)$$

The transition probability in equation 1.11 –and hence in 1.12– is real: the imaginary parts cancel pairwise to leave only cosines. One can explicitly break apart the sum into those terms where $i = j$, which do not change with time, and those where $i \neq j$, the *quantum interferences* that provide time dependency. Indeed, the squared norm of a complex number $z = z_1 + z_2 + \dots$ being $|z_1|^2 + |z_2|^2 + z_1 z_2^* + z_1^* z_2 + \dots$, each pair of interference terms are complex conjugates of one another. This allows one to re-express the transition probability with explicitly real-valued components.

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2 + 2\Re \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} e^{-\frac{i(E_i - E_j)}{\hbar} t} \right\} \quad (1.13)$$

This expression showcases clearly that, in the presence of *quantum decoherence* whereby the interference terms are lost, the transition probability is akin to neutrinos following the "classical" probability composition law 1.1, with the "intermediate" states 1, 2 and 3. In addition, the uncertainties on the distance L and the resolution on the energy E average out the interference terms. Expression 1.14 will then also effectively dictate the transition probabilities measured at Earth [28]. Because neutrino mass eigenstates have different masses, their respective wavepackets will propagate at different group velocities even if they hold the same energy. This means they will end up separating after a sufficiently long time, also reducing the probability of the incoherent expression [28, 29]:

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \sum_{i=1}^3 |U_{\alpha i}|^2 |U_{\beta i}|^2 \quad (1.14)$$

Because at $t = 0$, equation 1.13 reduces to $\delta_{\alpha\beta}$, one can typically express the transition probability using trigonometric functions.⁶

$$P_{\nu_\alpha \rightarrow \nu_\beta}(t) = \delta_{\alpha\beta} - 2\Re \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} \right\} + 2\Re \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} e^{-\frac{i(E_i - E_j)}{\hbar} t} \right\} \quad (1.15)$$

$$= \delta_{\alpha\beta} - 2\Re \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} \left[1 - \cos \left(\frac{E_i - E_j}{\hbar} t \right) \right] \right\} + 2\Im \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} \sin \left(\frac{E_i - E_j}{\hbar} t \right) \right\} \quad (1.16)$$

$$= \delta_{\alpha\beta} - 4\Re \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} \sin^2 \left(\frac{E_i - E_j}{2\hbar} t \right) \right\} + 2\Im \left\{ \sum_{i,j}^{i < j} Q_{ij}^{\alpha\beta} \sin \left(\frac{E_i - E_j}{\hbar} t \right) \right\} \quad (1.17)$$

This expression depends on the components of the PMNS matrix through the $Q_{ij}^{\alpha\beta}$ factors. When considering the antineutrino transition probability $P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta}(t)$, the formalism stays the same with $U \rightarrow U^\dagger$ and hence $Q_{ij}^{\alpha\beta} \rightarrow Q_{ij}^{*\alpha\beta}$. Thus CP-symmetry, the statement that the laws of the Universe are the same for particles and antiparticles in a mirrored Universe, can in principle be violated by the imaginary part of $Q_{ij}^{\alpha\beta}$.

⁶Where we used the fact that $\Re\{z_1 z_2\} = \Re\{z_1\} \cdot \Re\{z_2\} - \Im\{z_1\} \cdot \Im\{z_2\}$ as well as the Carnot formula.

1.1.3 Parametrization of the PMNS matrix

Explicit transition probability in terms of the neutrino mixing parameters necessitates a convention for the expression of the PMNS matrix for a given number of neutrino flavors. Assuming neutrinos to be Dirac particles, the standard parametrization uses three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and a charge-parity-violating phase δ_{CP} . The five other phases can be reabsorbed as overall phase shifts in the lepton fields.

$$U_{\text{PMNS}}^{(2)} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (1.18)$$

$$U_{\text{PMNS}}^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.19)$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (1.20)$$

with $c_{ij} = \cos(\theta_{ij})$ and $s_{ij} = \sin(\theta_{ij})$.

Naturally, a two-flavor system exhibits one squared-mass difference Δm^2 , while a three-flavor system exhibits three, related by the condition $\Delta m_{21}^2 + \Delta m_{32}^2 = \Delta m_{31}^2$, so that only two are independent.

Neutrinos oscillate in vacuum because the mass of each eigenstate are different and these differences drive the interference term. Thus only the mass-splittings –not the absolute masses– can be retrieved from oscillation experiments. This leaves unconstrained the so-called *hierarchy* or *ordering* of the neutrino masses. The two possibilities are labeled Normal Ordering (NO) and Inverted Ordering (IO) and are shown in Figure 1.1.

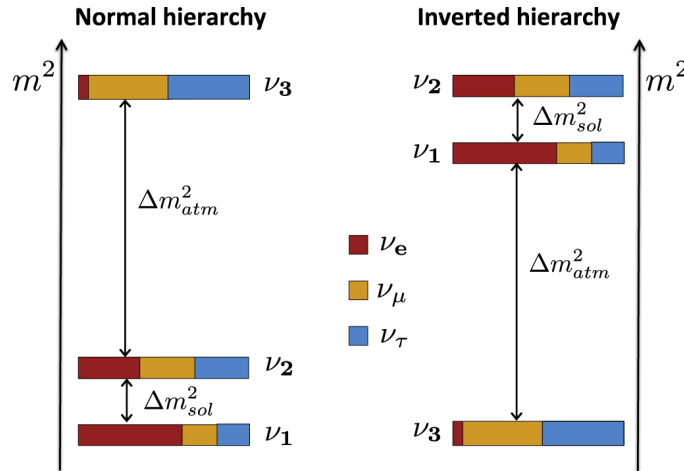


Figure 1.1: The possible orderings or "hierarchies" given two neutrino squared-mass difference. Normal ordering (NO) sees the heaviest neutrino mass eigenstate be almost orthogonal to the electron neutrino state, while the lightest takes this role in case of Inverted ordering (IO). Figure by Adrián-Martínez et al. (2016) [30].

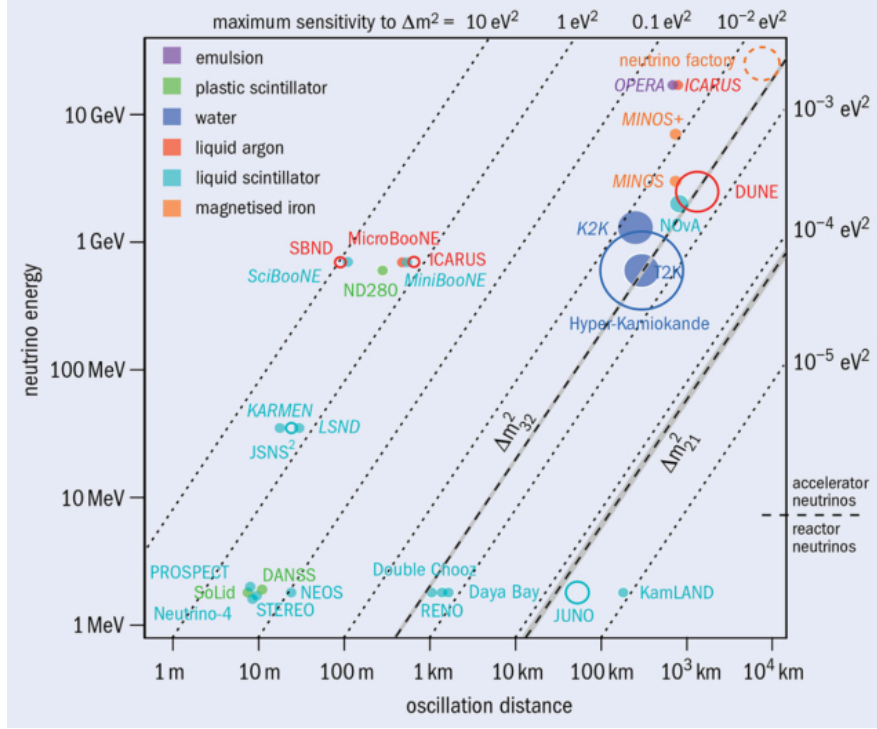


Figure 1.2: Maximally probed squared-mass difference as a function of the distance neutrinos propagate through and the energy they propagate with. Reactor and accelerator experiments are represented. M. Rayner, CERN Courier, 2020, <https://cerncourier.com/a/tuning-in-to-neutrinos/>

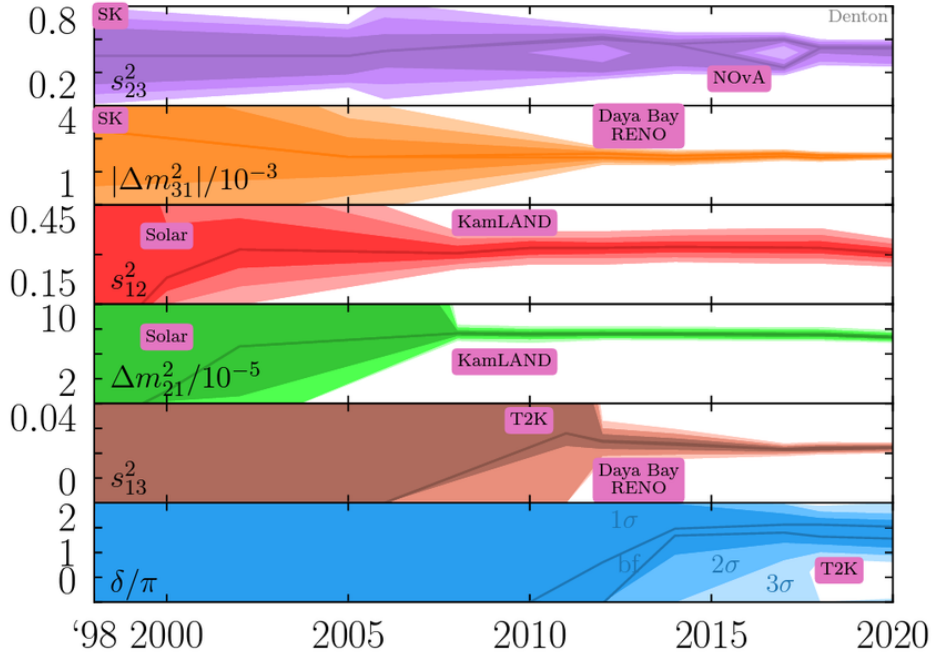


Figure 1.3: Summary of the main breakthroughs in the determination of oscillation parameters in the 21st century, with fainter colors indicating larger confidence intervals. Figure by Denton et al. (2022) [31], seen first in [32].

Neutrino oscillation parameters are inferred from a plethora of different experiments. Indeed, a

choice of propagation baseline L and neutrino energy E will mainly probe a given value of Δm^2 . Thus reactor, accelerator, solar and atmospheric neutrinos that are utilized to such ends each shed light on differing *sectors* —parameters— of the theory, graphically represented in Figure 1.2. A given experiment will typically focus on a specific sector of the theory, oftentimes reducing the problem statement to an effective 2-flavor oscillation system. An almost exhaustive account of these can be found in [33].

These studies are combined into *global analyses* of neutrino oscillation parameters. These meta-analyses of sorts alleviate the otherwise low statistics of individual experiments to increase the precision of the fit ; they greatly reduce the size of the parameter space by overlapping the different experimentally allowable ranges ; they expose tensions between different types of experiments aimed at studying a common parameter, hinting at errors induced by the intrinsic physics of the detectors [32].

The main contributors of such global fits are the NuFIT Collaboration, the Valencia Group and the Bari Group. Biennially, the main results from these groups are also presented in the Particle Data Group neutrino review [33]. The NuFIT 6.0 [34] and the sub-percent precision fits of the Bari Group [35] are the most up-to-date global neutrino parameter analysis as of writing these words. We use the reported values of NuFIT 6.0 throughout this work, presented in Figures 1.4 and 1.5.

	NuFIT 6.0 (2024)		
$ U _{3\sigma}^{\text{IC19 w/o SK-atm}} =$	$(0.801 \rightarrow 0.842$	$0.519 \rightarrow 0.580$	$0.142 \rightarrow 0.155)$
	$0.248 \rightarrow 0.505$	$0.473 \rightarrow 0.682$	$0.649 \rightarrow 0.764)$
	$0.270 \rightarrow 0.521$	$0.483 \rightarrow 0.690$	$0.628 \rightarrow 0.746)$
$ U _{3\sigma}^{\text{IC24 with SK-atm}} =$	$(0.801 \rightarrow 0.842$	$0.519 \rightarrow 0.580$	$0.142 \rightarrow 0.155)$
	$0.252 \rightarrow 0.501$	$0.496 \rightarrow 0.680$	$0.652 \rightarrow 0.756)$
	$0.276 \rightarrow 0.518$	$0.485 \rightarrow 0.673$	$0.637 \rightarrow 0.743)$

Figure 1.4: 3σ ranges for the PMNS matrix elements. The upper (lower) sections are inferences without (with) the data on atmospheric neutrinos provided by the Super-Kamiokande and the latest IceCube observations. The elements are correlated given the parametrization in 1.19 and the unitary condition, so that fixing one element restricts the interval of others.

Typically, it is assumed that each neutrino state holds the same, definite, momentum. This assumption can be relaxed by the introduction of a wave-packet formalism without changing the result [28]. In the highly relativistic regime, the respective energy of each mass eigenstate can then be expressed as:

$$E_i = pc \cdot \sqrt{1 + \left(\frac{m_i c}{p}\right)^2} \sim E + \frac{m_i^2 c^4}{2E} \quad (1.21)$$

where $E = pc$.

The oscillation probability in the 2-flavor case is then simply:

$$P_{\nu_\alpha \rightarrow \nu_\beta}\left(\frac{L}{E}\right) = \sin^2(2\theta) \sin^2\left(\frac{\Delta m_{ij}^2 c^3}{2\hbar} \cdot \frac{L}{E}\right) = \sin^2(2\theta) \sin^2\left(1.27 \Delta m_{ij}^2 \cdot \frac{L(\text{km})}{E(\text{GeV})}\right) \quad (1.22)$$

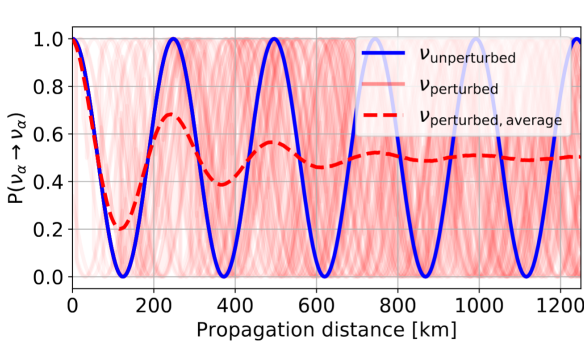
NuFIT 6.0 (2024)

IC19 without SK atmospheric data		Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2 \theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$
	$\sin^2 \theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \rightarrow 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \rightarrow 0.597$
	$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \rightarrow 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \rightarrow 50.6$
	$\sin^2 \theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \rightarrow 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \rightarrow 0.02397$
	$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \rightarrow 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \rightarrow 8.91$
	$\delta_{\text{CP}}/^\circ$	177^{+19}_{-20}	$96 \rightarrow 422$	285^{+25}_{-28}	$201 \rightarrow 348$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \rightarrow +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \rightarrow -2.438$	
IC24 with SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2 \theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \rightarrow 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \rightarrow 35.95$
	$\sin^2 \theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \rightarrow 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \rightarrow 0.584$
	$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \rightarrow 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \rightarrow 49.8$
	$\sin^2 \theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \rightarrow 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \rightarrow 0.02409$
	$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \rightarrow 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \rightarrow 8.93$
	$\delta_{\text{CP}}/^\circ$	212^{+26}_{-41}	$124 \rightarrow 364$	274^{+22}_{-25}	$201 \rightarrow 335$
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \rightarrow 8.05$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \rightarrow +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \rightarrow -2.421$	

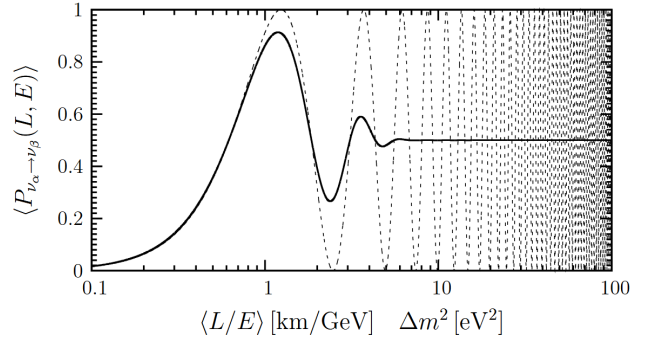
Figure 1.5: Three-flavor oscillation parameters retrieved by the NuFIT Collaboration as of September 2024. The upper (lower) sections are inferences without (with) the data on atmospheric neutrinos provided by the Super-Kamiokande and the latest IceCube observations. $\Delta m_{3l}^2 = \Delta m_{31}^2 > 0$ for NO and $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0$ for IO. Esteban et al. (2024) [34]

Oscillation experiments, because of baseline uncertainties and energy resolution, will inevitably bin neutrinos with different $\frac{L}{E}$ phases together, effectively averaging the square sine in Equation 1.22 as the number of cycles since emission increases. This is very similar to the effect of stochastic perturbations of the individual neutrino phases, as is shown in Fig. 1.6. In addition, wave-packet separation and neutrino decoherence also induce a damping of this oscillation up to the incoherent value 1.14 as cycles add up [28].

The maximum oscillation effect that can be observed is thus reached at the first maximum, when the phase is $\pi/2$ rad, or roughly when $\frac{L}{E}\Delta m^2 \sim 1$. This explains the notion of "maximally probed" Δm^2 present in Fig. 1.2. Much farther in distance—or at much lower energies—, transitions still occur but their probability does not oscillate anymore.



(a) Stuttard and Jensen (2020) [36].



(b) Giunti and Kim (2007) [28].

Figure 1.6: Left: Oscillation probability as a function of the distance traveled in a two-flavor system with $\theta = 45^\circ$. The blue line is the theoretical vacuum probability, the light lines are a collection of neutrinos experiencing stochastic phase shifts, and the dashed line is their average. Right: Oscillation probability as a function of the physical phase factors, averaged over a Gaussian distribution for $\frac{L}{E}$ with $\sigma_{L/E} = 0.2 \langle \frac{L}{E} \rangle$ to simulate detector resolution. Their final values both coincide with the incoherent result 1.14 one could also expect from neutrino decoherence or wave-packet separation.

The oscillation probability in the 3-flavor case, assuming $\delta = 0$ and $\alpha \neq \beta$ for the sake of simplicity is given by:

$$\begin{aligned}
 P_{\nu_\alpha \rightarrow \nu_\beta} = & -4[(U_{\alpha 1}U_{\beta 1}U_{\alpha 2}U_{\beta 2}) \sin^2(1.27\Delta m_{21}^2 \frac{L}{E}) \\
 & + (U_{\alpha 1}U_{\beta 1}U_{\alpha 3}U_{\beta 3}) \sin^2(1.27\Delta m_{31}^2 \frac{L}{E}) \\
 & + (U_{\alpha 2}U_{\beta 2}U_{\alpha 3}U_{\beta 3}) \sin^2(1.27\Delta m_{32}^2 \frac{L}{E})]
 \end{aligned}$$

The measured configuration of neutrino mass differences is such that $|\Delta m_{21}^2| \ll |\Delta m_{32}^2| \approx |\Delta m_{31}^2|$. This means that in good approximation the oscillations take place on only two "wavelength" scales. In the limit where $\frac{L}{E}$ is small, the Δm_{21}^2 oscillations are negligible, whilst the high $\frac{L}{E}$ limit sees the $\Delta m_{32}^2 \approx \Delta m_{31}^2$ oscillate so fast their squared-sine averages to $\frac{1}{2}$.

In parallel to this, the mixing angles are such that $\sin^2(2\theta_{13}) \sim 0$ at first order. This implies from 1.20 that $U_{e3} \sim 0$. The two latter terms in the oscillation probability thus drop when we consider electronic neutrinos. This motivates reducing general oscillations to two-flavor systems in some cases. Indeed:

- At first order in s_{13}^2 , when $\frac{L}{E}$ is small, electronic transitions have no significant contribution and the system reduces to a $\nu_\mu \Leftrightarrow \nu_\tau$ two-flavor oscillation scheme.
- At first order in s_{13}^2 , when $\frac{L}{E}$ is big, electronic transitions are the only ones whose $(U_{\alpha 1}U_{\beta 1}U_{\alpha 2}U_{\beta 2})$ factor depends only on θ_{12} , so that their case reduces to a $\nu_e \Leftrightarrow \nu_\tau + \nu_\mu$ two-flavor oscillation scheme.

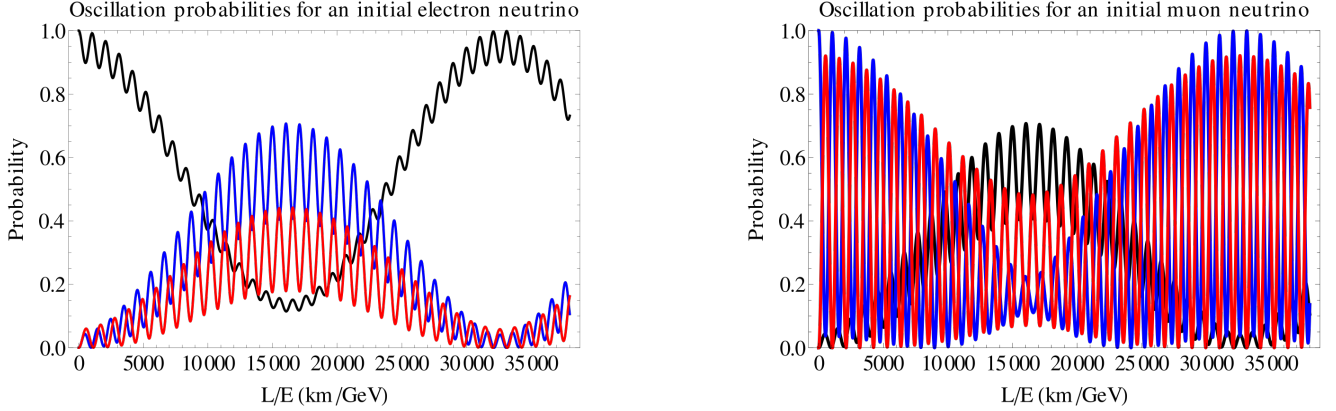


Figure 1.7: Oscillation probability from electron (left) or muon (right) to electron (black), muon (blue) or tau (red). For small $\frac{L}{E}$ (< 4000) electronic transitions are indeed negligible. For big $\frac{L}{E}$ the electronic flavor oscillates in a two-flavor fashion while the muon and tau exhibit a beating phenomenon. *Wikipedia*, based on Meszéna, Balázs. "Neutrino Oscillations". Consulted July 2nd, 2025.

1.2 The Z' boson: Hypothesis

1.2.1 Quantum Field Theoretic Setting

Giving a self-contained, rigorous description of the quantum theory of fields here is of course not feasible. Let us nonetheless recall the bare terminology to follow the remainder of this section.

Quantum field theory describes particles and their interactions as two sides of the same coin. Particles such as the electron are quantized excitations of a field governed by the Dirac equation, while interactions such as electrodynamics are fields governed by the Proca-Maxwell equation, whose quantized excitations are bosons, such as the photon. Quantized excitations of Dirac fields exhibit Fermi statistics while those of Maxwell fields hold Bose statistics.

Akin to traditional quantum mechanics, probability amplitudes associated with certain phenomena—initial and final particle states—are the sum of the amplitudes of each possible intermediate step, or particle path, resulting in said phenomenon. Two electrons may scatter by exchanging a virtual photon, or by exchanging a virtual photon which momentarily decays into an electron-positron pair, etc... The sum of each of these "paths" contains all the nuances of the coupled fields.

Interactions are somewhat peculiar insofar as they arise naturally by imposing that fermions be symmetric under the local action of specific groups. The electronic field, for instance, is invariant up to a change in phase $e^{i\phi}$, a well-known fact in quantum mechanics. If one amends the theory so that the dynamics of the field are left invariant up to a change in *local* phase $e^{i\phi(x,t)}$ too, one finds oneself

with a **charged** electronic field whose interactions are mediated by a photon field!

It is then said that the symmetry group $U(1)$, of which the $e^{i\phi}$ phases have been promoted to local phases $e^{i\phi(x,t)}$, has been **gauged**. The structure of the Standard Model of particle physics' (SM) gauge group is $U(1)_Y \times SU(2)_L \times SU(3)_c$.

The Standard Model is built from the gargantuan amounts of data retrieved from all areas of particle physics. It is the most precisely measured physical theory of all time, and accounts for almost all phenomena observed. Nonetheless, physics often calls for endeavors beyond the Standard Model (BSM). These are, for instance:

- Explanations to unsatisfactory or "coincidental" aspects of the theory: the number of families, apparent fine-tuning of some parameters, the non-quantum nature of gravity, etc.
- Empirically motivated extensions which stem from phenomena or measurements⁷ in conflict with the classical SM theory: neutrino oscillations, dark matter, the muon $g-2$ ⁸, etc.

Among these extensions lies the subgroup of those which, intentionally or not, mediate additional neutrino-neutrino interactions. Either way, neutrino physics can then play a crucial role in inferring bounds on the extension's parameters or reject it altogether. When discussed in a model-independent way, such interactions are typically called "secret". The case of interest for this thesis is the interaction that arises when gauging the $L_\mu - L_\tau$ -generated $U(1)$ symmetry. This interaction is mediated by a boson called Z' .

1.2.2 Motivations for a $L_\mu - L_\tau$ Z' -boson

A guiding principle when introducing a new boson is that the gauged symmetries it arises from must be *anomaly-free*: the conservation law associated with the symmetry –and thus the conserved charge– must still hold after the quantization procedure.

Straightforward candidates for Standard Model extensions are the $U(1)$ symmetries naturally arising from the lepton family numbers and baryon number conservation. The $U(1)$ symmetries stemming from these are global and not gauged. They are, unfortunately, anomalous. The only way to construct symmetries that are anomaly-free is to then cleverly combine these conserved quantities into linear combinations of exactly opposite charge, so as to cancel the anomalies. These are:

- The difference between lepton family numbers: $L_e - L_\mu$, $L_e - L_\tau$ and $L_\mu - L_\tau$,
- The difference between the baryon number and the total lepton number: $B - L$,
- The difference between the baryon number and three times any family lepton number "equivalent": $B - 3L_\alpha$, $B - \frac{3}{2}(L_e + L_\tau)$, ...

The first class of models, $L_\alpha - L_\beta$, is the simplest insofar as it does not require the introduction of right-handed neutrinos to cancel gauge anomalies [37], favoring them under Ockham's razor.

⁷No deviations have been greater than 5σ , the statistical significance threshold typically chosen.

⁸At least until June 2025.

Those including the electronic lepton number have long been sufficiently constrained by experiments to be excluded [37]. Thus the most straightforward extension that can be added to the Standard Model is the $L_\mu - L_\tau$ family lepton number difference.

Gauging the $L_\mu - L_\tau$ global symmetry leads to the appearance of a Z' boson mediating new interactions between the muon, the tau and their respective neutrino flavors. Such a Z' is parametrized by its mass, $M_{Z'}$, and its coupling constant, $g_{Z'}$. The interaction is governed by the Lagrangian density [19, 37]:

$$\mathcal{L}_{int}^\alpha = g_{Z'} \cdot Z'_\alpha \cdot [\bar{\mu}\gamma^\alpha\mu + \bar{\nu}_\mu\gamma^\alpha P_L\nu_\mu - \bar{\tau}\gamma^\alpha\tau - \bar{\nu}_\tau\gamma^\alpha P_L\nu_\tau] \quad (1.23)$$

With P_L the projector on the left-handed (resp. right-handed) component of the neutrino (resp. anti-neutrino) field. The sign difference naturally comes from the linear combination of family lepton numbers considered.

The reason the $L_\mu - L_\tau$ symmetry gauging has been a prevalent hypothesis in the last few decades is that some tensions in particle physics have seemed to be alleviated by its introduction.

The (historical) Δa_μ discrepancy

The anomalous magnetic moment a of the electron is an extraordinarily well measured physical quantity, in stark agreement with the theoretical value. It is one of mankind's deepest reach into the subatomic world.

The Dirac equation predicts a Landé factor value $g = 2$. Precise computations in Quantum Electrodynamics (QED) have revealed it to be slightly different, or *anomalous*. The anomalous magnetic moment for charged lepton l is then defined as:

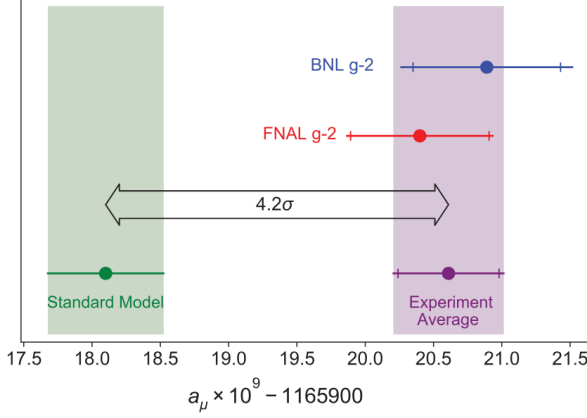
$$a_l = \frac{g_l - 2}{2} \quad \text{with} \quad \vec{\mu}_S = g_l \cdot \frac{\mu_B}{\hbar} \vec{S} \quad (1.24)$$

with μ_B the Bohr magneton and $\vec{S}$ the spin angular momentum.

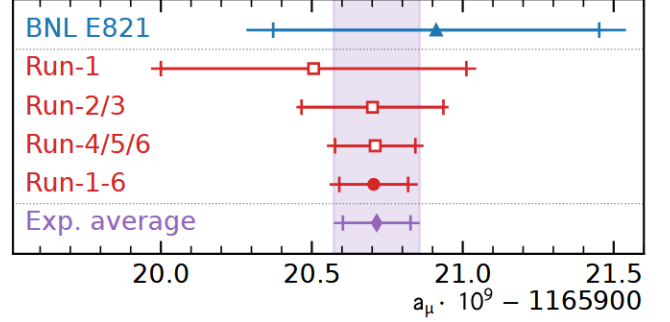
The muon anomalous magnetic moment, and even more so that of the tau, is a harder quantity to measure, due to their comparatively small lifetime with respect to the electron's. The experiments in which a_μ is measured typically feed muons into a magnetically confined loop. As muons circle this loop, their spins progressively precess until eventually they decay to electrons emitted along the spin axis. The knowledge of this axis and the magnetic field within the loop allows one to retrieve the muon's $g-2$, through the Larmor precession rate.

Figure 1.8 (a) shows the theoretical value as well as the value measured at the Brookhaven National Laboratory and the first run of the Fermi National Accelerator Laboratory's muon $g-2$ experiment [1]. The discrepancy measured was $\Delta a_\mu = (251 \pm 59) \times 10^{-11}$, at a 4.2σ significance. In June 2025, runs 2 to 6 at Fermilab were concluded, effectively measuring a_μ to 127 ppb, within previous experimental bounds. The combined results are shown in Fig. 1.8 (b).

Another breakthrough publication in June 2025 was the *Muon $g-2$ Theory Initiative's* publication of their 2025 White Paper [3], in which lattice QCD computations of vacuum polarization have refined the theoretical prediction of a_μ and considerably reduced the discrepancy to $\Delta a_\mu = (38 \pm 63) \times 10^{-11}$, effectively removing the tension and hence the need of a $L_\mu - L_\tau$ gauged symmetry to alleviate it.



(a) Abi et al. (2021) [1].



(b) The g-2 Collaboration (2025) [2].

Figure 1.8: Left: the old 4.2σ discrepancy between theoretical predictions and BNL E821 + FNAL-run-1 values of a_μ . Right: the newest measurements of a_μ as conducted by Fermilab (red). The latest theoretical measurements is $a_\mu \times 10^9 - 1165900 = 20.33(62)$, suppressing the tension [3].

The (historical) R_K and R_{K^*} anomalies

A lesser motivation for gauging $L_\mu - L_\tau$ was the branching ratio of the B-meson decay into $K^{(*)}\mu^+\mu^-$ relative to $K^{(*)}e^+e^-$, whose expected value is found to be in disagreement with measurements at 2.5σ at the LHCb experiment [38]:

$$R_{K^{(*)}} = \frac{\mathfrak{B}(B \rightarrow K^{(*)}\mu^+\mu^-)}{\mathfrak{B}(B \rightarrow K^{(*)}e^+e^-)} \quad (1.25)$$

If the Z' boson arising from $L_\mu - L_\tau$ couples, even slightly, to quarks, it is expected to provide additional interfering paths capable of amending $R_{K^{(*)}}$ to the measured value [39]. This tension also faded away in the light of new theoretical computations as well as higher statistics.

The Hubble tension

It is well-known that estimates of the Hubble constant H_0 converge to two separate values depending on the class of methods employed to measure it [40]. On the one hand, "late Universe" observations of astrophysical objects –typically standard candles–, yield $H_0 \approx 73.2 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$. On the other hand, "early Universe" measurements of the CMB yield $H_0 \approx 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

Among many different explanations for this tension, one consists in introducing a light gauge boson such as the one derived from gauging $L_\mu - L_\tau$ [41]. If such a Z' existed, the enhanced muon and tau annihilation in the early Universe would have led to a temperature surplus in ν_μ, ν_τ and Z' [14], and a higher ΔN_{eff} . This implies a higher radiation energy density than can be inferred from light alone, increasing the early expansion rate and thus the CMB-inferred H_0 value. This reduces the tension with late-Universe measurements of H_0 .

1.2.3 Feynman rules for the Z'

The experimentally relevant quantities computed in QFT are decay widths and cross-sections. In the same spirit as that of rules 1.2 and 1.3, to each event (such as the annihilation of particles a, b and subsequent emission of particles c, d) is attributed a probability amplitude that can decomposed into

the contribution of each possible "intermediate state" or more precisely each process that yields the overall $a + b \rightarrow c + d$ configuration.

Typical computations assume the initial and final states to be free quantum particles. As the unitary evolution of the process can be perturbatively expressed in powers of the coupling constant—the strength of the interaction—the n th power of that series can be interpreted as the contribution to the amplitude of all n point-like interactions in space-time separated by free propagation of the constituents.

Richard Feynman was the first to realize the power of this interpretation, lying in its consistency, as it allowed to enumerate, represent and interpret the amplitude-contribution of each possible process at a given order. These are the so-called Feynman diagrams. At leading order, the main contribution of neutrino-antineutrino scattering comes from the s-channel, the annihilation into a Z' and its subsequent decay, due to resonance:

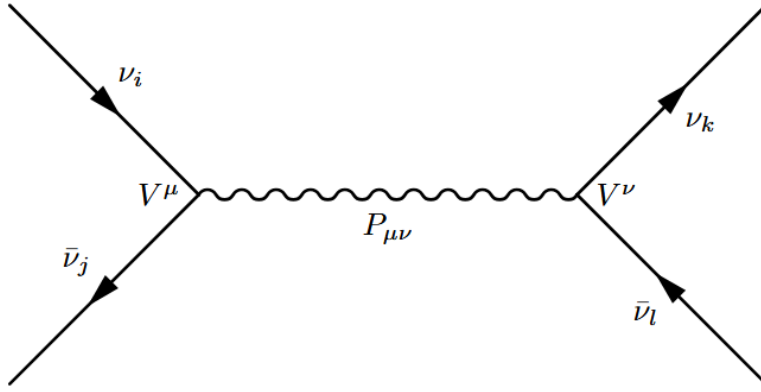


Figure 1.9: Tree-level s-channel Feynman diagram for the $\nu_\alpha + \bar{\nu}_\alpha \rightarrow \nu + \bar{\nu}$, mediated by the Z' boson.

The propagator and vertex associated with this interaction are respectively:

$$P_{\mu\nu} = \frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_{Z'}^2)}{s - M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}} \quad (1.26)$$

$$V_{\alpha\beta}^\mu = -\frac{i}{2}g_{Z'}Q_{\alpha\beta}\gamma^\mu(1 - \gamma^5) \quad (1.27)$$

with i the imaginary unit, μ, ν four-vector indices, $M_{Z'}$ the mass of the Z' , $\Gamma_{Z'} = \frac{g_{Z'}^2 M_{Z'}}{12\pi}$ the decay width of the Z' , s the center-of-mass energy squared of the kinematics, γ^μ and γ^5 the five Dirac matrices and $g_{Z'} \cdot Q_{\alpha\beta}$ the coupling constant between ν_α and ν_β . $g_{\mu\nu}$ is the spacetime metric while q_μ is the four-momentum of the virtual Z' .

These are the core quantities required to compute the cross-section for neutrino-neutrino scattering mediated by the Z' or the effective potentials neutrinos experience when propagating through a neutrino background. Computing these based on the above Feynman diagram is the aim of Chapters 2 and 4.

1.3 The $L_\mu - L_\tau$ gauged symmetry: Signatures

That a hypothesis explains an observation is not sufficient to validate it. While the existence of the Z' is motivated by the aforementioned physics, it also needs to be validated by experiments seeking for

its signature. As of today, no experiments have established its existence. Such inquiries instead yield constraints on the parameters of the interaction in the form of exclusion regions.

- Neutrino trident production, the scattering of a neutrino off a nucleus and subsequent production of a $\mu^+\mu^-$ pair, excludes too strong a coupling for a variety of different masses [42].
- BBN observations set bounds on the expansion rate of the early Universe and thus on the Z' whose existence increases it. Masses below the MeV-scale are typically rejected [41].
- Finally, too great a mass should create noticeable accelerator signals (such as those in [39, 43]) which can also be constrained. This is typically why gauged lepton number differences including the electronic flavor L_e are forbidden, due to too high a constraint.

In addition to these, the discrepancies mentioned above are solved by the Z' within specific regions of the parameter-space, orienting (historical) searches toward these. While the muon $g-2$ is no longer under tension, the typical values required to mimic the "historical" Δa_μ [19, 44] are very similar to those required to see the Hubble tension disappear [41]. These are light bosons with masses between a few MeV to a few dozen MeV with a coupling from 10^{-4} to 10^{-3} .

In order to stay within the bounds allowed by neutrino trident production while still retaining physical relevance, we focus our search on $M_{Z'} \in [1, 100]$ MeV, and $g_{Z'} \in [1, 10] \times 10^{-4}$. Extending the mass range to a few hundred MeV's would allow probing the domain inducing the (historical) corrections to the $R_K - R_{K^*}$ branching ratios [39].

In the past decades, the possibility of using very high energy astrophysical neutrinos to further constrain the existence of a $L_\mu - L_\tau$ gauge symmetry has progressively been explored [7–20]. This pursuit was further heightened with the advent of the IceCube Neutrino Observatory and its yet unparalleled reach to the high-energy tip of the neutrino spectrum.

Chapters 2 and 3 revisit such an endeavor by first independently reproducing the already existing results. In Chapters 4 and 5, I examine the effective potentials the Z' will introduce to neutrinos propagating through the C ν B and the related impact on the flavor ratios, an effect ignored up to now.

Let us first give a brief review of the literature pertaining to the effect of a light gauge boson on the astrophysical neutrino spectrum, the idea at the core of this master's thesis.

1.4 Probing ν -Secret Interactions with Astrophysical Neutrinos

The idea that astrophysical neutrinos may constrain new physics is not new. Already in 1987, Kolb & Turner [8] and Manohar [7] put forth the possibility that neutrino secret/self-interactions (ν SI) could be constrained given the first measurements of astrophysical neutrinos originating from a known-distance source: SN1987a. Indeed, too strong a secret interaction would effectively scatter all neutrinos on their route to Earth to lower energies and other directions. The observations of neutrinos associated with the supernova allowed the authors to place direct constraints on the intensity of such interactions.

Three decades later, in 2007, Dan Hooper [9] put forth the possibility that new gauge bosons, the likes of which had been proposed to account for the 511 keV SPI-Integral galactic bulge emission, could be constrained by VHE neutrino observations in upcoming detectors such as IceCube. The new gauge boson, mediating "secret interactions", would create an absorption feature in the spectrum's tail due to the resonance with the gauge boson's mass.

The author did not explore the particular case of the $L_\mu - L_\tau$. Instead, an "agnostic" approach to the nature of the coupling was taken, exploring ranges of mass and coupling constant for the a generic boson with a similar cross-section. Under the assumption that only the annihilation (s-) channel mattered in the interaction, the cross-section they computed has the traditional Breit-Wigner shape, albeit with a pre-factor differing from that which we find in Chapter 2.

While Hooper points out that the very narrow decay width of the boson leads to a very narrow line in the spectrum, he also remarks that the expansion of the Universe through which the neutrinos propagate slowly shifts the entire spectrum to lower energies, so that the depleted region is effectively broadened. This provides the necessary broadening, assuming sufficiently strong coupling, to make an observable dip in IceCube's data.

In 2014, Ioka et al. [10] and Ng et al. [11] independently gave an update on the topic, following the first results by the IceCube Collaboration in 2013. In the latter the collaboration fit the overall 28-events VHE spectrum with a power law of index $\gamma \approx 2$ with a noticeable, albeit statistically insignificant, gap between 500 TeV and 1 PeV.

To explain this gap, both 2014 papers seek to interpret it as a resonance with a MeV-scale unstable particle, as was predicted by Hooper (2007). They remain somewhat agnostic about the origin of such a mediator, not assuming scalar or vector-like nature, and adopt a general form for the interaction, thus constraining a general "coupling constant" and general "mediator mass". The effect of the alleged gauge boson is described by the resonant cross-section and related mean free path of the astrophysical neutrinos as they propagate through the $C\nu B$.

The authors of both studies describe the physics more finely by taking into account the possibility of neutrino cascades. The core idea is that scattered high-energy neutrinos may regenerate the spectrum at lower energies or up-scatter $C\nu B$ neutrinos to join the spectrum.⁹ This is done by numerically solving the computationally expensive Boltzmann equation for the neutrino co-moving density.

The same year, Ibe et al. [12] expanded on the latter works by exploring specific models that could bear such an interaction. They do not consider regeneration but instead explain that if some neutrino masses are below the temperature of the $C\nu B$ (in natural units), typically $\sim 10^{-4}$ eV, their thermal momentum spread cannot be ignored. This leads to a thermal broadening of the resonance, which they also compute. This possibility is also briefly mentioned in Ng et al., albeit only qualitatively at the end of their "Further Discussions" section.

It is in 2015 that the $L_\mu - L_\tau$ gauged symmetry's boson, Z' , enters the scene. In chronological order, Araki et al. [13], Kamada & Yu [14] and DiFranzo & Hooper [15] subsequently study this particular case.

Akin to our previous section, Araki et al. first motivate the Z' due to the g-2 discrepancy, still under tension at the time. Taking into account thermal broadening but not regeneration, they model the synthetic spectrum assuming enhanced scattering by the Z' .

DiFranzo & Hooper further note that in addition to the dip in the total flux measured at IceCube, deviations from the standard 1 : 1 : 1 *flavor ratios* measured at Earth could also be signatures of the new gauge boson. Unlike the latter two studies, they take into account neutrino cascades/regeneration when solving the Boltzmann equation. They further refine the problem by introducing a distribution in neutrino sources with redshift in direct analogy to the Star Formation Rate (SFR).

⁹A nuance is that while Ioka et al. consider regeneration to explain the observed PeV-flux close to the Waxman-Bahcall bound as being $C\nu B$ neutrinos up-scattered by EeV neutrinos, Ng et al. consider it to alter the shape of PeV-flux, not discussing its own origin.

With the 6-year HESE¹⁰ IceCube data published in 2020, endeavors supported with much higher statistics emerged. Adopting the agnostic approach to ν SI, Bustamante et al. [16] did not find statistical evidence for the presence of a new boson, the best fit in the (g_X, M_X) space being favored by the IceCube data at 2.7σ . With the exception of source distribution, they consider all the nuances to the problem statement mentioned previously.

Another paper, by Creque-Sarbinowski et al. [17], drops the time-consuming numerical approach to the Boltzmann propagation equation by using a so-called δ -approximation. Given the very narrow width of the —agnostic Breit-Wigner— cross-section, they approximate the resonance by a dirac delta function which allows for trivial integration.

The next year, in 2021, Esteban et al. [18] review the on-going efforts to seek traces of ν SI in the "high-statistic era" of astrophysical neutrino detectors. They criticize the universal assumption to only consider the s-channel annihilation contribution to $\nu + \bar{\nu} \rightarrow_{(Z')} \nu + \bar{\nu}$, showing other contributions become relevant beyond $E_\nu = M_{Z'}$.

They also showed the δ -approximation is dubious for sufficiently strong a coupling (typically $g \geq 0.001$) and studied the perspectives of IceCube-Gen2 in discerning such secret interactions. They noted the promising enhancements to IceCube's currently low statistics and limited energy range.

Let us end this review by mentioning Hooper et al. (2023) [19]. While solving the Boltzmann integro-differential equation with multiple configurations of sources, they explored the boson parameter space, neutrino masses and mass-hierarchy as well as the possibility that the Z' couples to dark matter. They did not consider matter potentials, however.

¹⁰High Energy Starting Events detected within the IceCube data. The IceCube data, detection methods and prospects are presented in detail in Appendix B.

Chapter 2

Interaction cross-section

The most straightforward way through which the existence of a light gauge boson will manifest itself in the VHE spectrum-tail measured at IceCube is by mediating increased rates of neutrino scattering off other neutrinos. The origins of the scatterers can be manifold, but the largely dominant contributions are those from the $C\nu B$. To properly quantify how an initial spectrum will be altered by such scattering—the aim of Chapter 3—in this chapter we compute the cross-section related with the scattering processes.

Neutrino interactions are notoriously scarce, and those mediated by the Z' boson are no exception. This means that the neutrino spectrum will be generally unaffected by its existence. However, as will become apparent by the end of this Chapter, a light gauge boson such as Z' generates a resonance in the cross-section sufficiently strong to be detectable within IceCube's energy range.

Such a resonance emerges because of the $\nu + \bar{\nu} \rightarrow \nu + \bar{\nu}$ annihilation channel depicted in Fig. 1.9. This is the dominant contribution at resonance by far, so that we limit ourselves to it. The validity of this hypothesis, as challenged in [18], is discussed in Chapter 6. Neglecting the ν masses, the cross-section is expressed as in 2.1, where $i \cdot M$ is the complex value associated with diagram 1.9.

$$\frac{d\sigma(\nu\bar{\nu} \rightarrow \nu\bar{\nu})}{d\Omega} = \frac{|M|^2}{64\pi^2 s} \quad (2.1)$$

Indices of the initial states are denoted by momentum p and spin s , while final states are denoted by momentum k and spin r , with primes on the antineutrino states. The u and v functions of (momentum, spin) are respectively the fermion wavefunction of the neutrinos and antineutrinos. While they are "column spinors", their barred versions are "line spinors". Slashed momentums are dotted with gamma matrices, e.g. $\not{q} = q_\mu \cdot \gamma^\mu$.

From there and diagram 1.9 one finds:

$$i \cdot \mathcal{M} = \bar{u}^r(k) \left(-\frac{i}{2} g_{Z'} \gamma^\mu (1 - \gamma^5) \right) v^{r'}(k') \cdot \left[\frac{-i \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_{Z'}^2} \right)}{s - M_{Z'}^2 + i M_{Z'} \Gamma_{Z'}} \right] \cdot \bar{v}^{s'}(p') \left(-\frac{i}{2} g_{Z'} \gamma^\mu (1 - \gamma^5) \right) u^s(p) \quad (2.2)$$

So that:

$$i \cdot \mathcal{M} = \frac{ig_{Z'}^4}{4} \cdot \frac{1}{q^2 - M_{Z'}^2 + i\Gamma_{Z'} M_{Z'}} \cdot (\bar{u}_k \gamma^\mu (1 - \gamma_5) v_{k'}) \cdot \left[\underbrace{g_{\mu\nu}}_{\rightarrow M_1} - \underbrace{\frac{q_\mu q_\nu}{M_{Z'}^2}}_{\rightarrow M_2} \right] \cdot (\bar{v}_{p'} \gamma^\nu (1 - \gamma_5) u_p) \quad (2.3)$$

The above expression can be split into two terms which we shall call $\mathcal{M}_1$ and $\mathcal{M}_2$.

$$\mathcal{M}_1 = \frac{g_{Z'}^2}{4} \cdot \frac{1}{s - M_{Z'}^2 + i\Gamma_{Z'} M_{Z'}} \cdot (\bar{u}_k \gamma^\mu (1 - \gamma_5) v_{k'}) \cdot (\bar{v}_{p'} \gamma_\mu (1 - \gamma_5) u_p) \quad (2.4)$$

$$\mathcal{M}_2 = \frac{-g_{Z'}^2}{4M_{Z'}^2} \cdot \frac{1}{s - M_{Z'}^2 + i\Gamma_{Z'} M_{Z'}} \cdot (\bar{u}_k \not{q} (1 - \gamma_5) v_{k'}) \cdot (\bar{v}_{p'} \not{q} (1 - \gamma_5) u_p) \quad (2.5)$$

Doing so allows one to compute the absolute squared value of $\mathcal{M}$ as the sum of three terms, $|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + \mathcal{M}_1 \cdot \mathcal{M}_2^*$. Given the identity $(\bar{u} \gamma^\mu (1 - \gamma_5) v)^* = (\bar{v} \gamma^\mu (1 - \gamma_5) u)$, these are respectively:

$$|\mathcal{M}_1|^2 = \frac{g_{Z'}^4}{16[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot (\bar{u}_k \gamma^\mu (1 - \gamma_5) v_{k'} \cdot \bar{v}_{k'} \gamma^\nu (1 - \gamma_5) u_k) (\bar{v}_{p'} \gamma_\mu (1 - \gamma_5) u_p \cdot \bar{u}_p \gamma_\nu (1 - \gamma_5) v_{p'}) \quad (2.6)$$

$$|\mathcal{M}_2|^2 = \frac{g_{Z'}^4}{16M_{Z'}^4[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot (\bar{u}_k \not{q} (1 - \gamma_5) v_{k'} \cdot \bar{v}_{k'} \not{q} (1 - \gamma_5) u_k) (\bar{v}_{p'} \not{q} (1 - \gamma_5) u_p \cdot \bar{u}_p \not{q} (1 - \gamma_5) v_{p'}) \quad (2.7)$$

$$\mathcal{M}_1 \cdot \mathcal{M}_2^* = \frac{-g_{Z'}^4}{16M_{Z'}^2[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot (\bar{u}_k \gamma^\mu (1 - \gamma_5) v_{k'} \cdot \bar{v}_{p'} \gamma_\mu (1 - \gamma_5) u_p) (\bar{v}_{k'} \not{q} (1 - \gamma_5) u_k \cdot \bar{u}_p \not{q} (1 - \gamma_5) v_{p'}) \quad (2.8)$$

Where we have left implicit the spin indices. Averaging over initial and summing over final spins, this results in the following traces.

$$|\overline{\mathcal{M}_1}|^2 = \frac{g_{Z'}^4}{16[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot \text{Tr} [k \gamma^\mu (1 - \gamma_5) k' \gamma^\nu (1 - \gamma_5)] \cdot \text{Tr} [p' \gamma_\mu (1 - \gamma_5) p \gamma_\nu (1 - \gamma_5)] \quad (2.9)$$

$$|\overline{\mathcal{M}_2}|^2 = \frac{g_{Z'}^4}{16M_{Z'}^4[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot \text{Tr} [k \not{q} (1 - \gamma_5) k' \not{q} (1 - \gamma_5)] \cdot \text{Tr} [p' \not{q} (1 - \gamma_5) p \not{q} (1 - \gamma_5)] \quad (2.10)$$

$$\overline{\mathcal{M}_1} \cdot \mathcal{M}_2^* = \frac{g_{Z'}^4}{16M_{Z'}^2[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot \text{Tr} [k \gamma^\mu (1 - \gamma_5) k' \not{q} (1 - \gamma_5)] \cdot \text{Tr} [p' \gamma_\mu (1 - \gamma_5) p \not{q} (1 - \gamma_5)] \quad (2.11)$$

Each of these is evaluated using the identities: $\begin{cases} \text{Tr} [\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] = 4(g^{\mu\nu} g^{\rho\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\rho} g^{\nu\sigma}) \\ \text{Tr} [\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^5] = -4i\epsilon^{\mu\nu\rho\sigma} \end{cases}$

At this stage, one could note that eq. 2.10 and 2.11 are nullified by the Ward identities. I omitted this fact as I derived the computation, however. Because of it, the only traces left to be evaluated are those arising from the $g_{\mu\nu}$ part of the propagator, i.e. that giving $\mathcal{M}_1$.

Because the evaluation of the traces and subsequent replacement of the latter by the Mandelstam variables is a lengthy and unenlightening process, it is not presented here. At the end of that process, one finds:

$$|\overline{\mathcal{M}}|^2 = \frac{4g_{Z'}^4 \cdot u^2}{[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \quad (2.12)$$

where we have made use of the Mandelstam variable u .

This is the omnipresent Breit-Wigner expression for an unstable Z' mediator with width $\Gamma_{Z'}$.

For a $2 \rightarrow 2$ scattering process:

$$a(p_1) + b(p_2) \rightarrow c(p_3) + d(p_4)$$

the Mandelstam variables are defined as:

$$\begin{aligned} s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \approx 2E_\nu \cdot m_\nu && \text{(center-of-mass energy squared)} \\ t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 && \text{(squared momentum transfer between } a \rightarrow c) \\ u &= (p_1 - p_4)^2 = (p_2 - p_3)^2 && \text{(squared momentum transfer between } a \rightarrow d) \end{aligned}$$

They satisfy the identity $s + t + u = \sum_{i=1}^4 m_i^2 = 0$ in the case of massless particles.

The differential cross-section is thus given by eq. 2.13, where the angle θ is measured in the center of mass frame as departure from the axis of collision.

$$\frac{d\sigma_1}{d\Omega} = \frac{g_{Z'}^4}{[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot \frac{s^2}{64\pi^2 s} (1 + \cos(\theta))^2 \quad (2.13)$$

Integrating over the final particles phase space to get the total cross-section, and given the cylindrical symmetry around the axis of collision (φ in $[0, 2\pi]$), one gets:

$$\int_0^{2\pi} d\varphi \int_0^\pi (1 + \cos \theta)^2 \sin \theta d\theta = 2\pi \int_1^{-1} (1 + \cos \theta)^2 (-d \cos \theta) = 2\pi \int_{-1}^1 1 + \mu^2 + 2\mu d\mu = 2\pi \cdot (2 + \frac{2}{3}) = \frac{16\pi}{3}$$

And thus the total neutrino-antineutrino cross-section is:

$$\sigma_1(\nu\bar{\nu} \rightarrow \nu\bar{\nu}) = \frac{g_{Z'}^4}{12\pi} \cdot \frac{s}{[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \quad (2.14)$$

Until now we have not specified the flavor-attribution of the neutrinos involved. If we wish to specify the initial neutrino flavors, note that the interaction Lagrangian density may be expressed as $\mathcal{L} = g_{Z'} \cdot Q_{\alpha\beta} \cdot \bar{\nu}_\alpha \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) \nu_\beta$ with $Q_{\alpha\beta} = \text{Diag}(0, 1, -1)$. Thus given the coupling constants arise in the amplitude through the vertices, we find:

$$\begin{cases} \sigma(\nu_\alpha \bar{\nu}_\beta \rightarrow \nu_\gamma \bar{\nu}_\delta) = \sigma_1 \cdot Q_{\alpha\beta}^2 Q_{\gamma\delta}^2 \\ \sigma(\nu_\alpha \bar{\nu}_\beta \rightarrow \nu\bar{\nu}) = \sum_{\gamma,\delta} \sigma_1 \cdot Q_{\alpha\beta}^2 Q_{\gamma\delta}^2 = 2\sigma_1 \delta_{\alpha\beta} \end{cases}$$

One may also express the cross-section for annihilation between different mass eigenstates. In that case, the Lagrangian density couplings are tilted by a pair of PMNS matrices, namely $\mathcal{L} = g_{Z'} \cdot Q_{ij} \cdot \bar{\nu}_i \gamma^\mu \left(\frac{1-\gamma_5}{2}\right) \nu_j$ with $Q_{ij} = U_{\alpha i}^\dagger Q_{\alpha\beta} U_{\beta j}$. The cross-sections become:

$$\begin{cases} \sigma(\nu_i \bar{\nu}_j \rightarrow \nu_k \bar{\nu}_l) = \sigma_1 \cdot Q_{ij}^2 Q_{kl}^2 \\ \sigma(\nu_i \bar{\nu}_j \rightarrow \nu \bar{\nu}) = \sum_{k,l} \sigma_1 \cdot Q_{ij}^2 Q_{kl}^2 \end{cases}$$

By conservation of the Frobenius norm, $\sum_{k,l} Q_{kl}^2 = \sum_{\alpha,\beta} Q_{\alpha\beta}^2 = 2$, which gives us the typical Breit-Wigner function adapted to our Z' model.

$$\sigma_{\nu_i \bar{\nu}_j \rightarrow \nu \bar{\nu}}(E_\nu) = \frac{g_{Z'}^4}{6\pi} \cdot \frac{2E_\nu \cdot m_\nu}{[(2E_\nu \cdot m_\nu - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \cdot Q_{ij}^2 \quad (2.15)$$

2.1 Cross-section Resonance

The influence of the new gauge boson on both the high energy spectrum and flavor ratios of astrophysical neutrinos will depend on the two dimensional parameter space of said gauge boson, as spanned by its coupling constant and mass ($g_{Z'}, M_{Z'}$).

The cross-section undergoes resonance when the center-of-mass energy is equal to the Z' mass: $s = E_{CM}^2 = M_{Z'}^2$. Given the relatively tiny neutrino masses, one can approximate the center of mass energy to $E_{CM} = \sqrt{2E_\nu \cdot m_\nu}$. The location of the resonance in the incidental energy space is thus:

$$E^{res} = \frac{M_{Z'}^2}{2m_\nu} \approx 5 \text{ PeV} \times \left(\frac{M_{Z'}}{10 \text{ MeV}} \right)^2 \times \left(\frac{0.01 \text{ eV}}{m_{\nu_j}} \right) \quad (2.16)$$

IceCube's astrophysical diffuse neutrino flux is fitted between 2.5 TeV and 6.3 PeV in energy [45]. Figure 2.1 plots the resonance location as a function of $M_{Z'}$. By doing so it shows that the current IceCube data is sensitive only to bosons between 0.5 and 50 MeVs in mass.

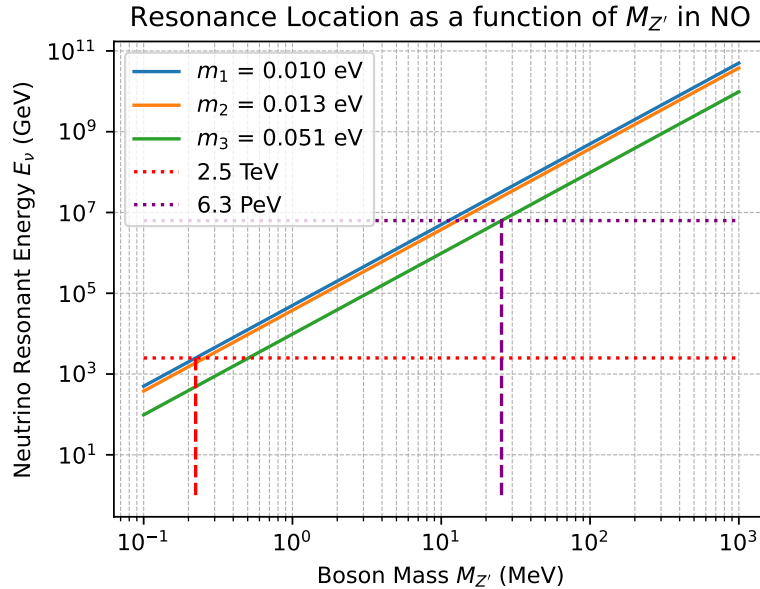


Figure 2.1: Resonant energy locations (GeV) as a function of the mass of the Z' boson (MeV) which is also the center of mass energy given resonance. The red and purple line are the lower and upper bound on the very high energy events observed at IceCube, and the associated vertical lines are the required boson masses to induce a resonant absorption at these energies.

The resonance that is observed in cross section 2.15 will give rise to a peak that is of the order:

$$\sigma^{res} = \frac{24\pi}{M_{Z'}^2} \approx 9.7 \times 10^{-27} \text{ m}^2 \times \left(\frac{10 \text{ MeV}}{M_{Z'}} \right)^2 = 97 \text{ b} \times \left(\frac{10 \text{ MeV}}{M_{Z'}} \right)^2 \quad (2.17)$$

when $E_{CM} = \sqrt{s} = M_{Z'}$ and using $\Gamma_{Z'} = \frac{g_{Z'}^2 M_{Z'}}{12\pi}$.

As expressed above and seen in Fig. 2.3, to each neutrino mass eigenstate of the $C\nu B$ will be associated one resonance in the spectrum. This is because, while it is the center of mass energy which "resonates" at $M_{Z'}$, the incident energy E_ν will give rise to such $E_{CM} = M_{Z'}$ at values dependent on the scatterers' masses.

The smaller the neutrino masses considered the higher the energy of the resonance. These peaks would coincide were we to plot the cross-section with respect to the center of mass energy $E_{CM} = \sqrt{s}$ instead.

Although these are strong resonances, they are compensated by the very small decay width of the Z' boson [19, 37] appearing in the propagator. Because Z' does not couple to electrons and we consider masses $M_{Z'} < 2m_\mu \approx 210 \text{ MeV}$, it may not decay to charged leptons.

$$\Gamma_{Z'} = \frac{g_{Z'}^2 M_{Z'}}{12\pi} \approx 2.65 \times 10^{-12} \text{ GeV} \times \left(\frac{g_{Z'}}{10^{-4}} \right)^2 \times \left(\frac{M_{Z'}}{10 \text{ MeV}} \right) \quad (2.18)$$

The Q_{ij}^2 factors are shown in eq. 2.19, where the j^{th} column corresponds to the $C\nu B$ neutrino mass m_ν . The $\nu_1\nu_3$ and $\nu_2\nu_3$ couplings are orders of magnitude higher than the others, so that at the resonance regions associated with m_{ν_1} and m_{ν_2} it is ν_3 that is most absorbed and *vice versa*. This is also observed in Fig. 2.3.

$$|Q_{ij}|^2 = (U_{\mu i}^\dagger U_{\mu j} - U_{\tau i}^\dagger U_{\tau j})^2 = \begin{bmatrix} 8.85 \times 10^{-3} & 8.37 \times 10^{-5} & 3.10 \times 10^{-1} \\ 8.37 \times 10^{-5} & 5.45 \times 10^{-2} & 6.49 \times 10^{-1} \\ 3.10 \times 10^{-1} & 6.49 \times 10^{-1} & 1.95 \times 10^{-2} \end{bmatrix} \quad (2.19)$$

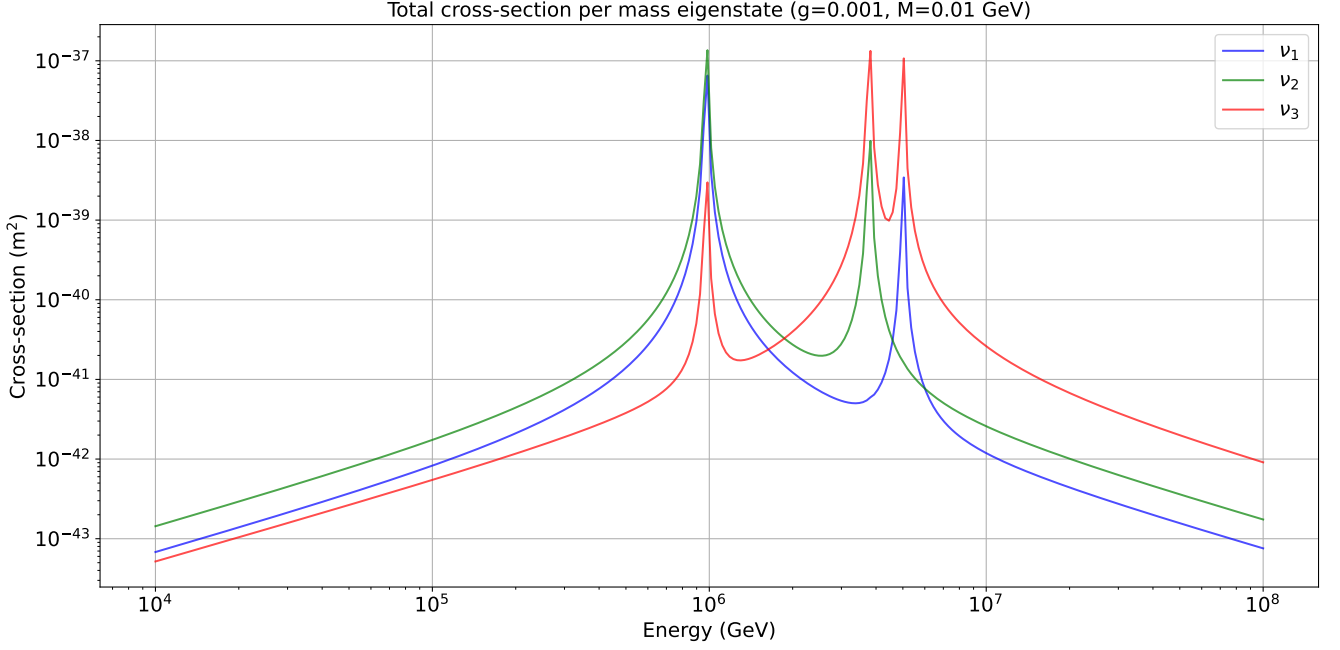


Figure 2.2: Total cross-section each neutrino mass eigenstate experiences while propagating, as a function of incident energy, in NO. The three peaks, which correspond to the three neutrino masses, do not scatter each eigenstate identically. Instead, the couplings 2.19 dictate that the heaviest state (the left-most peak) mostly scatters the two lightest and *vice-versa*. This figure is plotted with a log-grid that doesn't resolve the very narrow width and thus does not reach the maximum value. It is shown here for sake of lisibility.

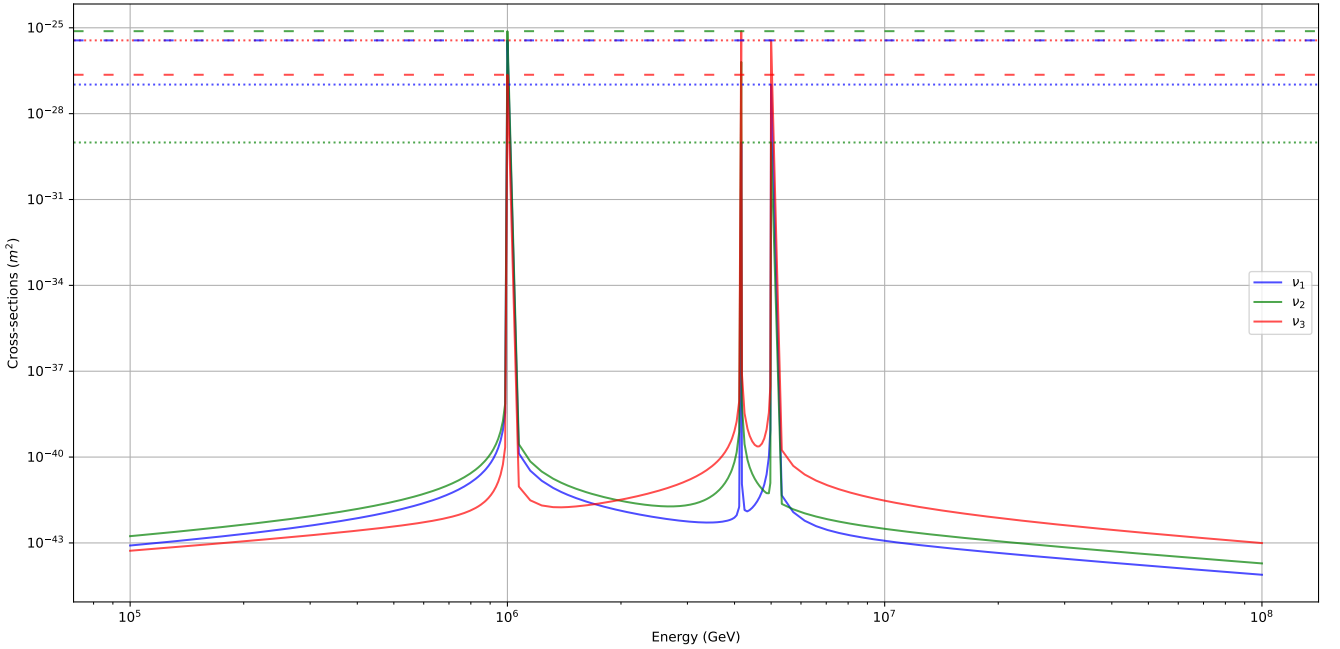


Figure 2.3: The same total cross-section than in Figure 2.2. The left peak is at 1 Pev and the two others are at roughly 5 PeV. The maximum value is around 10^{-26} m^2 . Color legend is identical to the previous figure.

Chapter 3

Synthetic Spectra without Z' Potentials

With the neutrino-antineutrino Z' -induced cross-section computed in Chapter 2, the ground is set to reconduct the analysis of the impact of such new physics on the neutrino spectrum. Following the footsteps of previous work on the topic [7–20], the approach is to define and solve the integro-differential propagation equation ruling neutrinos crossing an expanding Universe permeated by the $C\nu B$ in the presence of the hypothesized $L_\mu - L_\tau$ gauged interaction.

Rapidly, it becomes clear that the integro-differential equation that arises from the physics of the problem can not be analytically solved. To work around this, a python code which uses the `solve_ivp` function from the `scipy.integrate` package to solve the propagation equation has been implemented.

This section is organized in the following way:

- First, the physics captured by the Boltzmann transport equation solved here is described. This includes diffused neutrino production throughout the Universe, cosmic expansion and interactions with the $C\nu B$ mediated by the Z' , namely scattering and regeneration. Neutrino coherence length and oscillations are considered at this stage.
- Second, the python implementation of the integro-differential equation is described. The trickier numerical aspects of the problem, convergence and stiffness, are addressed. To that extent, special care is given to the binning process, which is performed analytically to take care of the extremely narrow decay width.
- Third, the results are presented, namely the synthetic spectrum for a range of boson parameters, neutrino masses, ordering choice and varying redshifts.

To explore the dependencies of the problem on the various physical parameters while keeping the computation time within reason, an intermediate description –typically, one without regeneration– that does not require the integral part of the propagation equation is adopted for most figures.

The rationale behind this choice is that the main goal of this thesis is to explore the effects of matter potentials mediated by the Z' on the IceCube spectrum. To the extent where IceCube does not currently provide enough statistics to compare potential-less with potential-full models in a statistically sound manner, the qualitative features of the spectra are what is relevant in this work.

Furthermore, the complete integro-differential solutions prove to be qualitatively similar to their regeneration-less counterparts and the extensive literature described in Chapter 1 arrives to the same results. Thus, extrapolating the discussions of this section to the regenerated case wherever necessary is straightforward.

Note that to preserve physical intuition, natural units are dropped momentarily.

3.1 Physics of the Propagation Equation

Neutrinos, once emitted by their many astrophysical sources, propagate through space until eventually some reach Earth. During this propagation, different physical phenomena alter their energy spectrum. To capture these effects, the evolution of the neutrino spectra is governed by a Boltzmann-like integro-differential equation.

Comoving Spectral Density

The ν_i spectrum evolving during propagation can be described by means of the comoving spectral number density, $\tilde{n}_i(t, E_\nu)$. Unless otherwise specified, "neutrinos" refer to both neutrinos and antineutrinos of a given kind.

$$\tilde{n}_i(t, E_\nu) = \frac{dn_i}{dE_\nu} \text{ [GeV}^{-1} \text{ cm}^{-3}] \quad (3.1)$$

The latter quantity is related to the observable neutrino flux $J(E_\nu)$ at Earth¹ by [11]:

$$J(E_\nu) = \frac{dN_i}{dA d\Omega dt dE_\nu} = \frac{c}{4\pi} \tilde{n}_i(0, E_\nu) \text{ [GeV}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}] \quad (3.2)$$

Coherence and Density Mixing

The comoving spectral number densities in the mass basis and the flavor basis are related. In the specific case where the oscillations are effectively decoherent, these relations stem directly from the PMNS matrix elements $U_{\alpha i}$ squared (and are also valid for the fluxes).

$$\tilde{n}_i(t, E_\nu) = \sum_{\alpha=1}^3 |U_{\alpha i}|^2 \tilde{n}_\alpha(t, E_\nu) \quad (3.3)$$

$$\tilde{n}_\alpha(t, E_\nu) = \sum_{i=1}^3 |U_{\alpha i}|^2 \tilde{n}_i(t, E_\nu) \quad (3.4)$$

While it seems intuitive that the astrophysical distances these neutrino travel are much larger than the neutrino coherence length, as has been claimed by some authors [17], their very high energies make this a dubious justification. Indeed, the neutrino coherence length (i.e. the distance required for two mass eigenstates wavepackets to separate and induce a loss of interference and thus oscillations) is approximated by [14, 29]

$$L_{coh}^{ij} \approx \frac{4\pi E_\nu^2}{\Delta m_{ij}^2} \sigma_x \quad (3.5)$$

where σ_x is the size of the wave packet at production.

Assuming the traditional pion production of the neutrinos, one finds [14]

$$L_{coh}^{23}(1 \text{ PeV}) \approx 100 \text{ Gpc} \gg \text{Diameter of the Observable Universe!} \quad (3.6)$$

Thus, the wavepackets do not have time to separate as they reach Earth. This being said, the other mechanisms getting rid of the interference terms described in Equation 1.14 are sufficient on their own. Recall the limited energy-resolution of the detector and the uncertainty on the distance traveled since production means that after multiple oscillation lengths, neutrinos of greatly differing phases are binned together: the oscillations are averaged out and the interference terms die out after a few oscillation lengths $L_{osc} \sim \frac{4\pi E_\nu}{\Delta m_{32}^2} \sim 0.0018 \text{ pc}$. Relations 3.3 and 3.4 are thus perfectly valid.

¹More rigorously, J is a spectral intensity.

Neutrino Sources

Massive neutrinos are generated at all redshifts up until a conventional cutoff z_{max} beyond which it is assumed that the contributions become negligible. These sources contribute to the spectrum via the following "emmissivity" term in the master equation:

$$\frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} = J_i(z(t), E_\nu) \quad (3.7)$$

Because sources may not have been constant throughout the history of the Universe, a dependency on time, expressed through the redshift z takes the form of a source distribution function $W(z)$. At a given redshift, the dependence on energy is given by a differential number luminosity $J_z(E_\nu)$.

This differential number luminosity is taken to follow a power law:

$$J_z(E_\nu) = A E_\nu^{-\gamma} \cdot W(z)$$

where the spectral index γ and the normalization coefficient A are chosen to match IceCube's data, and with $W(z)$ the redshift distribution.

Three modes of neutrino production are considered.

- Firstly, the spectrum is generated at a given redshift z_{max} , the boundary value of the differential equation and never replenished throughout propagation. $W(z) = \delta(z_{max})$. This is called the "boundary" mode. It is passed to the solver in the form of $3N_{bins}$ initial conditions $y(0)$ to the coupled system. It serves as a good sanity check as it allows for an easier, analytical, solution.
- The second mode is that of a constant diffuse source throughout the Universe. It was mostly implemented as an intermediate case before the rather empirical SFR-distribution and doesn't hold much purpose analysis-wise. $W(z) = 1$.
- The latter SFR distribution, the third mode, is the main source type used throughout the analysis. The redshift dependency, shown in Figure 3.1 is thus chosen to match the Madau et Dickinson (2014) [46] cosmic star formation history (valid for $z < 8$), that is:

$$W_{SFR}(z) \propto \frac{(1+z)^{2.7}}{1 + (\frac{1+z}{2.9})^{5.6}} \quad (3.8)$$

The idea that neutrinos follow roughly the same redshift distribution than the star formation rate (SFR), while theoretically justified in the case of $\sim$ MeV energy neutrinos created at stellar death, is less certain a hypothesis at the very high energy tail of the neutrino spectrum. Yet, it remains prevalent in the literature wherever a naive "boundary" mode is not sufficient, or convincing (see e.g. [15]).

This is not to say that SFR distribution is a baseless hypothesis in the modern literature. IceCube's VHE spectrum has been shown to be largely isotropic, indicating extragalactic origin, and while there is no solid consensus yet on the specific origin of these neutrinos, the protons necessary for pion production need to be accelerated in environments akin to those of starburst galaxies, at first order well tracked by the star formation rate.

AGNs such as BL Lac objects are also a strong contender, which is why their distribution (low redshift skewed relative to the SFR) has also been used to track the cosmic history of neutrino production (see [19]).

Because the point of this thesis is to explore the matter effects signatures in the flavor ratios, and because the difference between distributions has been studied previously (see above literature), this thesis sticks with SFR distribution as an acceptable proxy for the purpose of its main goals and does not model other arguably realistic distributions such as that of BL Lacs [19].

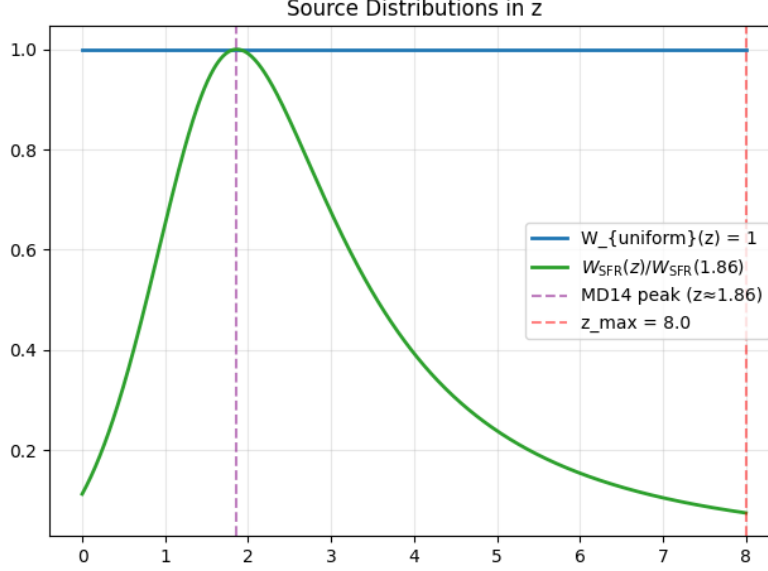


Figure 3.1: Neutrino production profiles in the uniform case (blue) and the SFR-derived case (green), as a function of redshift.

Cosmological Expansion

As the neutrinos propagate, the Universe expands. The relevant parameters to quantify the effect of this expansion are the Hubble rate, $H(z)$, a function of the redshift z . $H(z)$ is given by

$$H(z) = H_0 \sqrt{(1 - \Omega_m) + \Omega_m(1 + z)^3} \quad \text{with } H_0 = 100h \text{ [km s}^{-1} \text{ Mpc}^{-1}] \quad (3.9)$$

where $\Omega_m = 0.315$ and $h = 0.674$ as given by the Planck Collaboration (2018) assuming standard- Λ CDM cosmology [47].

The effect of this cosmic expansion is to shift the entire spectrum to lower energies, with time, as well as to progressively dilute the comoving density. To this end $\tilde{n}_i(t, E_\nu)$ will evolve by virtue of [11, 13, 16–18]

$$\frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} = \frac{\partial}{\partial E_\nu} [H(z) E_\nu \tilde{n}_i(t, E_\nu)] = H(z) \tilde{n}_i(t, E_\nu) + H(z) E_\nu \frac{\partial \tilde{n}_i(t, E_\nu)}{\partial E_\nu} \quad (3.10)$$

where $H(z) E_\nu$ is the energy loss rate.

The cosmic expansion and source terms of the master equation alone are called the *free-streaming* case [11, 18]. Taking into consideration neutrino self-interactions mediated by the Z' boson is done through two additional terms, representing scattering and regeneration.

Scattering

If the $L_\mu - L_\tau$ symmetry is in fact gauged and a fifth force —mediated by the Z' boson— exists in our Universe, then VHE neutrinos on their way to Earth will inevitably scatter off the $C\nu B$. The $C\nu B$ density is $n_\alpha^{C\nu B}(z) \approx 2 \times 56 \cdot (1+z)^3 [\text{cm}^{-3}]$ [18] per flavor, to take into account both neutrino and antineutrino targets. This convert to the same value for $n_i^{C\nu B}(z)$ given the unitarity of the PMNS matrix.

This introduces the following scattering term to the master equation:

$$\frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} = - \tilde{n}_i(t, E_\nu) \sum_{j=1}^3 n_j^{C\nu B}(z(t)) \cdot \sigma_{ij}(E_\nu) \quad (3.11)$$

Where the cross-section σ exhibits one resonance per neutrino mass eigenstate. Indeed recall that:

$$\sigma_{ij}(E_\nu) = \frac{g_{Z'}^4}{6\pi} \cdot \frac{s \cdot (U_{\mu i}^\dagger U_{\mu j} - U_{\tau i}^\dagger U_{\tau j})^2}{[(s - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \quad (3.12)$$

While this thesis considers specifically the case of the Z' stemming from the $L_\mu - L_\tau$ SM extension, the general Breit-Wigner form of cross-section 2.15 makes for an agnostic search for any interaction of the same kind. The scalar mediators, while yielding a different prefactor, will also harbor such a shape and thus induce similar phenomenology. This will not be the case anymore when discussing matter potentials.

Said form takes into account the possibility of resonance while its left (resp. right) tail converges to the heavy (resp. massless) mediator limit and thus to the simpler cross-sections:

$$\sigma(\nu_i \bar{\nu}_j \rightarrow \nu \bar{\nu}) \longrightarrow \begin{cases} \frac{g_{Z'}^4 Q_{ij}^2}{6\pi s}, & s \gg M_{Z'}^2 \\ s \frac{g_{Z'}^4}{M_{Z'}^4} \frac{Q_{ij}^2}{6\pi}, & s \ll M_{Z'}^2 \end{cases} \quad (3.13)$$

where the heavy mediator case has a degeneracy in $\frac{g_{Z'}}{M_{Z'}}$.

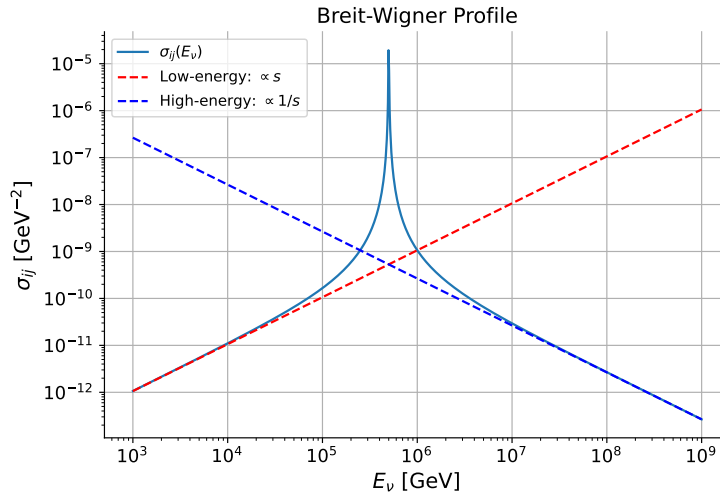


Figure 3.2: Cross-section 2.15, with typical Breit-Wigner profile, as a function of incident energy E_ν , for $M_{Z'} = 10 \text{ MeV}$, $g_{Z'} = 0.001$ and $m_\nu = 0.1 \text{ eV}$. The asymptotes correspond to cases 3.13. Note that these limits are beyond the region where it is safe to assume only the s-channel (resonance) contributes significantly to the total cross-section. This is further discussed in Chapter 6.

Given the expression of the cross-section and the density, the Mean Free Path (MFP) of incident neutrinos as they propagate through the CνB is expressed as:

$$\lambda_i(E_\nu) = \frac{1}{\sum_j n_j^{\text{C}\nu\text{B}} \sigma_{ij}(E_\nu)} \quad (3.14)$$

Taking into account the cosmological expansion with $\frac{dL}{dz} = \frac{c}{H(z)(1+z)}$, the optical depth $\tau(E_\nu)$ is also rather straightforward [11, 13].

$$\tau_i(E_\nu) = \int_0^{L_{\max}} \frac{1}{\lambda_i(z(L))} dL = \int_0^{z_{\max}} \frac{\sum_j n_j^{\text{C}\nu\text{B}}(0)(1+z)^2 \sigma_{ij}(E_\nu(1+z))}{H(z)} dz \quad (3.15)$$

Thus the cosmic expansion effectively broadens the otherwise extremely narrow (see Equation 2.18) absorption features in the spectrum in a range $E_\nu \in \left[\frac{M_{Z'}^2}{2m_\nu(1+z)}, \frac{M_{Z'}^2}{2m_\nu} \right]$. This range in energy will be efficiently depleted if a neutrino's mean free path at resonance $\lambda_i(E_\nu^{\text{res}})$ is significantly smaller than the distance it will remain at resonance given the expansion rate of the Universe, ΔL^{res} .

This distance is such that the associated expansion has shifted the spectrum by more or less a resonance width, which in the E_ν domain is given by $\frac{\Gamma_{Z'} M_{Z'}}{2m_\nu}$, i.e.

$$\frac{E_\nu^{\text{res}}}{1 + \Delta z} - E_\nu^{\text{res}} = \frac{\Gamma_{Z'} M_{Z'}}{2m_\nu} \Leftrightarrow \frac{\Delta z}{1 + \Delta z} \sim g_{Z'}^2 / 12\pi \Leftrightarrow \Delta z \sim \frac{g_{Z'}^2}{12\pi - g_{Z'}^2} \sim g_{Z'}^2 / 12\pi \quad (3.16)$$

where we used the fact $g_{Z'} \in [10^{-4}, 0.01]$ so that $g_{Z'}^2 \ll 1$.

Given $\Delta z = \frac{H(z)\Delta L}{c}$, we get the following criterion for depletion of the broadened absorption feature ($\Delta L^{\text{res}} \gg \lambda(E_\nu^{\text{res}}) \sim \frac{M_{Z'}^2}{24\pi n_{\text{C}\nu\text{B}}}$), with z the redshift at which the neutrino had energy E_ν^{res} :

$$g_{Z'} \gtrsim \sqrt{\frac{12\pi H(z) M_{Z'}^2}{24\pi c n_i^{\text{C}\nu\text{B}}(z)}} \approx 1.63 \times 10^{-4} \times \left(\frac{M_{Z'}}{10 \text{ MeV}} \right) \times \left(\frac{\sqrt{(1 - \Omega_m) + \Omega_m(1+z)^3}}{(1+z)^3} \right)^{1/2} \quad (3.17)$$

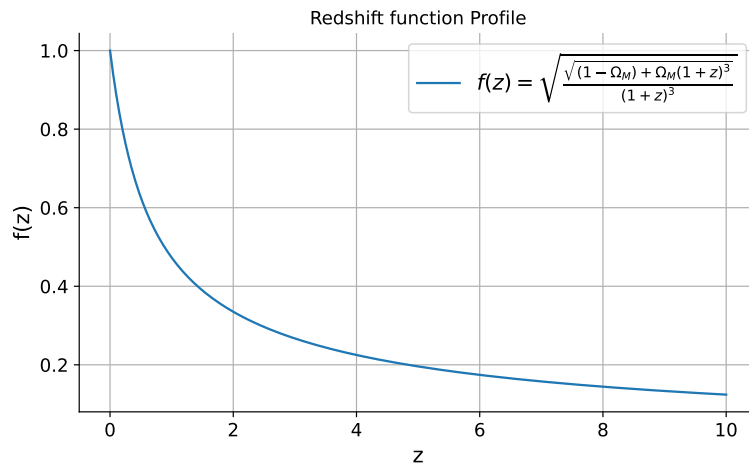


Figure 3.3: Redshift dependency in the depletion criterion 3.17. Neutrinos produced at higher redshift are more easily depleted because the CνB used to be denser and the Hubble rate greater.

Regeneration and Neutrino Cascades

The neutrinos which are scattered may leave the astrophysical beam or be forward scattered and remain on track towards Earth. The regeneration of final state neutrinos at lower energies in the spectrum also takes into account the possibility of neutrino cascades [10, 11], where the secondary products are immediately re-scattered within the beam.

In either scenario, some energy may have been lost to the $C\nu B$ scatterer, so that the interaction regenerates the spectrum at equal or lower energies than that of the neutrino removed. The master equation then gets amended by the integral term [15, 18]:

$$\frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} = \sum_{jkl} n_j^{C\nu B} \int_{E_\nu}^{\infty} \tilde{n}_k(t, E') \cdot \frac{d\sigma_{kj \rightarrow il}}{dE_\nu}(E', E_\nu) dE' \quad (3.18)$$

where E' is the incident neutrino energy regenerating the spectrum at E_ν .

Because the mediator is a vector field, the differential cross-section $\frac{d\sigma_{kj \rightarrow il}}{dE_\nu}(E', E_\nu)$ is not isotropic (it would've been with a scalar field ϕ in place of the Z' [17]). To ease up the numerical integration, this cross-section is approximated as in [15, 19] by:

$$\frac{d\sigma_{kj \rightarrow il}}{dE_\nu}(E', E_\nu) \approx \sigma_{kj \rightarrow il}(E') \cdot g(E', E_\nu) \quad (3.19)$$

$$\text{with } g(E', E_\nu) = \frac{3}{E'} \left[\left(\frac{E_\nu}{E'} \right) + \left(1 - \frac{E_\nu}{E'} \right)^2 \right] \Theta(E' - E_\nu) \quad (3.20)$$

The function $g(E', E_\nu)$ is normalized to 2 when integrated over E' to reflects that each Z' decay produces two neutrinos.

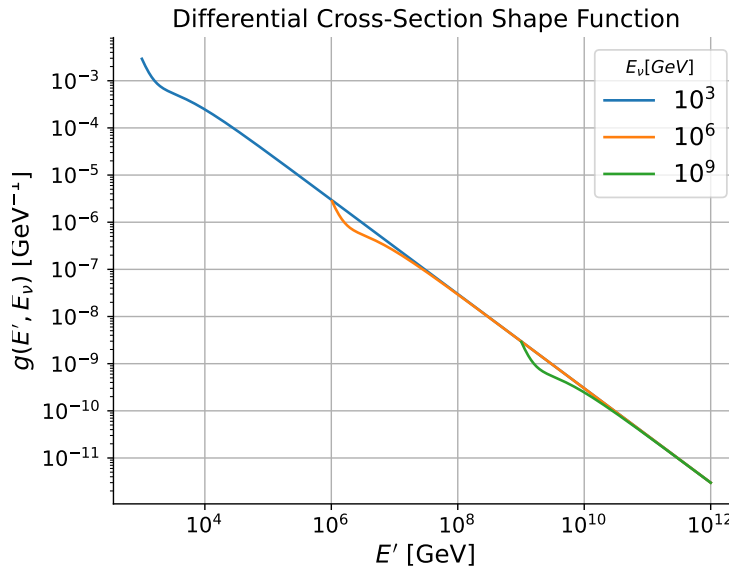


Figure 3.4: Function $g(E', E_\nu)$ as a function of the incident energy E' for different values of the regenerated energy E_ν .

3.1.1 Variable change from t to z

The integro-differential transport equation for the comoving spectral number density of neutrinos mass eigenstate i is thus given by:

$$\begin{aligned} \frac{\partial \tilde{n}_i(t, E_\nu)}{\partial t} = \frac{\partial}{\partial E_\nu} [H(z) E_\nu \tilde{n}_i(t, E_\nu)] + J_i(z(t), E_\nu) - \tilde{n}_i(t, E_\nu) \sum_{j=1}^3 n_j^{\text{C}\nu\text{B}}(z(t)) \cdot \sigma_{ij}(E_\nu) \\ + \sum_{jkl} n_j^{\text{C}\nu\text{B}} \int_{E_\nu}^{\infty} \tilde{n}_k(t, E') \cdot \frac{d\sigma_{kj \rightarrow il}}{dE_\nu}(E', E_\nu) dE' \end{aligned} \quad (3.21)$$

This equation can be recasted in terms of the redshift z rather than time t . In addition, one can get rid of the first term of Eq. 3.21 by means of the dependent variable $Z_i(z, E_\nu)$ rather than $\tilde{n}_i(t, E_\nu)$ [18]

$$\frac{dt}{dz} = \frac{-1}{H(z)(1+z)} \quad (3.22)$$

$$Z_i(z, E_\nu) = (1+z) \tilde{n}_i(z(t), E_\nu(1+z)) \quad (3.23)$$

where the Earth solution for $Z_i(0, E_\nu)$ is the same as that for $\tilde{n}_i(z=0, E_\nu)$.

By virtue of 3.22 and 3.23 and because the double explicit dependence on z invokes the chain rule, it follows that:

$$-H(z) \frac{\partial Z_i}{\partial z}(z, E_\nu) = -H(z) \left[\tilde{n}_i(z, E_\nu(1+z)) + (1+z) \frac{\partial \tilde{n}_i(z, E_\nu(1+z))}{\partial z} \right] \quad (3.24)$$

$$= -H(z) \tilde{n}_i + \left[\frac{\partial \tilde{n}_i}{\partial t} - H(z)(1+z) \frac{\partial \tilde{n}_i(z, E_\nu(1+z))}{\partial E_\nu(1+z)} \cdot \frac{\partial E_\nu(1+z)}{\partial z} \right] \quad (3.25)$$

$$= \frac{\partial \tilde{n}_i}{\partial t} - H(z) \tilde{n}_i - H(z) E_\nu \frac{\partial \tilde{n}_i}{\partial E_\nu} \Big|_{(z(t), E_\nu(1+z))} \quad (3.26)$$

$$= \frac{\partial \tilde{n}_i}{\partial t} - \frac{\partial}{\partial E_\nu} [H(z) E_\nu \tilde{n}_i] \Big|_{(z(t), E_\nu(1+z))} \quad (3.27)$$

Finally, by injecting in the latter expression the master equation as a function of time 3.21, for $E_\nu \rightarrow E_\nu(1+z)$, one gets the equivalent master equation for $Z_i(z, E_\nu)$.

$$\begin{aligned} -H(z) \frac{\partial Z_i}{\partial z}(z, E_\nu) = J_i(z, E_\nu(1+z)) - \frac{Z_i(z, E_\nu)}{1+z} \sum_{j=1}^3 n_j^{\text{C}\nu\text{B}}(z) \sigma_{ij}(E_\nu(1+z)) \\ + \sum_{jkl} n_j^{\text{C}\nu\text{B}}(z) \int_{E_\nu}^{\infty} Z_i(z, E') \cdot \frac{d\sigma_{kj \rightarrow il}}{dE_\nu(1+z)}(E'(1+z), E_\nu(1+z)) dE' \end{aligned} \quad (3.28)$$

With this differential equation, the Z_i neutrinos at energy E_ν are not redshifted anymore, but are instead evolved, at redshift z , as if they actually held an energy $E_\nu(1+z)$. E_ν can be interpreted as the observed energy at detection.

3.2 Implementation of the Propagation Equation

3.2.1 Energy Domain and Numerical Integration

The IceCube very high energy spectrum spans over 6 orders of magnitude, from dozens of TeV to almost 10 PeV. A logarithmic sampling of the energy-space therefore makes perfect sense and is utilized almost everywhere throughout this work. This being said, initially, an additional `res_num` points linearly sampled within $\pm\Gamma_{Z'}$ around each resonant energy is overlapped with the logarithmic grid. An additional `lin_num` linear grid from $E^{res}/(1+z_{max})$ to E^{res} were also added to sample the feature once broadened by redshift. This peculiar sampling is shown in Fig. 3.5.

This non-uniform grid, despite proving useful to showcase the cross-section structure and sample it in easier, analytical cases, turned out to hinder the numerical resolution because of very unequal bin widths. For this reason, it is not used beyond the cross-sections and analytical comparisons. Most if not all of the actual results simply use a logarithmic sample of varying bin number N_{bin} .

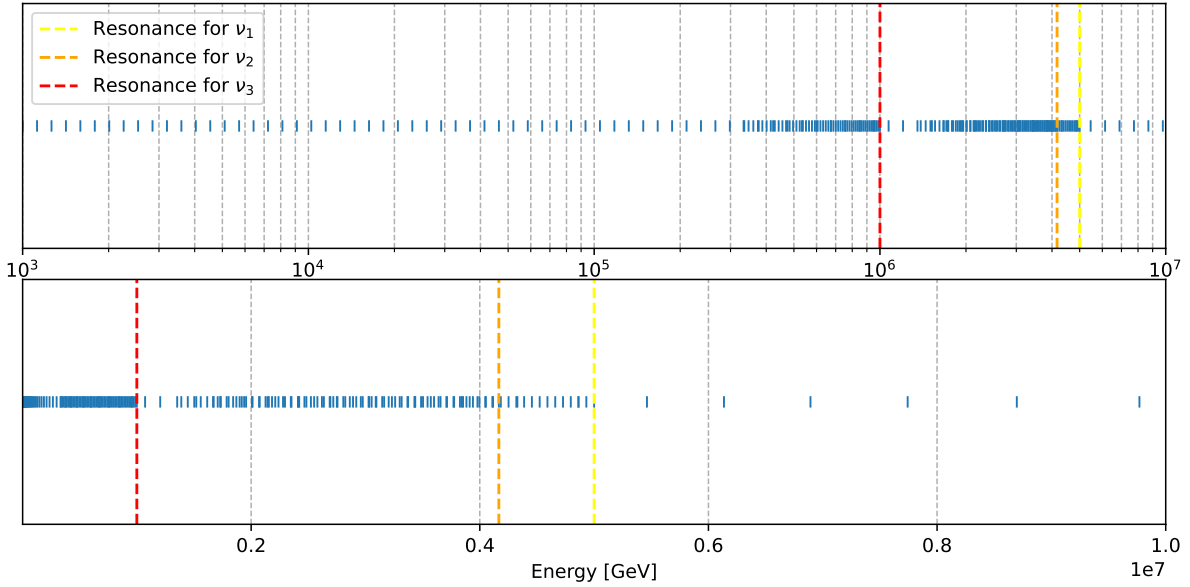


Figure 3.5: Energy Grid in GeV with `log_num` = 100, `res_num` = 10 and `lin_num` = 50 in log-scale (upper frame) and lin-scale (bottom frame) for $z_{max} = 3$. The log-points account for the continuum, the res-points for the resonance width and the lin-points for the broadened absorption region as a consequence of redshift. Resonant energies are given $M_{Z'} = 10$ MeV in normal ordering.

The implementation makes use of the function `solve_ivp` in the `scipy.integrate` python module. The logic of the solver is to numerically integrate a system of (possibly coupled) ordinary differential equations from an initial value $y(x=0)$ given $f(x, y) = \frac{dy}{dx}$, by incrementally adding differentials dy along the way. The step dx is flexible and calculated by the solver by estimating the error at each step and keeping it within relative (`rtol`) and absolute (`atol`) tolerances fixed by the user.

In this case, the $3 \times N_{bins}$ functions $Z_i(E_n)$, one per neutrino flavor and one per energy bin, must be integrated. The Jacobian of the system, whose sparsity may be hinted at to accelerate convergence, is diagonal in the regeneration-less scenario and block-diagonal when regeneration is taken into account.

3.2.2 Diffuse Source Implementation

In the boundary source case, all neutrinos are effectively created at the same redshift $z_{\max}$ through the initial condition $y(0)$ passed to the solver. This mode, although clearly erroneous given the diffuse nature of the extragalactic spectrum at hand, serves two purposes.

First, it allowed to debug the overarching code more easily. Second, it allows for an analytical solution when taking into account redshift through the computation of the optical depth. This solution is then used to sanity-check the numerical one by comparing them together. In principle, diffuse sources could also be analytically solved by convolution with the source's Green function, albeit at the cost of an additional integration. This is not done here.

In the uniform and SFR modes, the initial conditions are set to zero and replaced by an emissivity $J_i(z, E_\nu)$ for detected energy E_ν .

The general source term is thus of the form:

$$J_i(z, E_\nu) = J_0 \cdot W(z) \cdot (1+z)^3 \cdot \left(\frac{E_\nu(1+z)}{E_p} \right)^\gamma \quad (3.29)$$

with $W(z)$ respectively $\delta(z_{\max})$, 1 and $W_{\text{SFR}}(z)$ for the three productions modes, J_0 a normalization constant taken so that the spectra are equivalent in the unattenuated ($g_{Z'} = 0$) scenario, γ the power-law index fitted at IceCube (~ -2.5) and E_p the reference pivot energy (1 TeV) at which the fluxes are normalized, fitted at IceCube.

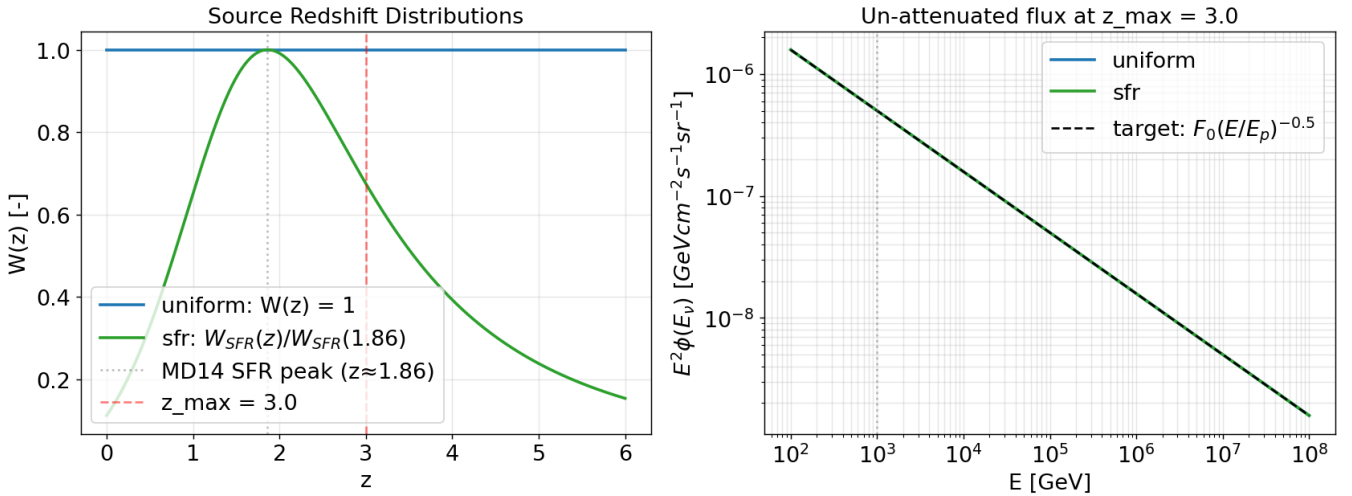


Figure 3.6: Unattenuated fluxes in the three neutrino production modes implemented when the normalizing factor J_0 is chosen to make them match. All three spectra (evidently) coincide. Y-axis is rigorously $E^2 \phi(E_\nu)$.

By recognizing that, in the unattenuated case,

$$F^{\text{obs}}(E_\nu) = \int_0^{l_{\max}} J(z(l), E_\nu) dl = J_0 \left(\frac{E_\nu}{E_{\text{pivot}}} \right)^\gamma \int_0^{z_{\max}} W(z)(1+z)^{3+\gamma} \frac{c}{H(z)} dz = J_0 \left(\frac{E_\nu}{E_{\text{pivot}}} \right)^\gamma I(z_{\max})$$

It is easy to obtain the normalization factor after numerical integration of $I(z_{\max})$, knowing that $J_0 = F(E_{\text{pivot}})/I(z_{\max})$. The flux at 1 TeV is also taken to match IceCube's fit ($\sim 5 \times 10^{-7} [\text{GeV cm}^{-2} \text{s}^{-1} \text{sr}^{-1}]$).

3.2.3 Absorption and Redshift Implementation

The cross-section derived in the previous section exhibits an extremely narrow resonance (more than 10 orders of magnitudes below the GeV). Meanwhile, the energy domain can span multiple orders of magnitudes in GeVs.

A first naive implementation consisted in simply evaluating the attenuation term of the propagation equation at each energy point sampled throughout the domain. This failed tremendously as soon as cosmic expansion got taken into account.

Indeed, neutrinos of observed energy E_ν crossed a resonance region at $z_{res} = \frac{E_{res}}{E_\nu} - 1$ if $z_{res} < z_{max}$. This is modeled in the differential equation by evaluating the cross-section for the E_ν point at $E_\nu(1 + z_{current})$ throughout the solver. Effectively, it is as if the neutrinos did not redshift but the cross-section slid through the spectrum from $E_{res}/(1 + z_{max})$ at $z = z_{max}$ to E_{res} at $z = 0$.

However, the finite Δz steps taken by the solver to integrate the solution correspond to finite ΔE "slides" of the cross-section. Given the infinitesimal width of the cross-section with respect to the energy domain ($\sim 10^{-10/16}$ relative width), it is effectively impossible for the solver to have fine enough a z -grid that it can correctly model every energy point's resonance z -region.

Furthermore, because the Δz steps are dynamically chosen by the solver to keep error estimates within preset tolerance values, it is impossible –at least with `solve_ivp`– to manually decide which redshift values would be mandatory to resolve.

To solve this issue, two solutions seemed feasible. The first was to separate the ODE's z -domain in as many sections as they were energy points, N_E . Rather than solving the ODE in one go in the $[z_{max}, 0]$ interval, repetitive calls to `solve_ivp` would have been made in the $[z_{max}, z_{res,1}] \dots [z_{res,i}, z_{res,i+1}] \dots [z_{res,N_E}, 0]$ successive intervals, with every iteration's initial condition being the last's solution.

This had the drawback of necessitating many successive calls to an already time-consuming solver-pipeline, and did not guarantee that the entire structure of the resonance would be correctly taken into account.

The fix that was instead implemented consists in a *semi-analytical binning approach*. Rather than evolving the n th energy point with the cross-section evaluated at its specific value, the cross-section is analytically averaged over the entire *energy bin* corresponding to the energy point.

By doing so, so long as the Δz increments do not brutally skip over an entire bin (which is guaranteed by comparatively large bin-size with respect to the solver steps²), the cross-section will have an impact on the bin-averaged flux.

While each individual absorption is damped by the averaging, they are more numerous to contribute to a given bin-averaged flux's absorption. The proportions cancel and one retrieves the overall absorption features: the resonance width resolution issue has been bypassed.

At this point, one could trivially simplify the analytical averaging of the cross-section over the bin by assuming it is so thin it may be well approximated by a Dirac distribution.

While this first seems a reasonable approximation, the review by Esteban et al. [18] has concluded that a simple δ -approximation for the cross-section led to qualitatively different (wrong!) results as soon as the interaction strength became greater than $g_{Z'} \sim 0.1$, a parameter range that I do not wish to discard. The relevant figure of their review is presented and discussed in Chapter 6 (Fig. 6.2a).

²It is in this sense that the non-uniform "flex" grid presented at the start of this section did not prove efficient.

The cross-section can be rewritten without approximation as

$$\sigma(E) = \frac{C_{ij} s(E)}{4m_j^2 [(E - E_{\text{res}})^2 + \gamma_E^2]} \quad (3.30)$$

with $C_{ij} = \frac{g^4 |Q_{ij}|^2}{6\pi}$, $E_{\text{res}} = \frac{M_{Z'}^2}{2m_\nu}$ and $\gamma_E = \frac{\Gamma_{Z'} M_{Z'}}{2m_\nu} = \frac{\Gamma_{Z'} E_{\text{res}}}{M_{Z'}}$

Near resonance, the slowly varying numerator $s(E) = 2m_j E$ can be well approximated by its value at the lower bin edge, $s(E_{\text{lo}}) \simeq 2m_j E_{\text{lo}}$. Under this single approximation, the bin integral becomes analytic:

$$\langle \sigma \rangle = \frac{1}{\Delta E} \int_{E_{\text{lo}}}^{E_{\text{hi}}} \sigma(E) dE \quad (3.31)$$

$$\langle \sigma \rangle = \sigma_{\text{max}} \left(\frac{\gamma_E}{\Delta E} \right) \left[\arctan \left(\frac{E_{\text{hi}} - E_{\text{res}}}{\gamma_E} \right) - \arctan \left(\frac{E_{\text{lo}} - E_{\text{res}}}{\gamma_E} \right) \right] \quad (3.32)$$

with the on-resonance peak cross section:

$$\sigma_{\text{max}} = \frac{g^4 |Q_{ij}|^2}{6\pi \Gamma_{Z'}^2}. \quad (3.33)$$

In particular, in the limits where the bin are, respectively, extremely narrow and extremely wide (with respect to γ_E), $\langle \sigma \rangle$ converges to the point value and $\sigma_{\text{max}} \frac{\pi \gamma_E}{\Delta E}$ respectively, so that the total absorption over the bin, $\langle \sigma \rangle \cdot \Delta E$ is bin-independent.

3.2.4 Interpolation and Broadcasting

At this point, it is interesting to note that, in order to accelerate the numerical integration, one imperatively needs to get rid of the costly Python calls/loops within each solver step. To this end, two features are implemented wherever possible:

- Repetitive calls to cross-sections, source terms or regeneration integrals, which may depend on energy as well as neutrino flavor, are performed *prior to* the solver's integration: they are passed to it as arguments. However, because these values depend on redshift, rather than calling my expensive (and poorly optimized) Python functions to compute them at each z-step, a grid of N_z such values are precomputed for a given number N_z of redshifts spanning the $[0, z_{\text{max}}]$ range, and the solver steps consists in **linear interpolations** between said grid values at z_{current} .
- When precomputing such multi-dimensional arrays (for instance, the cross-section(i, j, E_ν , z) is $3 \times 3 \times N_{\text{bins}} \times N_z$), special care is given to vectorize the call in broadcasted numpy arrays so as to leverage the much more efficient C implementations of array operations.

I want to stress that these two points, and especially how to broadcast to/vectorize through numpy in order to loop the successive calls to the physical functions in C rather than in python, was suggested to me by an LLM (ChatGPT) after I queried it for ways to optimize the implementation. I also stress that it has not helped me implement the physics described above, nor has it directly written the code in the files: I have only used it in a "stackexchange-like" fashion. It has of course not written any text within this manuscript, though it might've helped translating a couple of expressions.

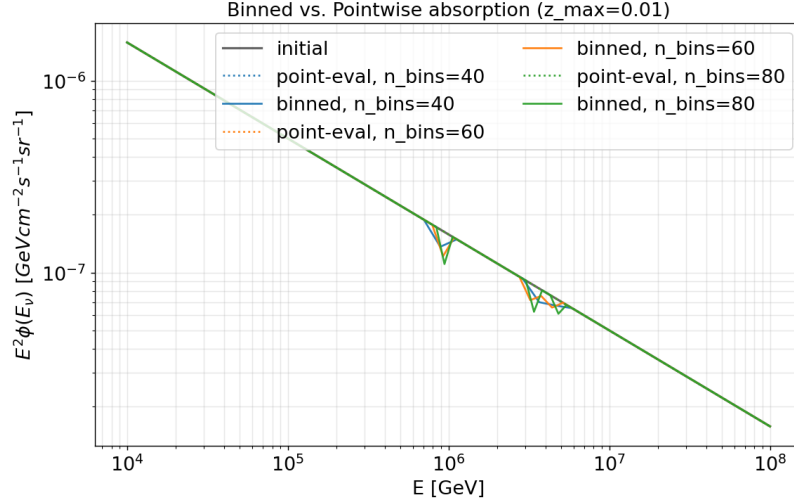


Figure 3.7: Comparison between point-wise and bin-averaged attenuation for different N_{bins} with a low $z_{max} = 0.01$. The point-wise attenuated spectrum completely misses the absorption feature while the bin-averaged ones resolve it.

3.2.5 Regeneration

After neutrinos $(\nu_k, \bar{\nu}_j)$ annihilated, the branching ratio of the Z' decay into the specific pair $(\nu_i, \bar{\nu}_l)$ is

$$\text{Br}(Z' \rightarrow \nu_i \bar{\nu}_l) = \frac{|Q_{il}|^2}{\text{Tr}[|Q|^2]} = \frac{|Q_{il}|^2}{2} \quad (3.34)$$

where the trace is basis-invariant and equals 2 because $\text{diag}(0, 1, -1)^2 = \text{diag}(0, 1, 1)$. Summing over the unobserved partner l , the expected number of ν_i produced per decay is:

$$B_i = \sum_l \frac{|Q_{il}|^2}{2}$$

where the relation $\sum_i B_i = 1$ enforces total neutrino conservation.

The regeneration term is thus the energy-integral over all parent neutrinos at $E' \geq E_\nu$ that scatter and produce a ν_i at E_ν :

$$R_i(E_\nu, z) = B_i \cdot n_{\text{C}\nu\text{B}}(z) \int_{E_\nu}^{\infty} g(E', E_\nu) \left[\sum_{j,k,l} Z_k(E', z) \sigma_{k,j \rightarrow i,l}(E'(1+z)) \right] dE' \quad (3.35)$$

which becomes a sum when the equation is discretized for the ODE solver.

$$R_i[n] = B_i \cdot n_{\text{C}\nu\text{B}}(z) \sum_{e \geq n} G[e, n] \Delta E_e \left[\sum_{j,k,l} F_k[e] \sigma_{k,j \rightarrow i,l}(E_e(1+z)) \right] \quad (3.36)$$

with $G[e, n] := g(E_e, E_n)$ a precomputed triangular matrix ($G[e, n] = 0$ for $e < n$) and $\Delta E_e = E_{e+1} - E_e$. The vectorized evaluation of $n_{\text{C}\nu\text{B}}(z)$ and $\sigma(z)$ on the z -grid allows the interpolation of the said values at z_{current} for each solver step, as described in the previous section.

3.3 Error Tolerance and Convergence

The accuracy of the numerical solution is controlled by two pairs of parameters: the solver's step-error tolerances (`rtol` and `atol`), and the discretization parameters introduced (the energy bin count N_{bins} and the redshift grid size n_z). Both must be chosen carefully because the default solver parameters (`rtol` = 10^{-3} and `atol` = 10^{-6}), and a poorly resolved redshift grid are inadequate for the physics of interest.

3.3.1 Solver tolerances

`solve_ivp` default parameters are trivially insufficient: the astrophysical flux at the pivot energy is itself $E_p^2 \phi(E_p) \sim 10^{-6} \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$, of the same order of magnitude as the default absolute tolerance. The solver would then admit errors comparable to the signal itself, producing oscillations on the scale of the spectrum and returning small negative fluxes inside saturated absorption regions.

Both tolerances are thus set to the physical flux normalization:

$$\text{rtol} = 10^{-6}, \quad \text{atol} = 10^{-12} \max(F_0, J_0 \cdot l_{\max}) \quad (3.37)$$

The relative tolerance is set six orders of magnitude below the flux scale, ensuring that the solver remains physically coherent throughout the deep dips. The absolute tolerance is set twelve orders of magnitude below the maximum between the initial flux F_0 (relevant for the boundary mode) and the normalization source emissivity (over the entire distance) $J_0 \cdot l_{\max}$.

The purpose of `atol` is to define the "zero floor" where values below it are dominated by noise and lose meaning. They are set to zero when making flavor ratio plots in order to not show error-dominated ratios in completely depleted regions of the spectrum.

3.3.2 Energy grid: N_{bins} and Redshift grid: n_z

After semi-analytical binning (binning and interpolation) was implemented, the non-uniform sample of the energy domain became obsolete and was discarded. The domain is instead simply sampled logarithmically, which keeps the ratio $\Delta E_e / E_e$ constant. The number of bins is the number of sample points minus one. For an energy range $[10^4, 10^8] \text{ GeV}$ with $N_{bins} = 300$, $\Delta E / E \approx 0.031$.

The redshift grid calculated from n_z and $z_{\max}$ is used to make vectorized calls to the physical quantities that change with redshift: the cosmic neutrino density $N_\nu(z) = N_0(1+z)^3$, the cross-section 4D tensor $\sigma_{ij}(E(1+z))$ and, when matter effects will later be taken into account, the PMNS angles used to retrieve Q_{ij} factors.

3.3.3 Convergence

Although it is not rigorously analytical, one can also retrieve a solution when taking into account cosmic expansion by numerically integrating the optical depth given in Eq. 3.15 and bin-averaging for consistency. This is what is presented in Figures 3.8 and 3.9. In both figures convergence is clear for n_z values resolving each bin, but in "boundary" source mode the three $z_{\max}$ -resonant bins diverge.

They are artifacts of this point-source neutrino production scenario, whereby the flux is initialized at the boundary condition and no additional neutrinos are added during propagation. They disappear in the "SFR" and "constant" diffuse production scenarios.

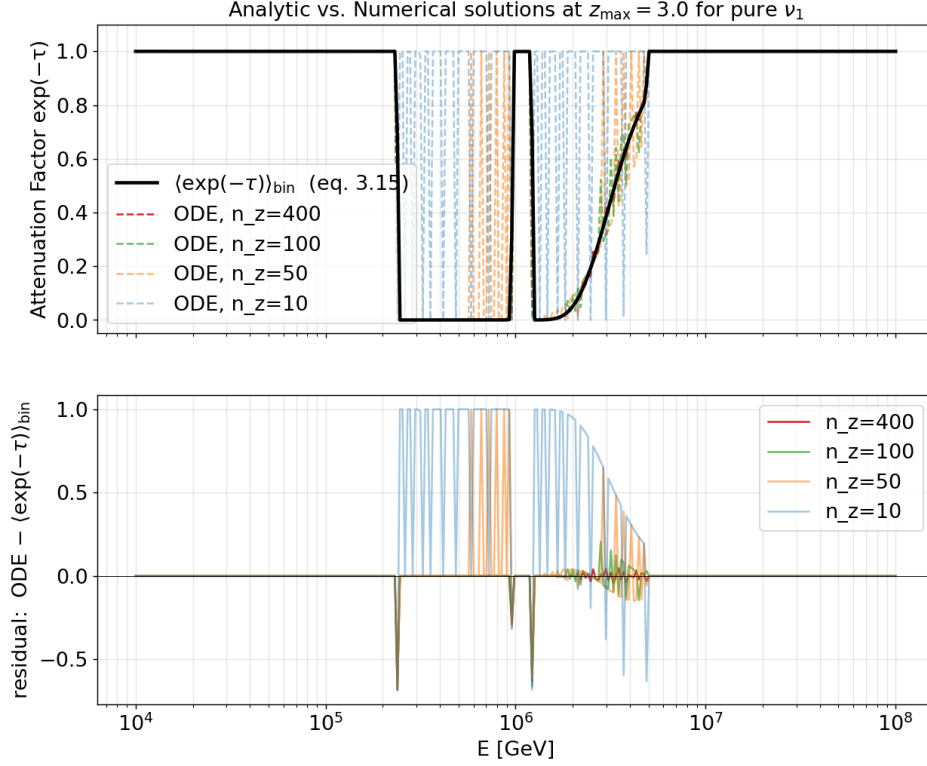


Figure 3.8: Comparison between the pure ν_1 "boundary" flux attenuation factor, with constant energy distribution for simplicity. This serves to showcase the solution oscillations within the absorption region for insufficient n_z , whereby the Δz steps do not resolve every few other bin.

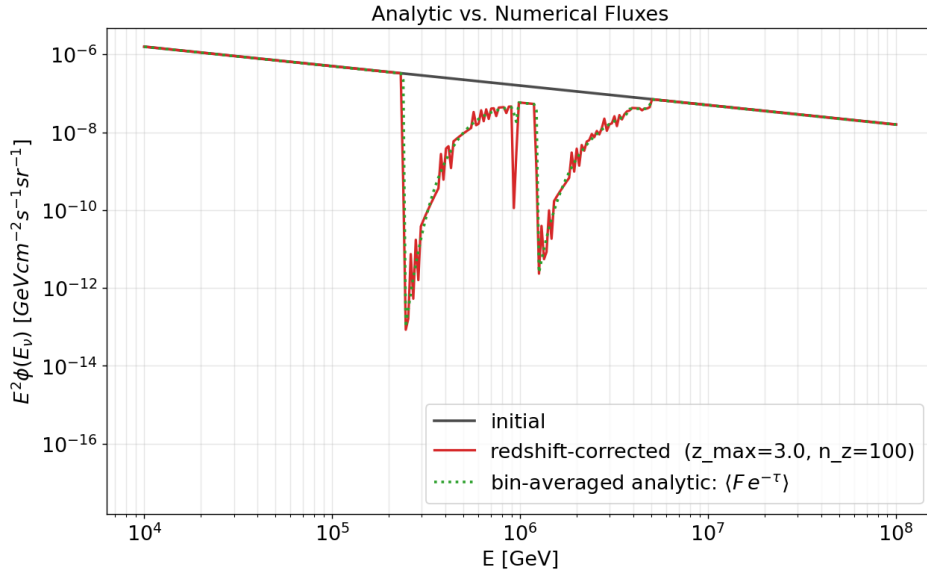


Figure 3.9: Comparison in the case of the actual spectra (boundary 1:2:0 production) for $g_{Z'} = 10^{-3}$, $M_{Z'} = 10$ MeV, $n_z = 100$, $N_{\text{bins}} = 300$. Both solutions superimpose with the slight oscillations of the numerical one (red) deviating slightly from the analytic one (dotted green).

3.4 Synthetic Spectra

This section aims at presenting the simulated spectra and how the characteristics of the physics reveal themselves within them. Indeed, the source distribution (point-source vs. distributed, z_{max}), the boson parameters ($g_{Z'}$, $M_{Z'}$), the neutrino properties (hierarchy, $\sum m_\nu$, mixing angles) all affect the spectrum or flavor ratios in quantitatively and qualitatively noticeable ways.

The graphics are presented in this way: first the effect of source distribution and regeneration, then of boson parameters and finally of neutrino physics constants.

3.4.1 Signature of the propagation physics

Although the point-source scenario proved useful to test the convergence of the solver in an expanding Universe, the neutrino spectrum observed at IceCube has been mostly isotropic, and thus affected an extragalactic, diffuse, origin. A spectrum such as Figure 3.9 is expected to be less markedly absorbed in a diffuse emission context. This is shown in Figure 3.10, where the constant and SFR shaped distributions yield a much less pronounced feature. The SFR distribution scenario will be kept as the norm throughout the rest of this thesis.

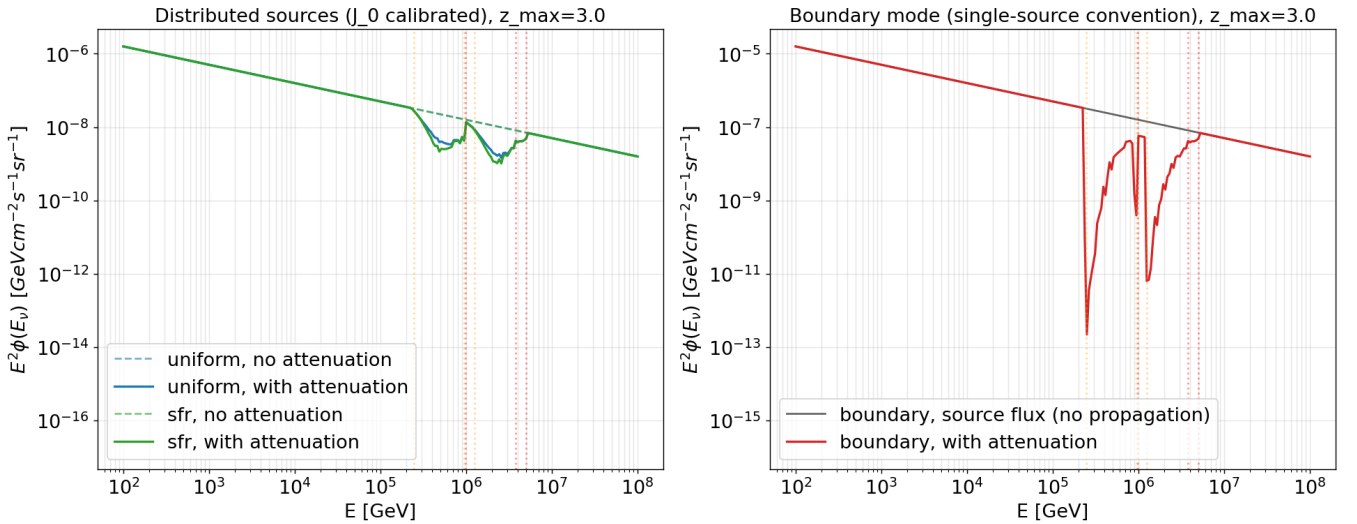
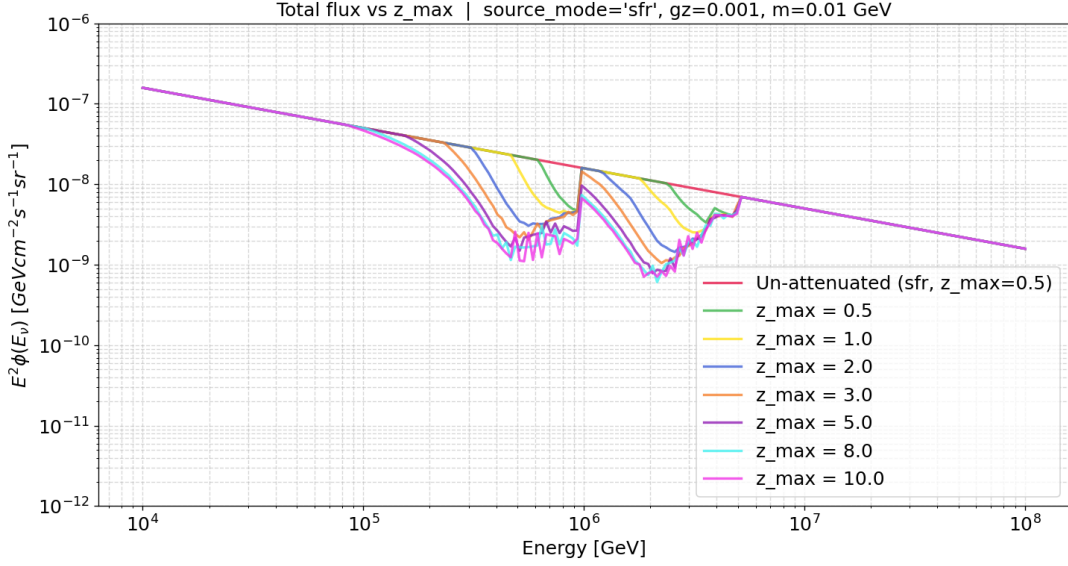
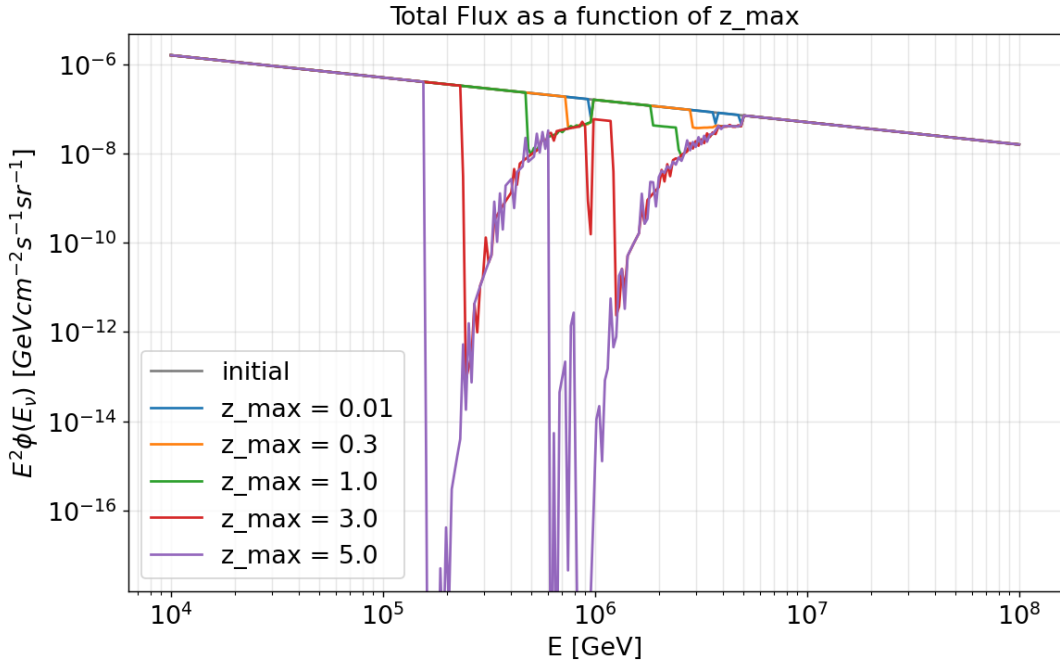


Figure 3.10: Comparison between different source conventions (point-source/boundary, uniform through z and SFR-like through z). Although the two distributed cases are very similar, the relative weight of redshift around ~ 1.86 in the SFR peak shifts slightly the location of strongest absorption in the spectrum. As predicted, these diffuse emissions get rid of the convergence anomaly of the boundary case, since there is no δ peak at the start of the numerical integration anymore. The locations of the resonance at $z = 0$ are marked by the dashed vertical lines. $g_{Z'} = 10^{-3}$, $M_{Z'} = 10$ MeV, $n_z = 150$, $N_{bins} = 300$, $m_{lightest} = 0.01$ eV.

Below, the effect of the maximum redshift value z_{max} the simulation starts at is shown, in boundary and SFR modes respectively. As expected, the higher the maximum redshift the broader the spectrum. This being said, the effect wears off at greater values of $z_{max} \gtrsim 8$ where the SFR-derived distribution becomes comparatively negligible (see Figure 3.3). This does not occur in boundary or constant diffuse emissions (the latter not showcased here).

(a) Effect of z_{max} in diffuse mode (SFR).(b) Effects of z_{max} in boundary mode.Figure 3.11: $g_{Z'} = 10^{-3}$, $M_{Z'} = 10$ MeV, $n_z = 150$, $N_{bins} = 300$, $m_{lightest} = 0.01$ eV.

The typical z_{max} kept throughout the remained of this thesis is 4. While it would be more physical to take into account the entire redshift range up to the validity limits of the SFR profile ($z = 8$), by $z = 4W(z) \sim 0.2$, so that no qualitative nor important quantitative corrections to the spectrum are observed (see Figure 3.11 above). It thus makes for a good balance between physical realism and convergence speed, allowing $n_z \in [100, 400]$ to be sufficiently accurate for the purposes of this thesis (see Figures 3.8, 3.9).

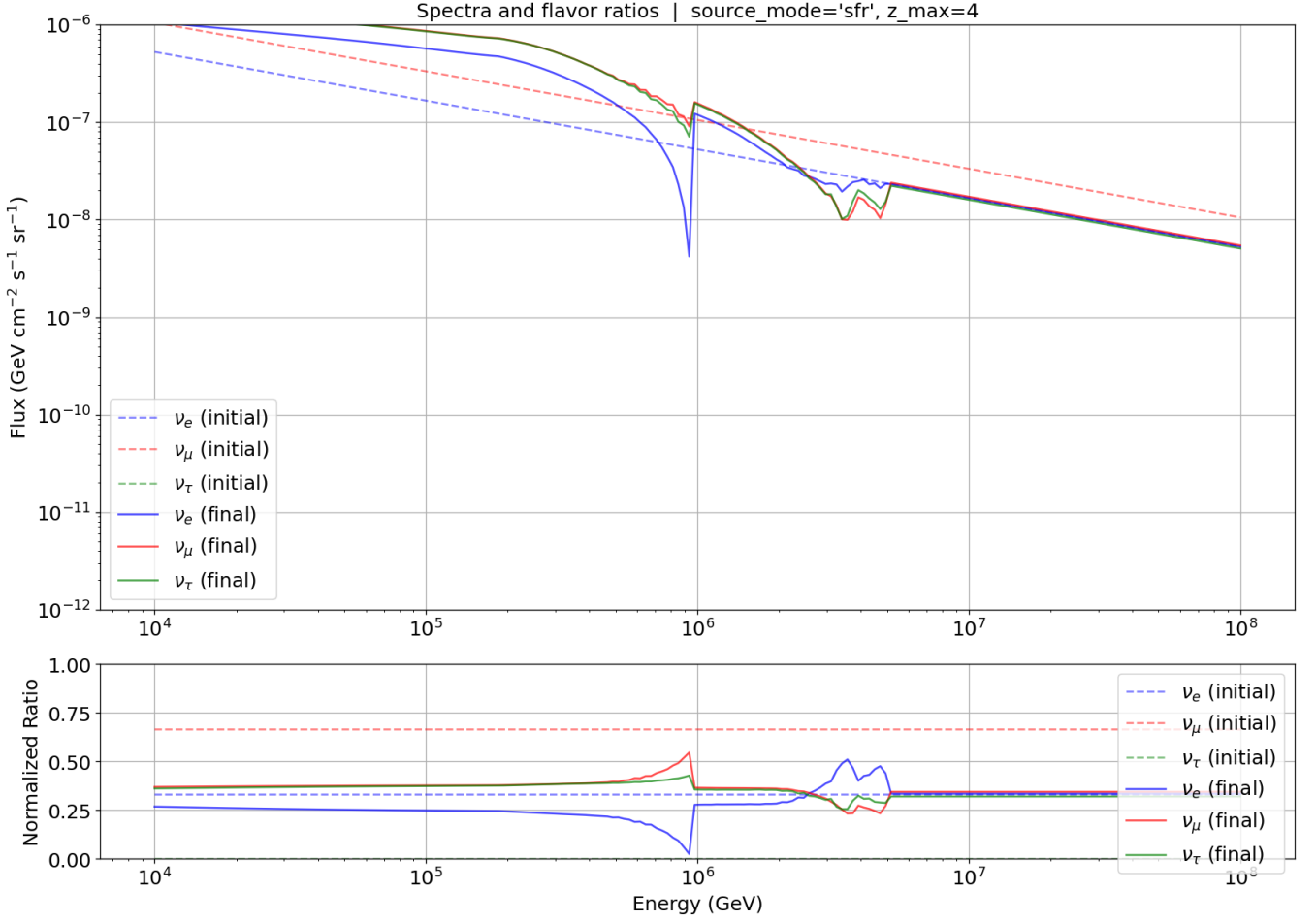


Figure 3.12: The effect of total regeneration (all outgoing neutrinos are kept in the beam and thus in the spectrum), as showcased here, is to significantly dampen the feature and smooth it. The curvature of the dips is "flipped", signaling that regeneration at low redshifts "fills in" the gap produced by the high- z absorption redshifted to lower energies. A slight excess is also noticeable below the absorption feature limit $E_{res}/(1+z)$.

3.4.2 Signature of the Boson Parameters

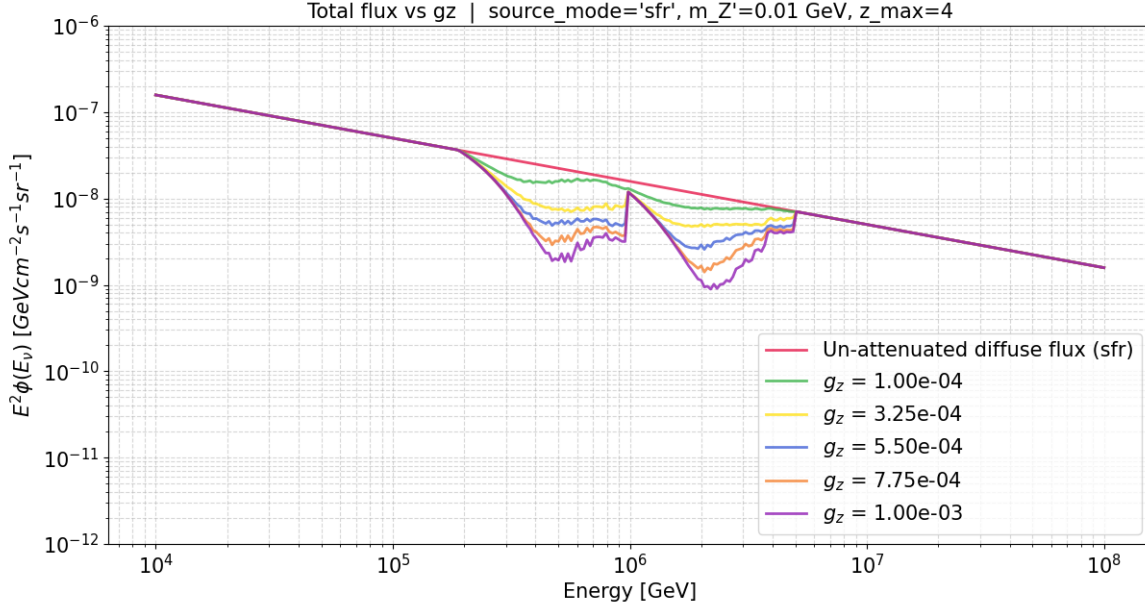


Figure 3.13: Dependence of the features on $g_{Z'} \in [10^{-4}, 10^{-3}]$ in SFR production mode. Normal Ordering with $M_{Z'} = 10$ MeV, $z_{max} = 4$. The lightest neutrino mass is taken as 0.01 eV ($\sum m_\nu \approx 0.074$ eV). As expected, when $g_{Z'}$ increases, the dips deepen, but remain localized at the same energies.

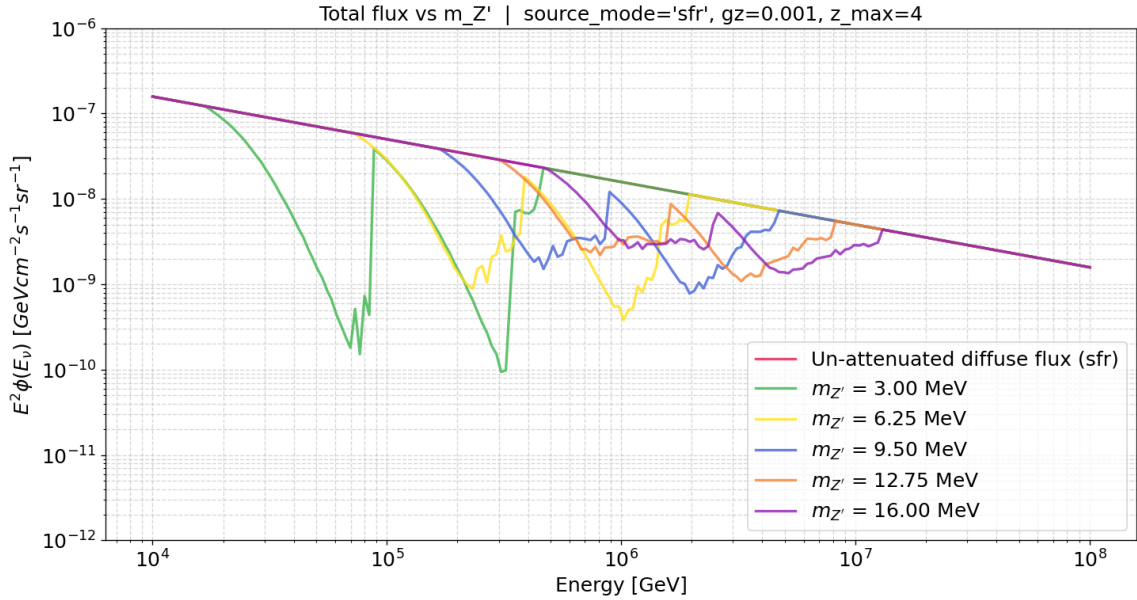


Figure 3.14: Dependence of the features on $M_{Z'} \in [3, 16]$ MeV in SFR production mode. Normal Ordering with $g_{Z'} = 10^{-3}$, $z_{max} = 4$. The lightest neutrino mass is taken as 0.01 eV ($\sum m_\nu \approx 0.074$ eV). As expected through the depletion criterion, when the gauge boson mass increases, the features drift towards higher energies and flatten. Indeed, the neutrino's mean free path scales with $M_{Z'}^2$.

3.4.3 Signature of Neutrino Physics

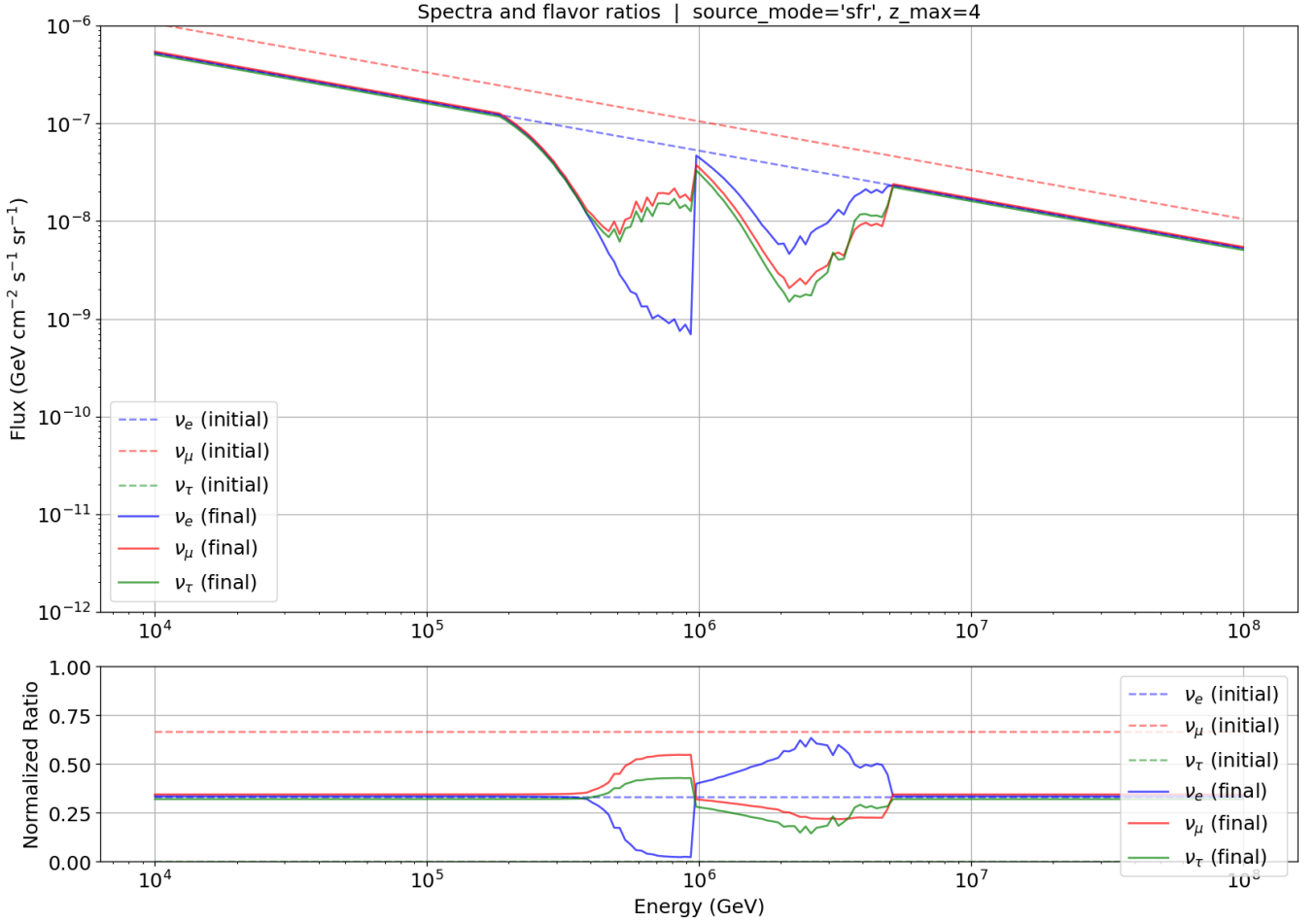


Figure 3.15: Normal Ordering. $g_{Z'} = 10^{-3}$, $M_{Z'} = 10$ MeV, $m_{lightest} = 0.01$ eV ($\sum m_\nu \approx 0.074$ eV)

Figures 3.15 and 3.16 respectively show the effect of the neutrino hierarchy on the spectrum features as well as the flavor ratios. Because the spectrum is an incoherent mixture of $\sim 1 : 2 : 0$ $\nu_e : \nu_\mu : \nu_\tau$ flavors at production (assuming standard pion decay), it is a roughly equal $1 : 1 : 1$ mixture of mass/propagation eigenstates. The flavor ratios have a particularly morphology: one dip is electron-depleted and one is electron-enhanced.

Scattering with each of the $C\nu B$'s ν_1, ν_2 or ν_3 neutrinos introduces a resonant feature at $E_{res}^i = \frac{M_{Z'}^2}{2m_{\nu_i}}$. As explained in Figure 2.3, because the Z' couplings 2.19 between two mass eigenstates differ between each pair, to each ν_i resonance feature corresponds different ν_j absorption strengths Q_{ij} .

This translates to the *flavor* plots above because electronic neutrinos ν_e contain mostly ν_1 , which in normal (resp. inverted) ordering is the lightest (resp. the heaviest) of neutrino mass eigenstate. In turn, the ν_3 Z' mixture is mostly ν_μ/ν_τ .

Then, because ν_3 interaction couplings with ν_1, ν_2 are 1-4 orders of magnitude stronger than any other (see 2.19), the ν_3 induced feature (the leftmost in NO and rightmost in IO), predominantly absorbs $\nu_1 \sim \nu_e, \nu_2$ and the blended ν_1/ν_2 features (rightmost in NO and leftmost in IO) predominantly absorb ν_3 , which is roughly an ν_e -less equal mixture of ν_μ and ν_τ .

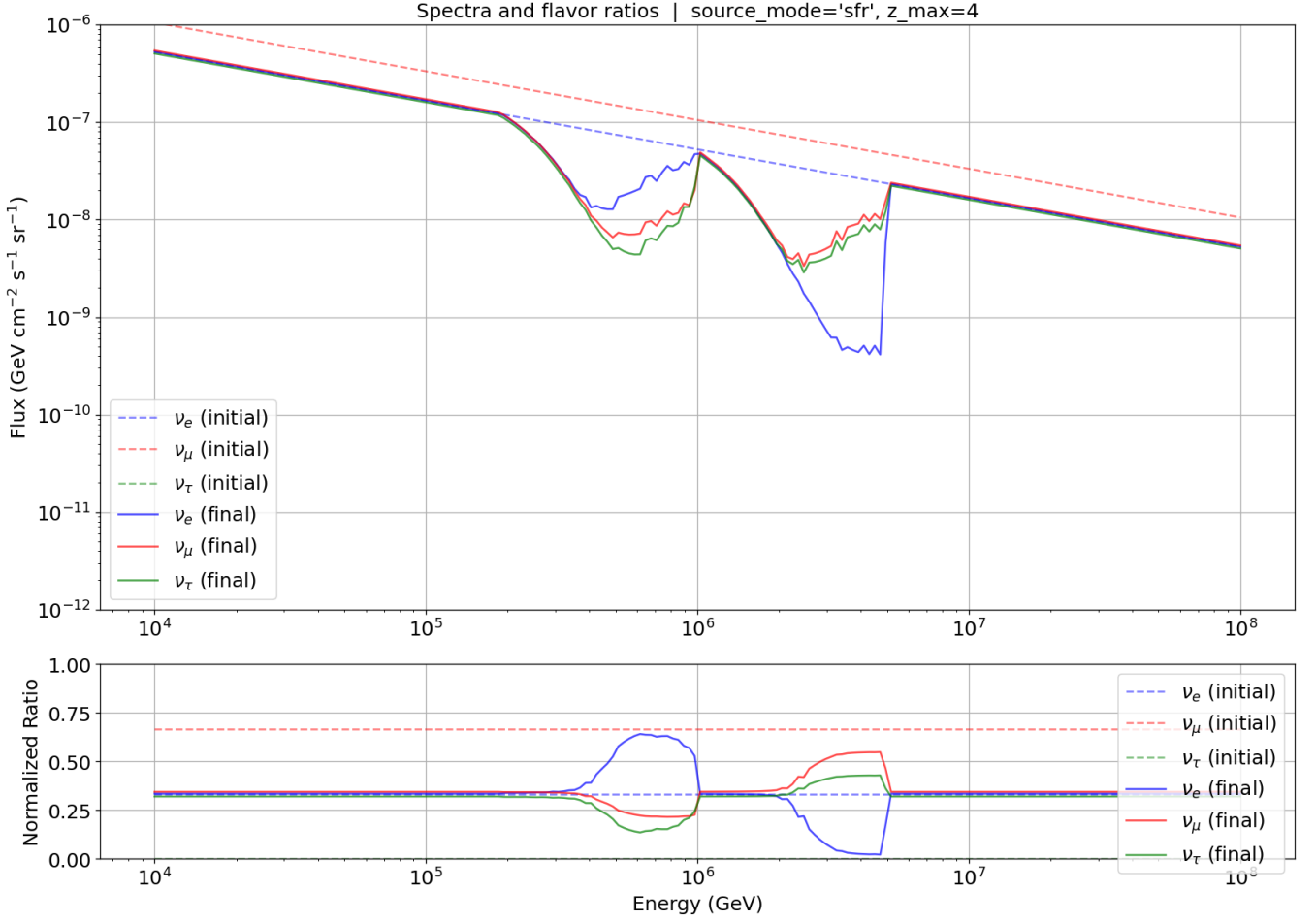


Figure 3.16: Inverted Ordering. $g_{Z'} = 10^{-3}$, $M_{Z'} = 10$ MeV, $m_{lightest} = 0.01$ eV ($\sum m_\nu \approx 0.074$ eV)

Finally, while the absolute location of the three dip structure is a signature of the mass of the interaction boson $M_{Z'}$, the masses of the neutrinos themselves are revealed in the relative location of the three dips with respect to one another.

The neutrino oscillation experiments constrain the three squared-mass splittings Δm_{ij}^2 , of which only two are independent. As such, the absolute scale of the neutrino masses is unconstrained, being specified by either the mass of the lightest neutrino $m_{lightest}$ or the sum of the three masses $\sum m_\nu$.

Because the squared-mass splitting fits depend on the assumption of a mass hierarchy (either NO or IO) for the neutrinos, the sum of mass as a function of the lightest neutrino mass depends on the hierarchy. The dependence is shown in Figure 3.17 below.

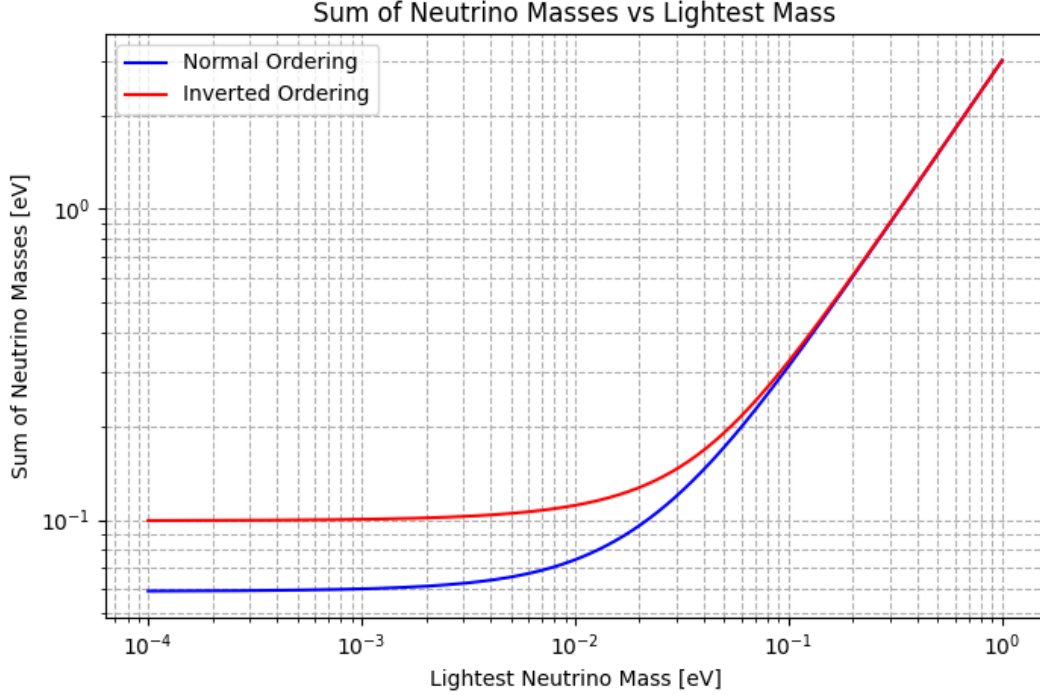


Figure 3.17: Sum of neutrino masses (given from the fitted squared-mass splittings) as a function of the choice of lightest neutrino mass. There is currently no lab constraint on the minimum value of $m_{lightest}$, yet $\sum m_\nu$ is minimally bounded by the fits of the squared-mass splittings assuming $m_{lightest} = 0$.

The regime used throughout this thesis needs to stay within the bound of cosmological constraints ($\sum m_\nu < 0.07 - 0.12$ eV [48]). As such, the high $\nu_{lightest}$ cases (> 0.1 eV) which completely overlap the three features, are discarded for the remainder of this thesis. Below 0.001 eV masses (up to a massless neutrino) would require taking into consideration thermal broadening of the C ν B in the scattering center of mass energy s . Although this broadens the feature slightly more [15], it is not of particular relevance to this thesis's objectives. The value kept is thus $m_{lightest} \sim 0.01$ eV.

This section made for an independent derivation of the literature exploring the absorption effects in levels of complexity ranging from an analytical integration of a point-source optical depth to the full resolution (with much more flare than here!) of the integro-differential equation ([9–19]).

It has been shown throughout that the physics of the problem leaves (more or less degenerate) signatures in the spectrum, ranging from the depth of absorption troughs to their absolute/relative locations and their number, with their morphology even indicating source distribution.

The flavor ratios are kept qualitatively and almost quantitatively similar in all cases, a strong reminder that the neutrino oscillation physics (the mixing angles) are a physical constant that is left unchanged throughout the propagation. The notable exception to this is the neutrino hierarchy, which swaps the order in which they are arranged in the flavor ratio plot.

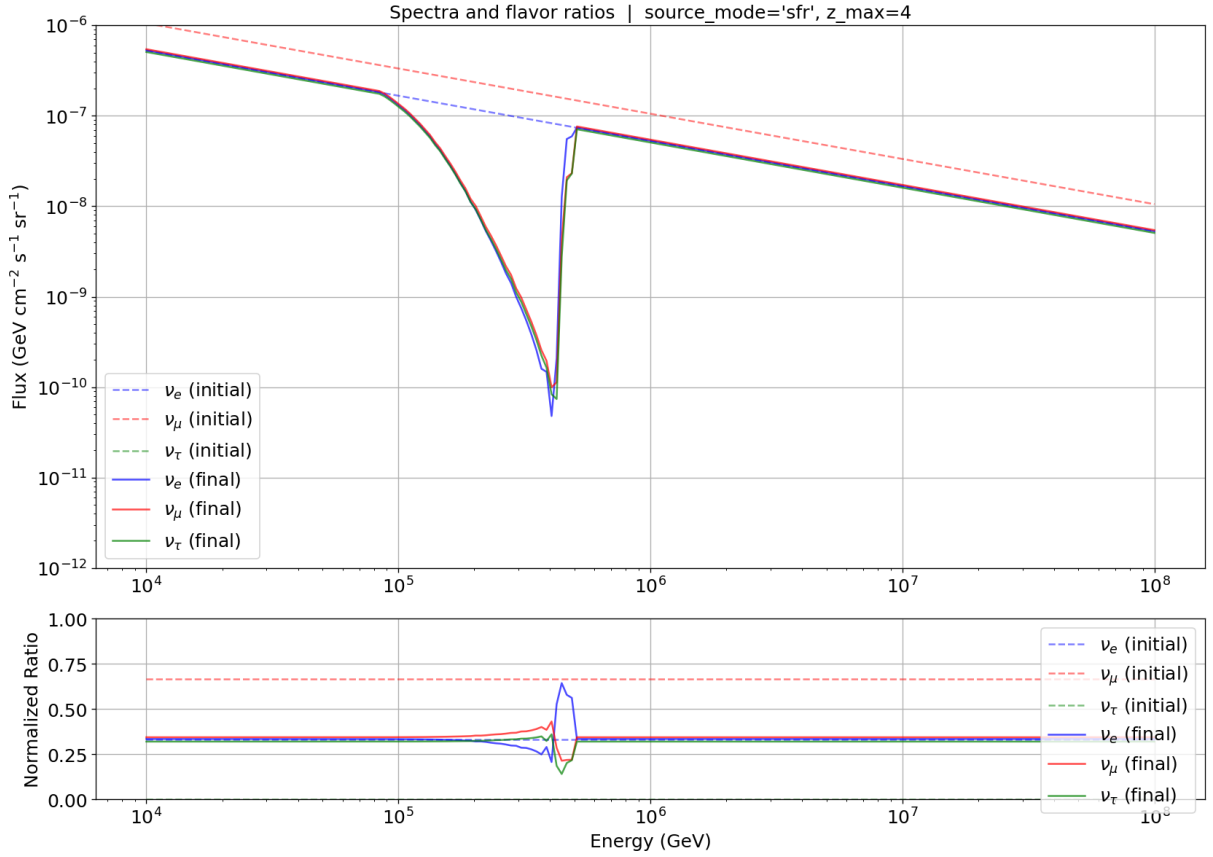
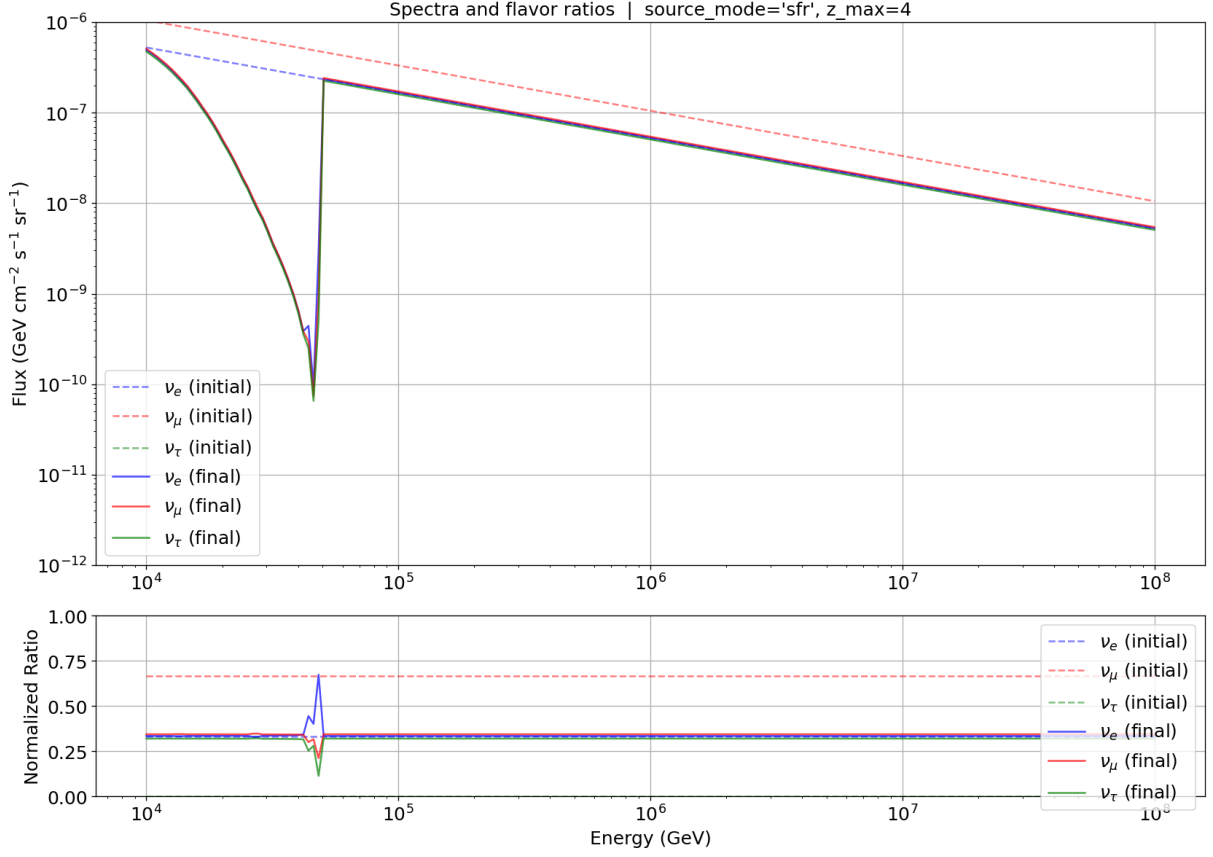
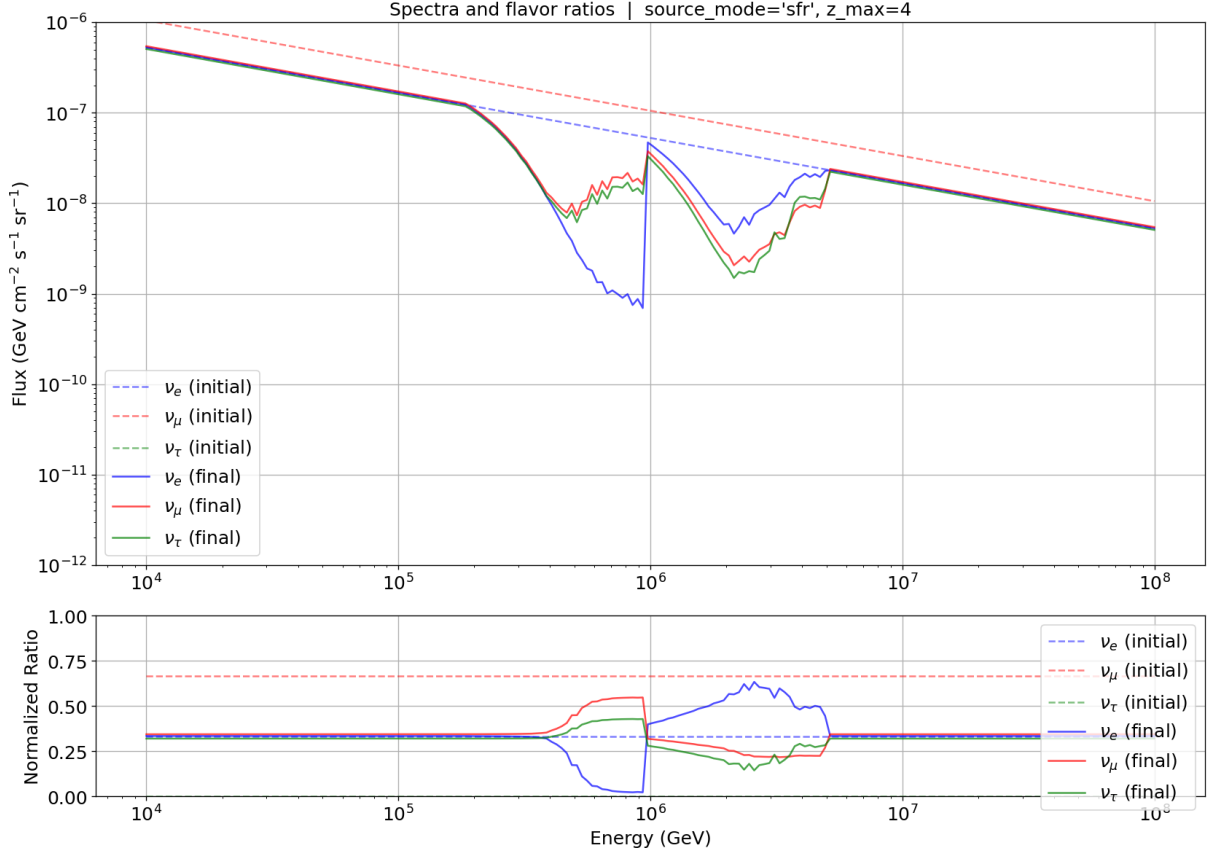
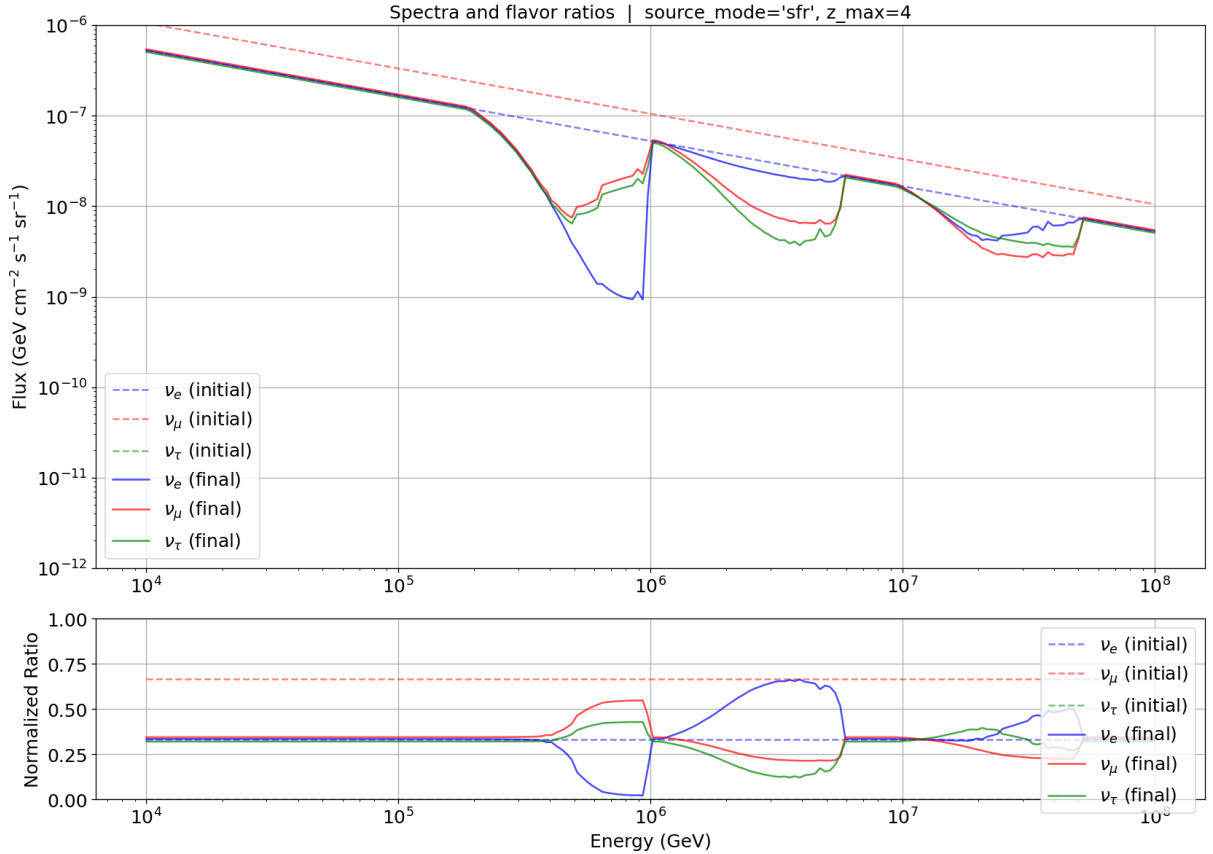


Figure 3.18: Spectra for different lightest neutrino mass m_L (Part 1/2). As its mass increases, the other neutrino masses reorganize with respect to the best-fit squared-mass differences used. At high masses all three features overlap while at lower masses all three separate.



(c) $m_L = 0.01 \text{ eV}$, $\sum^{NO} m_\nu = 0.074 \text{ eV}$, $\sum^{IO} m_\nu = 0.111 \text{ eV}$



(d) $m_L = 0.001 \text{ eV}$, $\sum^{NO} m_\nu = 0.059 \text{ eV}$, $\sum^{IO} m_\nu = 0.101 \text{ eV}$

Figure 3.18: Spectra for different lightest neutrino mass m_L (Part 2/2). As its mass increases, the other neutrino masses reorganize with respect to the best-fit squared-mass differences used. At high masses all three features overlap while at lower masses all three separate.

Chapter 4

Matter Potentials & Flavor Ratios

The flavor ratio structure derived in the last chapter are left qualitatively unchanged when tweaking the propagation/interaction/neutrino physics. This paves the way for the next step of the analysis: taking into account the modified neutrino oscillations that emerge when the hypothesized Z' generates, through forward scattering with the $C\nu B$, different phase shifts to each neutrino flavor.

These modified oscillations emerge as modifications to the PMNS matrix elements that rules flavor to propagation eigenstate change of basis and Z' couplings between these new propagation eigenstates.

These matter potentials and the issuing changes to the mixing angles have not been mentioned in the " Z' at IceCube" literature with the notable exception of Esteban et al. (2021) [18] who discard them by stating:

We ignore the neutrino matter potential induced by νSI [68-70 in citation]. A priori, it could affect the flavor composition at Earth, but for scalar νSI it just generates a negligible correction to neutrino masses [71 in citation]. For vector νSI , however, this effect might be measurable at higher energies than the ones we consider if the cosmic neutrino background has a large lepton asymmetry [68 in citation].

Their reference 68, [49] in this work, is a paper by Lunardini and Smirnov (2001), deriving the matter effects astrophysical neutrinos of extremely high energy experience when interacting with the $C\nu B$ via the Standard Model Z boson. Neither do they mention the IceCube spectrum (the facility was functional by 2011) nor do they study Z' boson signatures in possible spectra.

This Chapter is subdivided into three sections.

- First, a four-page literature review of matter potentials is presented, ranging from electronic matter potentials taking place in the Sun to new physics scenario, in order to introduce the reader to the topic, prepare the ground for the computation and highlight a few concepts that are relevant to the analysis, such as adiabaticity.
- Second, the derivations of the matter potentials per se are conducted, for two separate mediator mass regime. One where the Z' boson's mass is that of Chapter 3 and the matter potentials are confined to the resonant feature of the spectrum, supplementing the analysis of previous papers that overlooked them. One where the Z' boson's mass is much greater, so that only matter potentials appear in the IceCube spectrum and are analogous to the typical matter potential expression one obtains with W or Z bosons. This is mostly done for sake of comparison, as this mass regime is rejected experimentally for the Z' . It may nonetheless remain relevant when extrapolated to other new physics scenarios.

- Thirdly, the impact matter potentials have on the effective angles, is a derivation on its own. The resulting expression is what is fed directly to the numerical solver, in the form of a modified PMNS matrix, function of $(E_\nu, N_{\text{C}\nu\text{B}}(z))$. It can be derived either numerically (by diagonalizing the matter Hamiltonian) or by cleverly extrapolating an analytical approximation initially devised for the electronic matter potentials [50]. Both approaches are implemented.

4.1 Scientific Context

4.1.1 Electronic & Baryonic Potentials

Neutrinos, when propagating through vacuum, oscillate because of their non-diagonal Hamiltonian: the propagation or "mass" eigenstates ν_i are different from the flavor eigenstates ν_α , produced when electrons, muons and taus undergo weak interactions.

In matter, things change. Random, incoherent scattering of neutrinos with the medium attenuate the beam as it propagates and have been extensively addressed in the previous sections.

An additional effect arises from *coherent forward-scattering* interactions, where the momentum and direction of the neutrino is conserved as a result of the scattering. Nothing but a coherent change in the wavefunction's phase occurs. Neutrino oscillations are generated by the continuously changing phase shifts between species –cf. equation 1.11– as a result of their mass, and thus energy, differences.

This is equivalent to matter being permeated by effective, crucially flavor-dependent, **matter potentials** V_α . These effective matter potentials, being flavor-dependent and flavor-diagonal, combine with the free Hamiltonian, changing the propagation eigenstates from ν_i to ν_i^m and thus the elements of the PMNS matrix with them. Neutrino oscillations in matter are modified as if they were propagating in vacuum with modified, effective, fundamental parameters.

An analogy can be drawn with light propagating through a birefringent medium (the flavor-dependent neutrino interactions would make the C ν B "tri-refrington"). Just as the polarization content of the light may be changed after crossing such a medium, the flavor content of the neutrino beam can also be altered when crossing matter.

Let us briefly go over the history of matter potentials, in a way that allows to highlight the key ideas relevant to this work. The first part of this summary is based on that of Alexei Smirnov –the "S" of "MSW"–, which can be found in [51]. In addition to exposing the core mathematical notions, this article is full of historical anecdotes, insights on the exchange between the main actors at the time, etc.

In 1974, Opher [52] was the first to mention that coherent interactions induce a flavor-dependent "refractive index" to neutrinos in matter, addressing the possibility that the critical refraction point may be measured and used as a probe for the C ν B. No mentions were made about the modification to neutrino oscillations, which is now understood to be the most relevant aspect of this phenomena.

Wolfenstein, in 1978 [27], first realized the importance of these coherent processes when considering neutrino oscillations. He addressed two possibilities.

First, that massless neutrinos, which do not oscillate in vacuum, start oscillating in matter because of *flavor-changing* forward-scattering interactions, the effective potentials of which would be off-diagonal in the flavor-basis. Second, that massive neutrinos, which do in fact oscillate, find their oscillations modified by the potentials described above.

The complete, three flavor picture was put forth by Barger et al. (1980) [53], where they derived complicated analytical formulas for the effective, three-flavor, parameters. They consider the case of a constant density medium, and discuss the energy-dependence of the phenomenon.

An instrumental discovery in the study of matter potentials was that of their so-called "resonance" in a medium of varying density, an expression deemed misleading by some, such as Messiah [54]. This was first noticed independently by Cudell & Gaisser [55] (1985) and Mikheyev & Smirnov [56] (1986). As will be seen in section 4.2, at some "resonant" energy-density product $E_\nu \cdot N_{background}$, the modification to the effective oscillation angle becomes maximal. For small values of the –true– vacuum mixing angle, this happens when the oscillation length l_ν is equal to the length required for the phase shifts in matter to reach 2π , l_0 . Alternatively, as Smirnov puts it [51], "*the eigenfrequency of the [oscillating] system, $1/l_\nu$, equals the eigenfrequency of 'external' medium $1/l_0$* ".

This brought forward the now well-accepted solution to the solar neutrino problem. Indeed, consider a neutrino of energy E , produced at the core of our star, whose resonant density $N^R(E_\nu)$ lies at radius r^R . The neutrino is produced as a ν_e , which is effectively the *propagation eigenstate* ν_{2m} of highest mass, given the high potential at the core $V_e(0) \gg V_e(r^R)$.

What happens next is more nuanced than just enhanced oscillations as the neutrino crosses its resonance region, and revolves around the notions of *adiabaticity*. The exact picture is clarified by Hans Bethe the same year [57], but also expanded upon by a variety of authors including Mikheyev and Smirnov themselves.

Matter changes the oscillation picture to a time-varying Hamiltonian, whose eigenvalues are the effective $(\frac{m_i(N)^2}{2E_\nu})$, which change with density. The point of closest approach of both eigenvalues is the resonant density given a set neutrino energy, N^R .

The adiabatic theorem states that if the time-varying parameters of the Hamiltonian (in this case, $N(r)$) change slowly with respect to the eigenvalue-splitting $(\frac{\Delta m_i(N)^2}{2E_\nu})$, then a system which starts as an instantaneous eigenstate remains on that "eigenstate branch" as it slowly changes with time.

Thus the initial state $\nu_e \approx \nu_{2m}$ at the core of the Sun, exits as $\nu_{2m} \approx \nu_2$. If the mixing angle θ is small, which was assumed at the time, then $\nu_2 \approx \nu_\mu$ and the electronic neutrinos have been shifted to the muonic kind in a purely *non-oscillating* fashion!

Divergences from perfect adiabaticity, which is only ever an approximation, were discussed by Messiah [54] and Parke [58], amongst others, with the possibility that at closest approach between eigenstates, i.e. at resonance, the neutrino switches branch, $\nu_{2m} \rightarrow \nu_{1m}$. This is a Landau-Zenner transition in time-varying quantum mechanics [58]. This is where the review by Smirnov [51] ends.

The MSW effect in the Sun stems from the electronic neutrinos undergoing charged-current forward-scattering with electrons of matter. The neutral-current interactions with electrons, protons and neutrons affect each flavor in a similar way and only contribute to an overall, common phase for the neutrinos, which has no physical incidence. Figure 4.1 showcases the diagrams responsible for such interactions.

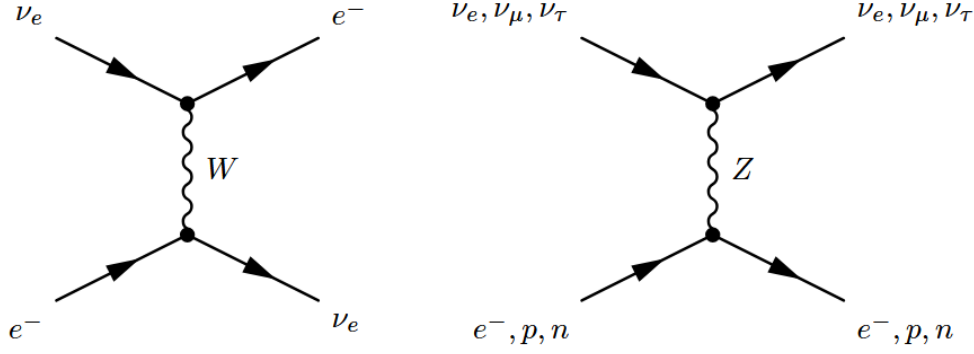


Figure 4.1: Diagrams for the two baryonic-matter-induced neutrino forward-scattering. Taken from [28]. Because solar neutrinos have energies ($\sim \text{MeV}$) much smaller than ($\sqrt{s} \approx 80 \text{ GeV}$), only the t-channel charged current diagram matters.

4.1.2 $C\nu B$ Potentials - Standard Model Mediator

A few years after matter potentials were envisaged to explain the solar neutrino problem, the possibility was raised that matter potentials may also arise from neutrinos coherently scattering with the $C\nu B$ background (*e.g.* Notzold (1987) [59]), that is, other neutrinos! Such events are mediated by neutral currents, *i.e.* the Z-boson.

Note that in the Proceedings of the 6th Moriond Workshop, where both Messiah's contribution on adiabaticity [54] and Mikheyev & Smirnov's talk on their resonant effect [56] were discussed, the latter already discuss briefly the possibility of neutrino-neutrino induced matter effects and highlight the asymmetry in $n_\nu - n_{\bar{\nu}}$ required for it to become significant.

When the scatterer are neutrinos themselves, both s, t and u channel diagrams (see Figure 4.2) contribute to the potential, because the background particle may or may not take the place of the incident neutrino by exchanging their momenta, if they are of similar flavor.

In 1992, Pantaleone [60] raises attention on the possibility that such neutral currents furthermore induce *off-diagonal* "potentials", although the term is somewhat improper. The idea is that the incident neutrino-background neutrino swap can take place even if both neutrinos hold different flavors.

This would be, *effectively*, yet another contribution to the matter potentials: one which is non-diagonal in both the flavor and vacuum propagation basis.

This is agreed upon by subsequent work, such as Sigl and Raffelt (1992) [61], where they take both the attenuation and matter potential into account at the same time. Attempts at reuniting both typically adopt a density-matrix formalism, so as to capture all coherence-related effects [61]. They note that while potentials are a first-order effect in the Fermi constant G_F , the attenuation is a second-order effect (the amplitude is squared). Importantly, the off-diagonal terms are directly related to off-diagonal terms of the density matrix.

This marks a first stark difference with this thesis. Because the point is to consider mediator mass ranges capable of generating a resonant absorption feature in the IceCube spectrum, the energy regime is not small enough to neglect the propagator structure and reduce the interaction to "Fermi theory". In addition, while the Z-boson couples with equal strength to each flavor of neutrino, this is not the case for the Z' . As we will see during derivation, the specific (0, 1, -1) coupling of the Z' make the t-channel contribution to matter potentials cancel.

The relationship between matter potentials and attenuation is profound. While the real-part of the forward-scattering amplitude accounts for the matter potentials, its imaginary part relates to the attenuation of the neutrino beam, *i.e.* the scattering cross-section, as per the optical theorem.

More recent work on the topic (*e.g.* [21, 62]), also taking the CνB as background, do not consider the off-diagonal contributions to the matter effect. Indeed, Diaz and Klinkhammer (2015) [62] account for the four Feynman diagrams given in Figure 4.2 responsible for $\nu - \nu$ matter potentials mediated by the Z' , but describe these matter potentials as "diagonal in the flavor-basis".

The possibility that off-diagonal potentials arise, especially as a result of diagram (b) in Figure 4.2 –beware of the typo– taken from their work, is not explicitly discussed.

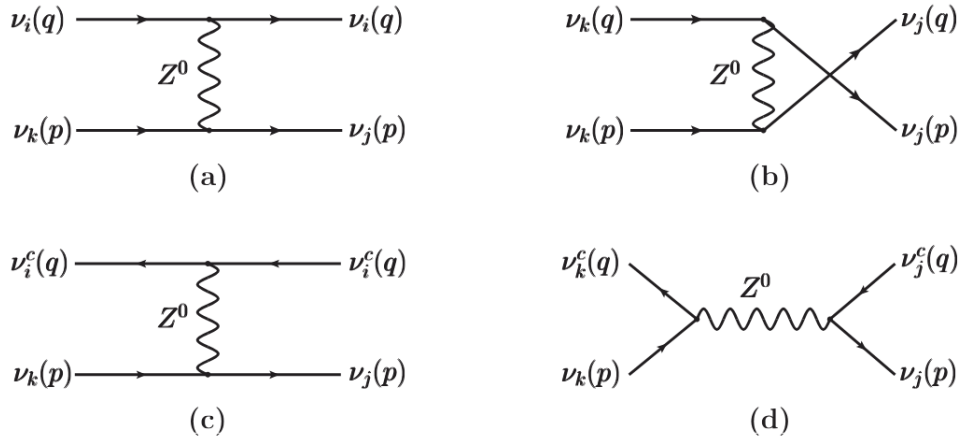


Figure 4.2: Scattering diagrams that contribute to the neutrino matter potentials induced by the CνB through the Z boson. According to Pantaleone [60], case (b), the u-channel, should also be expanded to any CνB neutrino flavor, and induce in the latter case off-diagonal potentials. Figure taken from [62]. There is a typo in diagram (b). Bottom subscript k and j, referring to momentum p particles, are inverted.

This is in fact quite an old controversy as to whether neutrino self-interactions have sizable off-diagonal terms [63]. To go into this debate is beyond the scope of this thesis, but let us note what seems to be the modern conclusion [63]: the off-diagonal potential terms depend directly on the off-diagonal elements of the density matrix describing the evolution of the neutrino ensemble. Because the CνB is thermalized and fully decoherent, these off-diagonal terms go to zero. I suppose this motivates Diaz and Klinkhammer's expression.

Intuitively, this makes sense. For the individual forward-scattering events to *coherently* shift the phase of the neutrinos, each even must leave the medium in the initial state. This is true for flavor-diagonal potentials, where each neutrino keeps its initial momentum and direction, but not for off-diagonal "potentials", where the composition of the background is altered. Following interactions do not occur in the exact same conditions and the coherence of the phase shifts suffer from it. The more decoherent the medium, the more diagonal the density matrix, the less off-diagonal phase shifts matter.

The second work, by Dev et al. (2025) [21], clearly states this time that they utilize the original potential by Notzold et al. (1987) [59], the fully diagonal expressions. They hold very interesting discussions on the possible overdensities and profiles of the CνB, a somewhat necessary feature to observe significant matter effect in IceCube. This is similar in spirit, albeit not quantitatively, for the reasons mentioned above, to the result of this work.

4.1.3 $C\nu B$ Potentials - New Physics Mediator

The scattering of astrophysical neutrinos, whether suppressive or cascade-like, is the main effect considered when probing the signatures of new fundamental interactions, such as the Z' , in the very high energy neutrino spectrum at IceCube.

As seen in Chapter 3, sufficient resolution of such features could not only unravel the existence of new physics, it could also constrain the related parameters ($g_{Z'}$, $M_{Z'}$) as well as the neutrino mass hierarchy [18].

The novelty in this thesis is to explore the impact the $L_\mu - L_\tau$ Z' -boson has on the IceCube flavor content through the *modified matter potentials* it generates with the $C\nu B$.

Potentials have been explored in the framework of the $L_\alpha - L_\beta$ gauged bosons in the context of ultralight mediators [64–66] in the range $[10^{-35}, 10^{-10}]$ eV. Such low masses translate to very long range interaction. The potentials considered are very different from those addressed here. They are Yukawa-like, long-range potentials, which couple neutrinos to the far-away cosmic reservoir of electrons (resp. to the particles in the Sun [64]), producing signatures in the IceCube spectrum [65] (resp. in oscillation experiments such as MINOS [64]). These are dissimilar to the matter potentials neutrinos undergo due to coherent forward scattering, and are not of particular relevance to this work.

The matter potentials that will be computed in Section 4.2 are of two kinds.

- First, $M_{Z'}$ is chosen such that resonance occurs in the IceCube spectrum –as in Chapter 3– and the energy-dependent propagator structure must be fully taken into account. In this case, the derivation is similar to that of Smirnov (2021) [67], with the notable difference that they consider a scalar mediator.
- Second, $M_{Z'}$ is chosen to be much greater than the $\sqrt{s}$ considered –hence no absorption resonance as in Chapter 3– and the effective low-energy field theory, Fermi interaction, is applicable. In this case, the derivation is very similar to that of the works mentioned in previous sections, W and Z bosons alike. The notable difference is the coupling matrix $Q_{\alpha\beta}$ of the Z' , as was discussed previously.

To the best of my knowledge, searching for the joint signatures in the IceCube flavor ratios of the Z' and the $C\nu B$ by taking into consideration both resonant absorption and modified oscillations –due to matter potentials– constitutes a cross-over between existing works that has not yet been looked at. In typical scientific jargon, this could be called "bridging the gap".

4.2 Z' - $C\nu B$ Potentials Derivation

4.2.1 Diagrams and Charge

Because the $C\nu B$ is fully decoherent, the interaction diagrams to consider exclude the flavor-swapping contributions. Those that remain for the V_α potential are the t-channel scattering with any kind of neutrinos, the u-channel exchange with the same specie ν_α and the s-channel scattering with the corresponding anti-neutrino $\bar{\nu}_\alpha$. These are presented in Figure 4.3.

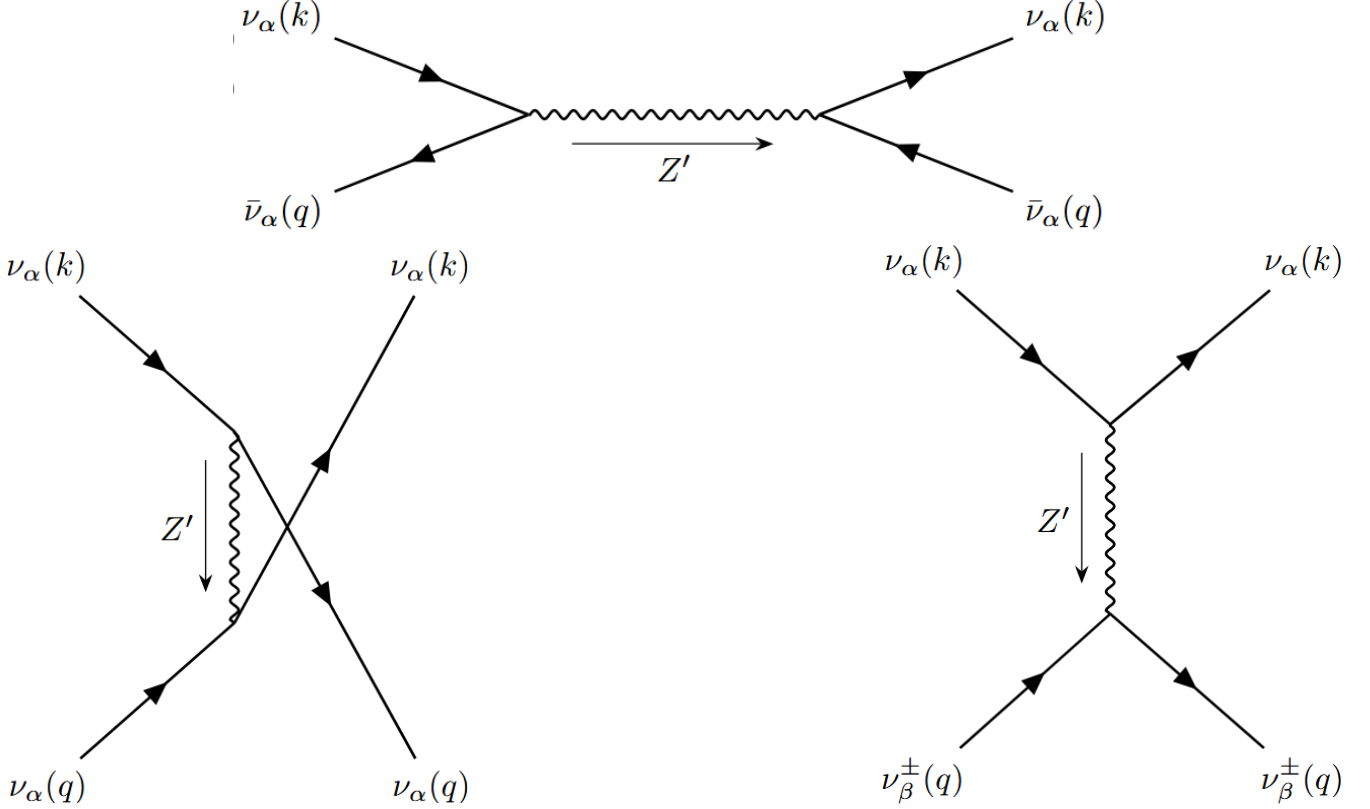


Figure 4.3: s-channel (up), u-channel (left) and t-channel (right) diagrams contributing to the ν_α potential V_α . Couplings from the Z' boson arise as $Q_{\alpha\beta}$ factors at the vertices. $\alpha, \beta = e, \mu, \tau$.

The Z' couplings —the charges of the different neutrino flavors under such fifth force— are given by $Q_{\alpha\beta} = \text{Diag}(0, 1, -1)$. Let us note two things.

- First, as with the Z -induced potentials, scattering with $C\nu B$ neutrinos give an opposite contribution to scattering with anti-neutrinos. The total contribution of the t-channel diagram will be $\propto (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})$.
- Second, unlike the Z -induced potentials, the difference in charge between ν_μ and ν_τ lead to the cancellation of their respective contributions in the t-channel diagram. The total contribution of the t-channel diagram will be $\propto (n_{\nu_\mu} - n_{\nu_\tau})$.

In multiple works studying Z -boson potentials, [21, 49, 61, 62], this was highlighted as the necessity for large matter-antimatter asymmetry in the $C\nu B$ to make sufficiently strong effects arise. This is also mentioned in Esteban et al. [18] as an *a priori* requirement for significant matter effects in the IceCube spectrum.

In this work, the couplings extinguish the t-channel contribution in proportion to the *flavor* asymmetry of the $C\nu B$. Here it is assumed $n_\alpha^\pm = 56 \text{ cm}^{-3}$, *i.e.* $C\nu B$ has flavor universality. Thus the t-channel diagrams do not contribute to the potentials computed here. This is a hypothesis that can in principle be relaxed, but all work must have scopes, and in this thesis departure from flavor universality is where they're set.

The u- (resp. s-channel) diagram requires a $C\nu B$ neutrino (resp. antineutrino) of the same flavor. As such, their respective contributions will be $\propto n_{\nu_\alpha}$ (resp. $\propto -n_{\bar{\nu}_\alpha}$).

Because the mass regime of interest is that where absorption features may be observed at IceCube, the energy-structure of the s-channel diagram will also enter resonance for matter potentials. As such, it is assumed that the s-channel potential dominates. Although I have not derived a quantitative argument for it, one can imagine this would also make the t-channel diagram "second order" in the event where the CνB would be flavor asymmetric.

The computations are conducted in the following way. First, the potentials are computed in the limit where $\sqrt{s} \ll M_{Z'}$: "Fermi" theory. Then, the complete energy-structure of the s-potential is addressed separately, in analogy to the scalar mediator scenario [67].

4.2.2 High Mediator Mass: Effective Fermi Potential

When $\sqrt{s} \ll M_{Z'}$, the propagator $P_{\mu\nu}$ and vertex $V_{\alpha\beta}^\mu$ reduce to:

$$P_{\mu\nu} = \frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_{Z'}^2)}{q^2 - M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}} \longrightarrow \frac{ig_{\mu\nu}}{M_{Z'}^2} \quad (4.1)$$

$$V_{\alpha\beta}^\mu = -\frac{i}{2}g_{Z'}Q_{\alpha\beta}\gamma^\mu(1 - \gamma^5) \longrightarrow -\frac{ig_{Z'}Q_{\alpha\beta}}{2} \quad (4.2)$$

The effective Hamiltonian density reduces to:

$$\mathcal{H}_{\text{eff}} = \frac{G_{Z'}}{\sqrt{2}} \sum_{\alpha\beta} Q_{\alpha\alpha}Q_{\beta\beta} [\bar{\nu}_\alpha\gamma_\mu(1 - \gamma_5)\nu_\alpha] \cdot [\bar{\nu}_\beta\gamma^\mu(1 - \gamma_5)\nu_\beta] = \frac{G_{Z'}}{\sqrt{2}} \sum_{\alpha\beta} Q_{\alpha\alpha}Q_{\beta\beta} J_\alpha^\mu \cdot J_{\mu,\beta} \quad (4.3)$$

with $G_{Z'} = \sqrt{2} \frac{g_{Z'}^2}{4M_{Z'}^2}$.

The derivation that follows is then much similar to that of the "baryonic" matter effects, which can be found for instance in [28]. One must average the latter Hamiltonian over the background, a so-called Mean-Field approach. Defining $Q_\alpha = Q_{\alpha\alpha}$, one finds:

$$\bar{\mathcal{H}}_{\text{eff}} = \frac{G_{Z'}Q_\alpha}{\sqrt{2}} [\bar{\nu}_\alpha\gamma_\mu(1 - \gamma_5)\nu_\alpha] \cdot \sum_\beta Q_\beta \langle \nu_\beta^\pm, \text{C}\nu\text{B} | \left(\int d^3p f_\pm(E_{\text{C}\nu\text{B}}, T_{\text{C}\nu\text{B}}) \times [\bar{\nu}_\beta\gamma^\mu(1 - \gamma_5)\nu_\beta] \right) | \nu_\beta^\pm, \text{C}\nu\text{B} \rangle \quad (4.4)$$

The background factor on the right is simplified by the fact the CνB is assumed to be isotropic and at rest. In analogy with [28], the normalization is conducted under a finite volume space V . Here + accounts for the neutrinos (t,u-channel) and − the antineutrinos (t,s-channel).

$$|\nu_\alpha(p, \text{C}\nu\text{B})\rangle = \frac{1}{2E_p V} a^\dagger(p) |0\rangle \quad \text{and} \quad \int d^3p f_\pm(E_{\text{C}\nu\text{B}}, T_{\text{C}\nu\text{B}}) = n_{\nu^\pm} \times V \quad (4.5)$$

Thus the background factor can be expressed:

$$\int d^3p \frac{1}{2E_p} \left[f_{\nu_\beta}(E_p) \sum_s \bar{u}_s(p) \gamma^\mu (1 - \gamma_5) u_s(p) - f_{\bar{\nu}_\beta}(E_p) \sum_s \bar{v}_s(p) \gamma^\mu (1 - \gamma_5) v_s(p) \right] \quad (4.6)$$

Using the standard spin sums,

$$\sum_s \bar{u}_s(p) \gamma^\mu (1 - \gamma_5) u_s(p) = \frac{1}{2} \text{Tr}(\not{p} \gamma^\mu (1 - \gamma_5)) = 2p^\mu, \quad \sum_s \bar{v}_s(p) \gamma^\mu (1 - \gamma_5) v_s(p) = 2p^\mu \quad (4.7)$$

one obtains

$$\int d^3p [f_{\nu_\beta}(E_p) - f_{\bar{\nu}_\beta}(E_p)] \frac{p^\mu}{E_p} \quad (4.8)$$

Replacing the background in the expression of the effective Hamiltonian density and integrating under the assumption of a isotropic medium at rest, only the time-component (γ_0) survives. Indeed, integrating $\vec{p} \cdot \vec{\gamma} d^3p$ from $-\infty$ to ∞ gives 0.

$$\bar{\mathcal{H}}_{\text{eff}} = \sqrt{2} G_{Z'} [\bar{\nu}_{\alpha L} \gamma_0 \nu_{\alpha L}] \cdot Q_\alpha \sum_\beta Q_\beta (n_{\nu_\beta} - n_{\bar{\nu}_\beta}) \quad (4.9)$$

where the factor of two come from the definition of P_L . This is equivalent to the potential:

$$V^{(t)} = \sqrt{2} G_{Z'} Q_\alpha \sum_\beta Q_\beta (n_{\nu_\beta} - n_{\bar{\nu}_\beta}) \quad (4.10)$$

This direct term is the t-channel contribution. As we've mentioned the charges Q_β are such that for the Z' this contribution cancels.

Beyond the direct (also called Hartree) term, exchange (also called Fock) contributions arise when both flavors are identical ($\beta = \alpha$). While we have acknowledged this does not produce coherent off-diagonal potentials with the decoherent CνB, it does add to the diagonal potential.

In this case, the spin-sum cannot be factorized. Instead, the bilinear forms are crossed when sandwiched between the background kets and one finds:

$$\frac{1}{2E 2E_p} \sum_s [\bar{u}(k) \gamma^\mu P_L u_s(p)] [\bar{u}_s(p) \gamma_\mu P_L u(k)] = \frac{1}{2E 2E_p} \text{Tr}(\not{k} \gamma^\mu P_L \not{p} \gamma_\mu P_L) = \frac{-2u}{2E 2E_p} \quad (4.11)$$

with $u = (k - p)^2$. For a rest target $p^\mu = (E_p, 0)$ and a relativistic incident neutrino, $u \simeq -2EE_p$, this ratio cancels to one. Therefore each background ν_α contributes:

$$V_\alpha^{(u)} = \sqrt{2} G_{Z'} Q_\alpha^2 n_{\nu_\alpha} \quad (4.12)$$

Analogously, with v -spinors one finds the opposite sign, yielding

$$V_\alpha^{(s)} = -\sqrt{2} G_{Z'} Q_\alpha^2 n_{\bar{\nu}_\alpha} \quad (4.13)$$

The total potential reads

$$V_\alpha = \sqrt{2} G_{Z'} \left[Q_\alpha \sum_\beta Q_\beta (n_{\nu_\beta} - n_{\bar{\nu}_\beta}) + Q_\alpha^2 (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}) \right] \quad (4.14)$$

Or, given, the Z' charge/coupling matrix:

$$V_\mu = V_\tau = \sqrt{2} G_{Z'} (n_\nu - n_{\bar{\nu}}) \quad (4.15)$$

4.2.3 Low Mediator Mass: Energy Structure Near Resonance

We now go on to compute the near-resonance cross-section for the s-channel contribution, this time in analogy to that done in the case of a scalar mediator [67].

In this case, one cannot make use of the Fermi-like constant $\frac{G_{Z'}}{\sqrt{2}} = \frac{g_{Z'}^2}{4M_{Z'}^2}$. The factor $\frac{g_{Z'}^2 Q_\alpha^2}{4}$, which stems from the vertices, remains, and is multiplied by the factor of 2 elsewhere compensated by reconstructing P_L from $(1 - \gamma_5)$.

Then, as discussed earlier, the real part of the propagator is what contributes to the potential.

$$\Re \left[\frac{1}{q^2 - M_{Z'}^2 + iM_{Z'}\Gamma_{Z'}} \right] = \frac{s - M_{Z'}^2}{(s - M_{Z'}^2)^2 + (M_{Z'}\Gamma_{Z'})^2} \quad (4.16)$$

Let us define, just like [67], $\xi = \frac{\Gamma_{Z'}}{M_{Z'}}$, $y = \frac{E_\nu}{E_R}$ with $E_R = \frac{M_{Z'}^2}{2m_\nu}$. For our Z' these values are of the order: $\left\{ \begin{array}{l} \xi \simeq 2.65 \times 10^{-10} \times \left(\frac{g_{Z'}}{10^{-4}} \right)^2 \\ E_R \simeq 5 \text{ PeV} \times \left(\frac{M_{Z'}}{10 \text{ MeV}} \right)^2 \times \left(\frac{0.01 \text{ eV}}{m_\nu} \right) \end{array} \right.$

Again, assuming a background at rest, this factor enters the expression of the potential alongside the antineutrino density of the $C\nu B$, $n_{\bar{\nu}_\alpha}$. Thus one finds:

$$V_\alpha^{(res)}(y) = \frac{n_{\bar{\nu}_\alpha} g_{Z'}^2}{4m_\nu E_R} Q_\alpha^2 \frac{y - 1}{(y - 1)^2 + \xi^2} = \sqrt{2} G_{Z'} n_{\bar{\nu}_\alpha} Q_\alpha^2 \cdot \left[\frac{y - 1}{(y - 1)^2 + \xi^2} \right] =: V_0 \cdot f(y) \quad (4.17)$$

In the $y \ll 1$ limit, the potential reduces to the low-energy effective case, constant with energy.

In the $y \gg 1$ limit, $V_\alpha \simeq \frac{g_{Z'}^2}{4Em_\nu} Q_\alpha^2$. At energies much higher than the resonance, the potential decreases as $\propto 1/E$.

A couple of things might be said about this function. Except extremely near resonance, it is basically $|\frac{1}{1-y}|$. Exactly at resonance, it is zero, as expected from the purely imaginary on-shell propagator, given the potentials arise from the real part.

It has two maxima located at $1 \pm \xi$, spaced by an energy gap $2\xi E_R$, which reach $\propto V_0/\xi$. In practice however, this resonant feature of the potential will be broadened by different effects [67], such as the thermal distribution of the $C\nu B$ or the uncertainty on the neutrino's energy. When the broadening $\sigma_y \gg \xi$, it effectively replaces it in the expressions for the width and the enhancement $1/\sigma_y$. The former scenario ($T \simeq 1.945 \text{ K}$, $m_\nu \simeq 0.05 \text{ eV}$) leads to $\sigma_y \simeq 10^{-4}$ while in the latter $\sigma_y \simeq \frac{\sigma_E}{E_R}$.

4.3 Effect on the mixing parameters

These potentials modify the matter Hamiltonian governing neutrino oscillations in the following way:

$$H_{\text{mat}}^{\alpha\beta}(E) = \frac{1}{2E} \left(U_{\alpha i} M_{ij}^2 U_{\beta j}^\dagger + A_{\alpha\beta} \right) = \frac{1}{2E} \left(U'_{\alpha i} M_{ij}^2 U_{\beta j}'^\dagger \right) \quad (4.18)$$

Where U' is the matter-modified PMNS matrix, M^2 is the propagation-basis free Hamiltonian, $\text{Diag}(0, \Delta m_{21}^2, \Delta m_{31}^2)$, and $A_{\alpha\beta} = 2EV_{\alpha\beta}$ is the matter potential contributions.

One can then either solve numerically the eigenvalue problem, in the optimized case of a Hermitian matrix, or adopt analytical approximations.

4.3.1 Analytical Solutions

We are free to rescale the Hamiltonian by any term proportional to the identity, which will only contribute to an overall phase-factor to the wavefunctions of each neutrino flavor. This is in fact why

one is allowed to neglect the Z boson contributions to the "baryonic" matter effects: they are the same for each flavor.

As such, the remarkable result which we obtain is that the $L_\mu - L_\tau$ fifth force, which does not couple to the electron, ends up generating an effective electron-only potential. The main difference with normal matter is that it is effectively negative, which means the phenomenology between the electronic neutrino and antineutrino will be exchanged. To the extent where one does not discriminate between each at IceCube, this is not very important.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \equiv \begin{bmatrix} -a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (4.19)$$

Analytical two-level system solution

In close analogy to the relations between two basis on a Bloch Sphere, whose separation happens because of the introduction of an off-diagonal coupling between "vacuum" eigenstates —coupling which is diagonal in the flavor basis—, the effective parameters are:

$$\Delta m_m^2(E) = \Delta m^2 \sqrt{\left(\cos 2\theta - A/\Delta m^2\right)^2 + \sin^2 2\theta} \quad (4.20)$$

$$\sin^2 2\theta_m(E) = \frac{\sin^2 2\theta}{\left(\cos 2\theta - A/\Delta m^2\right)^2 + \sin^2 2\theta} \quad (4.21)$$

As such, the "resonance" happens for

$$A_{\text{res}} = \frac{V_{\text{res}}}{V_{\text{osc}}} = \cos 2\theta \quad (4.22)$$

where $V_{\text{osc}} = \frac{\Delta m^2}{2E}$. The resonance terminology was argued to stem from the equality between both the matter and oscillation "energies", when $\theta \simeq 0$, a common assumption at the time.

Denton–Minakata–Parke (DMP) Approximation

The DMP approximation for a three-neutrino system is a viable alternative to the numerical solution when dealing with purely electronic matter potentials, which is effectively the case here for the Z' in u- and s- (same-flavor) channels. The core idea is that the θ_{23} sector is left unaffected to first order and its rotation matrix commutes with A . The two other sectors have their own 2-flavor approximations.

To compute the parameters, first one defines the "effective atmospheric splitting", Δm_{ee}^2 , which can be used to refine the first-order guess on the θ_{13} sector.

$$\Delta m_{ee}^2 \equiv \cos^2 \theta_{12} \Delta m_{31}^2 + \sin^2 \theta_{12} \Delta m_{32}^2 = \Delta m_{31}^2 - \sin^2 \theta_{12} \Delta m_{21}^2 \quad (4.23)$$

13 sector:

$$\tan 2\tilde{\theta}_{13} = \frac{\sin 2\theta_{13}}{\cos 2\theta_{13} - A/\Delta m_{ee}^2} \quad (4.24)$$

$$\Delta \tilde{m}_{ee}^2 = \Delta m_{ee}^2 \sqrt{\left(\cos 2\theta_{13} - A/\Delta m_{ee}^2\right)^2 + \sin^2 2\theta_{13}} \quad (4.25)$$

Once this is done, an effective potential is computed for the solar sector, based on the first-order approximations for the θ_{13} above.

$$A_{12} \equiv A \cos^2 \tilde{\theta}_{13} + \Delta m_{ee}^2 \sin^2(\tilde{\theta}_{13} - \theta_{13}) \quad (4.26)$$

12 Sector:

$$\tan 2\tilde{\theta}_{12} = \frac{\sin 2\theta_{12}}{\cos 2\theta_{12} - A_{12}/\Delta m_{21}^2} \quad (4.27)$$

$$\Delta \tilde{m}_{21}^2 = \Delta m_{21}^2 \sqrt{\left(\cos 2\theta_{12} - A_{12}/\Delta m_{21}^2\right)^2 + \sin^2 2\theta_{12}} \quad (4.28)$$

Unmodified sector:

$$\tilde{\theta}_{23} \simeq \theta_{23}, \quad \tilde{\delta} \simeq \delta \quad (4.29)$$

For antineutrinos, the potential just changes sign. Note that given a Δm^2 (either positive or negative) for a given 2-flavor approximation, neutrinos and antineutrinos cannot experience resonance of the matter potential at the same time.

Figures 4.4-6 plot both the numerical solution and DMP approximation in the range of energies of interest to us for a density value at resonance roughly in the middle of the energy range. Negative densities indicate the antineutrino is the one to undergo maximal mixing, or that the antineutrino background dominates in the case of the effective, low-energy potential.

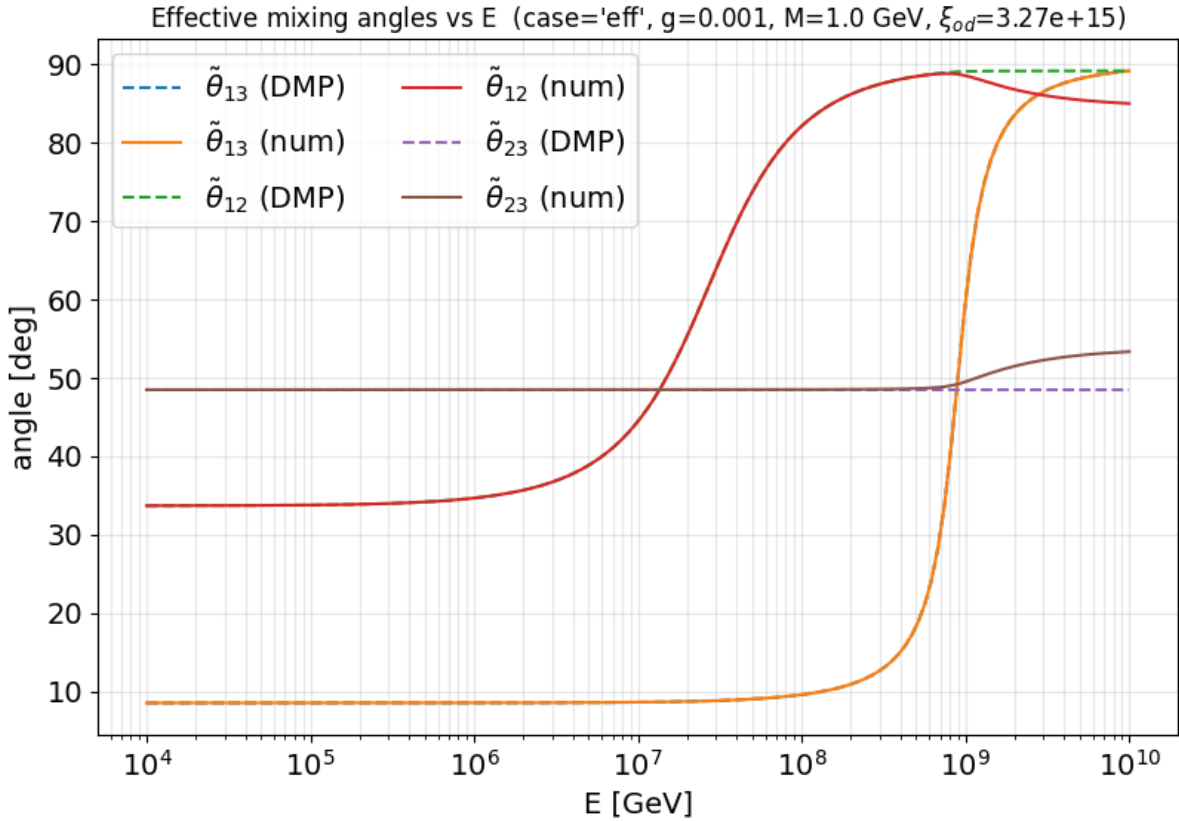


Figure 4.4: Variations of the mixing angles with energy, $g_{Z'} = 0.001$ for the low effective energy potentials ($M_{Z'} = 1$ GeV). The density difference (labeled N above the graph) is chosen to make θ_{12} resonate roughly in the middle of the graph. One sees deviations from the analytical approximations occur above 1000 PeV, as expected when approximating the θ_{23} -sector as unchanged.

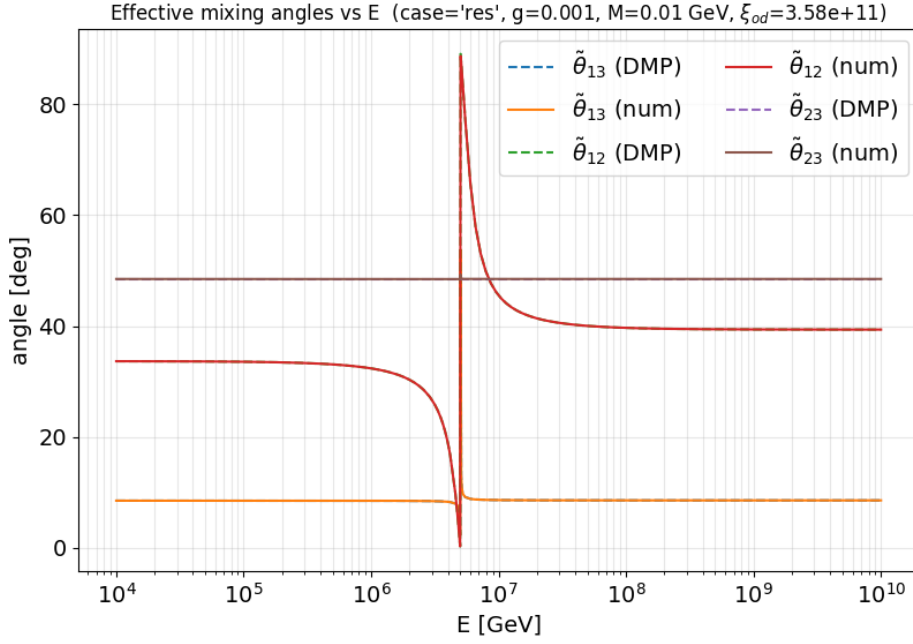


Figure 4.5: Variations of the mixing angles with energy, $g_{Z'} = 0.001$ for the resonant potential ($M_{Z'} = 10$ MeV). The density difference (labeled N above the graph) is chosen to make θ_{12} resonate roughly in the middle of the graph. The analytical expressions, in this case, do not showcase any deviations from the numerical computations (finding the eigenvalues of $\mathcal{H}_{\text{mat}}$).

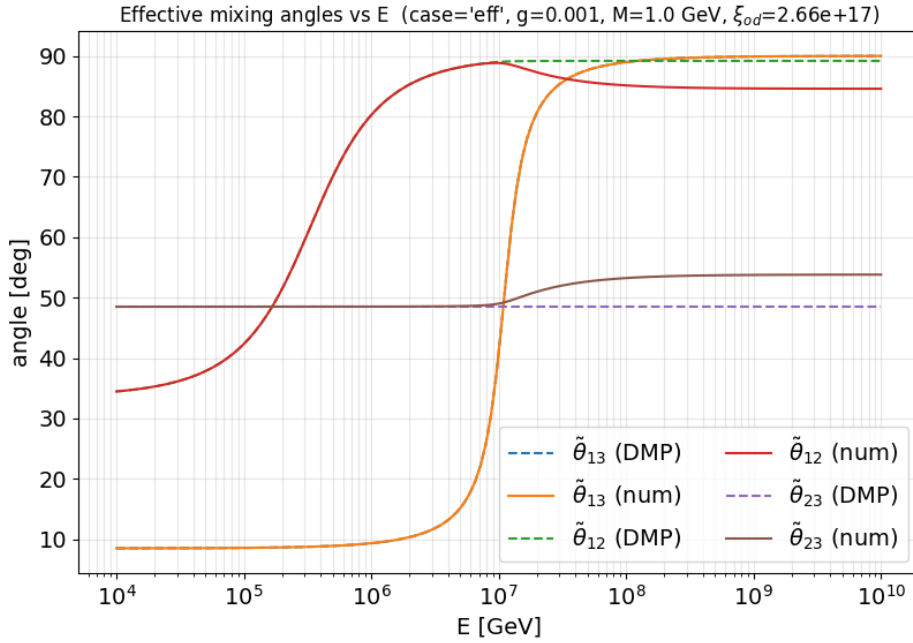


Figure 4.6: Variations of the mixing angles with energy, $g_{Z'} = 0.001$ for the effective, low-energy potential ($M_{Z'} = 10$ MeV). The density difference (labeled N above the graph) is chosen to make θ_{13} resonate roughly in the middle of the graph. Again, the DMP deviations from numerical values account for the perturbation theory errors of the order of $\epsilon \sim \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$.

It is particularly enlightening to observe the different profiles for the angles as the density varies around the value that makes them resonate at the center of the energy range. This is plotted in Figures 4.7-10.

As expected from the low-energy, "Fermi-like" potentials, when $\sqrt{s} \ll M_{Z'}$, the resonant energy changes location as density changes. Indeed, their product, $A \propto E \cdot N$, is constant at resonance. This is evident from the 2-flavor approximations expressions from the previous section.

Just like with normal matter effects, at very low densities (the maximum mixing energy is way higher up) the matter effects are negligible, while at very high densities, the matter effects completely destroy the mixing (the matter Hamiltonian is effectively diagonal in flavor-basis) as the angles reach $0/90^\circ$.

In the resonant case (which is simultaneous to the absorption effects in Chapter 3), the shape function $f(y)$ changes things. While it is still true, given the expressions seen in the DMP approximations, that too low a density produces negligible matter effects and too high a density kills the mixing, this is only the *enveloppe* that is modulated by the shape-function $f(y)$. This shape function effectively kills the potential far away from the *actual* resonance $\sqrt{s} \simeq M_{Z'}$, so that matter effects only occur near this energy. Too high a density and the resonance region is but a transition from 0 to 90 degrees (each equivalent to no oscillations). Too low a density and the resonance region sees angles extrema very close to the vacuum value.

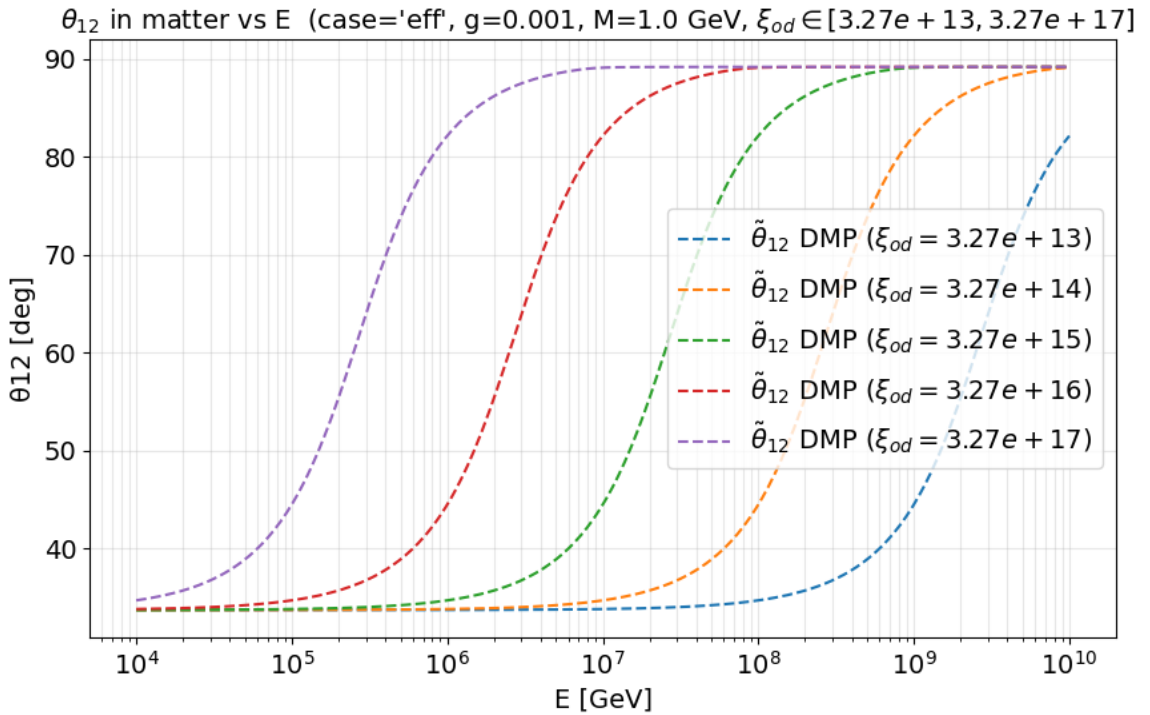


Figure 4.7: Variations of θ_{12} under DMP, for low-limit potentials, for values ranging between $[0.01, 100]$ times the critical density at roughly the center. Different densities lead to shifted profiles in energy. The variation of θ_{13} in the effective case is exactly similar, albeit shifted in energy due to the difference in resonant density.

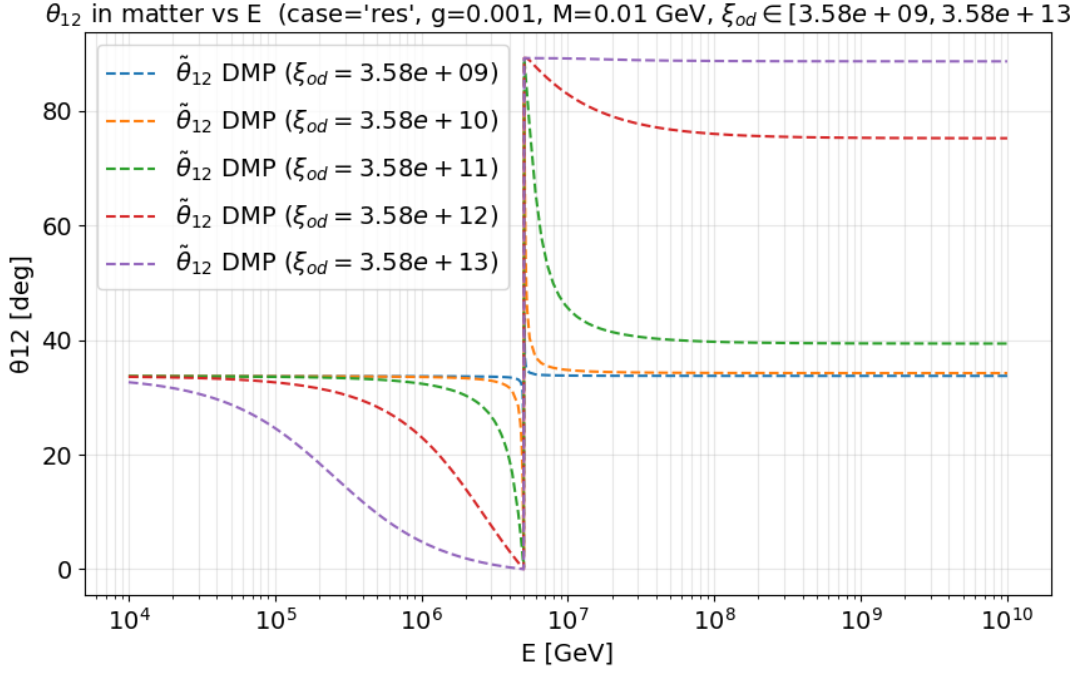


Figure 4.8: Variations of θ_{12} under DMP, for resonant potentials, for values ranging between $[0.01, 100]$ times the density at roughly the center. In this case they are all centered on the resonance $\sqrt{s} \simeq M_{Z'}$, and modulated by the envelope accounting for how far the resonance is from maximal mixing.

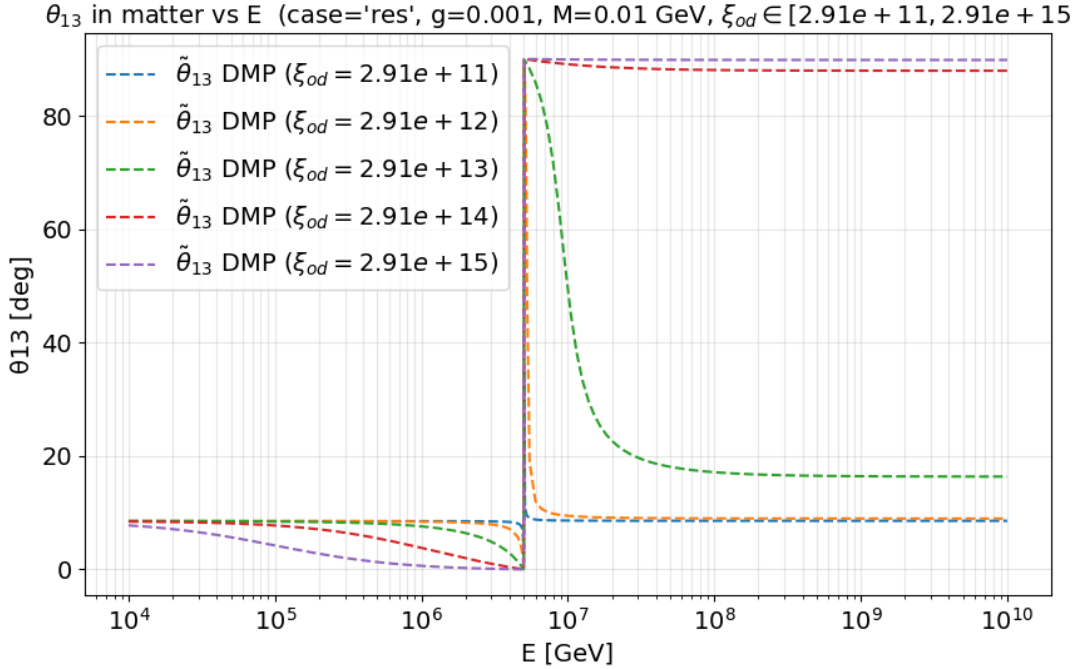


Figure 4.9: Variations of θ_{13} under DMP for values ranging between $[0.01, 100]$ times the resonant density at roughly the center. In this case they are all centered on the resonance $\sqrt{s} \simeq M_{Z'}$, and modulated by the envelope accounting for how far the resonance is from maximal mixing.

Chapter 5

Flavor Content with Z' Potentials

IceCube's increasingly large statistics unravel the possibility of probing the *flavor ratios* of astrophysical neutrino fluxes [68]. Having computed the effective mixing angles in the presence of the $C\nu B$ and the Z' boson, it is possible to derive the effects on the flavor ratios that can possibly add a layer of "signatures" within the spectrum, further signaling the physics of both the Z' and the $C\nu B$.

Let us spoil this section so as to not create ambiguity on the results throughout: in the absence of significant $C\nu B$ overdensities, the matter effects are utterly negligible. In the presence of such overdensities however, the matter effects can become maximal: the effective angles ruling matter oscillations can reach 45 degrees.

Unfortunately, this doesn't guarantee observability of the effect. Indeed, the overdensities required may in turn lead to an absorption so large in the observed energy range that it makes the flavor ratios meaningless (zero divided by n is still zero). In a nutshell, they are self-defeating.

Quite obviously, such extreme absorption scenarios are trivially ruled out by the observation of a diffuse extragalactic neutrino flux at IceCube. Whether this means there is no Z' or no $C\nu B$ overdensities (or both) is a question left open by degeneracy of the effects.

This section thus serves in deriving the above results, which in spite of being negative, can avoid someone else the hassle of deriving them. To reach these conclusions, first the notion of adiabaticity is presented and quantified in order to justify keeping the implementation without having to take into account complicated stochastic quantum transitions in the propagation process. Then a short notice is made about the way the code is amended to take into account matter potentials, before quantifying the overdensity requirements on the observability of matter potentials in the spectrum.

5.1 Adiabaticity

Whenever matter potentials are studied in a context of varying density (such as the Sun's MSW effect), the general notions of adiabaticity may come into play. This occurs when the physical parameters ruling the system may vary on time-or-length scales much greater than that of the system itself.

Because of this, the neutrinos are expected to remain in their respective "eigenstate" even though these slowly but steadily rotate in the Hilbert space throughout cosmological propagation. The initial and final (eigen-)states will have nothing to do with one another! Deviations from adiabaticity introduce a non-zero probability of "jumping" between eigenstate branch, called Landau-Zenner transitions.

In the case of matter potentials, adiabaticity can be estimated in two ways. First by comparing the length scale of variation of the neutrino background density, r^R with the typical length of neutrino oscillations in matter, l_m^R . This is the expression used by Smirnov in Ref. [51] used when discussing the unchanging-energy neutrinos escaping the Sun.

Equivalently, the more general expression applicable in varying-energy, redshifted neutrinos can be found for instance in Ref. [28] (p. 334, eq. 9.80) or [49] in the form of the *adiabaticity parameter*:

$$\gamma_{adiab} = \frac{\Delta m_m^2 / 2E}{2 \left| \frac{d\theta_m}{dt} \right|} \Big|_{res} = \frac{(\Delta m_m^2)^2}{2E \sin(2\theta_m)} \left| \frac{d(A)}{dt} \right|_{res}^{-1}$$

The matter potential term scales as $A_{11} \propto (1+z)^4 f(y)$, with three powers from the comoving density dilution, one from the redshifting energy and the resonant shape-factor $f(y)$. As such r^R is set by the Hubble constant, $r^R \sim [H(z)]^{-1}$, a cosmological distance. The in-medium oscillation length l_m^R , by contrast, is microscopic. At PeV-scale energies, one finds $\gamma_{adiab} \sim 10^{10}$!

This contrasts vastly with the low adiabaticity values of Lunardini & Smirnov (2001) [49], who consider values $E \sim 10^{20-23}$ eV, $\Delta m^2 \sim 10^{-10}$ eV² reducing the adiabaticity parameter considerably.

The adiabatic regime is thus assumed throughout this section, and no Landau-Zener correction are applied to model jump transitions between eigenstate. The neutrinos are described as stable populations of matter eigenstates: the diffuse source flavor fluxes are projected onto the eigenbasis using the local angles $\theta_{ij}^m(z, E)$, the populations are absorbed along the line of sight with charges $Q_{ij}^m(z, E)$, and the result is projected back to flavor states at $z = 0$.

5.2 Effects of Matter Potentials on Flavor Ratios

With the mixing angles having changed, the neutrino propagation eigenstates $|\nu_i^m\rangle$, are now separate from the mass eigenstates $|\nu_i\rangle$. These modified mixing angles depend both on redshift, because of the density increasing with the look-back time, and on energy.

The first way these affect the solution is when transforming the produced (resp. detected) fluxes from the flavor (resp. mass) to the mass (resp. flavor) bases. Adiabaticity guarantees rotating the fluxes at the start and the end is sufficient to capture "stand-alone" matter effects.

However this analysis also considers direct absorption of the spectrum. The matter potentials continuously rotate the coupling constants responsible for scattering of the matter propagation eigenstates, because their PMNS mixing with the flavor couplings, $\text{diag}(0, 1, -1)$, changes with time.

Let us also recall that, until now, neutrinos and antineutrinos were treated as one and the same in the implementation, by virtue of the alleged CP-symmetry of the CνB and because they are not resolved separately at IceCube. This hypothesis is relaxed in this Chapter, and each flux is evolved separately, in order to account properly for the difference in matter potential each one is subject to.

The implementation is thus amended by vectorizing the theta's into (z, E) dependent arrays interpolated at $z_{current}$ and computing these prior to the $\sigma(i, j, E, z, \theta)$ and $J(i, j, E, z, \theta)$ arrays. The numerical integration is then performed twice for the distinct neutrino and an antineutrino components, to properly treat their respective matter potentials, before recombining them at $z = 0$.

5.3 Density Requirements

For the matter effects to leave a noticeable imprint on the flavor ratios, the potential must be of the order of the vacuum energy splitting. This is verified when:

$$|A_{11}| = 2EV \sim \Delta m_{ij}^2 \cos(2\theta_{ij}), \quad V_{11} \propto \left(\frac{g_{Z'}}{M_{Z'}} \right)^2 N_{\text{C}\nu\text{B}}^\alpha(z) \quad (5.1)$$

$$\Rightarrow \left(\frac{g_{Z'}}{M_{Z'}} \right)^2 \cdot N_{\text{C}\nu\text{B}}^\alpha(z) \cdot \frac{2E}{\Delta m_{ij}^2} \approx 1 \quad (5.2)$$

But for the typical values assumed throughout this thesis for the boson parameters, the conventional value of the C ν B at $z = 0$ of 56 cm^{-3} ($\sim 10^{-40} \text{ GeV}^3$), and the best-fit neutrino parameters in the 12-sector, one retrieves a value $\sim 10^{-13}$!!! This was already in fact shown in Figures 4.4-6 of Chapter 4, where the resonant densities in the θ_{12} case were such orders of magnitude above the typical C ν B value accepted.

If one expects traces of the matter potentials in the spectrum at IceCube, an inverse factor must be supplied elsewhere. This is in fact not unheard of in the literature, with one particular 2025 reference (published in the middle of the completion of this thesis), Dev et al. [21], who undertook the study of the Standard Model's Z-boson-induced matter effects with the C ν B. The analysis focuses on theoretical pseudo-Dirac scenarios for neutrino mass generation with extremely small $\delta m^2 \sim 10^{-17} \text{ eV}^2$ values. Akin to the Z' , these are purely hypotheticalal.

The natural scapegoat for such a huge factor is hypothetical C ν B overdensities –clusters of dense C ν B regions throughout the Universe the neutrinos could cross–, parametrised by the overdensity factor ξ_{od} multiplying the total density:

$$N_{\text{C}\nu\text{B}}^\alpha(z) = 2n_0 \xi_{od} \eta_\alpha (1+z)^3 \quad (5.3)$$

with $n_0 = 56 \text{ cm}^{-3}$ the accepted C ν B density at zero redshift and $\eta_\alpha = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_{\nu_\alpha} + n_{\bar{\nu}_\alpha}} \in [-1, 1]$, a CP-asymmetry parameter taken to discuss the neutrino-antineutrino difference in matter effects, and how it can be leveraged to hunt for C ν B CP-asymmetries through the flavor ratios.

This asymmetry matters especially in the low-energy –i.e. high mediator mass– regime, where the matter effects derive from t-channel scattering and are proportional to the difference $n_\alpha - \bar{n}_\alpha$, which vanishes without a strong CP-asymmetry. It is a known result that such asymmetries are required in the low-energy limit ("Fermi-like") matter potentials [49].

The s -channel matter potential of higher interest in this thesis –because it coalesces with the previous endeavors on absorption features in the IceCube spectra–, on the other hand, is species-dependent: a propagating ν_α forward-scatters (through annihilation) with the antineutrino population $\bar{\nu}_\alpha$ and vice-versa. Hence it is not vanishing under a CP-symmetric background, and the η in eq. 5.3 should be replaced by: (this is done in the code but the notation is kept as η for sake of simplicity in the figures)

$$V_\nu \propto +n_{\bar{\nu}} \propto (1 - \eta_\nu), \quad V_{\bar{\nu}} \propto -n_\nu \propto (1 + \eta_\nu) \quad (5.4)$$

with the negative sign emerging because of the matter-potential sign flip with respect to antimatter.

5.4 Final Results

By virtue of the optical theorem, the matter potential (the real part of the forward amplitude) and the absorption cross-section (imaginary part) are two aspects of the same Z' -mediated amplitude. They cannot be chosen independently. The s -channel process $\nu_\alpha + \bar{\nu}_\alpha \rightarrow Z' \rightarrow \nu_\alpha + \bar{\nu}_\alpha$ that generates the resonant potential is also a resonant annihilation that removes flux. The optical depth neutrinos observed at E_ν and emitted at z_{max} crossed is:

$$\tau(E_\nu) \propto n_0 \xi_{od} \eta_\alpha \int_{z_{max}}^0 (1+z)^3 \sigma(E_\nu(1+z)) dz \quad (5.5)$$

At the overdensity required for resonant effective matter mixing, the optical depth is so large that the propagation flux, already absorbed significantly as seen in Chapter 3, is completely extinguished.

To nonetheless exhibit the matter-modified mixing signature in isolation, and for sake of comparison with Z-boson matter potentials, the flavor-ratios changes with respect to a (3x3) list of (ξ_{od}, η) parameters, which sum up all the physics of this section, are shown in Figure 5.1. To make them visible, the matter density that enters the scattering-regeneration is kept the standard n_0 one, while that which generates matter effects is combined with the overdensity parameter, as in eq. 5.3.

This answers the question "given a matter mixing, what flavor ratio would emerge?", but it is **NOT** a physical scenario, because the two densities are one and the same in reality.

The alternative case, that of effective potential, could seem more promising at first glance. Indeed, because the boson mass is now taken to be much higher, the absorption, which scales as $\sigma(E) \propto s \cdot \frac{g_{Z'}^4}{M_{Z'}^4}$ in the heavy mediator case ($E \ll M_{Z'}$), becomes negligible.

This case is much more analogous to Z-boson potentials that also neglect resonant absorption (due to the ~ 90 GeV mass of the Z).

However, the required overdensity factor for resonance will also scale with $M_{Z'}^2$, as seen in eq. 5.2. For a boson mass similar to that of the Z, ~ 100 GeV, the overdensity in θ_{12} reaches $\sim 10^{21}$ while in θ_{13} it reaches $\sim 10^{23}$. In addition to this, the values motivated by experimental tensions when introducing the Z' revolved around $\sim 10 - 100$ MeV.

This time the problem is not the absorption extinguishing the flux, but the overdensity factor keeping the scenario completely unrealistic. Indeed, the KATRIN experiment (monitoring tritium beta-decay to set laboratory constraints on the neutrino masses) has been shown to also constrain the C ν B overdensities at Earth around $\xi_{od} \approx 10^{11}$ [69]. While one could keep arguing about other line-of-sight overdensities outside of the Earth C ν B halo, arguments such as these can keep stacking and stacking up to infinity, and this thesis fixes its scope up to the present point.

This means that the flavor ratio grids 5.1-4 have no true scientific relevance in using the IceCube spectrum to trace the joint signature of potential C ν B overdensities and Z' -boson physics. More precisely, assuming the Z' signature is observed through an absorption feature of the VHE neutrino spectrum, C ν B overdensities are **not** traceable in further flavor ratio analysis of the said spectrum, precisely because the overdensity requirements seem far too unrealistic given the Z' boson parameters.

It is instructive that a closely related scenario keeps the overdensity within reason. Dev et al. (2025) [21] consider a pseudo-Dirac active-sterile splitting $\delta m^2 \sim 10^{-17}$ eV² –twelve orders of magnitude below the solar splitting Δm_{21}^2 – with the matter potential induced by the SM Z-boson. The visibility condition 5.2 involves a comparatively microscopic splitting. The required potential, and hence the required overdensity, is thus smaller: $\xi_{od} \sim 10^6$, which falls within the KATRIN bound.

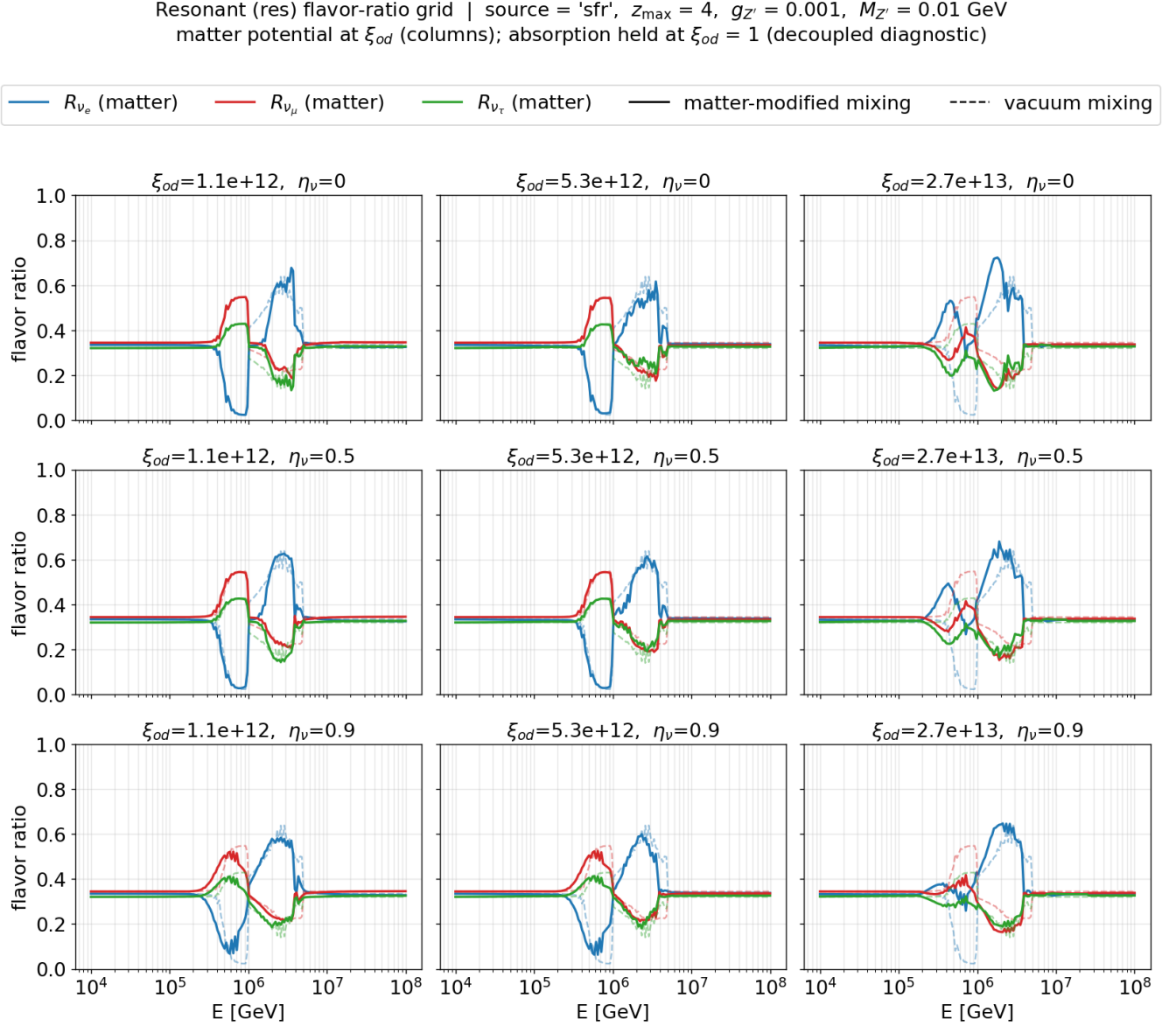


Figure 5.1: Flavor Ratios for a grid of 3 CP-asymmetry values $\eta = [0, 0.5, 0.9]$ (lines) and 3 overdensity factors $[0.2, 1, 5] \times N_{res}^{12}$ (columns). N_{res}^{12} is such that θ_{12} is maximal *at energy* $E_R(\nu_{lightest})$, which in the present NO hierarchy corresponds to the rightmost feature. As the density increases, the energy where matter angles are maximal shifts to the left ($A_{11} \propto EV$), and the higher energies become effectively flavor-diagonal (hence the strong electronic flavor boost). The dependence of the effect on η is minimal, because as stated earlier the s -channel matter potentials are proportional to $1 - \eta$ and $1 + \eta$ and thus both cannot be zero at the same time for a given η . Recall these do **NOT** represent physical scenarios, as such overdensities would completely extinguish the spectrum.

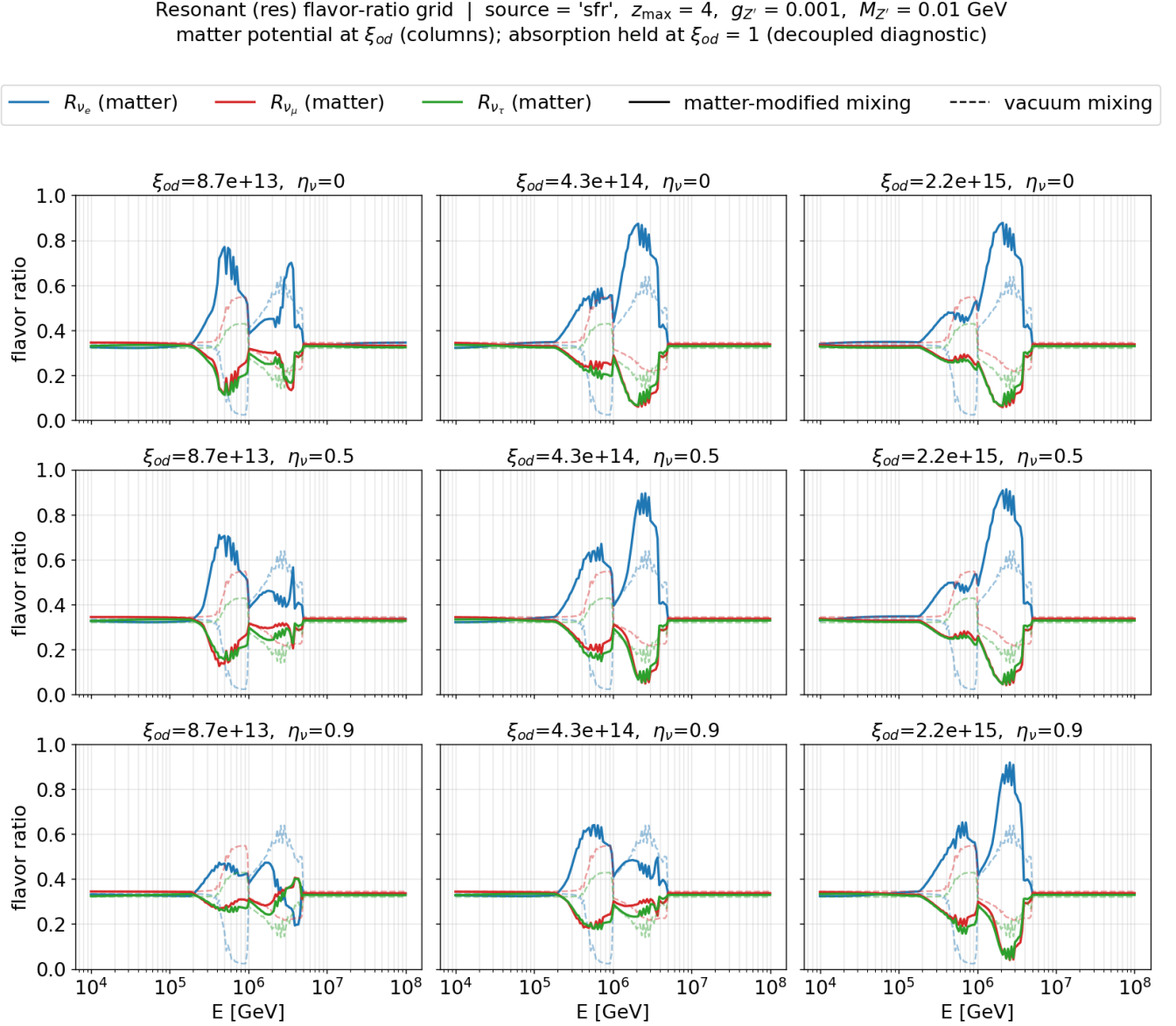


Figure 5.2: Flavor Ratios for a grid of 3 CP-asymmetry values $\eta = [0, 0.5, 0.9]$ (lines) and 3 overdensity factors $[0.2, 1, 5] \times N_{res}^{13}$ (columns), for $M_{Z'} = 10$ MeV. Numerical angle solutions. Because the required density is higher by a factor of $\Delta m_{31}^2 / \Delta m_{21}^2 \sim 33$, the matter potentials arise in both features of the spectrum (the mixing onset is shifted to lower energies), because $A_{11} \propto EV$ resonates at a given value set by the oscillation parameters. Recall these do **NOT** represent physical scenarios, as such overdensities would completely extinguish the spectrum.

Heavy-mediator (eff) flavor-ratio grid | source = 'sfr', $z_{\max} = 4$, $g_{Z'} = 0.001$, $M_{Z'} = 100$ GeV
 absorption and matter potential propagated together from one CvB density (no decoupling)

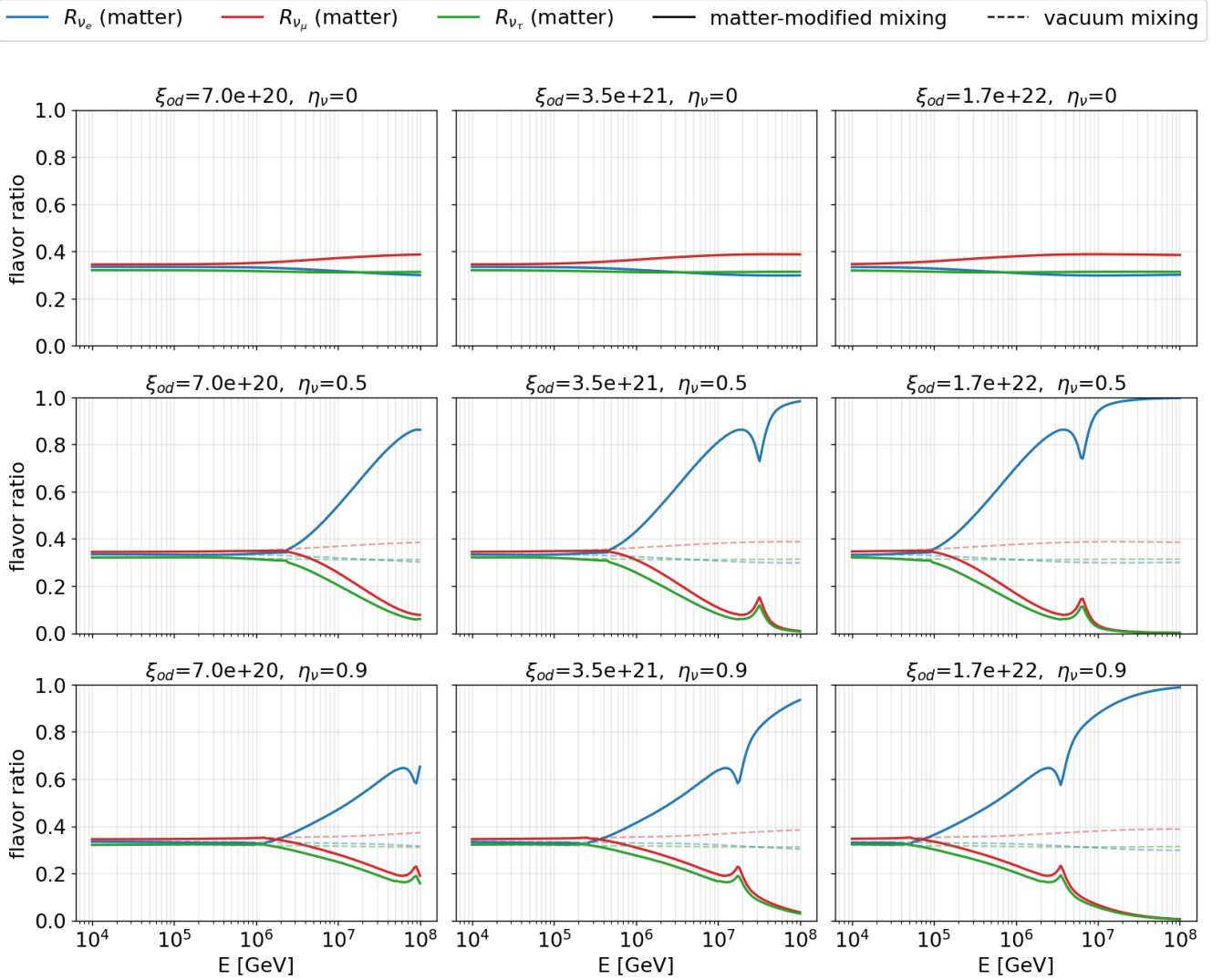


Figure 5.3: Flavor Ratios for a grid of 3 CP-asymmetry values $\eta = [0, 0.5, 0.9]$ (lines) and 3 overdensity factors $[0.2, 1, 5] \times N_{res}^{12}$ (columns), in the heavy mediator limit (100 GeV). Numerical angle solutions. Because there is no energy resonant structure, and as expected from the angles variation graphs of Chapter 4, in this case the variation is smooth. As the density increases, the location at which the effects become visible shift to lower energies. One can note an electronic dip at specific energy values which shifts with the density: it corresponds to the onset of θ_{23} -sector mixing, as was also observed in the angles plot. This interpretation is confirmed by comparison with the last figure, which is the same but performed using DMP-approximated angles, which ignore the θ_{23} -sector—and indeed show no such dip.

Heavy-mediator (eff) flavor-ratio grid | source = 'sfr', $z_{\max} = 4$, $g_{Z'} = 0.001$, $M_{Z'} = 100$ GeV
 absorption and matter potential propagated together from one CvB density (no decoupling)

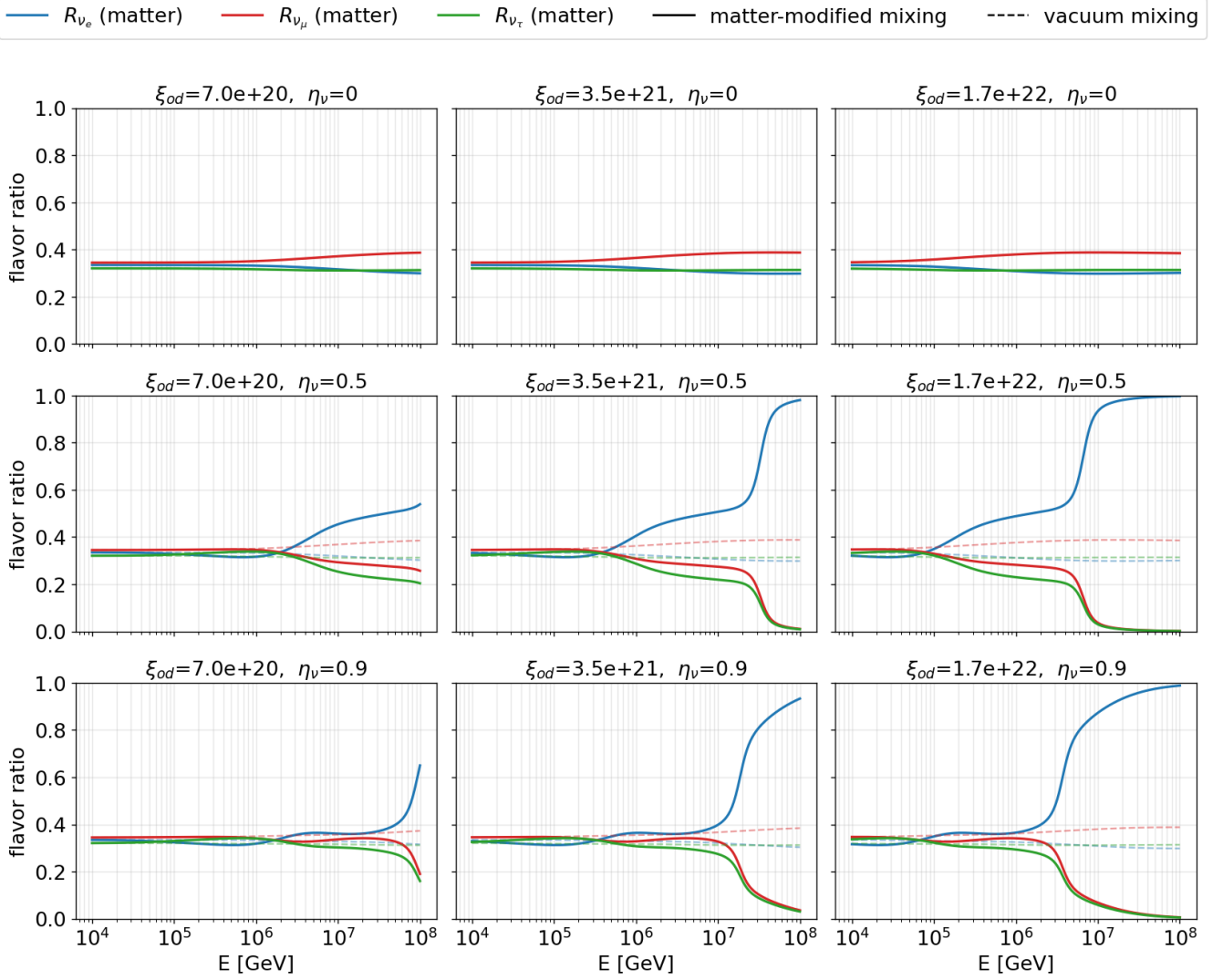


Figure 5.4: Flavor Ratios for a grid of 3 CP-asymmetry values $\eta = [0, 0.5, 0.9]$ (lines) and 3 overdensity factors $[0.2, 1, 5] \times N_{res}^{12}$ (columns), in the heavy mediator limit (100 GeV). DMP angle solutions. As expected, the θ_{12} and θ_{13} mixing onset happen separately, as with the angles. In this DMP approximated scenario, the 13-onset is not accompanied by the dip associated with θ_{23} mixing, which momentarily reverses the complete ν_e dominance, only to disappear at even higher resonance. In the eff cases, unlike the res case, the matter effects are not limited to the absorption features anymore.

Chapter 6

Discussions

6.1 To what extent are the hypotheses made valid?

6.1.1 s-channel absorption cross-section

Throughout this work, we have assumed the interaction cross-section of the $\nu_i \bar{\nu}_j \longrightarrow \nu \bar{\nu}$ was only generated by the resonant annihilation s-channel, described by diagram 1.9. Furthermore, the s-channel contribution to the matter potentials was deemed dominant and kept as the only contribution for the low mediator mass regime;

Naturally, this hypothesis is supported by the fact that, at resonance, this process is amplified by the energy structure. The Breit-Wigner form of the cross-section is also a much welcomed simplification to the otherwise lengthy, albeit more precise, expression. This biases works in favor of rejecting these additional contributions.

One can ponder at the validity of this assumption throughout the range of energies considered here and, while the resonant regions are found to be those experiencing the strongest effects, this is merely a *consequence* of the assumption made, not evidence for its validity.

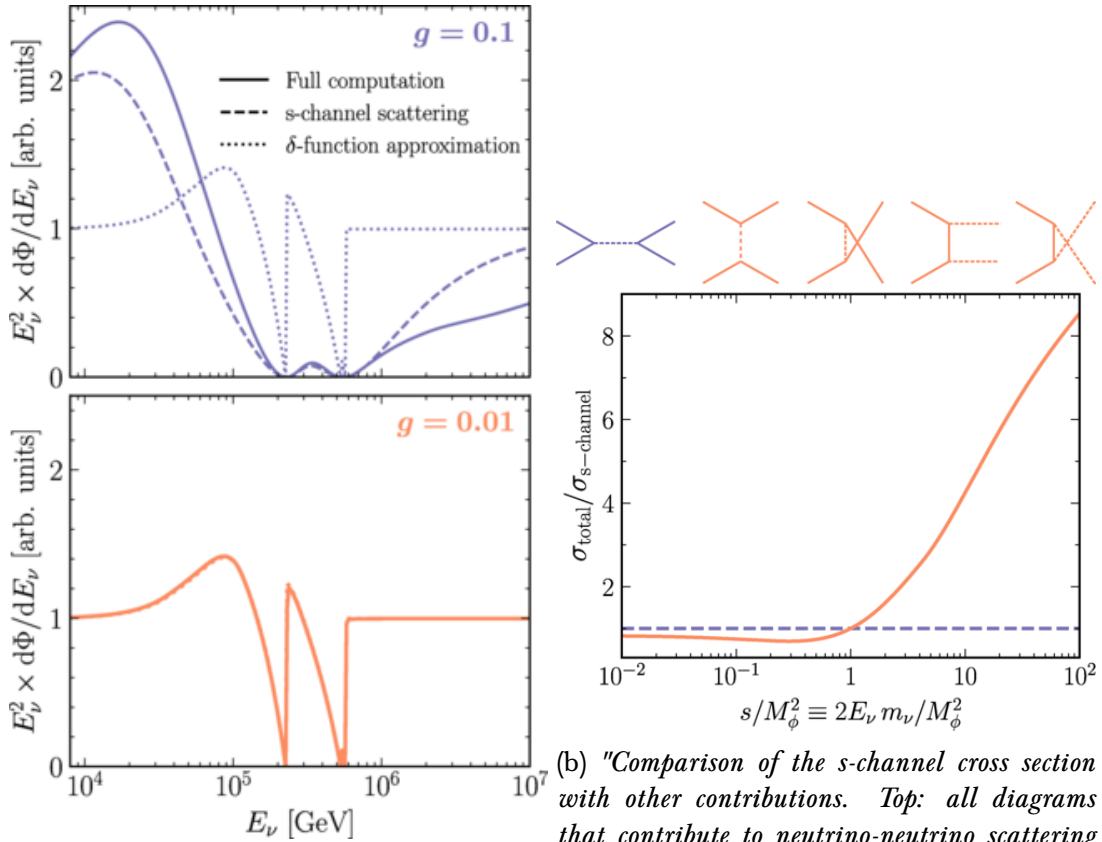
The extensive literature on the subject (to quote those I have built upon, [7–20]) almost exclusively adopt this very hypothesis. Only [18] has deviated from it, computing the subdominant contributions and showing that, outside of the resonant regions —and to the largest extent at greater energies— these contributions are in fact not subdominant at all, provided the coupling is sufficiently strong. Figures 6.2a and 6.2b, taken from their paper, substantiate these claims. The complete cross-section they derived is presented in 6.1 for both neutrino-antineutrino and neutrino-neutrino scattering, with initial states i, j . Double boson production is also a possible contribution.

How does this affect the results pertaining to the absorption feature? To the extent where the absorption effects remain strongest at resonance, given the current IceCube resolution and the parameter range for $g_{Z'}$ considered, not greatly.

For matter potentials, in principle the new contributions do not further affect the effective potentials, but rather switch the Q_{kl} couplings considered. The t-channel diagram for $\nu_i \bar{\nu}_j \longrightarrow \nu \bar{\nu}$, for instance, will be in proportion to $\sum_{kl} Q_{ik}^2 Q_{jl}^2$ because of the configuration of the diagram rather than the $\sum_{kl} Q_{ij}^2 Q_{kl}^2 = 2Q_{ij}^2$ factor of the "annihilation" configuration considered up to now. This is shown in Figure 6.1.

$$\begin{aligned}
\sigma_{ij}(s) = & \frac{1}{16\pi} \sum_{k,l} \left[g_{ij}^2 g_{kl}^2 \frac{s}{(s - M_\phi^2)^2 + M_\phi^2 \Gamma^2} + \left(\frac{3}{2} g_{ik}^2 g_{jl}^2 + \frac{1}{2} g_{il}^2 g_{jk}^2 \right) \left(\frac{s + 2M_\phi^2}{s(s + M_\phi^2)} + 2 \frac{M_\phi^2}{s^2} \log \frac{M_\phi^2}{M_\phi^2 + s} \right) \right. \\
& - g_{ij} g_{kl} g_{ik} g_{jl} \left(\frac{M_\phi^2}{s} - 1 \right) \frac{s + M_\phi^2 \log \frac{M_\phi^2}{M_\phi^2 + s}}{(s - M_\phi^2)^2 + M_\phi^2 \Gamma^2} + \frac{1}{2} g_{ik} g_{jl} g_{il} g_{jk} \left(\frac{1}{s} + \frac{2M_\phi^2 (M_\phi^2 + s)}{s^2 (2M_\phi^2 + s)} \log \frac{M_\phi^2}{M_\phi^2 + s} \right) \Big] \\
& + \frac{1}{64\pi s^2} \left(\sum_k g_{ik}^2 g_{jk}^2 \right) \left\{ \frac{s^2 - 4M_\phi^2 s + 6M_\phi^4}{s - 2M_\phi^2} \log \left[\left(\frac{\sqrt{s(s - 4M_\phi^2)} + s - 2M_\phi^2}{\sqrt{s(s - 4M_\phi^2)} - s + 2M_\phi^2} \right)^2 \right] - 6\sqrt{s(s - 4M_\phi^2)} \right\}.
\end{aligned}$$

Figure 6.1: Complete cross-section for neutrino-(anti)neutrino scattering or annihilation into ϕ -pair production. The last term accounts for $\nu\bar{\nu} \rightarrow \phi\phi$. Taken from [18] in the Dirac neutrino case.



(a) "Neutrino flux at Earth in the presence at lowest order. For double-scalar production, of νSI , using different approximation levels to the scalars quickly decay to neutrinos. Bottom: solve [the master equation]. We assume an astrophysical flux $\propto E_\nu^2$ and ν_τ self-interactions provided by the s-channel contribution as a function ($M_\phi = 5$ MeV). Top: for $g=0.1$, a full calculation of the scaled center-of-momentum energy is needed to obtain correct results. Bottom: squared. At high energies, the s-channel contribution for $g=0.01$, approximations may be made." [18]

(b) "Comparison of the s-channel cross section with other contributions. Top: all diagrams that contribute to neutrino-neutrino scattering" [18]

Figure 6.2: Figures from Esteban et al. (2021) [18]. Captions are taken from their article too. These substantiate the claim that, with sufficiently strong a coupling and outside of resonance, the complete cross-section 6.1 must be taken into account to correctly model the total spectrum.

This is of course overshadowed by the overdensity requirements found at the end of Chapter 5. Because significant matter effects cannot be observed at mediator mass regimes allowing at the same time resonant absorption features to appear in the spectrum, choosing to neglect or not the other diagrams is scientifically irrelevant in the case of the Z' . One could imagine, however, that other scenarios with more clement overdensity requirements predict simultaneous signatures for which the complete diagram set should be considered.

6.1.2 1:2:0 initial flavor ratios

The idea that we could detect new physics in the flavor ratios measured at Earth assumes a precise understanding of the flavor ratios produced at the source. This requires a solid understanding of the sources themselves, the main neutrino production mechanisms and the degree to which these mechanisms are dominant over other, subleading, processes.

The larger the effects of the new physics on the flavor ratios, the lesser the precision required to probe them.

In the past two decades, neutrino telescopes such as AMANDA and IceCube have pushed the frontiers of the field up to the petaelectronvolt (PeV) scale. Astrophysical sources produce VHE neutrinos when accelerated protons or nuclei collide with the ambient matter ($p + p$) or radiation field ($p + \gamma$), subsequently decaying into pions and eventually neutrinos (see Fig. 6.3). This process results in a flavor ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ [4]. Neutrino oscillations, given the best fit parameters, are then expected to lead to a flavor ratio at Earth of $\nu_e : \nu_\mu : \nu_\tau \approx 1 : 1 : 1$.

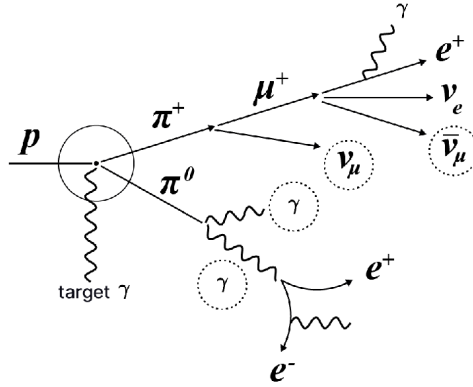


Figure 6.3: Flow chart of a cosmic ray proton decaying after collision with a photon. While neutral pions do not produce any neutrinos, charged pions end up emitting electronic and muonic flavors in 1 : 2 proportions. Proton-proton collisions produce all three pion charges in equal amounts. Figure from Halzen F. (2023) [70].

Beyond the pion decay scenario (1 : 2 : 0), subdominant processes may produce neutrinos in exclusively one flavor [68]: muon-damping by high magnetic or radiation field in the region of emission may significantly lower the rate of muon decay to $\bar{\nu}_e + e^-$ [71], leading to 0 : 1 : 0 ratios while neutron decay to $p + e^- + \bar{\nu}_e$ leads to 1 : 0 : 0 ratios instead.

In the extreme environments where astrophysical neutrinos are generated, charm quarks may be a produce of pp collisions. The subsequent decay of the associated mesons may become a dominant source of neutrinos at very to extremely high energies, yielding a 1 : 1 : 0 ratio instead [6].

A notable conflict in this endeavor is that our understanding of the initial flavor ratios at the sources themselves is sought to be refined by using the very flavor ratios at Earth one hopes to find new physics in [72]. But ultimately playing both sides at once is dangerous! This creates an evident bias whereby the assumption of new physics made to seek the latter's signatures conflicts with the assumption of its absence when inferring the source flavor ratios.

This criticism is somewhat alleviated when considering that neutrino astrophysics is of course not the only way to study the neutrino flavor ratios in astrophysical sources. In the overarching landscape of multi-messenger astrophysics, while neutrinos may help electromagnetic astrophysics to grow, the reverse is true also —and certainly truer to this day—. All the data retrieved from the source, and our understanding of physics, helps to constrain the source production of neutrinos.

Yet, disentangling the higher order effects of new physics on the flavor ratios with the likes of matter effects at the production site, prompt neutrino productions or higher order corrections in our understanding of the sources, is an ambitious task in the light of present-day statistics. While experimentalists seek to determine the actual flavor ratios at Earth as precisely as possible [73], theoreticians seek to constrain the expected flavor ratios given the plethora of established physics and uncertainties mentioned above [72]. Deviations from these conservative ranges may then more significantly signal new physics [68], that further theoretical endeavors may seek to explain.

6.1.3 Hypotheses on the $C\nu B$

The cosmic neutrino background has never been directly measured. Hypothesis on very large overdensities have been made throughout this thesis. While settling their existence is out of the scope of current science, the KATRIN experiment constrains the $C\nu B$ density by searching for relic neutrino capture on its high-purity molecular tritium gas source. Their current bound is $\xi_{od} < 9.7 \times 10^{10}/\alpha$ ($1.1 \times 10^{11}/\alpha$) at a 90% (95%) confidence level with $\alpha = 1$ (0.5) for Majorana (Dirac) neutrinos [69].

In addition to this, one may wonder whether the $C\nu B$ can be considered collisionless on the timescales pertaining to gravitational reordering. Indeed, both their masses and weak interaction coupling are almost non-existent. If this is the case, then one may expect the $C\nu B$ to be influenced by the gravitational wells of galaxies and clusters.

6.2 What other approaches to model the spectrum are possible?

In this work, the effects on the spectrum at Earth are derived by numerically solving the Boltzmann integro-differential equation. This is the approach used overwhelmingly in the literature [7–20], but it is not the only one.

No approach, I believe, is intrinsically superior, they must be seen as complementary. Instead, each have their pros and cons and are often rather suited for a problem more than another. The goal of this section is to describe which and how.

Monte-Carlo simulations

To the best of my knowledge, the direct search for νSI in the IceCube diffuse spectra has not been pursued using Monte-Carlo simulations directly. The aim of these searches being to derive the signatures of νSI in a spectrum, computing the evolution of thousands of particles at once does not offer much

advantages compared to directly solving the evolution equation of the spectrum. The master equation also offers a semi-analytical to analytical solution (depending on the complexity of the physics involved) which offer additional clarity.

The Monte-Carlo approach, however, has proven fruitful when the trajectories of individual neutrinos become relevant. Carpio and Murase (2023) [74] have implemented a Monte-Carlo approach to compute the arrival time delays some neutrinos can exhibit relative to their electromagnetic counterpart, because of small angle scattering changing the distance traveled. Here, the Monte-Carlo approach becomes enlightening, modeling the travel times the transport equation is unable to properly capture.

In Carpio and Murase's case, the Monte-Carlo treats neutrinos as classical particles with classical trajectories. This fails to capture the quantum aspects of neutrino oscillations, wavepacket separation or the matter effects on mixing parameters. While this is no problem in their case, such an approach would be incompatible with this thesis's goals.

δ -approximation

While the Boltzmann transport equation offers a very nuanced picture of the neutrino spectrum's evolution, it comes with high computational cost and lengthy simulation times.

Creque-Sarbinowski et al. (2020) [17] put forth an analytical approach taking advantage of the extremely narrow width of the cross-section 2.17. By assuming that the latter is well approximated by a Dirac delta distribution, $\sigma(E_\nu) = \sigma^{res} E_\nu \delta(E_\nu - E_\nu^{res})$, the integrals making use of the cross-section, specifically that for the optical depth and regeneration, are then simply evaluated explicitly. This approach solves the aforementioned drawbacks by being particularly time-efficient.

However, it was shown that this approximation fails to describe the spectrum correctly for too strong a coupling [18]. Indeed, beyond a critical value of $g_{Z'}$, the solution does not depend on $g_{Z'}$ anymore. This is also shown in Figure 6.2a, taken from Esteban et al. [18].

This is explained by the total depletion criterion in Equation 3.17: when the δ approximation is made, the spectrum cannot be depleted beyond $g_{Z'}^{crit}$, the value at which the spectrum is totally depleted before the redshifts displaces it; when the cross-section is considered fully, out of resonance regions may still keep growing past the critical $g_{Z'}$ as established for the resonance.

6.3 How do these results translate to other ν SI scenarios?

Throughout this thesis, we have considered a particular case of neutrino self/secret interactions: the Z' boson created by gauging the $U(1)_{L_\mu - L_\tau}$ symmetry. We also have considered a particular case of background scatterers: $C\nu B$ neutrinos. Both can be dropped in favor of a more general, agnostic approach to the problem.

6.3.1 Other gauge bosons?

Other fields in fundamental physics call for additional gauge bosons. Describing them falls out of the scope of this thesis. But to quote DiFranzo et al. (2015) [15], one may mention those which

"[...]are predicted in Grand Unified Theories [...], little Higgs theories, dynamical symmetry breaking scenarios, models with extra spatial dimensions and many other extensions to the SM."

The easiest way to generalize the ν SI is to not specify a gauge group, or a particular model, from which to derive the associated boson. The general Breit-Wigner form for the cross-section may be kept [9] while one may explore the parameter space of couplings $Q_{\alpha\beta}$, set to $\text{Diag}(0, 1, -1)$ in the $L_\mu - L_\tau$ scenario.

Thus, what has been done in Chapter 3 extrapolates easily to any gauge boson, whether scalar or vector, of mass M . This is in fact the default case when dealing with ν SI, see *e.g.* [7–12, 18]. Specific interactions such as the Z' [13, 15, 19] are generally motivated by some other aspect than pure theory, as has been discussed in Chapter 1 - 1.2.2 "Motivations for a $L_\mu - L_\tau$ Z' -boson". If one were to gauge $U(1)_{L_\tau}$, for instance —ignoring the quantum anomalies thereof—, the same formalism, expressions and code could be used by simply updating the $Q_{\alpha\beta}$ coupling matrix to $\text{Diag}(0, 0, 1)$.

Things are different for matter potentials. A key difference must be noted between scalar and vector bosons. While the latter give rise to a potential akin to that of normal matter, the former does not, and the results from Chapter 4 and 5 do not apply. Instead, neutrinos coupling to scalar particles —a notable example of which is the mass-generating coupling with a non-zero v.e.v. field— give rise to "*a correction to the neutrino mass matrix rather than the matter potential*". [75]

An even more general search for new physics in the IceCube data, and in general in many areas of particle physics, is the so-called operator-based approach [68]. An effective field theory model is devised with a series expansion of the Hamiltonian, with the n th order term responsible for a different physical effect. In [68] for instance, the zeroth and first order terms of the Hamiltonian expansion respectively induce ν SI and equivalence principle violation.

This provides a useful common ground between experimentalists and theoreticians, where the former describe tensions in their results in terms of which operators preferentially relieve them, and the latter express the effect of their new models in terms of these conventional operators.

The approach of this thesis, unlike the "agnostic couplings", does not allow one to directly extrapolate the expressions and results that have been derived. Instead, it can be seen as an alternative framework to engage with the search for new physics in detectors such as IceCube. Whether one adopts an agnostic operator-based approach, or specifies a model (such as the Z' of this work) and directly computes the phenomenological implications is a matter of taste and convention, rather than a necessity. In my opinion, the common ground it provides comes at the cost of clarity and meaning when expressing the physics as increasing order terms in a Hamiltonian expansion.

Conclusion

*"At last,
My love has come along.
My lonely days are over,
And life is like a song.
Oh, yeah, yeah, at last,
The skies above are blue.
"*

–**Etta James**, "At last!"

This thesis was born out of the desire to explore the crossover between two separate endeavors in the field of neutrino astrophysics. On the one hand, the usage of VHE neutrinos observed at IceCube to hunt for signatures of new physics, in particular the $L_\mu - L_\tau$ gauge boson, namely resonant absorption features in the spectrum. On the other, the study of matter effects induced on astrophysical neutrinos propagating towards Earth.

The former endeavor used to be strongly motivated by laboratory discrepancies, most importantly the muon $g - 2$, whose statistical significance has recently been reduced below the threshold required to be considered as such. Because of the wide range of novel physics that can be probed using this method –any addendum to the SM that makes neutrino interact with one another–, this has not undermined the power of the VHE neutrino spectrum, only lessened the relevance of the specific Z' studied in this work.

The level of details of the propagation physics in the most modern literature took into account effects that made any analytical attempt at solving the differential system vain. Thus in order to capture these effects the biggest aspect of this thesis consisted in independently computing the propagated spectra in presence of the CνB with a hand-made python implementation of the differential equation. The details of the physics and notable aspects of the implementation were presented in Chapter 3.

The preliminary work leading to this implementation, namely the derivation of the Z' interaction cross-section, was also conducted manually in Chapter 2 in the typical quantum-field-theoretic framework, and lead to a Breit-Wigner-like cross-section expected from such an annihilation diagram.

Then the matter potentials to which the neutrinos would be subject throughout their propagation were derived. Two cases were clearly distinguished. The "res" scenario, with $M_{Z'} \sim 10$ MeV, corresponded to cases where both matter effects and absorption features could be expected in the spectrum. The "eff" scenario, with a heavy mediator generating resonances way above the spectrum, corresponded to cases where only the matter effects could be expected in the flavor ratios. Its purpose was mostly pedagogical.

CONCLUSION

The matter potential derivations, although intricate, are not unprecedented. The effective case ($M_{Z'}^2 \gg s$) is analogous to the typical matter effects neutrinos leaving the Sun experience, and the resonant structure has been explored by Smirnov & Valera in Ref. [67]. I want to stress that I initially derived it without knowledge of the latter paper (it is after all only the real part of the energy-structured s-channel propagator), but reading it the first time has prompted me to express it in the more formal $f(y)$ -adimensional way.

After dealing with multiple issues throughout the implementation (to name a few, stiffness of the original propagation equation, tolerance issues causing solution oscillations due to the extremely steep cross-section, the necessity of the analytical binning and subsequent interpolation to properly take into account the cosmic-expansion, the expensive nature of the regeneration integral to be computed at each step, etc.), the code was ultimately capable of showcasing the signatures of neutrino physics, source production, and Z' hypothesis in the synthetic spectra.

These results were compared with analytical solutions in the less detailed physical scenarios. This cemented the accuracy of the implementation, and the discussions on how the different physical parameters –of the sources, the neutrino physics or the Z' boson properties– emerged more-or-less degenerately in the spectrum and flavor ratios showcased how rich the latter can be in studying multiple areas of physics and astrophysics.

This paved the way in Chapter 5 to unravel the effects of the Z' -induced matter potentials on the spectrum. This was done in two mediator-mass regimes, both of which proved pathological:

- The "res" case required overdensities which, given the mass of the Z' in such a scenario, would have implied total extinguishing of the spectrum from energies $\frac{E_{res}}{1+z}$ to E_{res} . While physically possible, this meant one of two things. Either overdensities of that magnitude are indeed present throughout the line-of-sight, in which case the flux is totally absorbed and no matter effects can be seen, or such overdensities do not exist and no matter effects can be seen either.
- The "eff" case, analogous to the W or Z induced matter effects (respectively with the electrons or the $C\nu B$), bypassed the absorption-limiting constraint at the price of a much higher overdensity requirement. The key insight was that the ξ_{od} factor required to "maximize" the mixing scaled with $M_{Z'}^2$.

Matter potentials with the $C\nu B$ had almost exclusively been studied for the Standard Model's Z boson. This field of research has converged, by February 2025, to the paper by Dev et al. [21]. Like all Z-boson scenarios, their potentials hold no s-channel energy structure ($M_Z \sim 90$ GeV). This means that the inevitable overdensities required to see noticeable matter effects do not extinguish the spectrum.

The key difference in result is that Dev et al. explore a so-called "pseudo-Dirac" mechanism responsible for generating neutrino masses. This –hypothetical– new physics entails extremely small mass splittings $\sim 10^{-17}$ eV² that dramatically reduce the overdensity requirements down to "only" $\sim 10^6$, well below the loose 10^{11} KATRIN experimental bound.

A caveat to note is that the KATRIN bounds constrain the $C\nu B$ in the vicinity of Earth, not throughout the entirety of an extragalactic neutrino's path. Thus possible extrasolar or extragalactic $C\nu B$ overdensities are left unconstrained by the KATRIN experiment.

The natural extension to Dev et al.'s work and this master's thesis is to study the matter potential phenomenology (if any) of other new physics scenario in the VHE neutrino spectrum, whether IceCube or any forthcoming detector. Observability has been shown to be crucially intertwined with the overdensity requirement, meaning two things:

- First, any attempt to trace matter potentials signatures in the VHE IceCube spectrum will be intrinsically intertwined with probing the $C\nu B$ and its overdensities along the neutrino's path. In the event of a lack of signature, one cannot directly point to a culprit: the problem is somewhat degenerate.
- Second, in the event where one alleviates the above issue, the physical scenarios that have the less restrictive overdensity requirements –and hence the best chance of being constrained by the VHE spectrum–, by virtue of Equation 5.2, minimize the mass splittings, occur at higher energies and maximize the $\frac{g^2}{M^2}$ ratio.

Chapter 6 served the purpose of critical assessment of the work done. In the light of previous research, objectives set and practical considerations, broad questions such as "Are the hypotheses sound?", "What other approaches could've been taken to answer the same questions?", "How can the work be generalized?", "What to do next?" and the frightening "What could've been done better?" were explored.

While such questions, by virtue of their openness, often leave no room for definite answers, I have discussed as many alternate paths and potent hypothesis as I could, while retaining sight of the thesis's objectives so as to not fall astray too much on the winding road of self-criticism.

Recall from the Introduction this thesis sought **(a)** to reproduce the scattering-only analysis independently ; **(b)** to determine under which conditions the matter potentials are strong enough to matter in the flavor ratio analysis ; **(c)** to exhibit the signatures of neutrino parameters, Z' physics and crucially the $C\nu B$ within the spectrum and flavor ratios; **(d)** to give perspective on the role of current and upcoming neutrino detectors in unraveling such parameters and properties.

Objectives (a), (b) and (c) are answered directly. The theoretical derivation of the Z' - $C\nu B$ cross-section and matter potentials, together with the numerical resolution of the propagation equation, allowed the effect of matter potentials to be quantified and revealed the rich information the spectra and flavor ratios hold. The intricacies of the different signatures of the multiple parameters influencing the physics is laid throughout multiple plots and discussions.

The results show that an observable matter effect signature in the flavor ratio requires $C\nu B$ overdensities that are either self-defeating –extinguishing the very flux required to observe them– or physically implausible given existing constraints. This is a negative result, but not a deficiency in the analysis: it holds independently of the method used to derive it.

Objective (d) is answered only philosophically. Under the light of the said negative result, matter effects, assuming that KATRIN bounds apply to the entire Universe, are unobservable in the case of the Z' , independently of the statistics. This made for the computation of effective counts at IceCube or any next-generation/forthcoming detector (such as KM3NeT or IceCube-Gen2) pointless. This motivates pushing the descriptive details on the IceCube detector to the Appendix.

Within the parameter space consistent with current $C\nu B$ densities, the gauged $L_\mu - L_\tau$ boson Z' does not produce an observable matter-effect signature in the VHE neutrino flavor ratios.

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Appendices

The IceCube Neutrino Observatory

"[...] a particle that cannot be detected."

– Wolfgang Pauli

The Neutrino Detector

To quote J. A. Aguilar for the IceCube collaboration [76]:

The IceCube Neutrino Observatory is a cubic-kilometer detector located at the South Pole and buried in the Antarctic ice. The glacial ice serves as both a target for neutrino interactions and as a Cherenkov radiator for the secondary charged particles produced during these interactions. Cherenkov photons are collected by a three-dimensional array of 5,160 photo-multipliers deployed along 86 strings following an octagonal footprint and buried between 1,450 meters to 2,450 meters in depth. Each string supports 60 Digital Optical Modules (DOMs) arranged vertically, with a spacing of 17 meters between DOMs and 125 meters between strings.



Figure 6.4: Photograph of the IceCube Laboratory, hosting the computers that collect and preprocess the data in real-time. *"The IceCube Lab under the stars – 2013"*, Credit: Felipe Pedreros, IceCube/NSF

As the natural successor to the AMANDA project —deployed throughout the 90’s in order to assess the possibility of observing high energy neutrinos in the deep permafrost of Antarctica—, IceCube is located near the Amundsen-Scott station (see Figure 6.10 in the appendices).

The IceCube Laboratory, shown in Figure 6.4, collects the raw data and filters it in real time to only send the significant events to UW-Madison, where they are treated through an more extensive offline pipeline.

In addition to the main array of optical modules, IceCube also harbors a denser sub-array at the base of the structure, *DeepCore*, capable of bringing IceCube’s energy range down to 10 GeV [77], compared to the initial 100 GeV.

At surface level, 162 ice-filled tanks following the same layout as the main array, are monitored to record cosmic-ray induced atmospheric showers [78]. This structured is called *IceTop*.

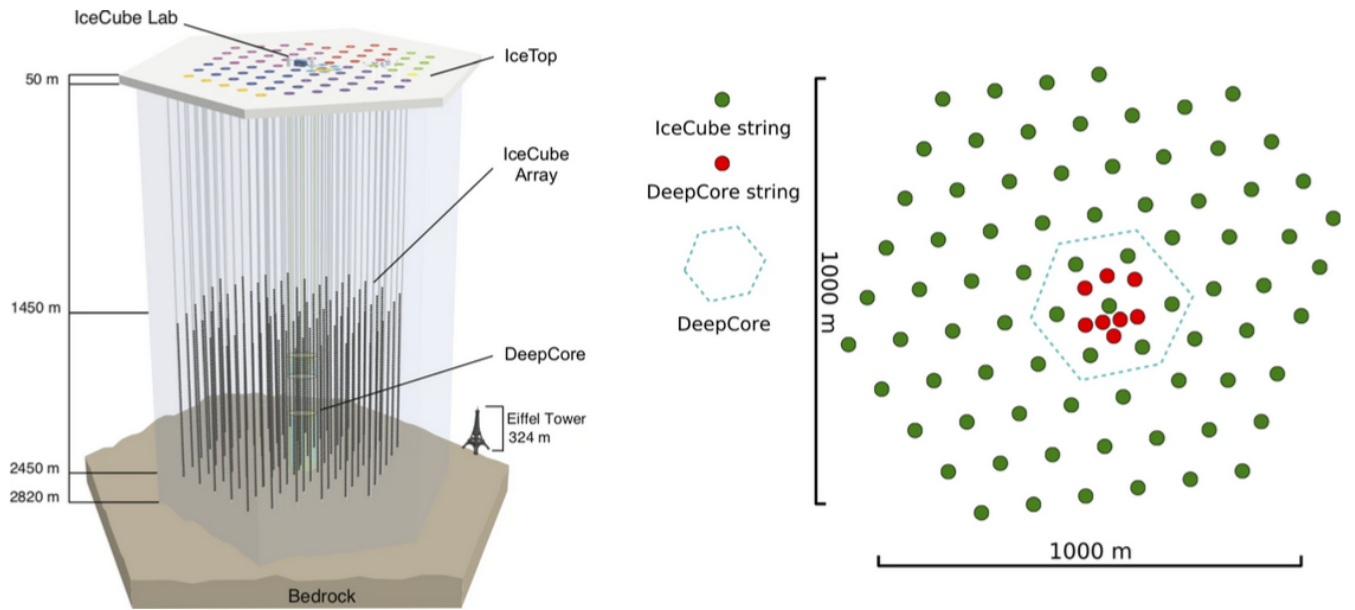


Figure 6.5: The configuration of IceCube main array, alongside the DeepCore and IceTop sub-arrays. Between 2000 and 2100 meters deep, no monitoring is made due to the *dust layer* reducing the quality of the data. Figure taken from Alhers et al. (2018) [79]

The raw data consists of the DOM waveforms, signals over time, and integrated charges, the total amount of light received quantified in numbers of photoelectrons (NPE). These digital pulses are related to the amount of Cherenkov radiation through precise calibration, whereby the response and inhomogeneities of the ice core and individual photomultiplier tubes (PMT) are corrected for [78]. This calibration is performed using *in-situ* LED flashers embedded in the DOMs alongside each PMT. [78].

The offline data analysis performed to reconstruct the incident neutrino best fit is a so-called forward-fold [80], i.e. the events are reconstructed by means of Monte-Carlo simulation assuming a range of neutrino parameters [80], and then compared to the actual event by means of a likelihood test. Starting with initial guesses tuned by only hit times and positions –to reduce the computation time–, the general likelihood maximization performed offline retrieves the neutrino’s flavor, energy and initial direction that best fit the observed event.

Given the hypothesis H_0 on the incident neutrino (E_ν, θ , morphology), these simulations synthesize the light fields, and thus the NPEs, as a function of time and DOM. To this end, one must synthetically compute the light's propagation throughout the ice-core, taking into account complicated physics such as wavelength dependent Mie-scattering, absorption or ice anisotropy [80].

Events Morphology: Reconstructing Flavor

The DOMs distributed throughout the ice core allow for the reconstruction of the high energy neutrinos tridimensional signatures. As astrophysical neutrinos ($E_\nu \sim \text{TeV-PeV}$) undergo deep inelastic scattering with the ice, they will interact with a nucleon either through charged or neutral currents (exchange of a W or Z boson, respectively). These are:

$$\nu_\mu + N \rightarrow_W \mu^- + X \quad (6.1)$$

$$\nu_e + N \rightarrow_W e^- + X \quad (6.2)$$

$$\nu_\tau + N \rightarrow_W \tau^- + X \quad (6.3)$$

$$\nu_\alpha + N \rightarrow_Z \nu_\alpha + X \quad (6.4)$$

Charged current interaction convert the incident neutrino into the associated charged lepton. Neutral currents scatter the neutrino alongside the nucleon instead. Either way, the scattered nucleon induces hadronic showers at the interaction location, registered in the DOMs as a *cascade*. The different events that then occur may be classified into one of three morphologies [76]:

- *Cascades Events* occur when a ν_e (resp. ν_τ) interacts and the energetic e^- (resp. τ^-) quickly loses energy through electromagnetic (resp. hadronic) showers. Neutral current interactions with any flavor, only energizing the scattered nucleon, also lead to such cascade events.
- When ν_τ have sufficient energies, the τ produced may travel a sufficiently long distance so that the hadronic showers induced by its decay can be resolved from the initial showers at the initial interaction point. Then, two neighboring cascades are observed, an event which is called a *Double-Bang*.
- In the specific —albeit dominant— case where a ν_μ interacts via charged-currents, the muon produced is too massive to be dampened through electromagnetic showers and too long-lived to decay into hadronic showers. It exits the detector leaving behind it a straight, Cherenkov *track*¹.

These morphologies are presented in Figure 6.6. Track-like events are easily imputable to muon-flavored neutrinos. To determine which flavor caused a cascade is a subtler task. While there is virtually no way to distinguish neutral-currents induced by each flavor, charged-current are either single cascades or double cascades depending on the neutrino flavor. The average distance between each cascade in a Double-Bang will depend on the ν_τ 's decay length and thus its energy [81]

$$\langle L_\tau \rangle \approx 50 \text{ m} \left(\frac{E_\nu}{\text{PeV}} \right) \quad (6.5)$$

To detect a tau neutrino, a given event will be fitted by a maximum likelihood test to each morphology [81, 82]. A preselection first examines the possibility that the event might be described by a

¹Sometimes, a ν_τ -induced τ will decay into a μ , leaving behind a track as well.

double cascade. A necessary condition is typically that $E_\nu > 60$ TeV and the inferred distance between cascades $L_{dc} > 10$ m [81].

Events that pass preselection are then further analyzed using different heuristics to guarantee the tauic nature of the neutrino. Quoting [82], these are, in addition to L_{dc} , "the energy of each cascade in the double-cascade event, the sum of the energies of the two cascades, the sum of the energies deposited within 40 m of each cascade vertex compared to the total energy [and] the relative energy asymmetry between the two cascade".

For instance, when the energy is significantly spread beyond 40 m, events are typically tracks, whereas when the first cascade holds almost all the energy and L_{dc} is rather small, events are typically single-cascades [81]. Interestingly, in [82], the classification of events using a Deep Neural Network ended up having an 88% efficiency on true cascades, 97% on true tracks while it classified most true double cascades as single ones, hence the need for the classification scheme described above.

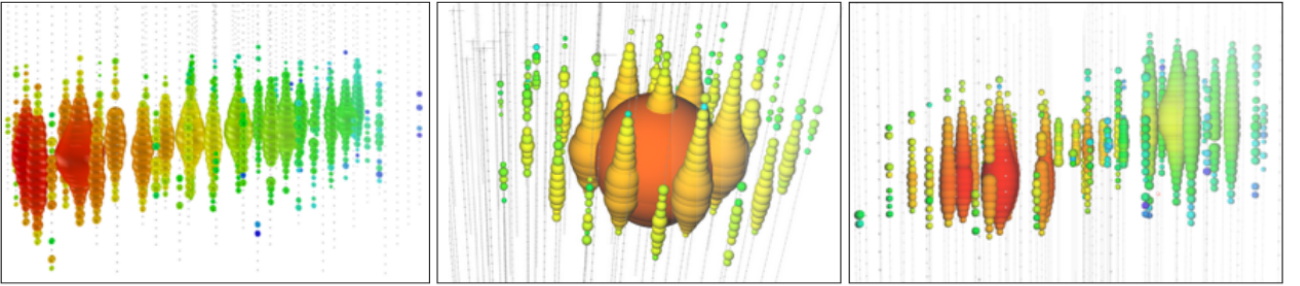


Figure 6.6: Typical track-like (left), single Cascade (middle) and double-bang (right) event morphologies in IceCube. The redder a DOM's sphere the earlier it received photons, the bigger a DOM's sphere the more photons it received. Figure taken from the IceCube-Gen2 Collaboration (2021) [83]

Typically, instead of assuming a neutrino flavor for each instance of the likelihood search, a morphology (either track-like or cascade-like) is assumed.

Events Luminosity: Reconstructing Energy

Evidently, the intensity of the light received, as well as the scale of the event throughout the ice core will inform one about the neutrino's energy. The integrated Cherenkov radiation is proportional to the deposited energy [84].

Charged-currents cascades deposit most of the incident neutrino's energy into the detector (upwards of 90%, increasing with E_ν), while in neutral-currents interactions the outgoing neutrinos has preserved most of its energy (upwards of 70%, increasing with E_ν) [84]. Neutral-current interactions are typically statistically inferred, not individually identified. The largest systematic uncertainties stem from the ice's absorption [84] ($\sim 10\%$).

Track-like events upwards of 100 GeV do not monitor the muon's entire energy deposition. As such, a purely proportional calorimetric approach is forbidden, making uncertainties on the parent neutrino larger than in the cascade case [84]. Rather, the differential energy loss rate $\frac{dE}{dx}$ is used to infer the muon's energy [84].

As an order of magnitude, cascade events have uncertainties $\mathcal{O}(10 - 20\%)$, and tracks are higher, depending on the stochastic losses the muon undergoes [84].

Events Incidence: Reconstructing Zenith Angle

Conventionally, the direction of the neutrinos is described by means of the zenith angle θ at the South Pole, with $\theta = 0^\circ$ (resp. 180°) when the neutrino is down-going (resp. up-going), *i.e.* comes from the sky zenith (resp. the center of Earth). Indeed, at energies below ~ 100 TeV, the Earth is effectively transparent to neutrinos. The zenith angle is schematically described in Figure 6.7, taken from [?].

The angular resolution one can hope to obtain from the morphologies is naturally much greater in the track-case ($\sim 0.1^\circ - 1^\circ$, better at higher energies), while for cascade events the asymmetry in the light emitted hints at the direction with a resolution of $\sim 15^\circ$ [76].

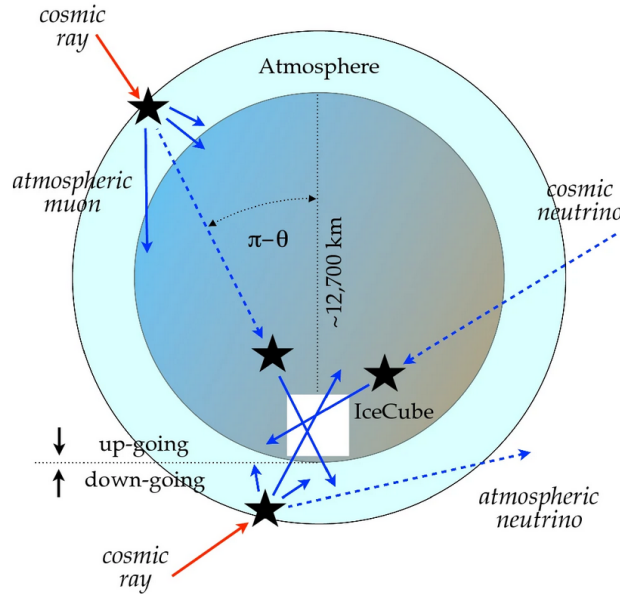


Figure 6.7: Definition of the zenith angle. Taken from Ahlers et al. (2018) [79]

The pipeline described above is the generic and backbone processing, which will be amended or revised depending on the authors. In particular, neural networks approach trained on Monte-Carlo simulations are a prominent part of the state-of-the-art literature and have not been mentioned here [76].

Astrophysical Neutrino Spectra

Background and Event Selection

The IceCube detector records events at a frequency of ~ 2500 Hz, of which one in a million is a neutrino [83]. The remainder is composed of muons created by cosmic ray showers in the atmosphere. Out of these $\sim 7.7 \times 10^4$ events per year, ~ 30 are believed to be of astrophysical origins [83]. The others are atmospheric neutrinos produced alongside the muons in cosmic ray showers.

With these numbers in mind, it is clear how a strict selection process must be put in place to infer the astrophysical nature of the events observed. To this end, the main strategies put in place are [76, 83]:

- Atmospheric muons can effectively be dealt with by either only considering events with sufficiently big zenith angle or by only considering so-called *Starting-Events*, which exhibit no signature in

the outermost layers of ice, utilized as veto regions. Each approach has its drawbacks. While the former effectively discards the entire Southern sky, the latter does not allow the usage of muons generated by astrophysical neutrinos outside of the inner ice core, called *through-going events*. Taking these into account —but only for upgoing events— allows for larger statistics about the Northern sky [85].

- Cascade events of sufficient energies may not have been caused by atmospheric ν_μ that have oscillated to other flavors on their way to the detector, and are therefore good indicators of astrophysical sources, the associated atmospheric background being comparatively low. In fact, the first detection of astrophysical neutrinos with IceCube, in 2013, were such events [86].
- These approaches may never fully discard the possibility of atmospheric neutrinos of any flavor for a sufficiently large baseline. A notable exception to this are events where a down-going atmospheric neutrino coincides with particles from down-going atmospheric showers, as can be monitored using IceTop). In general, the atmospheric contribution to Starting Events are addressed statistically, via appropriate models for atmospheric cascades, cosmic ray distributions and residual muon contaminations.

Typically, for DOM hits to be deemed interesting, they must also pass noise rejection criteria and be compatible with causal light propagation patterns coherent with the propagation of relativistic particles.

Attributed Astrophysical Sources

Since it's first measurements of two astrophysical neutrino events in 2013 [86], at a significance of 2.8σ when comparing to the background levels, the IceCube Collaboration regularly reports on the best fits to the astrophysical neutrino spectrum given the ever-increasing amount of data available. These typically consider high energy starting events (HESE), which have energies > 60 TeV, but recent endeavors have extended these to medium energy starting events (MESE), i.e. > 1 TeV, greatly improving the statistics.

Starting events fits to the spectra include, non-exhaustively, [87] with a 3-year dataset of 37 events, [88] with a 6 years dataset of only cascade events, [89] with a 7.5 year dataset composed of 102 HESE events and [90] with a 10.3 year Enhanced Starting Track Event Selection (ESTES) of 680 neutrinos starting at 3.6 TeV. In [82] authors have used the 11.4 year dataset of ~ 10000 MESE events to study the flavor content at IceCube specifically. Some studies have also included through-going muons from the Northern hemisphere, i.e. for $\theta > 85^\circ$, such as [85, 91], and find it in agreement with the Southern hemisphere results.

The search for neutrinos coming from known astrophysical objects has been an arduous task. The first light-correlated neutrino source ever measured —excluding the Sun, of course— is Supernova 1987a, three hours before which a 13-seconds burst of 25 neutrinos was detected in three detectors around the world.

In September 2017, neutrino "IceCube-170922A", with as much energy as a (fat) ant dropped from the first floor (in vacuum), 290 TeV. It coincided with flaring blazar TXS 0506+056, cementing blazar as a VHE neutrino source, as was predicted.

Monitoring of non-transient sources are also ongoing, such as active galaxy NGC 1068, which has shown an excess of 80 ± 20 events at the TeV scale over a ten year period, at a significance of 4.2σ [92],

the most prominent by far.

Beyond this search for point sources, the other astrophysical neutrinos form a diffuse —or at least unresolved— sky background that is roughly isotropic, indicating extragalactic origins [76]. The interactions between cosmic rays and the interstellar medium (ISM) are expected to yield a galactic contribution superimposed within it, non-negligible albeit subdominant between 400 GeV and ~ 60 TeV. The fits to these contributions, dependent on cosmic ray models, are corroborated by the IceCube data at 2.7σ [45]. In this case only track-like events were used, due to their enhanced angular resolution.

The diffuse astrophysical neutrino spectrum, at rough equipartition [76] with the Ultra-High-Energy Cosmic Ray Background (UHECRB) and the Extragalactic Gamma Ray Background (EBG), is usually fitted by a Single Power Law (SPL)

$$\Phi_{\alpha}(E_{\nu}) = \Phi_0^{\nu+\bar{\nu}} \left(\frac{E_{\nu}^{-\gamma}}{100 \text{ TeV}} \right)$$

with Φ_0 its normalization factor in and γ its spectral index.

Figure 6.8, taken from [76], presents the fits from different analyses, some mentioned above, as well as the combined global fit stemming from these.

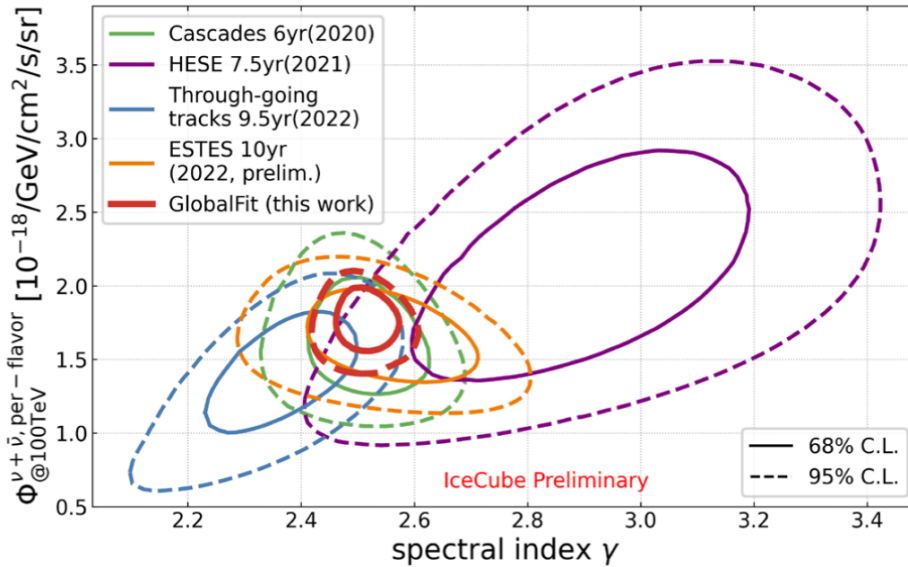


Figure 6.8: Global fit (red) of cascade [88], HESE [89] and through-going [91] datasets as well as the ESTES [90] best fit. This figure does not include the MESE dataset from 2025, being its contemporary. Figure taken from [76].

Deviations from the SPL distribution have also been tested. For instance, the SPL distribution was allowed to have a different normalization in each energy bin, revealing no specific features [76]. In the first HESE astrophysical spectrum fit [87], a gap feature appeared between 400 TeV and 1 PeV at low significance, notably sparking the resurgence of ν SI models in 2014-2015 [10–14]. No mentions of it remain in the current literature, indicating it has been resolved by statistics into the range of statistical fluctuations.

Detector Response: Effective Areas

We have already mentioned that the individual events are reconstructed by a "forward fold fit", whereby simulated events are recreated for a range of possible parameters by Monte-Carlo simulations, and then artificially fed to the detector by using its response functions.

The same spirit allows one to feed a synthetic spectra through the adequate "detector response" to obtain event rates per energy bins at IceCube, effectively quantifying the effects of the new physics described throughout this thesis using IceCube's language.

The IceCube Collaboration puts at the disposal of researchers tables of effective areas $A_{eff}(E_\nu, \Omega)$ which take into account the detector's volume and importantly Earth's absorption as a function of the zenith angle [89]. It is presented in Figure 6.9.

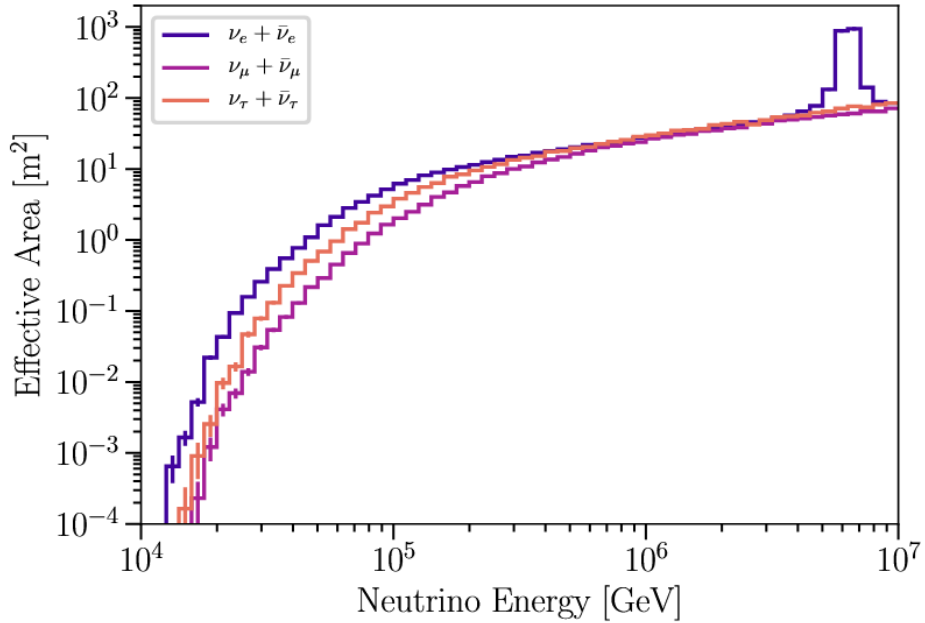


Figure 6.9: Effective Area for each neutrino specie in the IceCube astrophysical spectrum's energy range, in the case of the 7.5 years HESE data [89] The latter data and this Figure are from [89].

Given a synthetic flux $\phi_\alpha(E_\nu, \Omega)$ [$\text{GeV}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}$], the true number of events expected at IceCube is given by [93]

$$N = T \int dE_\nu \sum_{l=e,\mu,\tau} A_l^{\text{eff}}(E_\nu) \frac{d\Phi_{\nu+\bar{\nu}}}{dE_\nu} \quad (6.6)$$

with T the lifetime of the observation.

To get an number of events by bin from these real interactions, one proceeds using response matrices, which give the conditional probability of finding observables $\hat{x}$ ($\hat{E}, \hat{\theta}$, event morphology) given intrinsic neutrino parameters x (E_ν, θ, α). Integrating said probability over the range of heuristics $\hat{x}$ associated with a given bin, b , one obtains the expected event number per bin.

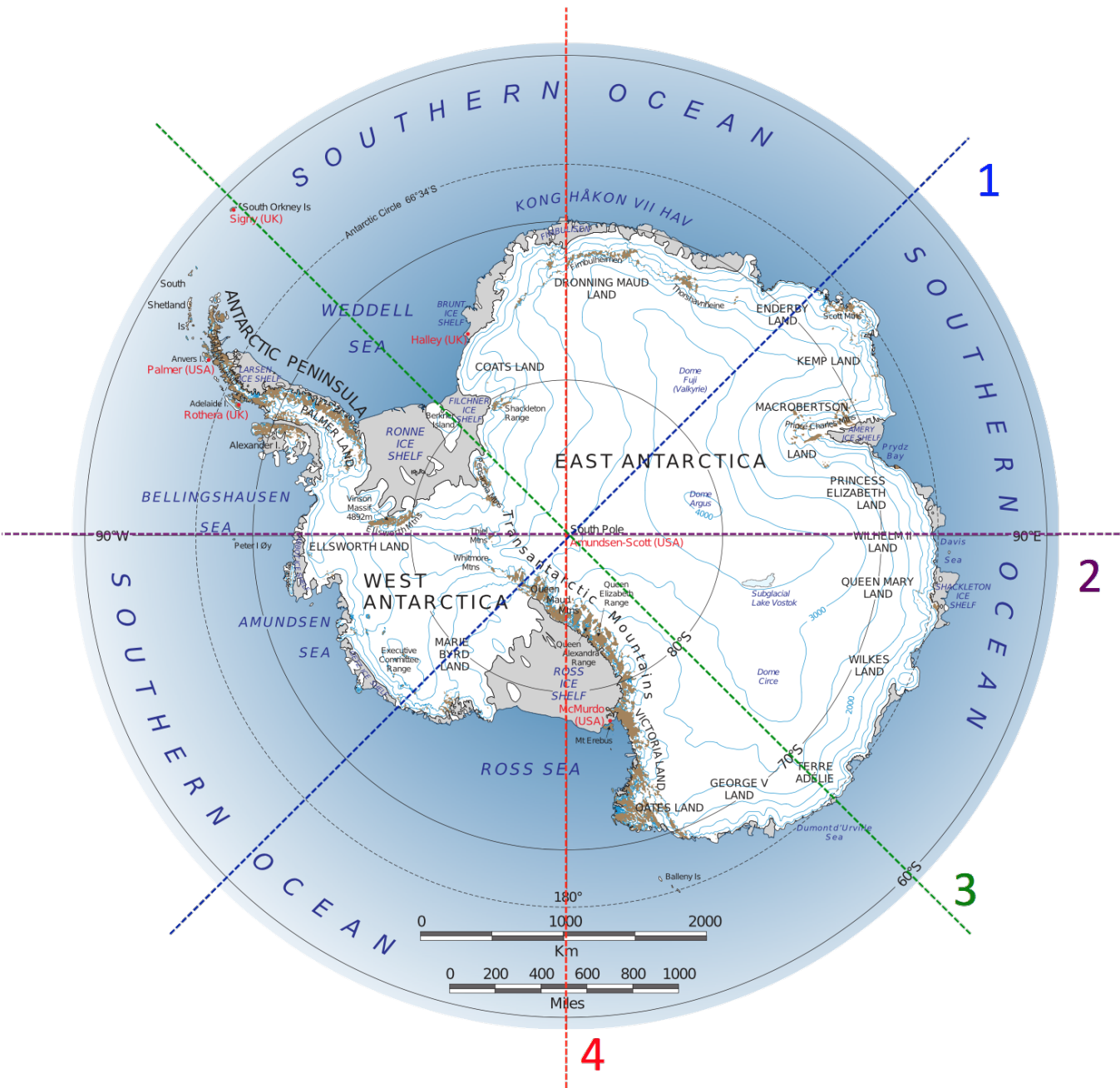


Figure 6.10: "Encompassing A Cubic Kilometer Of Ice, Icecube Searches - Physical Features Of Antarctica Map", NicePNG, Under License: Personal Use.

Map of Antarctica, with the Amundsen-Scott station, home to the IceCube Observatory, at its center, 250 meters away from the South Pole. Roald Amundsen and Robert Falcon Scott were respectively Norwegian and British explorers, who first reached the South Pole in separate expeditions, with the latter arriving a month after the former, in the (southern-hemisphere) "summer" of 1911-1912. Scott's team perished of exhaustion, cold and hunger on the way back. (Wikipedia)