Method for Prediction of Sea Ice Thickness Based on the Blowing Air Temperature and Speed

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Master Thesis

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SUMMARY

Maritime traffic with ice breaking vessels as well as offshore activities in polar regions depend a lot on the freezing of the sea water; hence the importance of the sea ice determination, i.e. its thickness and its distribution over the sea area. This work studies the problem of the ice thickness determination. Generally, some information about sea ice thickness is provided by remote sensing data or from climatological models for large arctic regions. However, these data are not so accurate. Due to the lack of existing reliable measured data, the ice thickness must be often estimated.

Various methods and models are able to predict the ice thickness to a certain extent. Some are empirical, but the most physically reliable are rather based on thermodynamic processes that occur during the ice growth. Nevertheless, some environmental conditions, such as snowfall, solar radiation, different heat fluxes coming from the atmosphere, the sea, salinity etc. complicate a lot the sea ice formation processes making equations difficult to be solved.

This Master Thesis presents a thermodynamical approach of the ice growth based on the blowing air characteristics, more precisely its temperature and speed. The model is rather constructed for first-year ice. Some physical phenomena such as snowfall and the interaction between the water and the surrounding environment has been taken into consideration. Unlike other climatological models, this model treats the salinity distribution over the depth of the ice sheet, using more recent and well adapted data. The different heat transfer equations between the water, the ice, the snow if any and the atmosphere are solved numerically using finite differences methods. A simple forward differencing scheme has been applied with adequate space and time steps to provide stable solutions.

The results found with the above theoretical approaches have been compared with the ones found during the experiments that have been performed in the ice facilities of the Hamburg Ship Model Basin (HSVA). Some conclusions comprised of results discussion and propositions are presented as well for any further studies.

RESUME

La glace constitue la caractéristique principale des mers polaires. La connaissance de sa répartition sur la l'étendue de la mer et de son épaisseur dans ces régions est d'une importance capitale pour toute activité maritime. Généralement, des informations sur l'épaisseur de la glace de mer sont fournies par des données de télédétection ou des modèles climatologiques pour de grandes régions arctiques. Cependant, le manque de précision dans ces données contraint souvent à une estimation de l'épaisseur de glace.

Il existe diverses méthodes pouvant prédire l'épaisseur de la glace, mais les plus fiables sont celles qui tiennent compte de procédés thermodynamiques qui ont lieu au cours du processus de la formation de la glace. En outre, certains paramètres tels que la neige, le rayonnement solaire, la salinité de l'eau, les différents flux de chaleur provenant de l'atmosphère et de la mer devraient être considérés dans un modèle se réclamant sophistiqué.

Ce travail présente une approche thermodynamique de la formation et la croissance de la glace se basant sur la température et la vitesse de l'air froid qui souffle sur la surface de l'eau. Ce modèle, étudiant seulement la glace nouvellement formée, prend en compte la présence de la neige et l'interaction entre l'eau et l'atmosphère, tout en appliquant la distribution verticale de la quantité de sel dans la glace. Les équations mathématiques résultantes ont été numériquement résolues par la méthode des Différences Finies et un maillage approprié a été employé pour garder la solution stable.

Enfin, des expériences ont été effectuées dans le laboratoire et le bassin arctique du HSVA (The Hamburg Ship Model Basin) et leurs résultats ont été comparés avec ceux fournis par l'approche théorique.

ΠΕΡΙΛΗΨΗ

Θαλάσσιες δραστηριότητες (πλωτές κατασκευές, παγοθραυστικά πλοία καθώς και άλλες υπεράκτιες διαστηριότητες) σε πολικές θάλασσες έχουν άμεση εξάρτηση από την πήξη του θαλασσινού νερού. Κατά συνέπεια, ο προσδιορισμός του πάχους και της κατανομής του πάγου καθίσταται μεγάλης σημασίας. Η παρούσα διπλωματική εργασιά μελετά μόνο το πρόβλημα του πάχους. Γενικά το πάχος του πάγου πρέπει να υπολογίζεται συχνά λόγω έλλειψης αξιόπιστων μετρούμενων δεδομένων.

Στην βιβλιογραφία, υπάρχουν διάφορες μέθοδοι οι οποίες μπορούν να προβλέπουν το πάχος πάγου ως κάποιο βαθμό. Κάποιες από αυτές είναι εμπειρικές, αλλά οι πιο ενδιαφέρουσες και αξιόπιστες είναι εκείνες οι οποίες βασίζονται σε θερμοδυναμικές διεργασίες που λαμβάνουν χώρα κατά την διάρκεια της ανάπτυξης και της εξέλιξης του πάγου. Παρ'όλα αυτά, ορισμένοι περιβαλλοντικοί παράγοντες όπως η αλατότητα του θαλασσινού νερού, το χιόνι, η ηλιακή ακτινοβολία και οι διαφόρων πηγών θερμότητες (ατμόσφαιρα, θάλασσα, κλπ) περιπλέκουν το πρόβλημα καθιστώντας τις εμπλεκόμενες μαθηματικές εξισώσεις δύσκολες να επιλυθούν.

Η παρούσα εργασία παρουσίαζει μια θερμοδυναμική μελέτη της εξέλιξης του πάγου βάσει των χαρακτηριστικών του αέρα, δηλαδή της θερμοκρασίας και της ταχύτητάς του. Μερικοί παράγοντες όπως η παρουσία του χιονιού, η κατανομή της αλατότητας επί του πάχους καθώς και οι αλληλεπιδράσεις μεταξύ ατμόσφαιρας και νερού έχουν ληφθή υπ'όψην. Το πρόβλημα επιλύθηκε αριθμητικά με τη μέθοδο Πεπερασμένων Διαφορών.

Στην συνέχεια εκτελέστηκαν κάποια πειράματα στις εγκαταστάσεις της εταιρίας The Hamburg Ship Model Basin(HSVA) και τα εμπειρικά αποτελέσματα συγκρίθηκαν με τα θεωρητικά.

DECLARATION OF AUTHORSHIP

I declare that this thesis and the work presented in it are my own and have been generated by me as the result of my own original research.

Where I have consulted the published work of others, this is always clearly attributed.

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I have acknowledged all main sources of help.

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This thesis contains no material that has been submitted previously, in whole or in part, for the award of any other academic degree or diploma.

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1. INTRODUCTION

Nobody can talk about offshore activities in polar seas or generally rivers without referring to the freezing of the water. The sea ice constitutes a defining aspect of the polar oceans. Nowadays, many firms involved in these kinds of activities are trying to improve their quality and develop the field as well. Hence the importance of understanding and projecting future sea ice conditions, such as the sea ice thickness, sea ice distribution, etc.

It seems important, before going deep on the main subject, to ask the following questions: “why do the ice conditions have to be known in advance?”, “what does sea ice influence in our work as naval, marine, ocean or offshore engineers?”

Offshores structures which are supposed to operate in arctic regions must be designed by taking into account the freezing of the water and the same applies for the performance of icebreaking vessels. Additionally, it is rather obvious that because the water passes from a liquid state to a solid one, it does not have the same properties. That leads to a conclusion that ice can induce some additional loads. These loads are referred as ice loads and they have an impact on the propulsion machinery system of vessels. In fact, for ship navigation in arctic waters, the ice influence depends on the ice properties and conditions, the hull shape, the propulsion system of the vessel. When a propeller operates in an arctic environment, a submerged ice in front of it disturbs the flow of the water and impacts its blades. This is known in the literature as *Ice Propeller Interaction* and has as consequence the modification of the propeller thrust. Thus, one could conclude that when designing a ship which is supposed to operate in arctic seas, the propulsion system design should take into account the sea ice conditions and properties. This implies that knowing in advance and the most accurately possible the sea ice conditions is of a great importance in the design process of this kind of vessels.

Some available information about ice thickness of sea is provided by remote sensing data or from climatological models for large arctic regions. Nevertheless, these data are not as accurate as to fulfill the requirements. Due to this fact and the lack of existing reliable measured data, the ice thickness must be often estimated.

The difficulty in predicting the sea ice thickness seems to reside in the lack of sufficient information. However, there are some existing approaches, statistical analysis analytic and numerical predictors, which unfortunately present some limitations. The ice sheet is a result of many complex physical processes, and logically, the most appropriate approach for predicting its thickness should consider all the parameters that have an effect on it.

Actually, the statistics based methods are simple. In this category, we can find empirical analyses based on the Freezing Degrees Days (FDD), which can be used at the simplest level and refined if needed in many ways, especially if sea ice thickness data are available for calibration. However, the experience showed that these methods are not so reliable. Considering the fact that many physical processes occur during the sea ice formation and sea ice growth, it is believed that a more reliable sea ice prediction method should take into account all these processes. The main phenomenon that defines the formation and the growth process of the sea ice is surely the “Heat Transfer”.

Therefore, the objective of the project is to develop a method to estimate sea ice thickness that results from heat transfer processes based on some meteorological data, such as the air temperature and velocity.

First of all, it is important to focus on the sea ice formation and development processes and discuss the physics behind them. Afterwards, before presenting the Model developed in this Master Thesis, i.e. its theory, methods of solution and results, some important models based on the thermodynamic processes are provided. Finally, the experimental results are compared to the theoretical ones.

2. SEA ICE GROWTH

2.1 Sea Ice Formation

Ice is one of the three states (phases) of the water, it is obtained when water freezes. It is also well known that water freezes when its temperature decreases and gets lower than a critical temperature called the freezing point. The freezing of water to ice is a classic problem in applied mathematics, involving the solution of a diffusion equation. However, we are dealing with sea water and this water contains salt; the presence of salt introduces some interesting changes.

2.1.1 Water Phase Diagram

A phase diagram is a graphical representation of the physical states of a substance under different conditions of pressure and temperature, i.e. it illustrates the variations between the states of matter. Typically, these diagrams have pressure on the y-axis and temperature on the x-axis. The curves on the diagram represent the coexistence of two different states. In the diagram in Figure 1, these curves are in black. All the important details concerning the variations of water states can be seen on the figure below. It is worth mentioning that the diagram below applies for fresh water.

The roman numerals represent the different ice phases according to molecular structure. Generally, they are classified into two categories: low and high pressures ices. Nevertheless, we will not focus on that at this stage.

As for the diagram, it is obvious that although, the temperature seems to be the major feature which regulates the water phases, but the pressure plays also an important role in the change of water states. Our focus will be on the liquid and solid phases. We observe that the curve of equilibrium between the two phases is around 0ºC (~273 K) at lower pressures and starts decreasing from pressure higher than 10 MPa until it reaches almost 21ºC. Since 10 MPa of pressure means that we are approximately at 1 km under the sea free surface, this curve is almost parallel to the y-axis for depth lower than 1 km.

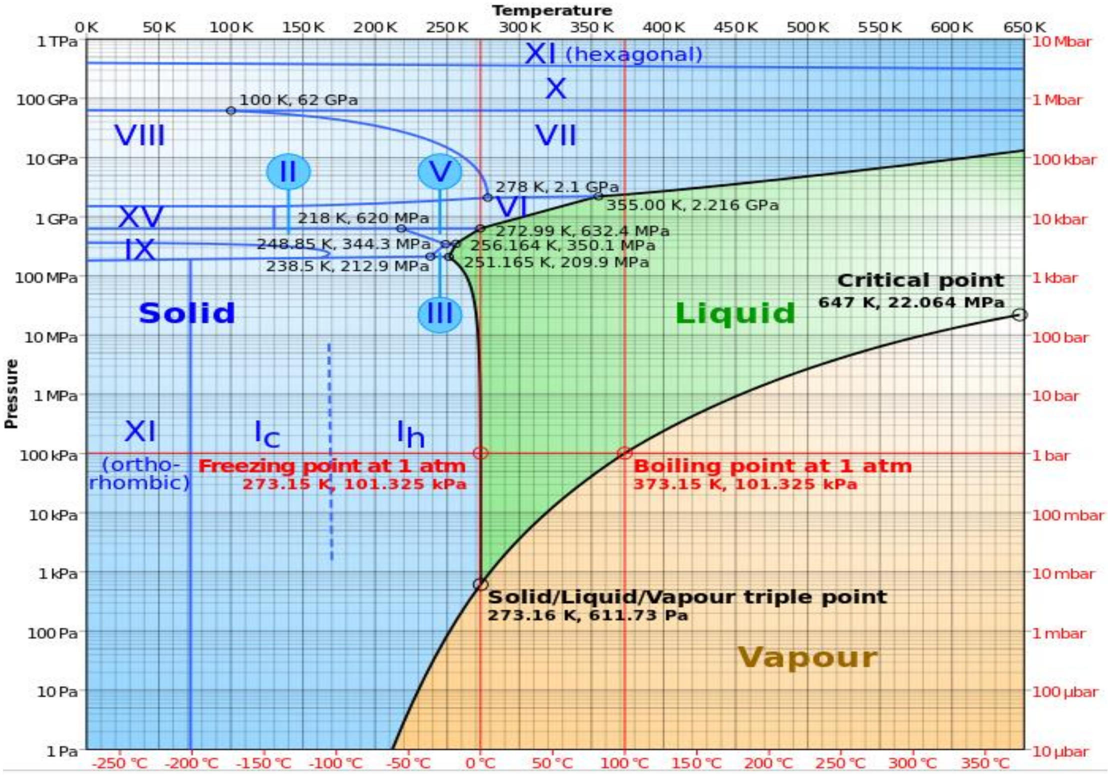


Figure 1 Pressure - Temperature Phase Diagram of Water (from Wikipedia)

Therefore, this curve is different in the case of sea water, which is salty. Let us consider salinity and see how this curve varies under the influence of salt in the water.

2.1.2 Salinity

Salinity is a measure of the concentration of dissolved salt in water. Nowadays, salinity is usually described in practical salinity unit – psu[1]. Before explaining the process of transformation from sea water to sea ice, we have to talk about salinity and its effects on the under discussion problem. In fact, the temperature at which sea water starts freezing depends on its salinity. As it can be seen in Figure 2, the saltier the water, the lower the freezing point is.

[1] 1 psu = 1g of salt (Na+Cl) per kg of sea water

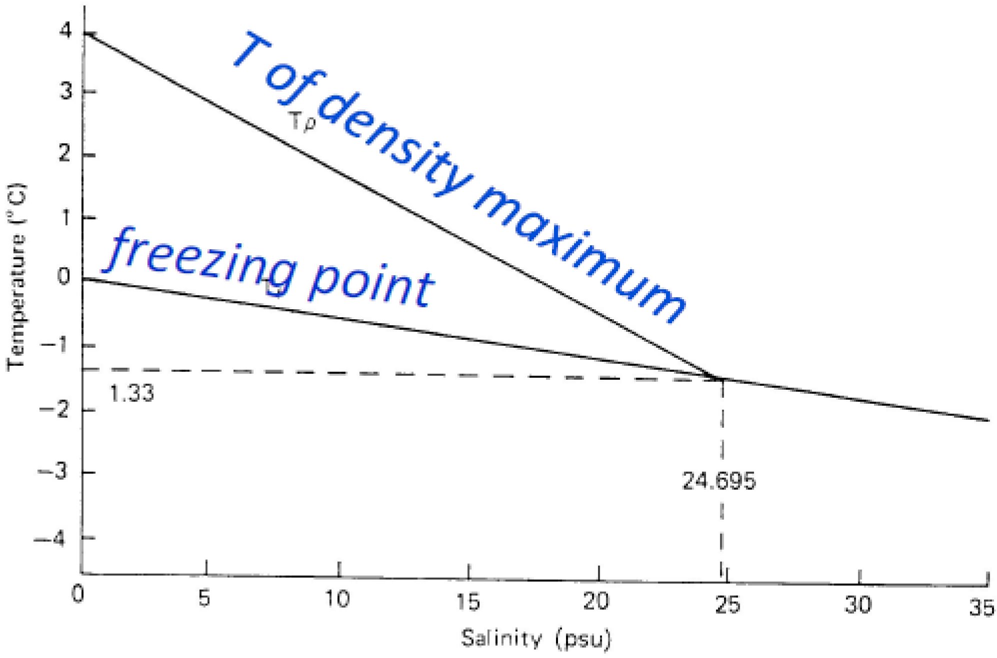


Figure 2 Freezing Point as Function of Salinity in Water (from GEOS615, 2000)

The information provided through the above diagram is that the freezing temperature of water with zero salinity is zero degrees Celsius (0oC), but this temperature varies with the presence of salt in water.

More specifically, for water of salinity less than 24.7 psu, the temperature of maximum density (T_P) is less than the freezing temperature (freezing point) although for the one that has salinity greater than 24.7 psu, the water freezes before it reaches its hypothetical maximum density, so that colder water is heavier. This temperature variation according to salinity is quantified as follows: For every 5 psu increase in salinity, the freezing point decreases by almost 0.28 degrees Celsius.

For information, most of seas and oceans have salinity between 32 and 36 psu, which means they start freezing at around -1.9ºC.

2.1.3 Description of the Formation Process

The freezing of sea water is caused by cold air blowing on the sea surface. When a sufficiently cold air blows on the sea, it incites the sea water to start freezing; then small ice crystals called Frazil Ice (see Figure 3) appear at the free surface and they are stirred by wind, currents and waves. These crystals are typically 3 to 4 millimeters in diameter. Since salt does not freeze, it accumulates into droplets and the crystals reject them into the water; hence frazil crystals are made of nearly pure fresh water. These droplets are referred as brine in the literature. They are saline, whereas the crystals are not. Some of the brines become trapped in pockets between the ice crystals, as it can be seen through the Figure 4.

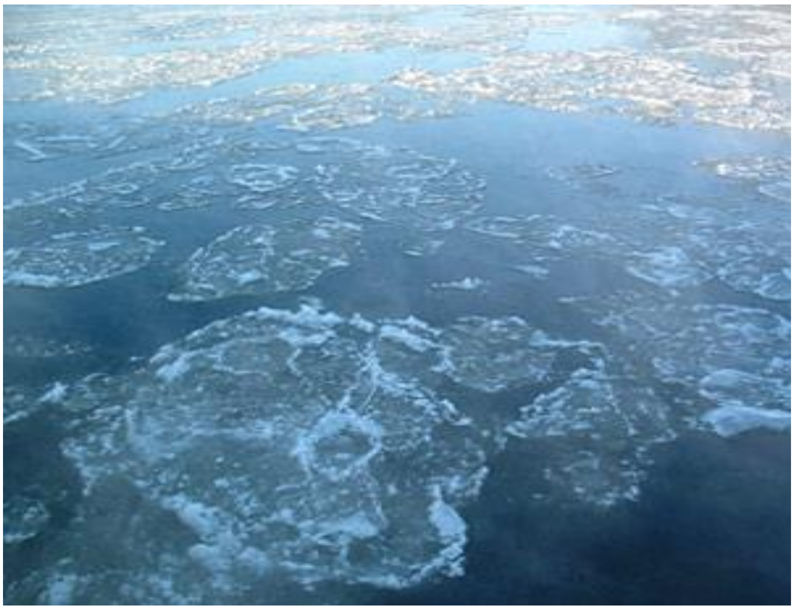


Figure 3 Frazil Ice (http://www.popularmechanics.com/)

The Figure 4 shows the way ice crystal and brines are arranged during their formation. Moreover, at the right-down corner of the picture, we can see indicated a bacteria as to realize the order of magnitude of the generated brine.

Then an increase of salinity can be observed on the near-surface water, whereas the brine is still liquid since it needs much lower temperatures to be frozen. Over the time, the brine leaves the air pockets and flows out of the sea ice. Most commonly, the brine passes through holes and channels in the ice, and moves downward under the action of gravity. Figure 5 illustrates the way brine drains out, leaving the air pockets.

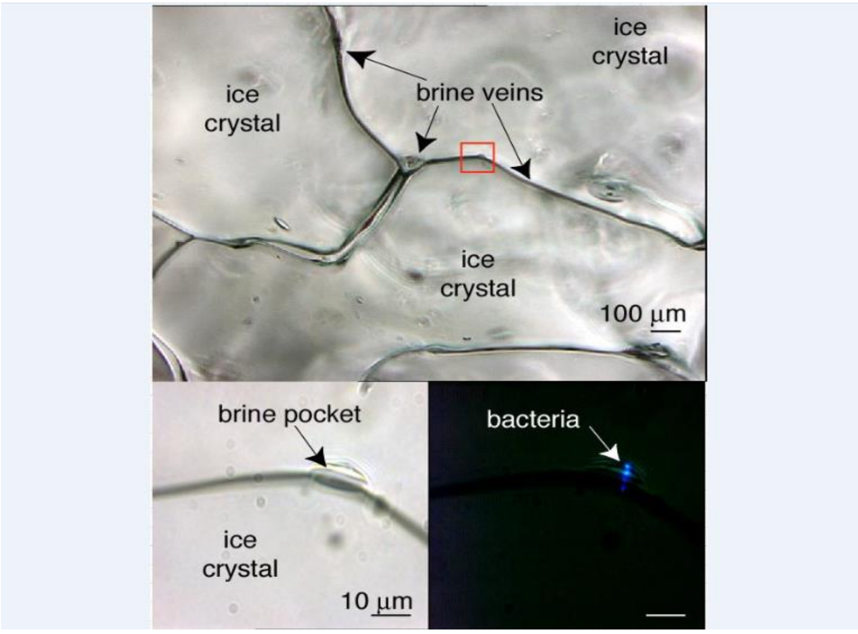


Figure 4 Ice Crystal and Brine Formation (http://oceanexplorer.noaa.gov/)

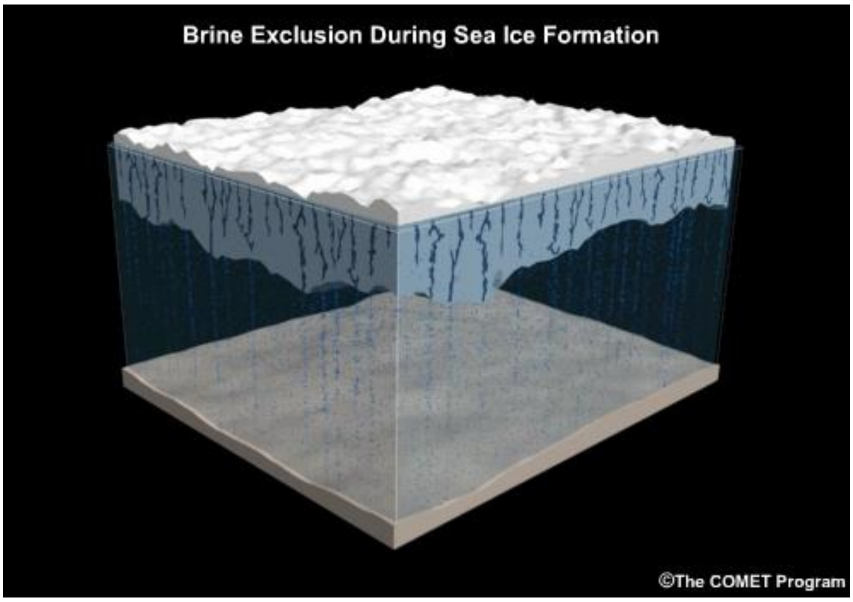


Figure 5 Brine Exclusion during Sea Ice Formation (www.meted.ucar.edu)

a. *Sea Ice Growth Process*

The growth stage is the period between the Frazil Ice and the Sheet Ice. When ice sheet is formed, it may thicken through heat transfer processes. The occurring of the sea ice growth depends on meteorological features. Therefore, relationships between ice conditions and weather combined with heat transfer theory may be useful for our quest to determine the sea ice thickness.

Sea ice growth is a very complex process, on the grounds that many factors such as the snowfall can have some influence on it. On the one hand, the buildup of a snow cover can delay the ice growth; on the other hand, snowfall adds insulation to the ice surface causing the ice to warm from the bottom up: this affects the ice growth as well.

Additionally, the “state” of the sea plays also an important role in the ice development. Thus, the created Frazil Ice can grow in two ways depending on the climatic conditions: It can develop from grease and congelation ice in calm waters or Pancake Ice in rough ocean. Both processes are schematized on the Figure 6.

Figure 6 Ice Growth Process (from NSIDC: http://nsidc.org/)

Under calm conditions at sea, the frazil crystals form a smooth and thin layer of ice known as Grease Ice, since it looks like an oil slick. Grease Ice develops into dark nilas, which is a continuous, still thin and dark sheet of ice. The dark nilas gets lighter as it thickens. Afterwards, the nilas undergoes rafting. Rafting refers to a process during which currents or winds push the nilas around so that they slide over each other. In the end, the nilas thickens enough to get more stable. This more table sheet of ice has a smooth bottom surface and is called congelation ice. At this stage, there is no possibility to get further frazil ice, so only congelation ice develops under the ice sheet can contribute to the continued growth of a congelation ice sheet. Congelation ice crystals are long and vertical because they grow much slower than frazil ice. The ice growth in calm waters is illustrated on the Figure 7.

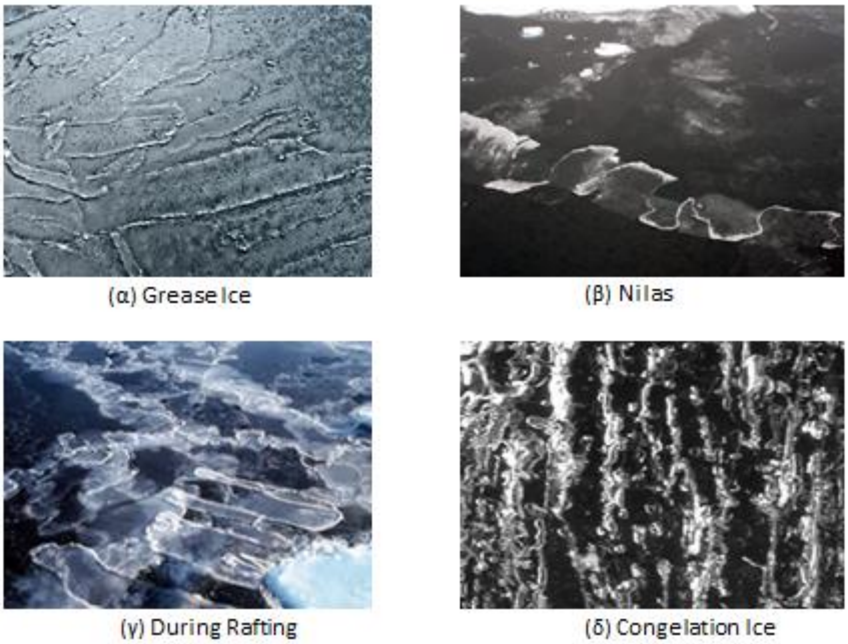


Figure 7 Ice Growth in Calm Water (http://www.popularmechanics.com/)

Unlike the calm waters, in rough ocean, the frazil crystals accumulate into slushy circulars disks, called pancake or pancake ice (see Figure 8). The main characteristic of the pancakes is either the raised edges or ridges on the perimeter, and that results from their collision with each other due to ocean waves. In case the motion is strong enough, rafting occurs; if the ice is thick enough, ridging occurs, where the sea ice bends or fractures and piles on top of itself forming lines of ridges on the surface. Eventually, the pancakes bond together and consolidate into a coherent ice sheet. Unlike the congelation process, sheet ice formed from consolidated pancakes has a rough bottom surface.

Then the sheet ice continues to grow during the winter. Since in spring and summer, temperatures are high, the "first-year ice" starts melting. This ice may survive until the next winter and if it is the case, it is referred as "Multiyear ice". In fact, if the sheet ice does not get thick enough in the winter, it will completely melt during the summer. However, if the ice grows enough over the winter, it thins during the summer but does not completely melt.

Figure 8 Pancake Ice (from https://antarcticfudgesicles.wordpress.com)

b. Sea Ice Thickness

As seen above, the ice growth process is very intricate. The fact that many external factors and phenomena interfere complicates the attempt to predict accurately the ice thickness. Field experience has shown that ice can be formed from the bottom downwards "Bottom Ice Growth", and also be on the surface "Surface Ice Growth". The growth of these two types of ice depends on the air temperature and the snow cover thickness (see Figure 9).

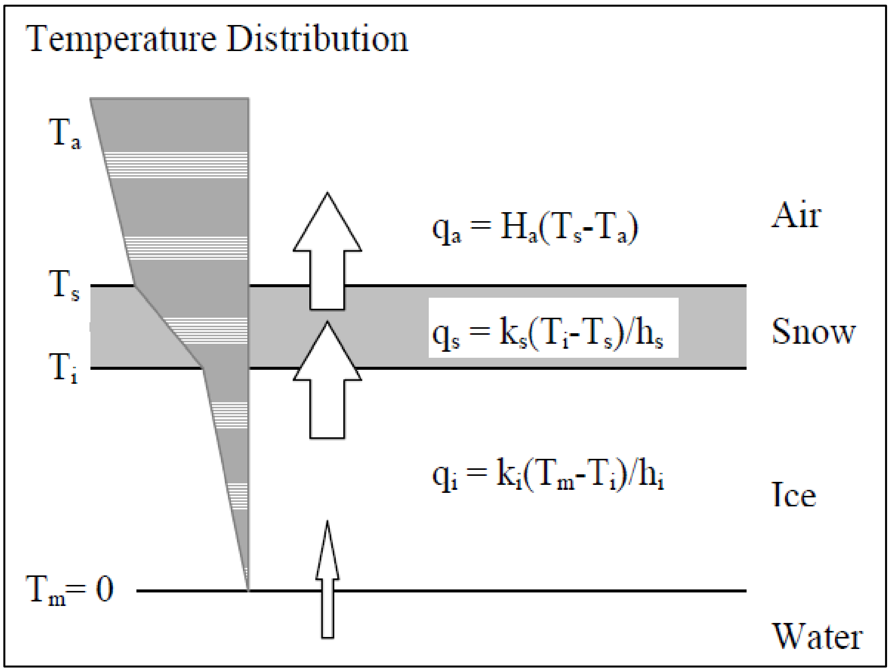


Figure 9 Heat Flux through an Ice Cover (Singh & Comfort, 1998)

- Bottom growth

This is a result of the estimation by balancing the heat flux from the ice surface to air with the heat flux required in the fusion of ice. The temperature is assumed to be distributed linearly in the ice and the snow cover. Moreover, the air temperature Ta was not taken to be equal to the snow surface temperature Ts.

- Surface Growth

Ice surface growth results from the freezing of slush formed on the ice surface as revealed after observations (Comfort, 2013). It can result from various mechanisms. In the study performed by Comfort and Abdelnour in 2013, rainfalls added water to the ice surface as well as melting the snow that was already there, and this water and slush subsequently froze

The total ice thickness at any given time is the sum of the “bottom” and the “surface” growth.

- Multiyear Sea Ice

Many properties distinguish multiyear ice from first-year ice: these properties result from the melting processes during summer, as it has already been stated in previous paragraphs. This makes yet harder our problem, because structurally, multiyear ice contains much less brine and more air pockets than first-year ice, which means that the former is stiffer than the latter. Consequently, multiyear ice makes the navigation and clearing more difficult for icebreakers.

However, even if multiyear ice is more and more common, especially in the Arctic, this master thesis will deal only with the simpler case, i.e. first-year ice.

- Albedo

Another factor which plays also an important role in the ice growth process is the *Albedo*. Albedo is a non-dimensional quantity which measures the reaction of a given surface towards the solar radiation. More specifically, it indicates how well a surface reflects solar energy. Its value varies between 0 and 1, and it is commonly referred to as the "whiteness" of a surface, with 0 meaning black and 1 meaning white. A value of 0 means the surface is a "perfect absorber" that absorbs all incoming energy. Absorbed solar energy can be used to heat the surface or, when sea ice is present, melt the surface. A value of 1 means the surface is a "perfect reflector" that reflects all incoming energy.

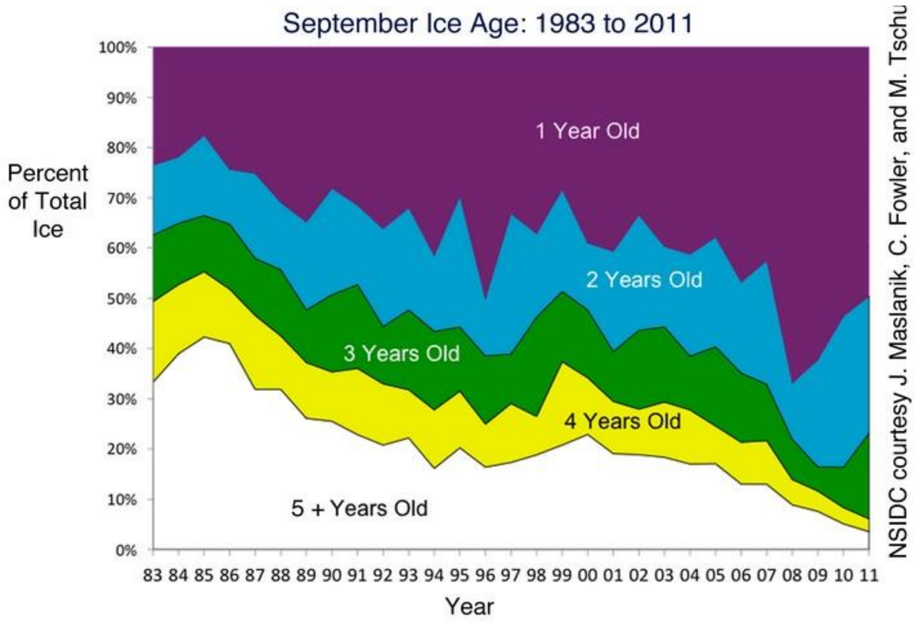


Figure 10 Multiyear Ice Constitution (from NASA)

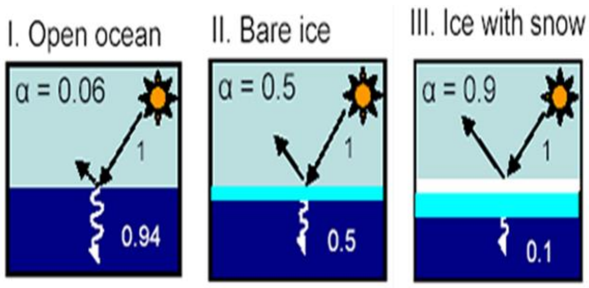


Figure 11 Albedo Values in Ocean (NSIDC: http://nsidc.org/)

According to the Figure 11 sea ice will have higher albedo than the surrounding ocean (0.5>>0.06), which means that the surrounding water reflects only 6% of the incoming solar radiation and absorbs the rest, while the sea ice reflects from 50% to 70% sometimes. As consequence, sea ice keeps its surface cooler. Additionally, the albedo explains better what has been said above about the insulating role of snow.

3. SEA ICE THERMODYNAMICS

In the previous chapter, the process has been described only from optical and structural point of view. In this one, we try to discuss the physics behind the process; this will be very helpful for the elaboration and the execution of the project.

In fact, when cold air causes the open sea-water to start freezing, there appear some ice crystals, which means sea ice begins to form. The ocean temperature then tends to the freezing point, the water density increases and the water sinks literally. In others words, in the beginning the ocean surface is warm, and when freezing, the water on the surface is replaced by warmer water, which was below the surface; this warmer water must also be cooled, and then it will be on its turn replaced: that is the basic idea behind the ice thickening process. Once ice begins to grow, it acts as an insulator between the atmosphere and the sea. Thus, the heat transfer from the sea to the atmosphere will occur through the sea ice. This heat will be conducted in the sea ice before being emitted to the atmosphere. At this stage, the ice growth slows as the ice thickens because it takes longer for the water below the ice to reach the freezing point.

The most important feature to keep here is that the ice becomes thicker as the heat transfer process continues between “warm” ocean and the cold atmosphere. Moreover, in the previous paragraph, we talked about the multiyear-ice and one might ask the following question: “Does multiyear ice becomes more and more thick?”
The answer is NO because sea ice eventually reaches what is known as “thermodynamic equilibrium”. In fact, since ice insulates the ocean from the atmosphere, and the insulation capacity is function of the ice thickness (the thicker the ice, the less heat is transferred); if the ice gets thick enough to inhibit any heat transfer, the ice growth stops. This is what scientists call the thermodynamic equilibrium thickness. For instance, in the Arctic, the thermodynamic equilibrium is around 3m.

The problem of modelling the sea ice growth has been being treated since the 19th century and constitutes a classical problem in geophysics. These approaches, initiated by the significantly valuable solution of Stefan, have rather been proved very relevant in modelling the thermodynamic mechanisms which take place during the ice formation and growth. Later on, many empirical approaches had emerged, but the most important scientific breakthrough is the introduction of numerical models to solve the problem. In this section, first we define the general problem of the thermodynamical modelling of ice growth and then some solution schemes are presented with their advantages and drawbacks.

3.1 Definition of the Problem

The phenomenon that governs the ice growth is the heat exchange between the involving environments, i.e. mainly atmosphere and sea water, but one can observe also the presence of snow, which plays also an important role as it will be discussed later. It has to be mentioned at this point that in this master thesis the general case of the problem covers the presence of the snow (see the Figure 12).

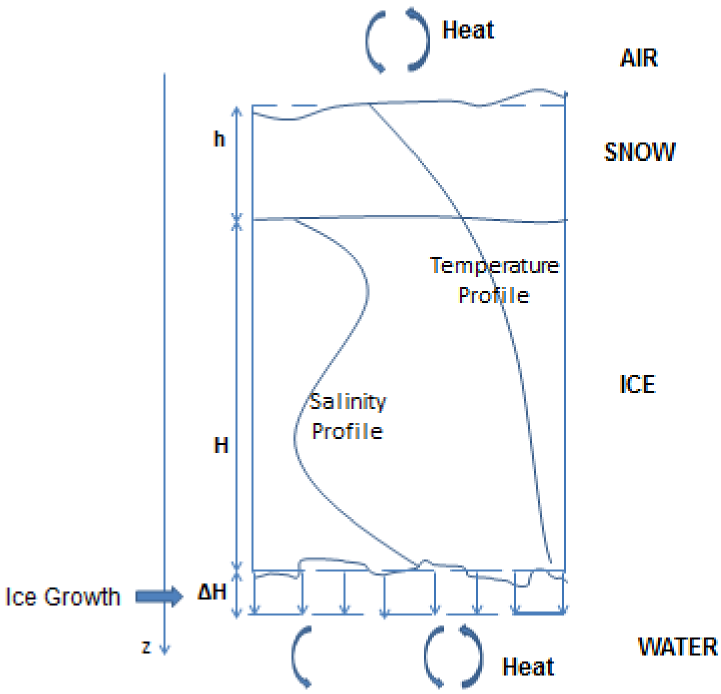


Figure 12 Schematic Illustration of the Thermodynamic Ice Growth

From the Figure 12, it can be seen that only the vertical exchange has been represented. In fact, traditionally this problem is treated as unidirectional, considering only the vertical heat exchange. The reason behind that is that the vertical heat exchange in the vertical direction is carried out by an important temperature gradient, much more important than the horizontal one. This led to an important assumption when solving the problem of the thermal ice growth which the “Horizontal Uniformity” Moreover, as stated in the previous paragraphs, the growth which is taken into account is the Bottom growth. Actually, this is very realistic, because ice grows mainly at the bottom: when the sea water starts freezing, some latent heat is released. This heat is conducted through the ice and snow if there is some, and transmitted to the atmosphere by radiation and turbulent heat fluxes.

3.2 Basic Equations

To tackle the above described problem, one needs to be able to describe physically what happens within the ice and within the snow (if any) as well as at the different interfaces such as water-ice, ice-snow, snow atmosphere (ice-atmosphere).

3.2.1 Heat Conduction in the ice

Within the ice, heat is transferred through conductive processes and the basic relationship that governs the thermodynamics of heat flow in solids is:

$$\frac{\partial(\rho_i\, c_i\, T)}{\partial t} = \nabla\,(\kappa_i\, \nabla T) + q \qquad (3\text{-}1)$$

Table 1 Heat Conduction in Ice - Terms and Units

symbol	quantity	units
t	time	sec
ρ_i	Ice density	g/cm^3
c_i	Specific heat of ice	cal / (g K)
T	Temperature	K
κ_i	Heat conductivity	cal / (cm sec K)
q	Internal source term	cal / (cm^3 sec)

Because of the assumption of "*One dimensional (vertical) heat transfer*", the equation (3-1) can be reduced to:

$$\frac{\partial(\rho_i\, c_i\, T)}{\partial t} = \frac{\partial\left(\kappa_i\, \frac{\partial T}{\partial z}\right)}{\partial t} + q \qquad (3\text{-}2)$$

Since the main ice growth occurs at the bottom, it is rather obvious that the temperature at the bottom of the ice is equal to the freezing temperature of the sea water. Then, the boundary conditions are:

At the top, we have:

$$\kappa_i \frac{\partial T}{\partial z} = Q_T \qquad (3\text{-}3)$$

Where Q_T is the sum of the heat loss (or gain) from the atmosphere and phase changes.

And at the bottom, we have moving boundary due to melting (or freezing):

$$\rho_\iota L \frac{dH}{dT} = \left(\kappa_i \frac{\partial T}{\partial z}\right)_{bottom} - Q_w \qquad (3\text{-}4)$$

Table 2 Lower Boundary Equation - Terms and Units

Symbol	**Quantity**	**Units**
L	Latent heat of freezing	cal / g
Q_w	Heat flux from water	cal / (cm^2 sec)
H	Ice thickness	cm

The above 4 equations describe fully at the same time, the heat conduction within the ice and the ice growth, i.e. the evolution of it thickness. Another major problem along with the main one is to determine quantitatively properties and the different heat fluxes. I t is really a serious difficulty to be considered on the grounds that many thermal properties depend on the salinity and pressure, although the salinity has a very unpredictable behavior in the ice. Additionally, the difficulties in defining the heat fluxes are due to the fact that they depend on the sea in which the ice is developing and its location.

3.2.2 Heat conduction in snow

To have ice growing, the blowing air temperature must be very low, much less than the freezing point of the sea water. Thus, we cannot always exclude the snow fall, since those temperatures are most of the time accompanied by a snow fall. In case of snow presence upon the upper ice surface, the heat conduction in snow has to be considered.

The equation governing the heat conduction within a snow layer is similar to the equation (3-2)

$$\frac{\partial(\rho_s c_s T)}{\partial x} = \frac{\partial\left(\kappa_s \frac{\partial T}{\partial z}\right)}{\partial z} + q \qquad (3\text{-}5)$$

Table 3 Heat Conduction in Snow - Terms and Units

symbol	quantity	units
t	time	sec
ρ_s	snow density	g/cm^3
c_s	Specific heat of snow	cal / (g K)
T	Temperature	K
κ_i	Heat conductivity	cal / (cm sec K)
q	Internal source term	cal / (cm^3 sec)

As for the boundary conditions, the upper surface of the ice is not in contact with the atmosphere any more, but we have now the interface ice-snow. Then, not only the temperatures of the snow and the ice must be equal at that interface, the continuity of the heat flux is also imposed as boundary condition:

$$\left(\kappa_i \frac{\partial T}{\partial z}\right)_{ice} = \left(\kappa_s \frac{\partial T}{\partial z}\right)_{snow} \qquad (3\text{-}6)$$

Thus, in the analysis, the equation (3-3) is replaced by:

$$\kappa_s \frac{\partial T}{\partial z} = Q_T \qquad (3\text{-}7)$$

Where κ_s is the heat conductivity of the snow.

Just as it has been said above for the ice, the properties of snow (mainly its density) vary also according to whether the latter is solid or starts melting.

The snow fall depends also on regions: in the Baltic Sea and the Arctic Seas for example, the snow fall is quite large, and when there is a huge amount of snow on the ice surface, that induces the formation of another layer referred as *snow-ice*. Lepparanta-1983, has attempted to model the ice growth taking into account the snow-ice layer. This case will not be analysed in the present work.

4. SEA ICE GROWTH MODELLING

4.1 Analytical Models

From the previous analysis, considering the variability of the ice and even snow properties, the difficulties in precising the different heat fluxes which have to be considered in the problem, it is obvious that solving analytically the above equations might lead to many complications. Curiously, the developed simple analytic models provide rather satisfactory results. In these models, many assumptions have been made, among others the thermal properties which have been taken as constants. These assumptions can be justified by the fact that during the growth period the ice is cold and the properties are not supposed to vary very much, and as for the fluxes, the solar radiation which is also a very determining feature is insignificant and the temperature variations are generally small.

4.1.1 Stefan's Law (1891)

The Stefan solution was initiated by a work made earlier by Neumann in the 1860s. Objectively, Stefan was the first to provide an analytical solution to the ice growth problem. His work has been recognized as the basic analytical model and has been used and extended by many scientists afterwards. His solution is based on some fundamental assumptions he made. In fact, the Stefan's law relies on that the temperature gradient during the conduction through the ice of the heat released by the bottom freezing is linear, i.e. linear temperature profile within the ice. More specifically, Stefan considered the problem described above and made the following assumptions:

(i) No thermal inertia[2], no internal heat source

(ii) No radiation

(iii) No snow

(iv) The temperature T_s on the top of the ice is known

(v) No heat flux from the sea

[2] the degree of slowness with which the temperature of a body approaches that of its surroundings and which is dependent upon its absorptivity, its specific heat, its thermal conductivity, its dimensions, and other factors.

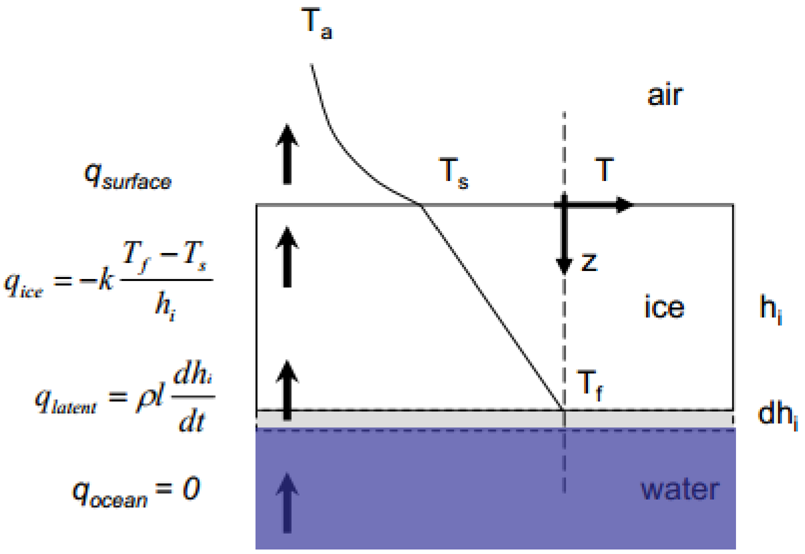


Figure 13 Schematic Illustration of the Stefan Law (Sébastien Barrault, 2008)

Because of the assumption (i), the equation (3-2) becomes:

$$\frac{\partial T}{\partial z} = constant \tag{4-1}$$

The boundary condition on the top surface is reduced to

$$T_o = T_o(t) \tag{4-2}$$

And finally because of the last assumption, the equation (3-4) (the ice growth equation) is reduced to:

$$\rho_i L \frac{dH}{dt} = \frac{\kappa_i (T_f - T_o)}{H} \tag{4-3}$$

The equation (4-3) can be solved analytically and its solution gives:

$$H^2 = H_o^2 + a^2 S \tag{4-4}$$

Where

$$a = \sqrt{\frac{2\,\kappa_i}{\rho_i L}} \tag{4-5}$$

And

$$S = \int_0^t \left(T_f - T_o(\tau)\right) d\tau \tag{4-6}$$

The equations (4-4) along with (4-5) and (4-6) are referred as the Stefan Law, with the time expressed in *days* and the value of *a* is approximately $3.3\sqrt{\frac{cm}{°C * days}}$. The factor *"S"* is well known as the negative degree-days.

Remarks

- The heat from the ocean slows to a certain extent the ice growth process. When it is not taken into account, it may sometimes result in extremely wrong results.
- The main restriction of the Stefan's solution is that the top boundary condition is not known with sufficient accuracy. Ideally, the ice surface temperature T_s must be estimated from the air temperature. This is fairly difficult, especially if the ice thickness is still small or in case of snow presence on the ice sheet.
- Stefan did not consider the specific heat of the ice (no thermal inertia),
- There are many other parameters which influence the ice growth--, such as the snow layer upon the ice, the existence of other energy sources such, e.g. turbulent heat transfer between the ice and underlying water or shortwave radiation absorption within the ice that Stefan neglected.
- It is rather obvious that the thermal properties of the ice depend on the temperature and they vary with the depth and the time, which is not the case in Stefan Model.
- All these simplifications tend to overestimate the ice thickness. As a consequence, the Stefan's predictions of the ice thickness are considered as upper boundary limits for favourable ice-growth conditions

All the above mentioned considered, some empirical formulas have been suggested in order to get more reasonable results. However, some more theoretical approaches, which are in fact the extension of the analytical models from the Stefan's law including the contribution of the atmosphere, the presence of snow, etc.

4.1.2 Ice-Atmosphere Coupling – Anderson 1961

Anderson was the first who presented a model that couples the ice and the atmosphere. The purpose of coupling the two environments is the estimation of the ice surface temperature. Unlike in the Stefan's solution, Anderson considered the fact that the ice surface temperature is not known, neither it is constant, but undergoes continuous variation. However, similarly to the Stefan's solution, no presence of snow upon the ice surface is treated.

A very simple approach has been used to express the heat exchange between the ice and the air (see equation (4-7)) to determine the ice surface temperature.

$$Q_a = \kappa_a(T_o - T_a) \tag{4-7}$$

Where κ_α is a heat exchange coefficient.
Then the continuity condition for the heat flux at the ice-air interface can be expressed as:

$$\kappa_a(T_o - T_a) = \frac{\kappa_i\left(T_f - T_a\right)}{H} \tag{4-8}$$

Solving for T_o the equation (4-3) transforms into (Lepparanta 1992):

$$\rho_i\, L\frac{dH}{dt} = \frac{\kappa_i\left(T_f - T_a\right)}{H + \frac{\kappa_i}{\kappa_a}} \tag{4-9}$$

The solution of the above equation for H is:

$$H = \sqrt{H_o^2 + a^2 + \left(\frac{\kappa_i}{\kappa_a}\right)^2} - \frac{\kappa_i}{\kappa_a} \tag{4-10}$$

The ratio κ_a/κ_a is estimated around 10 cm.

Remarks

- This approach presented values very different from the Stefan's ones and these differences are rather significant in the early stage of the growth.
- The Anderson's approach added two terms to pure Stefan's law: these terms provide a negative contribution to predicted thickness. In fact, as it has been stated above, the

Stefan's law overestimates the thickness, which implies that it also overestimates the speed of the ice growth. This fact is very considerable in the early stages of the ice growth. With the ratio κi/κa around 10%, the difference between the values provided by both approaches in the very early stages (ice thickness less than 5 cm) is more than 80%, and it is almost 10% for ice thickness of 1 m.

- Furthermore, as the thickness increases, the values provided by the Anderson's model is κ_i/κ_a or 10 cm lower than the ones deduced from the Stefan's law.

4.1.3 Considering the presence of snow upon the ice surface

In the previous paragraphs, it has already been stated that the presence of snow on the ice surface reduces the ice growth speed, since it acts as insulator. The fact that snow properties vary a lot with time makes the problem very complicated to be modelled. That is one of the main reasons why the models that have been discussed above did not take into account the snow influence.

For this case, the following assumptions have been made:

- As previously, no thermal inertia, neither internal heat source in snow is considered.
- The temperature on the snow surface is known T_s=T_s(t)
- The temperature profile within snow is taken linear

Considering the continuity condition for the heat flow at the ice/snow interface, the presence of snow transforms the equation (4-3) into:

$$\rho_i\, L\frac{dH}{dt} = \frac{\kappa_i\left(T_f - T_s\right)}{H + h\dfrac{\kappa_i}{\kappa_s}} \tag{4-11}$$

With $\boldsymbol{T_s}$ the snow surface temperature and $\boldsymbol{h}$ the snow thickness. Nevertheless, the equation (4-11) cannot be solved analytically, since normally T_s and κ_s vary with the time.

As a consequence, another assumption concerning the snow thickness has been made:

$$h = \lambda\, H \tag{4-12}$$

In addition, the thermal conductivity of snow is assumed to be constant.

Considering the above assumptions for H_o=0, we have:

$$H^2 = \frac{2\,\kappa_i\,S}{\rho_i L\left(1+\lambda\frac{\kappa_i}{\kappa_s}\right)} \tag{4-13}$$

The thickness values produced by this approach are less than the one's produced by Stefan, being reduced by the factor $1+\lambda\frac{\kappa_i}{\kappa_s}$.

It has to be underlined that, this formula is only valid as long as the snow is above the water surface. Sometimes, the snowfall is so large that the ice becomes submerged and then slush formation starts in the ice/snow interface. As consequence, we have another layer which is referred as ***snow-ice***. . As for the coefficient λ, according to the Archimedes' law, for no submergence, $\lambda \le \frac{\rho_w-\rho_i}{\rho_s} \approx 0{,}3$. For a typical situation $\kappa_i \approx 10\kappa_s$ the snow reduction factor varies between ½ and 1.

The major questions here are "Does the snow thickness depend on the ice thickness?" and "are they correlated?"

The answer is: Obviously not! Only numerical solutions have provided models that solve the problem considering both thicknesses totally independent to each other.

4.1.4 Considering the Heat Flux from the Water

The three approaches mentioned above do not take into consideration the heat flux coming from water. This heat reduces the ice growth rate. The thickness can be deduced by solving the equation (3-4), but due to its form, this equation cannot be solved with an analytical approach. Badgley-1966 and Maykut-1986 have agreed that typical values of heat flux from water Q_w are between 1 and 5 W/m^2 and then the reduction effect to differential ice growth $\frac{Q_w}{\rho_i L}$ varies typically between 0.1 and 1 cm per day (Lepparanta, 1992).

Table 4 Influence of the Ocean Heat Fluxes on the Ice Thickness (Lepparanta, 1992).

Ice thickness in [cm]		
Time [days]	Q_w=5 W/m^2	Q_w=0
5	15	23
10	21	33
100	64	104
∞	400	∞

4.2 Numerical Models

The analytic tools provided above, are good tools for first approximations of sea ice growth in different environmental conditions. They have provided the understanding of the physics of the problem. However, to quantify more correctly the ice growth features, one should use numerical methods. The numerical models offered the ability to overcome the complications introduced by the thermal properties which are expected to vary with time and depth. The full (vertical) sea-ice heat conduction equation was numerically solved first by Maykut and Untersteiner (1971), and five years later, Semtner suggested some simplifications. The idea behind these models is that starting from an arbitrary initial condition, the model is numerically integrated until annual equilibrium is reached. In other words, they deal with ice which is already formed and has a certain thickness, one could talk about multi-year ice.

Until today, the M&U[3] is considered as the reference model in ice growth modelling, even if Semtner is widely used because of the high simplicity it offers.

4.2.1 Maykut and Untersteiner Model 1971

The ice sheet is considered as an infinite, horizontally homogeneous slab, floating on its own liquid phase. As for the heat, it is transferred through the ice between the ocean and the atmosphere by conduction. The boundaries of the ice sheet are pictured as mathematically idealized planes and energy is assumed to be absorbed at these planes. The model includes surface and bottom growth, accretion and ablation depending on the balance of the energy

[3] Maykut and Untersteiner model (1971)

fluxes at the boundaries. The different heat fluxes considered are the incoming radiation and turbulent fluxes, oceanic heat flux; and along with the ice salinity, the snow accumulation and the surface albedo, they are all specified as function of time.

The governing equations are:

a. Heat conduction in ice

Untersteiner noticed that the equation (3-2) with q=0 does not completely define what happens in reality. M&U then used the following formulation:

$$(\rho\, c)_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial \mathrm{z}}\left(k_i \frac{\partial T}{\partial z}\right) + \kappa_i\, I_o\, e^{-\kappa_i z} \tag{4-14}$$

With the last term representing the heat generated by absorption of solar radiation per unit of time. The subscript "i" refers to ice. The most important feature is that, they let the parameters $(\rho\, c)_i$ and k_i vary in the ice, because the small pockets of brine during the freezing as referred above are considered to remain in the ice. Thus they suggested:

$$(\rho\, c)_i = (\rho\, c)_{i,f} + \frac{\gamma\, S(z)}{(T-273)^2} \tag{4-15}$$

$$k_i = k_{i,f} + \frac{\beta\, S(z)}{T-273} \tag{4-16}$$

With the subscript "f" for pure ice, S(z) the salinity which varies with the depth and $\gamma = 4100\, cal\frac{K}{g}$ & $\beta = 0{,}28\, cal\frac{cm^2}{g\, sec}$ constants.

They assumed a standard salinity profile with the shape in the Figure 14, which the approximated with the equation (4-17)

$$S(z,t) = A + B \sin\left[C\left(\frac{z}{H}\right)^{\frac{n}{\left(\frac{z}{H}+D\right)}}\right] \tag{4-17}$$

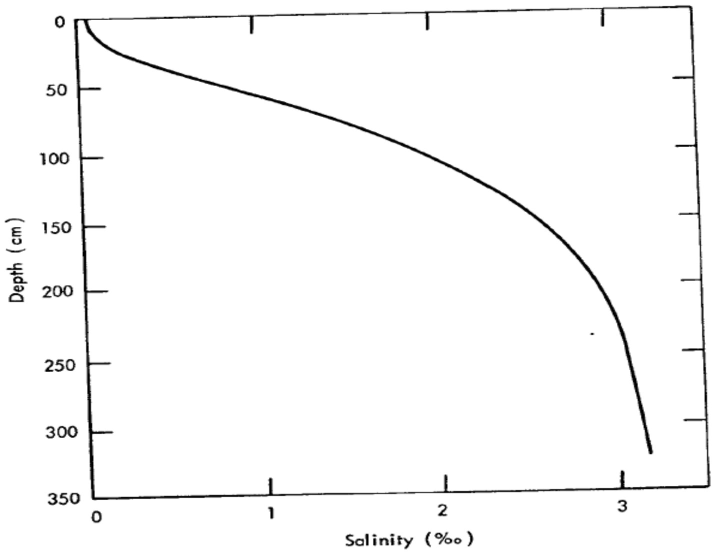


Figure 14 M&U Salinity Profile (Maykut & Untersteiner 1971)

b. Heat conduction in snow

The heat conduction equation in the snow is similar to the equation (4-17)

$$(\rho\ c)_s \frac{\partial T}{\partial t} = k_s \frac{\partial^2 T}{\partial z^2} \tag{4-18}$$

With the differences that the solar radiation absorption is not considered, and the $(\rho\ c)_s$ and k_s are constant in time and space.

The boundary conditions are as important as the heat conduction equation to determine the clear picture of the under studies problem, i.e. the surface temperature, the temperature profile within ice and snow, the mass changes at the bottom and/or on the top.

c. The boundary conditions:

- Upper surface

The major energy fluxes on the top considered in the model are the incoming long-wave radiation from the atmosphere and clouds F_L, incoming short-wave radiation F_r, reflected short-wave radiation from the surface (αF_r) and outgoing long-wave radiation ($\varepsilon_L\ \sigma T_o^4$). In addition to these, there are several smaller but important fluxes: the fluxes of sensible heat (F_S) and latent heat (F_l), heat conduction flux in the ice or snow (F_C), and flux of radiative energy through the surface into the ice (I_o).

$$(1-\alpha)F_r - I_o + F_L - \varepsilon_L \sigma T_o^4 + F_S + F_l + k\left(\frac{\partial T}{\partial z}\right)_o = \begin{cases} -\left[q\frac{d}{dt}(h+H)\right]_o & ,T_o = 273\ K \\ 0, & T_o < 273\ K \end{cases} \quad (4\text{-}19)$$

Which means that on the surface there might be either accretion (freezing) or ablation (melting).

- Ice-Water Interface

For this interface which is in fact easier to be dealt with than the previous one, M&U, unlike Stefan, considered the heat flux from the ocean. Nevertheless, this value is constant and does not depend to the ice growth.

$$k_i\left(\frac{\partial T_i}{\partial z}\right)_{h+H} - F_w = q\frac{d}{dt}(h+H)_{h+H} \quad (4\text{-}20)$$

- Ice-snow Interface

M&U posed these conditions at the ice-snow interface:

The temperature of the ice is equal to the temperature of the snow, i.e. at z=h, T_s=T_i.

The effects from penetration of short-wave radiation are negligible so that the heat conduction is continuous, i.e.

$$k_i\left(\frac{\partial T_i}{\partial z}\right)_h = k_s\left(\frac{\partial s}{\partial z}\right)_h \quad (4\text{-}21)$$

d. Resolution Method Used

These equations have been solved using a finite difference theory. Since the problem involves moving boundaries, only an explicit scheme could be applied. However, since the time increment of 1 hour was chosen knowing that the model is very complex on the grounds that 30 to 50 years of integration will be necessary to achieve the equilibrium, a Saul'yev method has been used, which not only is much more efficient than a simple forward difference scheme for that case, but also needs less computation time.

e. Limitations

Maykut and Untersteiner have admitted facing some limitations of conceptual and physical natures. The latters are mainly due to the uncertainties about the environmental data, such as the surface albedo, the ocean heat flux, the solar radiation, etc. The conceptual limitations are the following:

- It is well known that the wind can produce huge deformations, but the mechanical stresses have not been taken into account
- The models treats only the ice formed through thermodynamic process. In fact, ice can be made up of pressure ridges.
- Changes in the heat budget resulting from the variation of open water areas are not covered by the model.
- The turbulent fluxes at the boundaries. Actually, the flux of sensible heat F_s, the flux of latent heat F_l and the ocean heat flux are assumed to be independent of the ice growth, which is not realistic.
- The model present inaccurate results for ice of thickness less than 50cm. In fact, since relatively “thin” ice is mechanically weak, they generally show different drift deformation patterns along with loss of horizontal uniformity. Moreover, the assumption about the salinity profile and turbulent fluxes are probably invalid for young rapidly growing ice.

4.2.2 A.J. Semtner Model 1976

This model, which has been widely used in climate simulation, is a simplification of the M&U model. Its results have been compared with the M&U’s and showed high similarities.

The Semtner model is referred as the “3-layer model” because it considers one snow layer and two layers of equal thickness in the ice. The main assumption is that the temperature profile within each grid box is linear. The temperatures to be determined are the ones shown on the Figure 15.

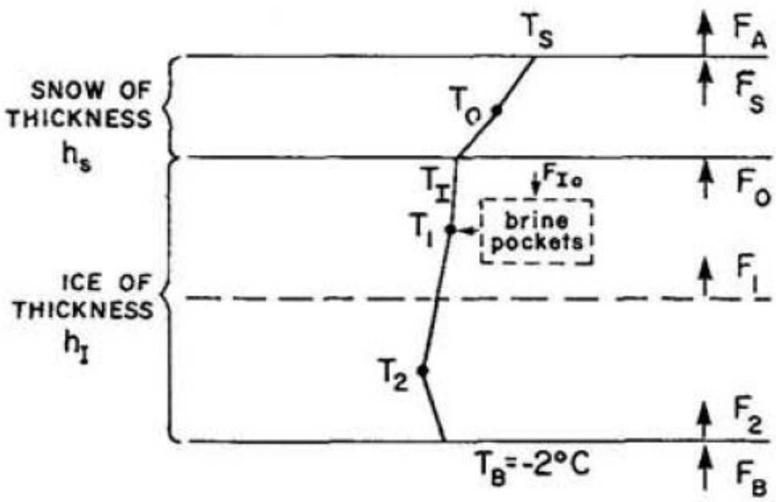


Figure 15 Semtner Model Illustration (A.J. Semtner 1976)

In this paragraph, the main differences between both approaches are presented, as well as the assumptions made by Semtner to solve the problem.

a. Heat conduction in ice

First of all, Semtner has not considered the penetrating solar radiation, stating that its energy does not cause immediate surface melting in summer. Moreover, when the ice is snow-covered there is no penetration of solar radiation.

In addition, the ice properties such as the thermal conductivity k_i and the density times the specific heat $(\rho\, c)_i$do not vary as function of time and the used salinity profile since the changes applied by M&U in the specific heat capacity $(\rho\, c)_i$especially, were dramatic when the temperature is close to the melting point. Then, he decided to use the values of pure ice for both values. Obviously there is no assumed salinity profile as it was the case in M&U model.

Thus the equation used is

$$(\rho\, c)_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial \mathrm{z}}\left(k_i \frac{\partial T}{\partial z}\right) \tag{4-22}$$

b. Heat conduction in snow

The equation (4-18) has been used without any change.

c. Boundary conditions

The boundary conditions are identical to the ones used by M&U. the only one thing worth mentioning is the idea behind the determination of the surface temperature T_s. Assuming that $T_s = T_p + \Delta T$, with T_p the surface temperature at the previous time step, ΔT can be found by solving the following equation, which is in fact the upper boundary condition used by M&U.

$$F_l + F_t + F_L + (1 - \alpha_s)F_r + \sigma(T_p)^4 + 4\sigma(T_p)^3\Delta T + k_s \frac{T_o - T_p - \Delta T}{\frac{h_s}{2}} = 0 \tag{4-23}$$

d. Resolution Method

The Semtner model presents a very rough resolution compared to M&U solution which is rather very sophisticated. Regarding the discretization, a simple forward-differencing scheme and the chosen grid time to keep the solution stable were 8 hours. One of the most determinant assumptions made by Semtner is that the temperature profile between grid points is linear. As a consequence, the conductive fluxes F_n across interior interfaces are based on the above so that

$$F_n = k_I \frac{T_B - T_n}{\frac{h_I}{2n}} \tag{4-24}$$

e. Remarks

The very rough solution of Semtner does not treat the internal heat storage by brine pockets as it is the case in M&U model (salinity profile) but present results that are in quite satisfactory agreement with M&U results. The main advantage presented by this model is the speed of calculation and the simplicity combined with the accuracy of the results.

5. NEW MODEL DEVELOPMENT

5.1 Introduction

The thermodynamical conception does not differ from the one described above, i.e. the heat transfer that is taken into account is only in the vertical direction. In addition, the ice sheet is also considered to be horizontally uniform with the upper and lower surfaces perfectly plane.

In order to avoid as many assumptions and simplifications as possible, we opted to solve the problem numerically. The model is not as sophisticated as the M&U's but is more close to the Semtner one, since we have opted to use less grid points to compute the temperature in the ice; Less than M&U but more than Semtner.

The model results are supposed to be validated by the experiments performed in the ice facilities of the HSVA. Thus, the conditions and energy fluxes used in the model must be as close as possible to experiments conditions.

First of all, in case of snow layer upon the ice, its thickness will be taken to be constant over the time, owing to the difficulty to model the snow fall or melting in the ice facilities. Moreover, since the under study case treats only the ice formation and development, no melting will be studied and a priori solar radiation will not be taken into account.

5.2 Theoretical Model Formulation

5.2.1 Heat Conduction in snow

The equation governing the temperature in snow is the one-dimensional heat conduction equation, as described in previous paragraphs.

$$\rho_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial \mathrm{z}}\left(k_s \frac{\partial T}{\partial z}\right) \tag{5-1}$$

Where ρ_s, c_s and k_s are constant and their respective values are: $\rho_s = 0.33\ g/cm^3$, $c_s = 0.499\ cal/(g\ K)$ and $k_s = 7.4x10^{-4} cal/(cm \sec K)$.

5.2.2 Heat Conduction in Ice

Within the ice layers, the temperature governing equation is similar to the one in the snow. It has to be reminded that the solar radiation penetration is not taken into consideration.

$$(\rho\ c)_i \frac{\partial T}{\partial t} = \frac{\partial}{\partial \mathrm{z}}\left(k_i \frac{\partial T}{\partial z}\right) \qquad (5\text{-}2)$$

In the Model-A, the parameters $(\rho\ c)_i$ and k_i are constant, i.e. they do not vary with the depth, neither with the time. Their values are the same with the ones of pure ice (ice made from fresh water), since in the Model-A, the salinity is taken to be constant over the depth, which means that it does not consider the fact that the small pockets of brine trapped during the freezing process remain in the ice.

Their values are $\rho_i = \rho_{i,f} = 0.9\ g/cm^3$, $c_i = c_{i,f} = 0.499\ cal/(g\ K)$ and $k_i = k_{i,f} = 4.86x10^{-3} cal/(cm\sec K)$.

In the model coupling the upper surface with the atmosphere, the Model-B, these values vary with the salinity and the temperature, and consequently with the depth. Then we have used the equations (4-15) and (4-16) provided by M&U.

Where $\gamma = 4100\ cal\frac{K}{g}$ & $\beta = 0{,}28\ cal\frac{cm^2}{g\ sec}$ are constants given by Ono (1966).

Inserting the above equations, i.e. (4-15) and (4-16) in the equation (5-2), we obtain the following:

$$\begin{aligned}\left[(\rho\ c)_{i,f} + \frac{\gamma\ S(z)}{(T-273)^2}\right]\left(\frac{\partial T}{\partial t}\right) \\ = \left[k_{i,f} + \frac{\beta\ S(z)}{T-273}\right]\left(\frac{\partial^2 T}{\partial z^2}\right)\end{aligned} \qquad (5\text{-}3)$$

5.2.3 Boundary condition

a. *Upper Boundary*

In the Model-A, no thermal inertia has been considered; which implies that the ice (or snow if any) surface temperature is equal to the blowing air temperature, $T_s = T_a$ (or $T_i = T_a$ in case of snow absence on the ice surface). This assumption is not realistic, since the upper surface is the interface between two environments and the temperature at that point must be resulted from a heat balance coming from both environments. As the growth progresses, the surface temperature tends to approach the air temperature, but will never reach it owing to the fact it is receiving an influence coming from the snow (or ice) which is warmer than the air in the freezing period.

In the Model-B, the thermal inertia has been taken into consideration. In fact, the temperature on the surface is determined by the heat balance between the atmosphere and the ice (or snow if present). Thus, if the sum of these fluxes is Q_T, then the balance is

$$Q_T + k\left(\frac{\partial T}{\partial z}\right)_o = \begin{cases} 0 & if\ T_s < 273\ K \\ -\left[q\frac{d}{dt}(h+H)\right]_o & if\ T_s \geq 273\ K \end{cases} \tag{5-4}$$

The second case means the temperature on the surface gets higher than the melting point, then the melting process starts. However, this work does not study the melting case.

b. Snow-Ice Interface

At the snow-ice interface, it is obvious that the ice and the snow temperatures must be equal. Additionally, the heat conduction in this interface must be continuous and the interface temperature T_i must verify this condition, i.e. T_i must satisfy the conductive heat fluxes balance on the interface:

$$k_i\left(\frac{\partial T}{\partial z}\right)_{h_s} = k_s\left(\frac{\partial T}{\partial z}\right)_{h_s} \tag{5-5}$$

c. Lower Boundary

At the bottom of the ice the boundary is moving and the main fluxes that are considered are the oceanic heat flux and the heat conduction in the ice. The growth is determined by an imbalance between the two fluxes.

$$q\frac{d}{dt}(h_s + H)_{h+H} = k_i\left(\frac{\partial T_i}{\partial z}\right)_{h+H} - F_w \tag{5-6}$$

The difficulty resides in the fact that the heat flux coming from the sea is difficult to calculate without a theoretical model of the ocean (Maykut and Untersteiner 1971). Normally, the term F_w is not constant during the ice growth, it varies with respect to the temperature, the depth, and consequently the salinity according to the relationship that follows:

$$F_w = (\rho\ c)_w k_w \left(\frac{\partial T_w}{\partial z}\right)_{h+H} \tag{5-7}$$

5.2.4 Used Parameters

A part from the main equations of the model, the other important problem to be solved is the determination of the external parameters that are to be used in the model. These are really key elements, because the temperatures and thickness will be changing based on them. Consequently, the wrongly determined or chosen parameters might lead to remarkably wrong results.

a. Heat fluxes

One of the most difficult problems faced when modelling the ice growth in a region is the determination of the different fluxes on the boundaries. The majorities of the existing models, especially the ones that are mentioned in this work have considered the following fluxes:

- On the upper boundary: The incoming short-wave radiation, the incoming long-wave radiation, the flux of sensible heat and the flux of latent heat.
- On the lower boundary: the flux coming from the ocean, or more generally the river.

M&U and Semtner models have used the average monthly values according to Fletcher (1965) for upper boundary fluxes.

The shortwave radiation comes from a hotter source like the sun, with high energy amount and the longwave comes from a cooler source. The environment of our experiments is totally closed and it does not allow any solar radiation penetration. Even if it would, we would not be able to compute the exactly values of these quantities for the conditions of our experiments. All these referred energy fluxes vary with the ocean and the location in the ocean. Since the conditions of our experiments were totally different from the ones of these provided values, another approach has been used.

In fact, as stated through the title, the purpose of this work is to estimate the ice thickness using the speed of the blowing air and its temperature. Ignoring the quantities of the different atmosphere heat fluxes, we considered that there occurs convection between the room atmosphere and the water. Then the term Q_T from the equation (5-4) is the convective heat and can be expressed as:

$$Q_T = H_a * (T_s - T_a) \tag{5-8}$$

Where $H_a$is the convective heat transfer coefficient, T_a and T_s the air and the surface temperatures respectively. Introduce the equation (5-8) into (5-4), and then the quantity to be determined is the surface temperature. However, before that, the heat convection coefficient $H_a$has to be found.

An empirical formula provides $H_a$as function of the speed pf the studied object through the blowing air (www.engineeringtoolbox.com).

$$H_a = 10.45 - w_s + 10 * w_s^{0.5} \tag{5-9}$$

Where w_s is the relative speed of the object through the air in m/s. The empirical formula has been developed for speeds from 2 to 20 m/s.

Thus, the following have been considered:

- Since the containers do not move, an observer moving with the air sees the containers moving with the air velocity.
- Interpolation in case of air velocity slightly less than 2 m/s. Nevertheless, a comparison with the experimental value has to be done.

b. The Heat coming from the ocean

The order of magnitude of the oceanic flux is not well known. Many experiences and studies have been carried out (Malmgren 1927, Timofeev 1958, Crary 190, Untersteiner 1964, etc.) and the values of this heat flux were always different from each other. Even direct measurements have been proved difficult (M&U 1971). However, most of the models more or less agree that the average value is approximately $1.2\,\frac{kcal}{year\ cm^2}$. This value is considered by many as a standard value to be used, and has been widely used.

Normally, we should not use it at all, but we did in some cases in order to be able to quantify the influence it could have in the results. This part is well commented in the results chapter.

c. Salinity

The salinity profile used here is the mean profile of the ones suggested by Wei Lv et al (PIM – Parameter Identification Method), Thesci and Eicken. (Figure 16).

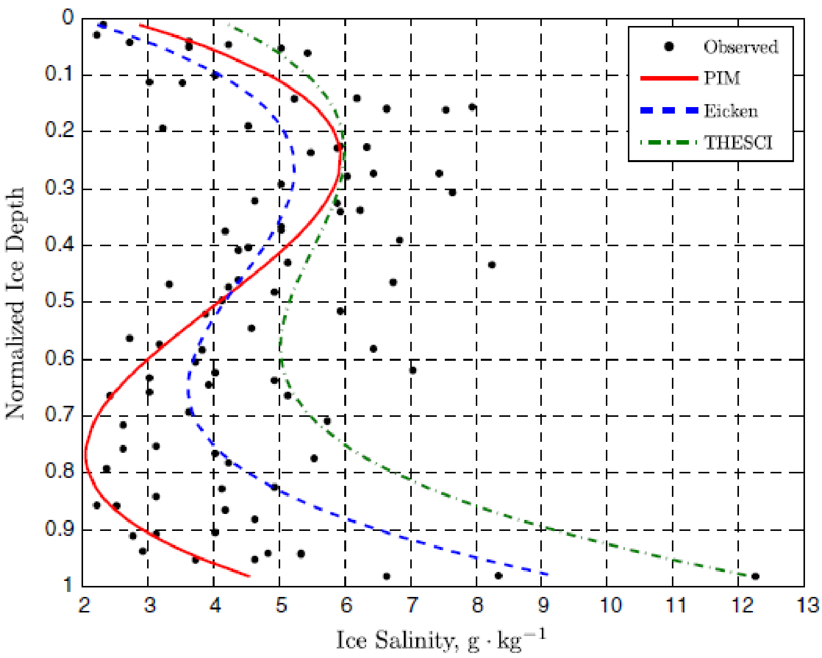


Figure 16 Salinity Profile (Xiaojiao Li, Enmin Feng, 2014)

5.3 Methods of Solution

5.3.1 Notions

The method chosen to solve numerically the above problem is the Finite Difference Method FDM. Therefore, all the derivatives are approximated by finite differences. The space which is in fact the snow thickness and the ice thickness is discretized using a mesh z_1, z_2... z_m and the time is partitioned using time mesh t_o, t_1… t_k. Obviously, the discretization of time and space will be uniform. That implies $z_{i+1} = \Delta z + z_i$ and $t_{j+1} = \Delta t + t_j$ where Δt is constant. However, owing to the fact that we have a moving boundary at the bottom (sometimes at the surface), the spacing discretization is not "constant" over the time, and depends on the thickness variation, i.e. as the ice gets thicker, Δz increases: $\Delta z = \Delta z(t)$.

The finite differences can be expressed under two schemes, the implicit or the explicit. It is proved that the implicit scheme is unconditionally stable, which means that the size of Δt and Δz does not have any importance in order the scheme to be stable. On the contrary, the explicit scheme can be either stable or unstable depending on the chosen space and time steps.

One could expect that we used the implicit scheme, but for this study the explicit scheme was used because of the simplicity it provides. Due to the moving boundaries and varying properties, the implicit scheme was not simple to implement, and since the used explicit scheme did not show any problem of stability as it can be seen through the provided results in next sections, the latter has been adopted.

5.3.2 Diffusion equation

The one-dimensional heat conduction equation is a parabolic partial differential equation. Thus, in order to be solved an initial temperature profile, at the time step t=0 is needed, as well as values of the temperature at both boundaries at each step. Therefore, the temperature profile at each time step will be based on this information. From the previous analysis, we know that at the lower boundary the temperature is equal to the freezing point of the sea water. As for the upper surface, a heat fluxes balance has to be made at the snow- or ice-atmosphere interface in order to determine the temperature.

Let us consider *z(t)* the space (thickness) subdivided into *m* intervals of length $\Delta z(t)$ because Δz varies with the time, and *t* the time subdivided into *k* intervals of constant length Δt.

Using the *Euler* Scheme, which is the 1st Order Upwind Scheme for the 1st term of the equation (5-2) , we obtained the following equations:

$$\left[(\rho\, c)_{i,f} + \frac{\gamma\, S(z)}{(T_z(t+\Delta t) - 273)^2}\right] \frac{T_z(t+\Delta t) - T_z(t)}{\Delta t} = \left[k_{i,f} + \frac{\beta\, S(z)}{T_z(t+\Delta t) -} \right. \quad (5\text{-}10)$$

Where $\Delta z(t+\Delta t) = \frac{H(t)}{m}$ with m the number of layer within the ice and H(t), the ice thickness at the previous time step.

It has to be mentioned that since the chosen time step is very small, the variation of the temperature is very small between two successive time steps. Therefore, to simplify the calculations and reduce at the same time computation time, we decided to use the following equation:

$$\begin{aligned} &\left[(\rho\, c)_{i,f} + \frac{\gamma\, S(z)}{(T_z(t) - 273)^2}\right] \frac{T_z(t+\Delta t) - T_z(t)}{\Delta t} \\ &= \left[k_{i,f} + \frac{\beta\, S(z)}{T_z(t) - 273}\right] \frac{T_{z+\Delta z}(t) - 2T_z(t) + T_{z-\Delta z}(t)}{\Delta z(t+\Delta t)^2} \end{aligned} \quad (5\text{-}11)$$

Then the temperature profile within the ice is fully described.

As for the snow, if any, the chosen places to describe the temperatures are on the top, in the middle and at the interface with the ice, T_s, T_o, and T_i.

T_o at time step ($t+\Delta t$) is calculated via the formulas that follows.

Where Δz_n is the grid space within the snow.

T_i and T_s are on boundaries (see boundary conditions analysis)

5.3.3 Boundary conditions

a. Upper Boundary

The upper surface is a determinant factor in the ice development process. In fact, as it has been stated above, there are many energy fluxes that contribute to the thickening (freezing) or the melting of the ice. The equation (5-9), which concerns this section does obviously not to be discretized with regard to the time, since an average wind speed has been considered, due to the difficulty not only to predict but also to measure its variation over the time.

b. Snow-Ice Interface

In case there is a snow layer on the ice surface, the snow and the ice temperatures at the interface must be the same, T_i. Moreover, this temperature must satisfy the conductive flux balance condition at the interface.

Thus, we have

T_1 is the temperature at the grid point that is just under the snow-ice interface.

$$\left[k_{i,f} + \frac{\beta\, S(z)}{T_i(t) - 273}\right] \frac{T_1(t+\Delta t) - T_i(t+\Delta t)}{\Delta z(t+\Delta t)} = [k_s] \frac{T_i(t+\Delta t) - T_o(t+\Delta t)}{\Delta z_{sn}} \tag{5-12}$$

c. Lower Boundary

At the bottom of the ice, the temperature is constant and taken to be equal to the freezing point of the sea water T_B. This temperature depends on the average salinity. For an average salinity of 34 psu, this value is approximately -2°C. The lower boundary moves at each time step by $\Delta H_B(t)$. therefore, the finite difference form of the equation (5-6) is

$$\Delta H_B(t+\Delta t) = \frac{\Delta t}{q_B}\left[\left(k_{i,f} + \frac{\beta\, S(z)}{T_i(t) - 273}\right) \frac{T_B - T_m(t+\Delta t)}{\Delta z(t+\Delta t)} - F_B\right] \tag{5-13}$$

Where T_m is the temperature in the lowest ice layer and F_B the heat flux coming from water.

5.3.4 Stability of the Chosen Scheme

The one dimensional heat equation used in this study have been solved using an explicit finite differences scheme as it can be seen in the paragraph 5.3.2. Let us consider a parameter $\boldsymbol{\lambda}$ with analytic expression:

$$\lambda(t,z) = \left[k_{i,f} + \frac{\beta\, S(z)}{T_z(t+\Delta t) - 273}\right] * \frac{\frac{\Delta t}{\Delta z^2}}{\left[(\rho\, c)_{i,f} + \frac{\gamma\, S(z)}{(T_z(t) - 273)^2}\right]} \tag{5-14}$$

The main advantages provided by the explicit scheme is the simplicity and the high speed of computation, however the stability is still an issue compared to the implict and other schemes, of which the solutions are unconditionally stable.

Thus, a stable solution is feasable if and only if :

$$\lambda(t,z) < \frac{1}{2} \tag{5-15}$$

The difficulty is due to the fact that the salinity and the temperature are varying at each time step. Moreover, since the ice is thickening, the space step Δz is increasing. Consequently, we managed to opt for the lower possible time step which is 1 second. Another point we had to care about was the initial thickness of the ice. In fact, through the equation (5-14) one could easily notice that for water of high salinity it is impossible to obtain a stable solution with a relatively small initial thickness (low thickness value implies small grid). As an illustration, the Figure 17 is provided. This figure concerns the case discussed in the next paragraph. It can be seen that before the first second, the parameter λ for the considered grid points is very high because of the chosen initial ice thickness. However, it immediately gets less that 0.5 after the first second of calculation and remains much smaller than that throughout the whole time of calculation.

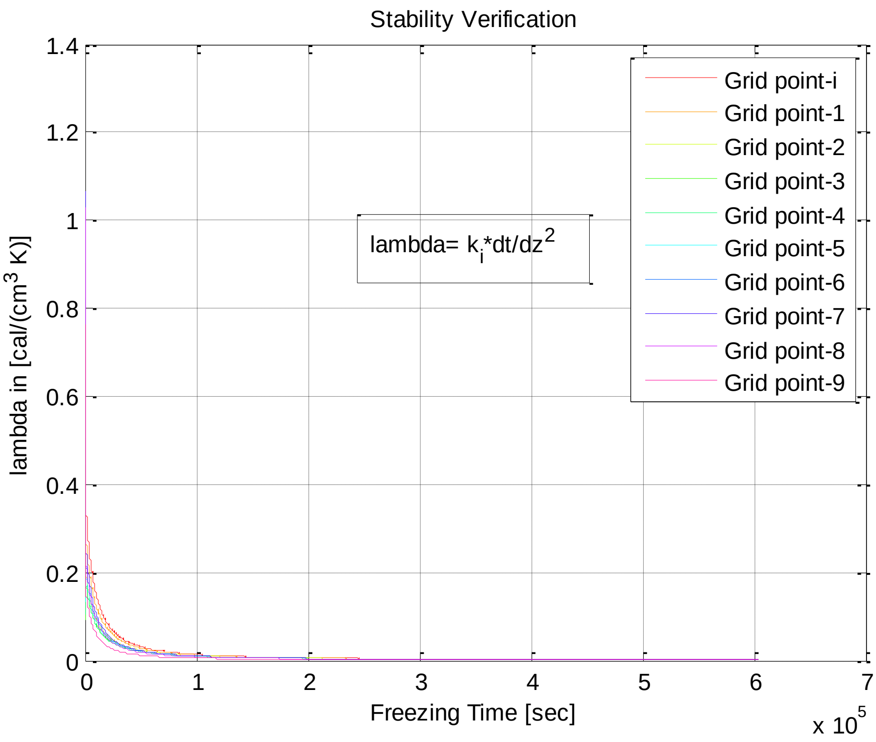


Figure 17 Scheme Stability

5.4 Method Concept

First of all we chose to perform all the numerical calculations in Matlab. Therefore, the processes described in section 5.3 have been implemented in Matlab. In this section, we provide the main features of the method and the Matlab code.

The first issue was to determine the number of layers, i.e. the number of grid points within the ice sheet and within the snow layer if any. In fact, this is the space discretization of the problem, so the accuracy of the results depend a lot of this issue.

After studying the convergence of the results and the computing time for different numbers of layers, we ended up to consider nine (9) layers within the ice. As for the snow sheet, only two layers have been considered because with that, the accuracy was already quite satisfactory.

The time discretization is also very important. As it has been developed in the previous paragraph, the time step was set to 1 second for the studied cases. In fact, the time step has a direct impact on the computing time: the smaller it is, the higher the computing time is. The cases studied in this project concern small ice thickness, and the computing time was insignificant. In case of bigger ice sheets, the Matlab code automatically increases the time step, checking at the same the scheme stability to minimize the computing time.

Figure 18 shows the architecture of the Matlab code.

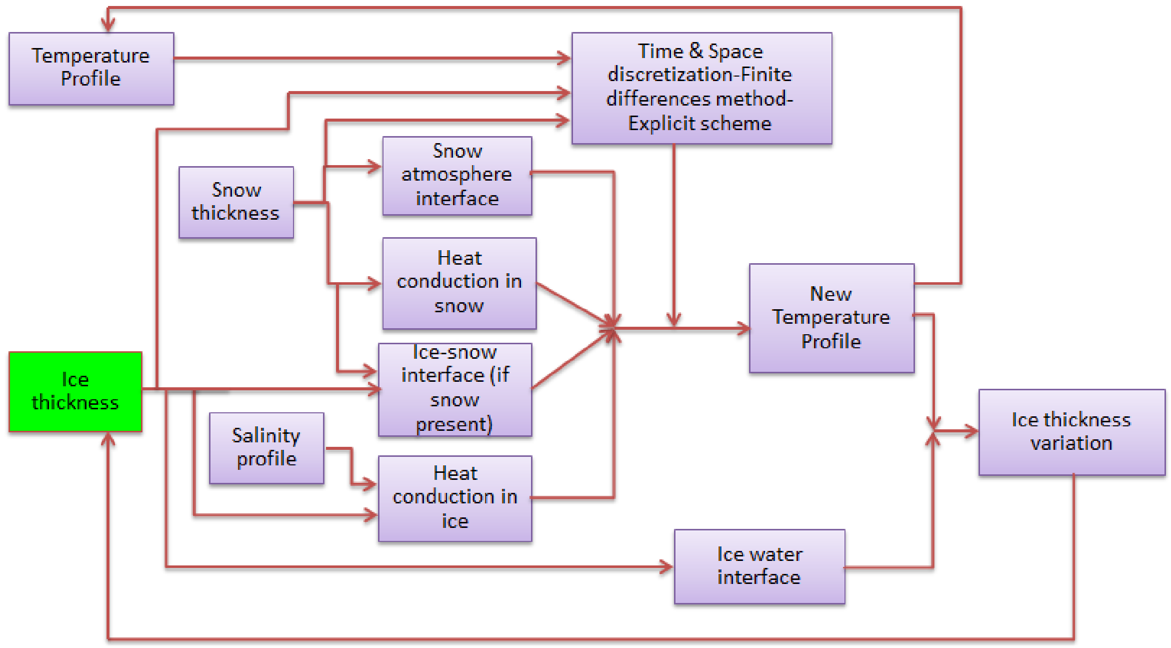


Figure 18 Matlab Code Architecture

As initial conditions, we have a temperature profile, the thickness of the snow layer and the thickness of the ice sheet. The initial ice thickness has been taken to 5mm; and the temperature profile has been taken constant and equal to the freezing temperature of the water, because in a small ice sheet the temperature variation is insignificant between the upper and the lower surfaces. The salinity profile over the depth of the ice sheet contributes as an external parameter.

We have then five equations to solve: *Snow-Atmosphere Interface*, *Heat Conduction in Snow*, *Ice-Snow Interface*, *Heat Conduction in Ice* and *Ice-Water Interface*.

Afterwards, the time and space discretization are applied and the first four (4) equations are solved at the first time step. As outcome we have a new temperature profile. Based on this new temperature profile, the fifth equation is solved and gives the variation of the ice thickness.

Having a new temperature profile and a new ice thickness, the above steps are performed at the second time step and so on.

5.5 Comparison with Stefan and Anderson's Results

Let us consider sea water with a mean salinity of 34.5‰. The blowing air has 1.5 m/s speed and a temperature of -15°C. What will be the ice thickness after 7 days? Firstly, without snow on the top of the ice and secondly with a 3.7 cm snow thickness upon the ice surface.

5.5.1 Ice Thickness

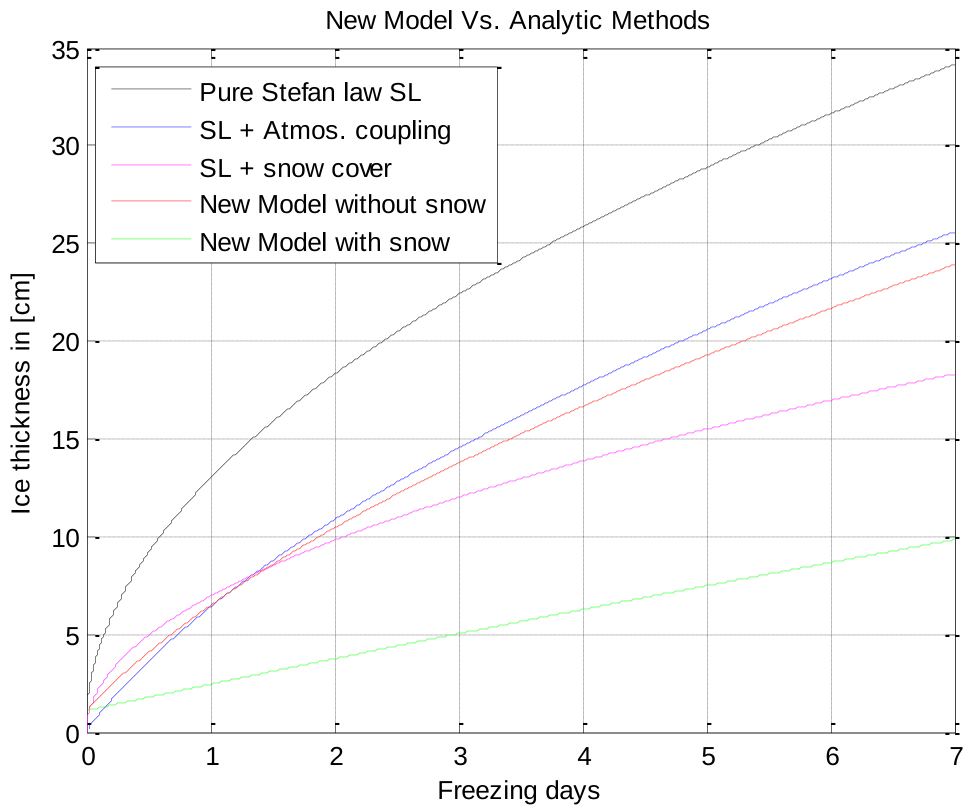
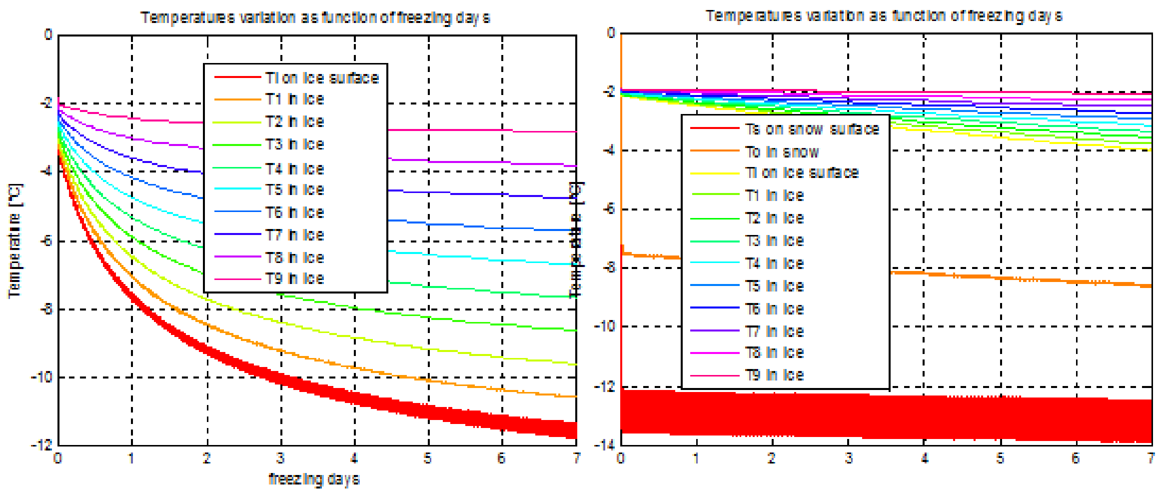


Figure 19 Thickness comparisons with the Stefan's Law

As already stated in previous paragraphs, the Stefan's law overestimates the ice thickness. This phenomenon can be seen on the above figure (Figure **19**). The Anderson's coupling between the atmosphere and the ice gives smaller thicknesses than the pure Stefan's law: slightly less than 10 cm. This is due to the fact that Stefan's law considers that the ice surface temperature is equal to the air temperature. In the next section, it is shown that these two temperatures cannot be equal. The Anderson's mistake relies on the "simplification" of the atmosphere's influence by considering a fixed value of the heat transfer coefficient. In fact, the influence of the atmosphere depends on many meteorological conditions, different heat fluxes.

As for the snow-covered ice, the dramatic difference is due to two reasons. First, the analytical solution estimates the snow thickness from the ice thickness, stating that they are correlated according to the equation (4-2). In reality the snow fall is not related to the ice growth process, otherwise one could say: in case of snow presence on the ice surface, there is no need to measure the ice thickness since it can be deducted from the snow thickness. The second reason concerns the atmosphere. Actually, the analytical solution considers that the snow surface temperature is equal to the air temperature, which is not accurate. That is the reason why the differences between the analytic and the numerical results have the same range as in the case of snow-free ice.

5.5.2 Temperature Issue



(a) Snow-free Ice (b) Snow-Covered Ice

Figure 20 Temperature Evolution throughout the Freezing Days

The above figure shows the temperature evolution over the freezing days for the two under studies cases (snow-free and snow-covered ice). The red ones are the surface temperatures (ice (a) and snow (b)). The curves seem to be thick because in the Matlab code, the air temperature was allowed to vary in a sinusoidal way. It is rather clear that none of them has reached the value of -15°C, which is the air temperature.

6. EXPERIMENTS

The results provided by the Matlab code are based on the theoretical model developed in previous chapters. It means that they are based on some assumptions that have been considered as good approximation of the real physical problem. Therefore they are to be validated by performing some experiments so that their quality and reliability can be assessed.

These experiments consist on modelling the ice growth by trying to model as good as possible the real ice growth conditions and environments (air, water and snow).

First of all, two containers made of polyethylene (see Figure 22), have been used for the experiments. The containers have different sizes. As for the measurements and samples, some instruments have been used to measure the temperature, the salinity, the wind speed and other quantities. This chapter provides all the important details about the tools and procedures which were applied in order to perform the experiments.

6.1 Facilities and Installations

6.1.1 Cooling Room

The experiments were performed in the ice laboratory of the *Arctic Technology Department at HSVA*. The room is well insulated from the outside environment, and its ambient temperature can reach -20ºC. Apart from the insulation system, the cooling system is automatically controlled to keep the temperature stable in case of any disturbance. The room contains three air blowers, situated in height.

6.1.2 Containers

As stated above, two containers (boxes) have been used for the experiments. As for their shape, they are quasi-perfect parallelepiped (Figure **21**). They are made of high density *polyethylene,* which has a thermal conductivity of *0.48 W/(m K)*. In fact, one of the most important assumptions we made in the theoretical part is that *any horizontal heat transfer is neglected*. Thus, during the experiments, we had to manage to avoid as perfectly as possible the heat coming horizontally from the boundaries. As a consequence, the boxes have been insulated on the boundaries (at the bottom and on the sides): Two 40mm-layers of polystyrene[4] have been used for each side and for the bottom as well, to avoid any heat exchange for the side and the ground (see Figure **21**). The total thickness of polystyrene insulation is 80 mm.

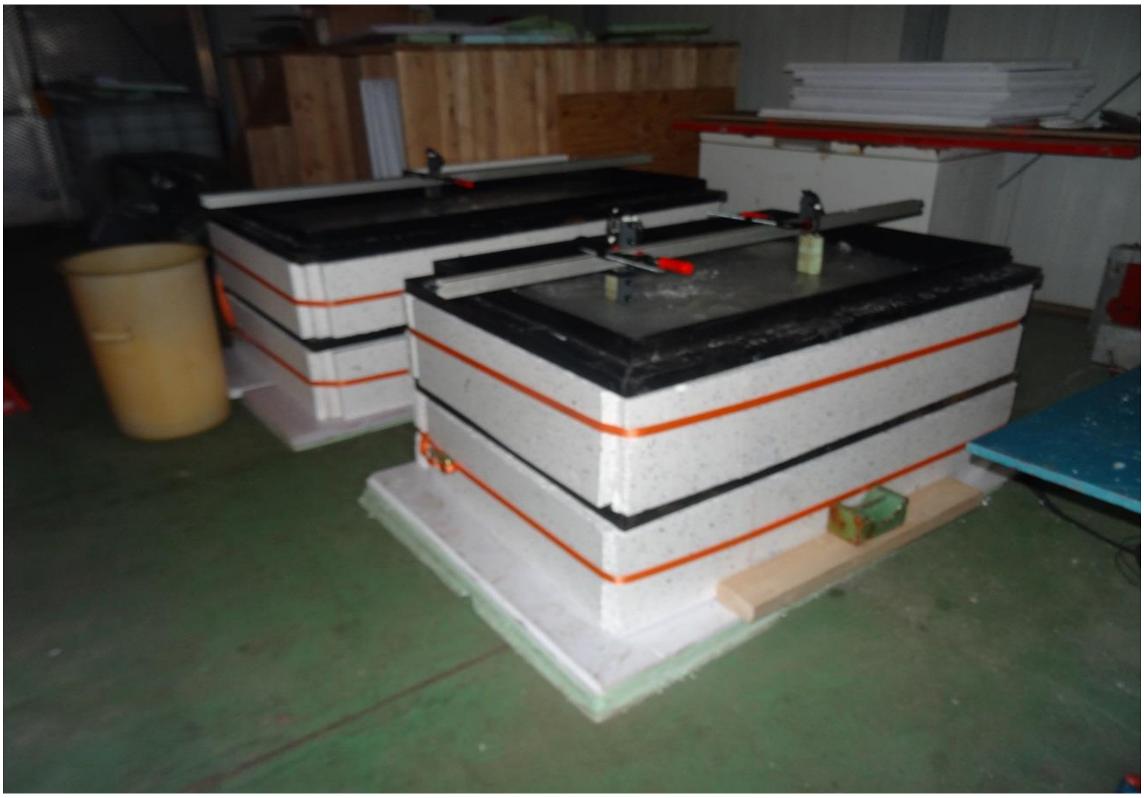


Figure 21 Experiments Set up

Let us, for instance, estimate the amount of heat coming from the side boundaries: we consider the larger edge of the biggest container. Since the area of contact is the same for both materials (polyethylene and polystyrene), the total heat transfer coefficient per unit area is given by:

$$\frac{1}{H_{eq}} = \frac{1}{H_a} + \frac{e_{PE}}{k_{EP}} + \frac{e_{PS}}{k_{PS}} \tag{6-1}$$

Where: H_{eq} represents the total heat transfer coefficient, H_a the air convection coefficient, e_i and k_i the thickness and the heat conductivity coefficient respectively. PS and PE symbolize respectively polystyrene and polyethylene layers. The thicknesses and heat

[4] The thermal conductivity of polystyrene is 0.035 W/(m K).

conductivity coefficients are all known and do not vary; however, for information purposes only, we consider $H_a = 19.45 \frac{W}{m^2K}$ for a speed of 1 m/s.

Finally, the equivalent overall heat transfer coefficient $H_{eq} = 0.425 \frac{W}{m^2K}$. This value corresponds to only 2.18% of H_a. Surely, it might have a certain influence on the results, but as for our assumption, we can neglect the flux coming from the side as they constitute only a small part of the total flux coming into the box.

The following sketch shows the dimensions of the boxes.

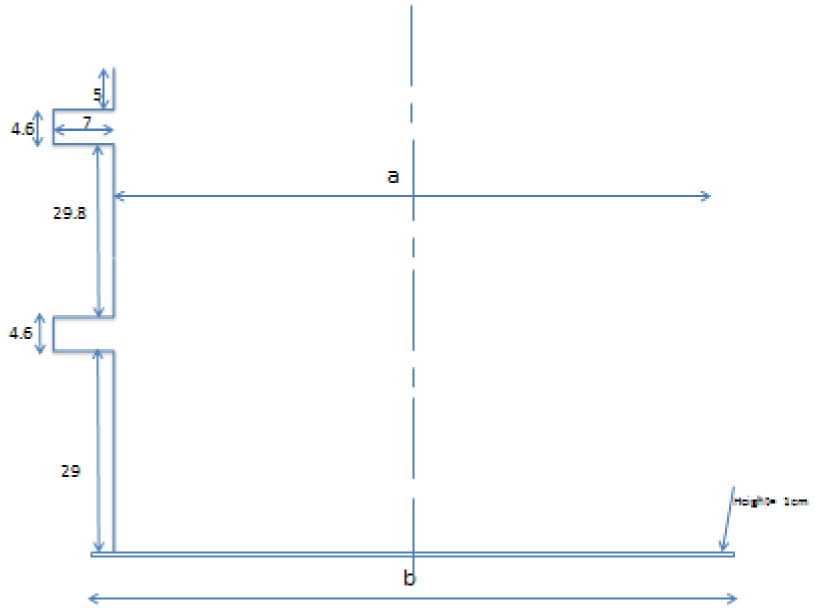


Figure 22 Schematic Illustration of the Containers Dimensions

Table 5 Containers Main Dimensions

Dimensions	Bigger Container **[cm]**		Smaller Container **[cm]**	
	Bigger edge	Smaller edge	Bigger edge	Smaller edge
a	**195.5**	**103.5**	**145.5**	**95.5**
b	**199.5**	**107.5**	**149.5**	**99.5**

The thickness of all the plates is constant and equal to *8 mm*. Last, the moulded lengths, widths and depths of the boxes are respectively *193.5cm x 102cm x 74.5cm* for the bigger box and *144cm x 94cm x 74.5cm* for the smaller one.

6.1.3 Temperature sensors

The instruments that can be seen on the top of the boxes are the temperatures sensors. The theoretical model has computed the temperatures at some grid points. From the description of the theoretical model, one could understand that our grid boxes are changing dimensions, and as consequence the grid points are also shifting from one place to a lower one (since the ice thickens). Even if the used temperature sensors have fixed points, it is rather interesting to see the general trend of the temperature profile over the depth of the ice. The most important grid point to determine the temperature is of the upper surface of the ice (or snow). That is why it has been managed to keep on sensor on the upper surface (approximately), and at least one above the surface in the air (see Figure 25) to see and demonstrate practically, what is the mistake in the simple model or generally in the Stefan law.

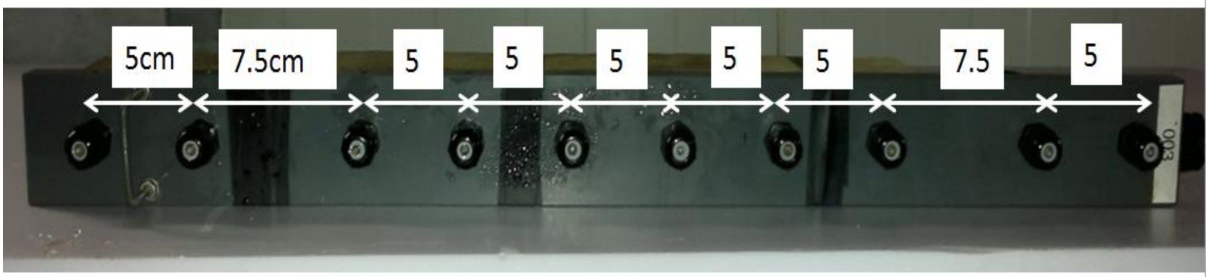


Figure 23 Sensors Disposition - PT100

Figure 24 Temperature Sensors Installation

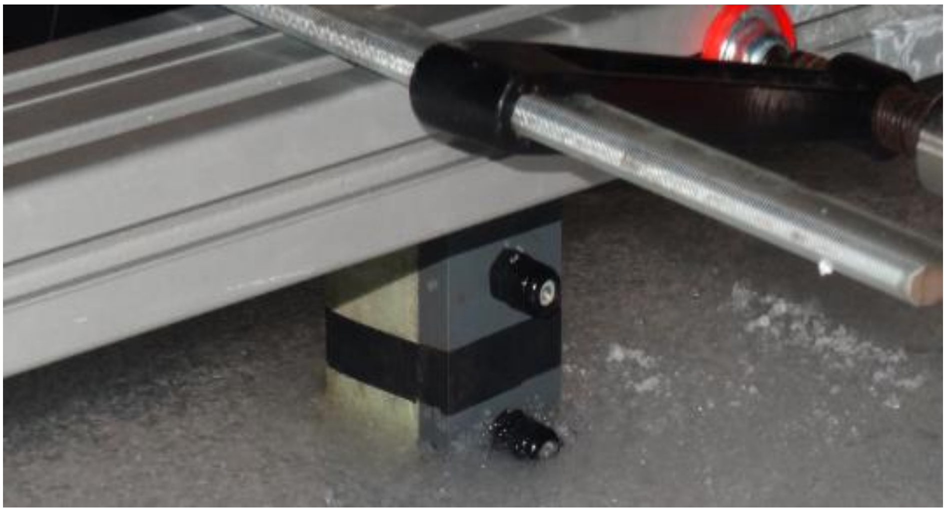


Figure 25 Sensors Arrangement

These temperature sensors are connected to a computer which records the whole time history of measurements captured by the sensors.

6.1.4 Different Used Instruments

The ambient temperature of the room, the mean salinity of the water as well as the mean ice salinity, the wind speed, and the humidity have to be measured and used in the theoretical model so that the different quantities we need could be compared.

c. Thermometer

Actually, the temperature of the laboratory is regulated by the cooling control system. However, since a thermometer was available to measure the ambient temperature and the humidity of the laboratory, it has been used for sake of comparison and reliability of the results.

Figure 26 Thermometer

d. Salinometer

In Figure 27, we provide the device we used during the experiment to measure the degree of salinity of the water and the ice. The value salinity of the salinity displayed is in ‰.

Figure 27 Salinometer

e. Anemometer

A very sensitive anemometer was needed, since we worked with relatively low wind speeds. The small device that is shown in Figure 28 satisfied our needs because the lowest wind speed it can measure with good accuracy is 0.1 m/s.

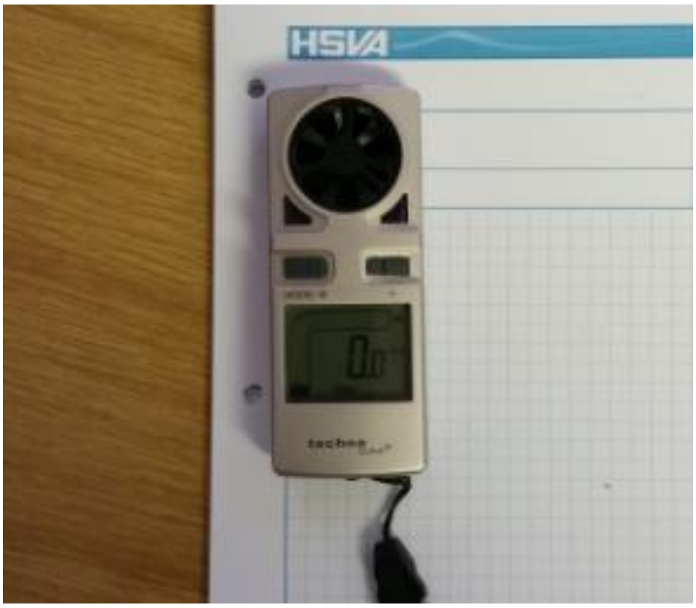


Figure 28 Anemometer

6.2 Procedures

After being insulated, the containers are filled with salty water. The water comes from the ice tank of the *HSVA* and the mean salinity measured with the Salinometer is approximately 7.5‰.

From the Figure 24, one can see how the temperature sensors were arranged. The arrangement shown in this figure concerns the first experiments, which were performed with snow-free ice. The laboratory was cooled in order to reach an air temperature value of *-15°C* most of the time. The temperature was sometimes increased to *-2°C*, but all the results are given in the next chapter. More precisely, the measured salinity of the water was about *7.5‰* in the larger container and *7.3‰* in the smaller one.

The second experiments were about a snow-covered ice. Nevertheless, it was not possible to model the snow fall in the laboratory. Due to the fact that most of the time, in nature, ice growth cannot be detached from the snow fall, we had to use an approach to consider the presence of the snow upon the ice surface. In the previous sections, we referred the influence snow can have on the ice growth: Hence, it is important to see physically how this phenomenon happens. Therefore the notion of *Thermal Resistance* R_t has been used. The thermal resistance of a material is the ratio between its thermal conductivity and its thickness.

$$R_t = \frac{e}{k} \qquad (6\text{-}2)$$

Where k is the thermal conductivity and e the thickness.

The insulating capacity of a material is correctly assessed by its thermal resistance R_t.Two different objects have the same insulating capacity if they have the same thermal resistance. In this study, the task was to use a material which could play the same role as the snow with the same thermal resistance. Therefore, layers of *Extruded Polystyrene (XPS)* have been used to play the insulating role of the snow.

Figure 29 Snow-Covered Ice Experiments Setup

In fact, because of the difficulty to model the snow fall, the snow has been taken to be constant. Thus, the problem is the following: for a given thickness of snow, how thick the layer of the extruded polystyrene must be so that it insulates the ice in the same way as the snow. For that, the condition is the equation (6-3).

$$\frac{e_s}{k_s} = \frac{e_p}{k_p} \tag{6-3}$$

With the subscripts *"s"* for the snow properties and *"p"* for the extruded polystyrene. The thermal conductivity of the extruded polystyrene is about 0.035 W/(m K).

The thermal conductivity of the snow k_s is approximately taken equal to 0.31 W/(m K). Thus for a snow thickness of e_s, the layer of the extruded polystyrene must have a thickness of :

$$e_p = e_s\left(\frac{k_p}{k_s}\right)$$

$$e_p = 0.11\ e_s \tag{6-4}$$

Thus, experiments have been performed for two different values of the thickness of the XPS layer: ***2 mm*** and ***4 mm***, which correspond to thicknesses of ***18.1*** and ***36.3 mm*** for real snow layers.

6.3 Ice Sampling and Measurement

The method used to sample and measure the thickness appears extremely important, contrary to what one might think. That is why a whole chapter is dedicated to that. Actually, the method used to measure the thickness was a destructive one. We had to drill and retract every time a piece of ice to measure manually the thickness using a *caliper* or a *measuring tape* for higher thicknesses.

Figure 30 Sampling & Measurement Processes

This method is not the ideal one in this case and it can be fatal for the results: after a couple of measurements, there is a great chance that the following ones are influenced. The hole, even filled will delay the ice growth in the vicinity. To avoid this phenomenon as much as we could the sampling places were distanced from each other. However, since the containers are not that large, in the end some holes were close to each other. As a consequence, the thicknesses sampled about the end of the process might have been influenced by the previous measurements.

6.4 Theoretical vs. Experimental Results

Before presenting the results, some important parameters (inputs and some measured quantities) have been given on the Table 6, Table **8** and Table 9.

6.4.1 First Experiments: Snow-free Ice

Table 6 First Experiments Summary

	Large Container	Small Container
Snow Thickness [mm]	0	0
Mean Water Salinity [‰]	7.5	7.4
Mean salinity of the Melted Ice [‰]	2.62	2.65
Number of Freezing days	12	12
Wind speed [m/s]	0.9	0.8
Cooling Temperature [°C]	-15 & -2	-15 & -2

As already stated before, the first experiment was about snow-free ice. Both containers full of salty water (7.5‰ for the larger and 7.3‰ for the smaller, see Figure 21), are cooled for about 12 days at the following temperatures:

- -15°C from day 1 to day 6
- -2 °C from day 6 to day 7
- -15°C from day 7 to day 9
- -2 °C from day 9 to day 12

a. Ice Thickness

The discrete points are the measurements from the experiments. We can see how well they fit with the theoretical curve.

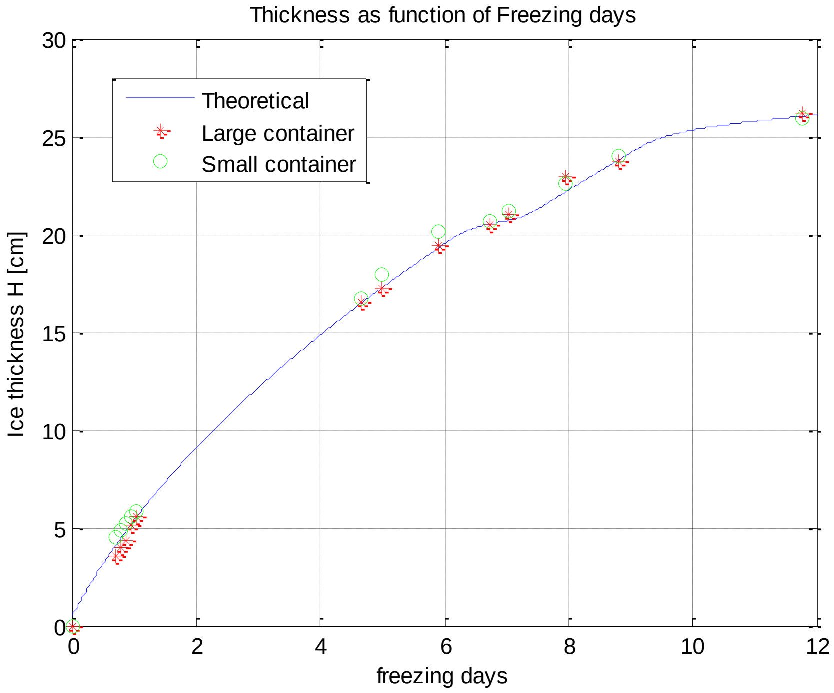


Figure 31 Snow-Free Ice Experiments – Thicknesses

One of the most important feature concerns the earlier stage of the growth. In fact, the methods mentioned in the previous sections overestimate the thickness at that moment of the process. This good fitting is mainly due to the approach we used to estimate the salinity profile: After melting a sample of ice, we measured the salinity of the resulted water. This value is taken as the mean salinity, and all the other values are computed based on this one using the salinity profile on the Figure 16, which has been used as a parametric curve.

b. Temperature Profile

As illustrated in Figure 32, one can clearly notice the assumption concerning the linearity of the temperature profile within grid boxes can be, without extremely significant error, extended to the linearity of the temperature profile within the ice, and consequently the calculation process could concern only one layer of ice, instead of nine as it is the case in this study. Nevertheless, this could be valid only if the following conditions are satisfied:

- Small ice thickness, i.e. preferably less than 30 cm.
- Constant air temperature: In Figure 32, the 10th day (when the temperature decreased from -15 to -2) shows a curvilinear temperature profile.

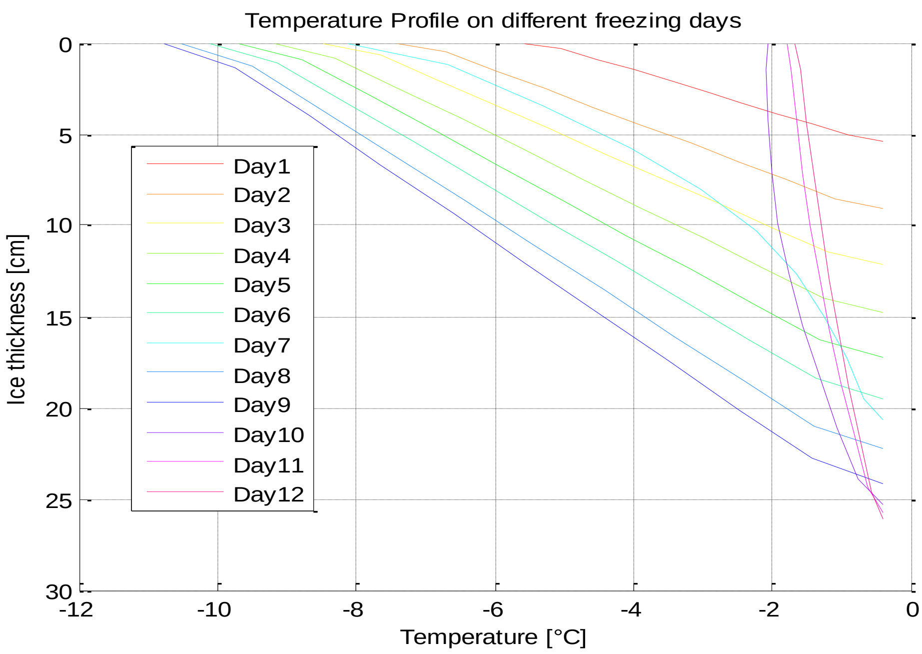


Figure 32 Snow-Free Ice Experiments - Temperature Profile on different freezing days

c. *Temperatures Evolution*

Most of the analytical approaches assume that the ice (snow if any) surface temperature is equal to the air temperature. The Figure 33 reflects what we assumed in our approach. The thicker curve, the red one is the ice surface temperature and is on any day equal to the air temperature which is in this case -15°C on some days and -2°C on others. This curve goes asymptotically towards the air temperature after a lot of days, but will not reach it.

However, the questions are "what happens in reality?" and "Does our assumption reflect the reality?" To answer these questions, we provide the Figure **34**, which contains the temperature records from the sensors.

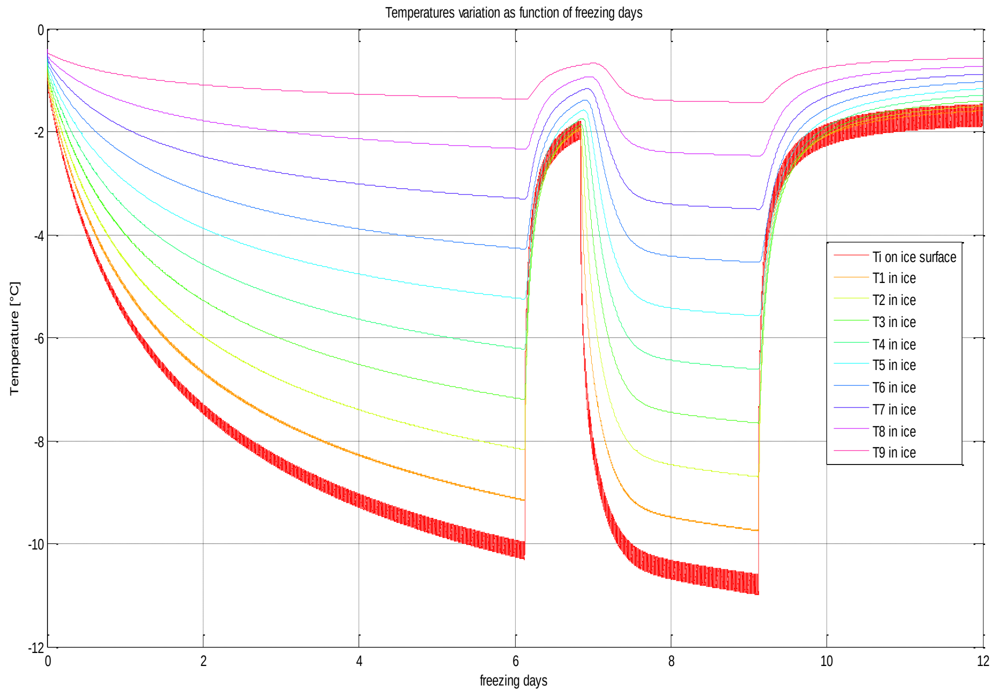


Figure 33 Snow-Free Ice Experiments - Temperatures changes

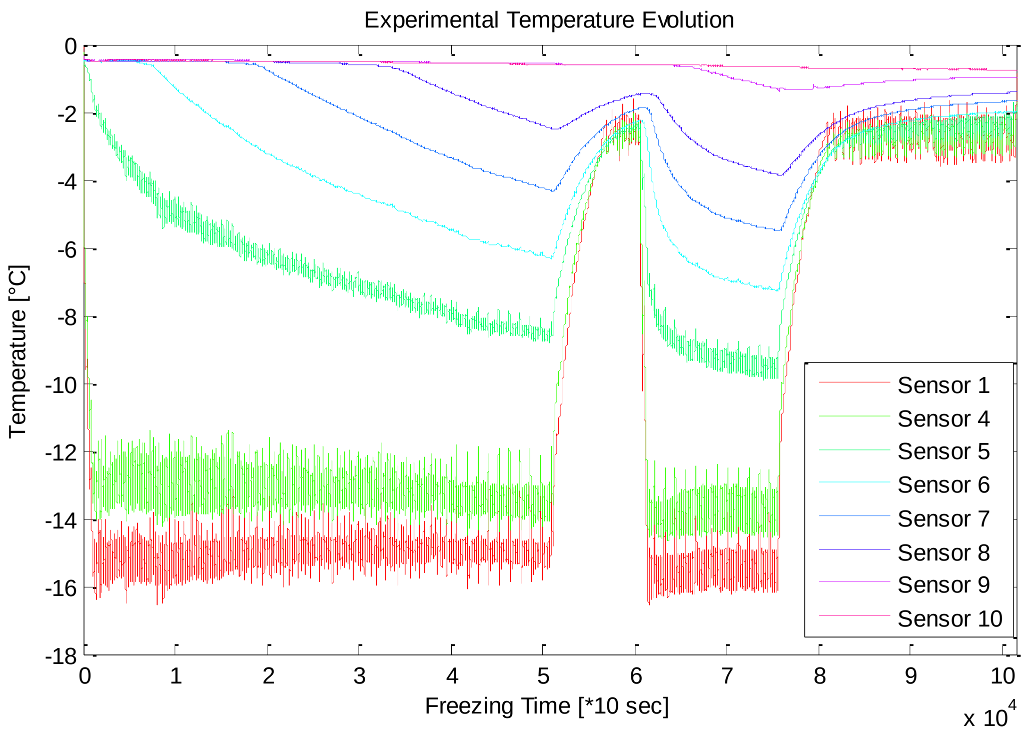


Figure 34 Snow-Free Ice Experiments - Temperature Sensors Outputs

When comparing the Figure 33 and the Figure 34, one must be very careful and consider the following difference, which important: The grid points move as the ice thickens i.e. their distances from the upper surface of the ice increase, although the temperature sensors remain at the same depth throughout the whole growth process. Based on Figure 23, Figure 24 and Figure 25, we create the following table with the sensors and their depths:

Table 7 Sensors Depths - Snow-free Ice

Sensor	1	2	3	4	5	6	7	8	9	10
Depth [cm]	-22.5	-17.5	-10	-5	0	5	10	15	22.5	27.5

The negative values of depth mean that the sensors are above the ice surface. The curve of interest for us is the ***sensor number 5***, which is exactly on the ice surface. We can see how similar this curve is with the red curve on the Figure 33. This fact simply reflects how well the atmosphere in the laboratory has been simulated. As for the other sensors, our observations will be limited in the shapes of the curves which are quite satisfactorily similar with the theoretical ones. The sensor 10 is out of the ice, and its temperature remained approximately constant throughout the whole ice growth.

d. Cooling Temperature

The temperature in the laboratory is controlled automatically. Because of some disturbances, the temperature could not always be kept constant. The variation range as indicated through the red curve on the Figure 34 is $\pm 1^{o}$C. Theoretical this curve was approximated as a sinusoidal function as provided on Figure 35 and Figure 36.

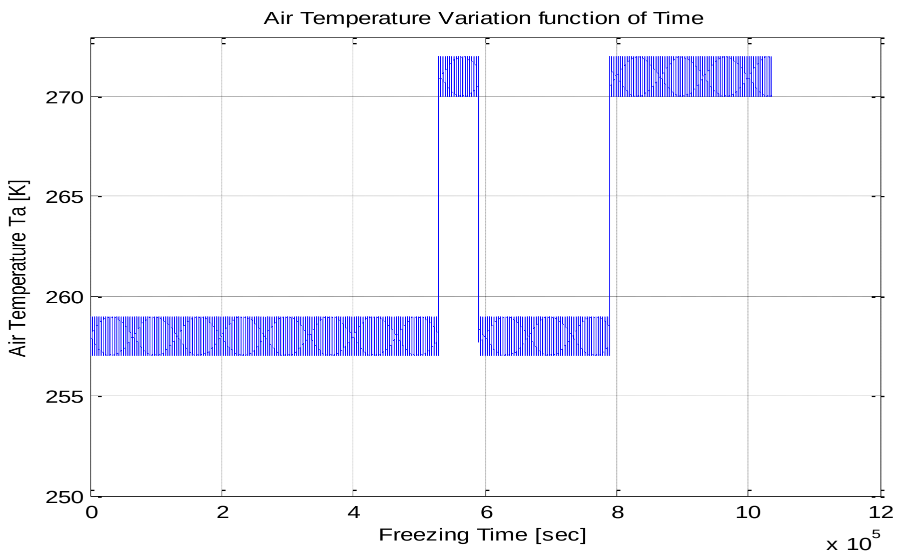


Figure 35 Snow-Free Ice Experiments - Air Temperature Variations over the freezing time

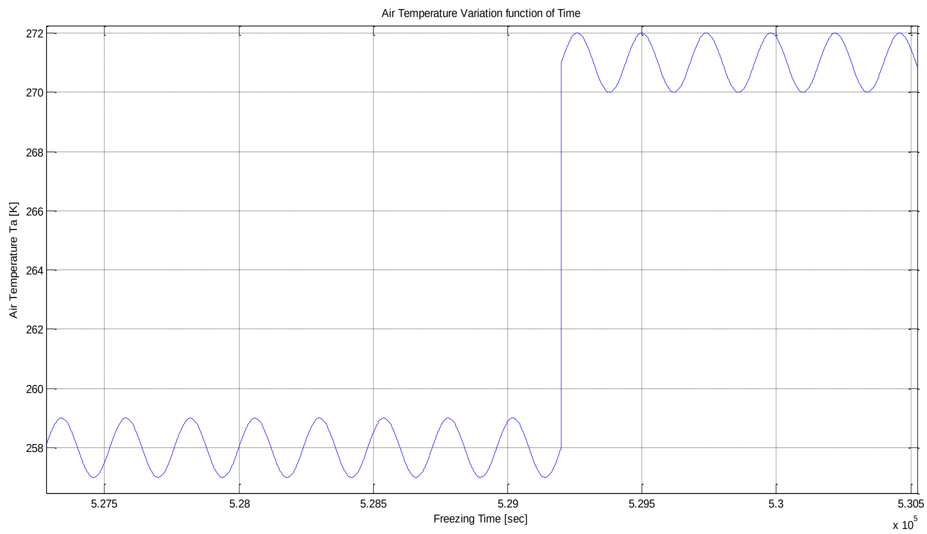


Figure 36 Snow-Free Ice Experiments - Air temperature Variation (zoom)

e. *Growth Rate*

Let us now verify some theoretical features that have been mentioned in previous sections. First of all, as the ice thickens it starts playing an insulating role. From Figure 37, we can observe that as the growth process passes of, the ice becomes an insulator and delays further growth, i.e. the growth rate decreases.

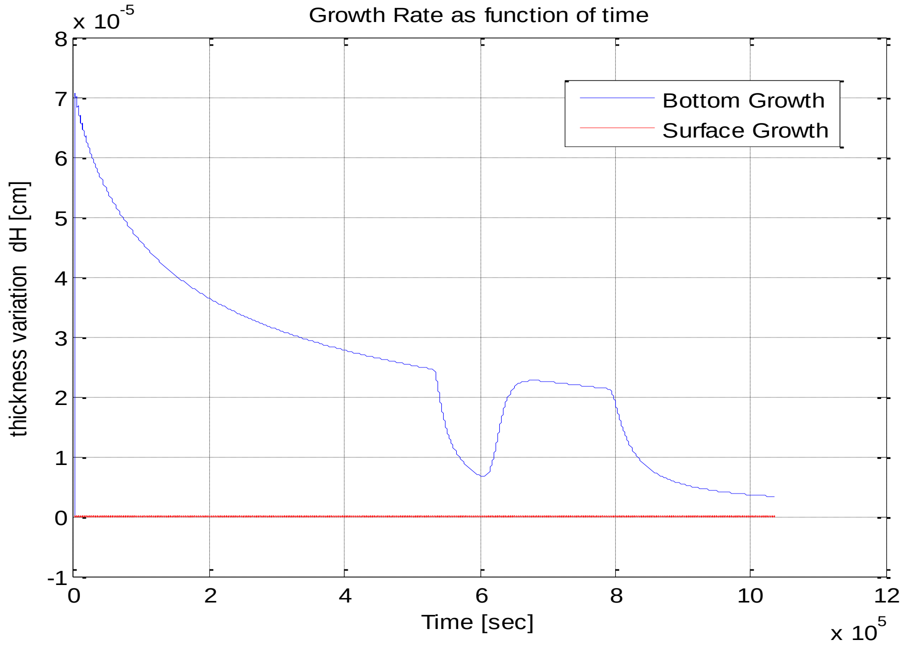


Figure 37 Snow-Free Ice Experiments - Growth Rate

Moreover, we can see how insignificant is the surface growth compared to the bottom growth. In this study, the hydrostatic equilibrium phenomena have not been considered. From the experimental point of view, this was expectable since the conditions in the laboratory could not allow any important surface growth. In fact, the extremities of the ice were limited by the boundaries of the containers, thus no vertical movement, no water on the ice surface.

f. Stability

The stability of the scheme for this calculations procedure can be assessed using the Figure **38**. Actually, the parameter λ is, at all the steps of the computation, much less than 0.5, apart from the first steps, where it is a little bit high but still less than the target value, because of the small initial thickness.

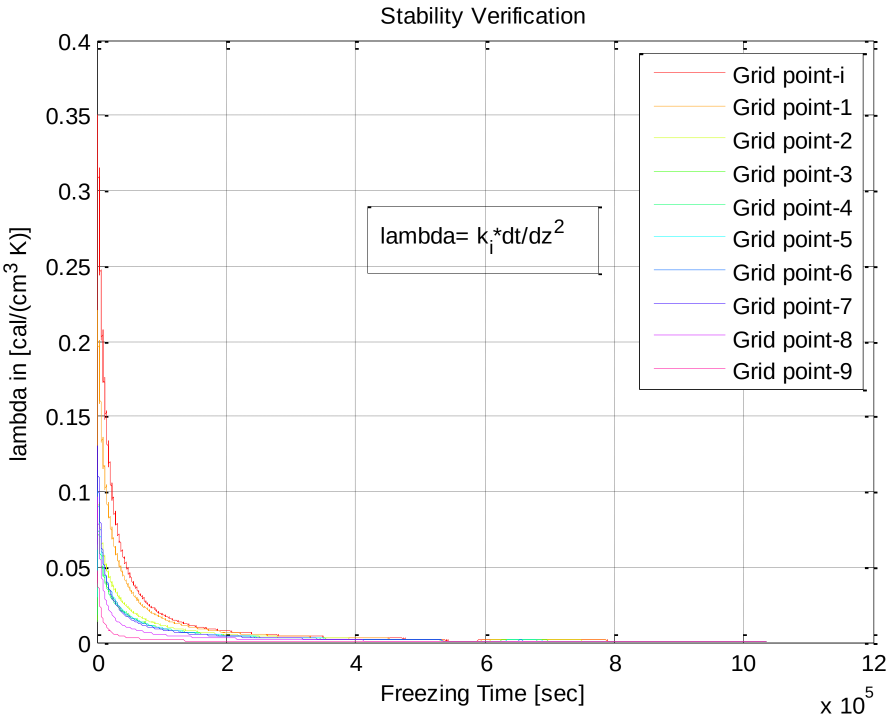


Figure 38 Snow-Free Ice Experiments - Scheme Stability

6.4.2 Second Experiments: Snow-covered Ice

Table 8 Second Experiments Summary

	Large Container	Small Container
Snow Thickness [mm]	20	40
Mean Water Salinity [‰]	9.54	9.50
Mean salinity of the Melted Ice [‰]	2.8	2.8
Number of Freezing days	12	12
Wind speed [m/s]	0.7	0.8
Cooling Temperature [ºC]	-20	-20

The second experiment was about snow-free ice. Both containers full of salty water (9.54‰ for the larger and 9.5‰ for the smaller) are cooled for about 12 days at the following temperatures:

First thing worth noticing is the different salinity, which means different freezing point, and consequently different behaviour. In fact, after performing the snow-free ice experiments, we did not melt the ice, but we destroyed and removed it. As a consequence, the remaining water in the containers was more saline because of the brine during the ice growth.

a. *Ice Thickness*

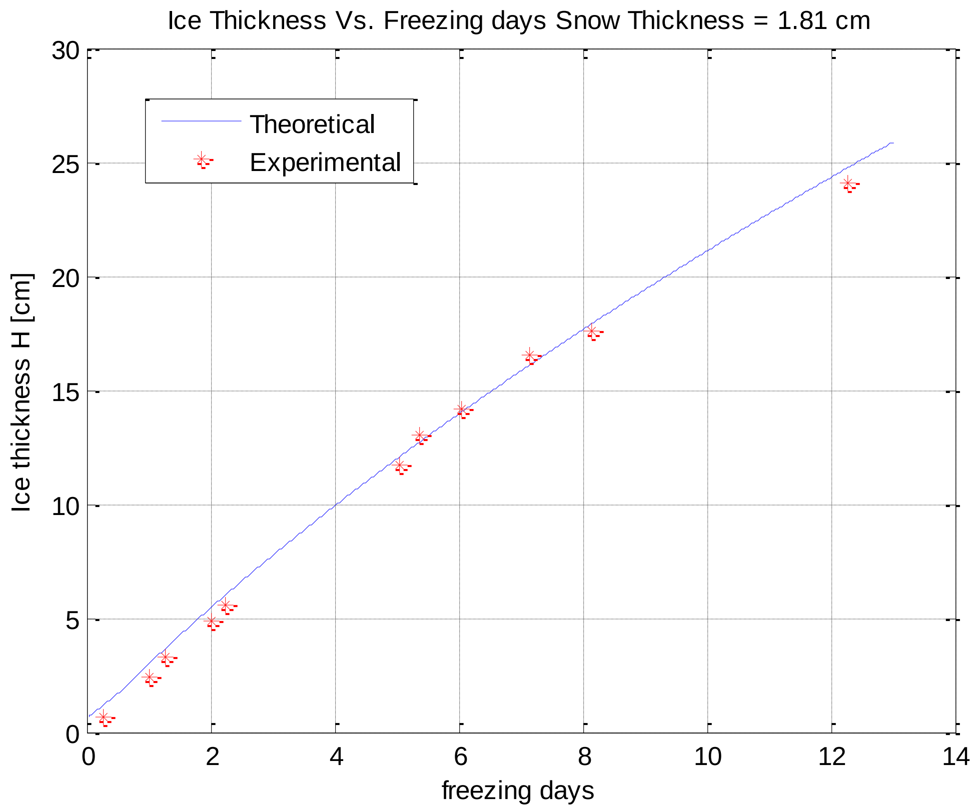


Figure 39 Ice Thickness _1.81 cm-Snow -covered Ice

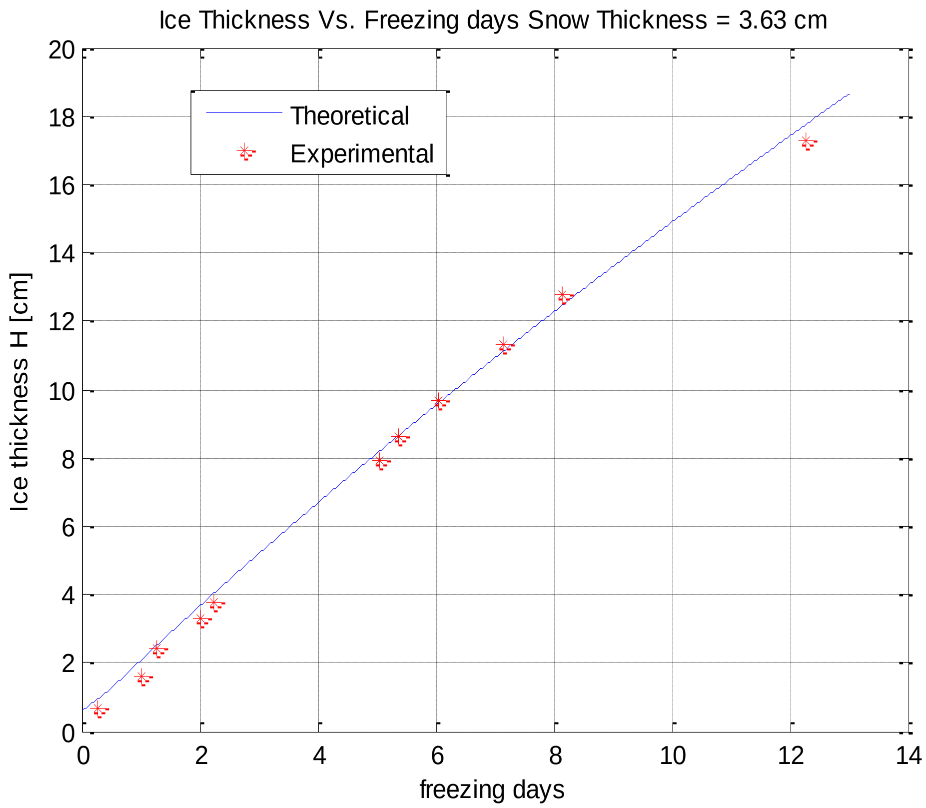


Figure 40 Ice Thickness _3.63 cm-Snow-covered Ice

As already stated above, to simulate the snow layer upon the ice surface, layers of extruded polystyrene have been used. Apart from the reasons we discussed in the case snow-free ice, here we also have a small problem caused by the extruded polystyrene (XPS). In fact, even if we agreed that the behavior is the same, we could notice the following:

- The layer of snow in nature is not uniform, but we had "more or less" the same XPS thickness upon the whole ice surface. Thus, this problem could not be solved.
- The XPS layer could not lay uniformly on the ice surface. As a result, we have some trapped air pockets in the interspace between XPS layers and ice surface, which resulted in another ice formation optically and "crystallographically" different from the normal ice.

b. Temperature Profile

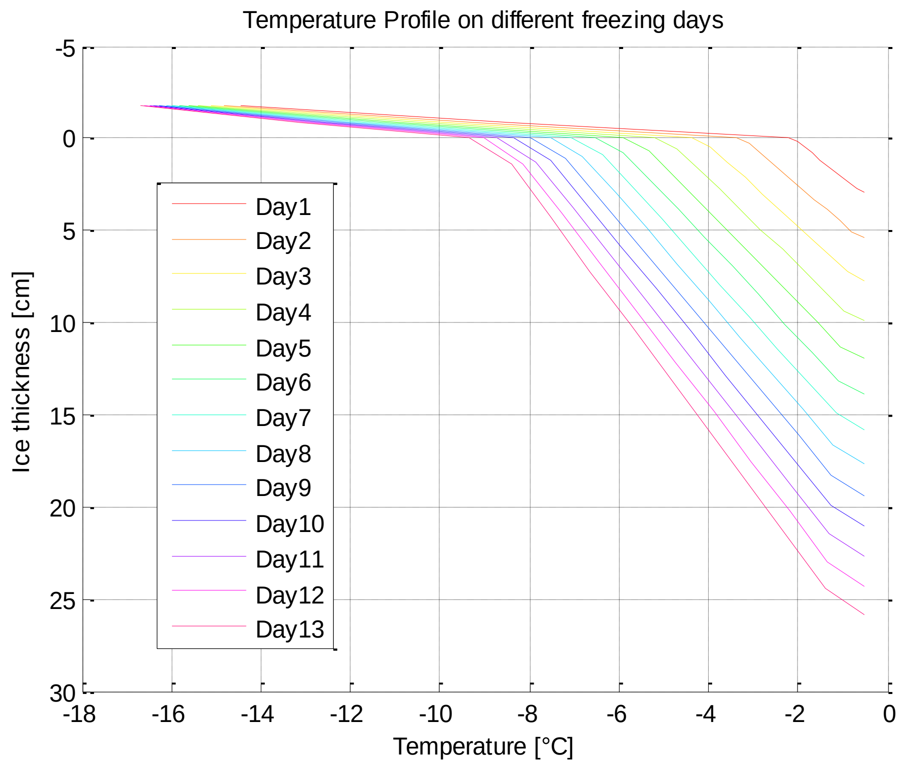


Figure 41 Temperature Profile _1.81cm-snow-covered ice

The most important feature to highlight from the Figure 41 and Figure 42 is the fact that within the snow, the temperature profile does not vary a lot over the days. Additionally, the difference between the temperatures on the upper and the lower surface of the snow is tremendous compared to what happens in the ice layer. This explains simply why snow on the ice surface plays an insulating role: In fact, since snow has low heat conductivity, it transfers a really small amount of heat from the atmosphere to the ice (or water in the beginning of the process).

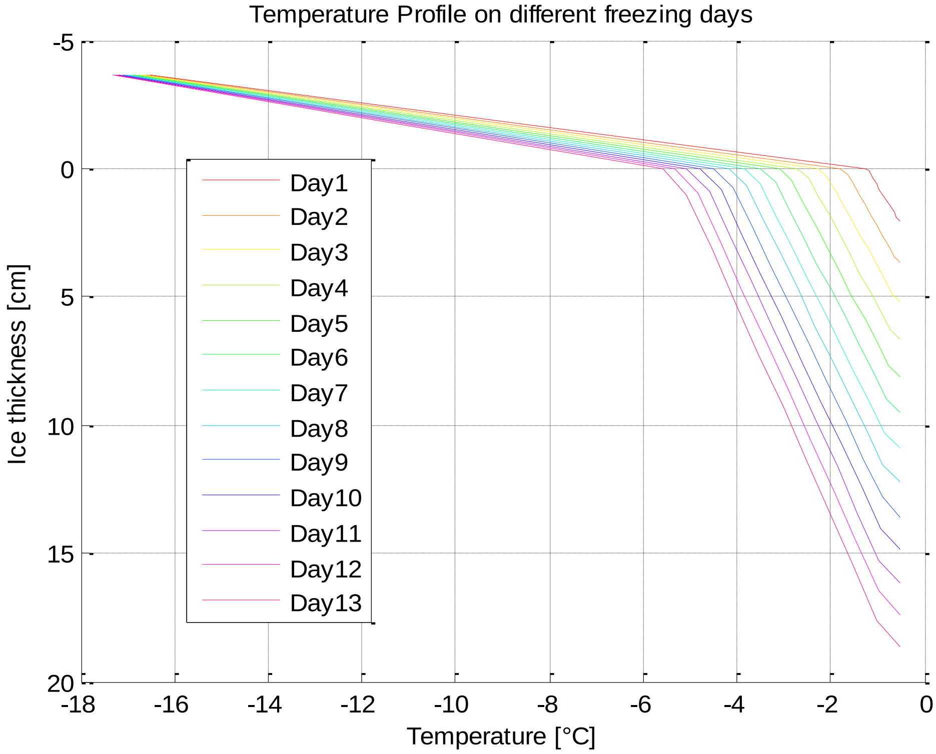


Figure 42 Temperature Profile _3.63 cm-snow-covered ice

c. *Temperature Evolution*

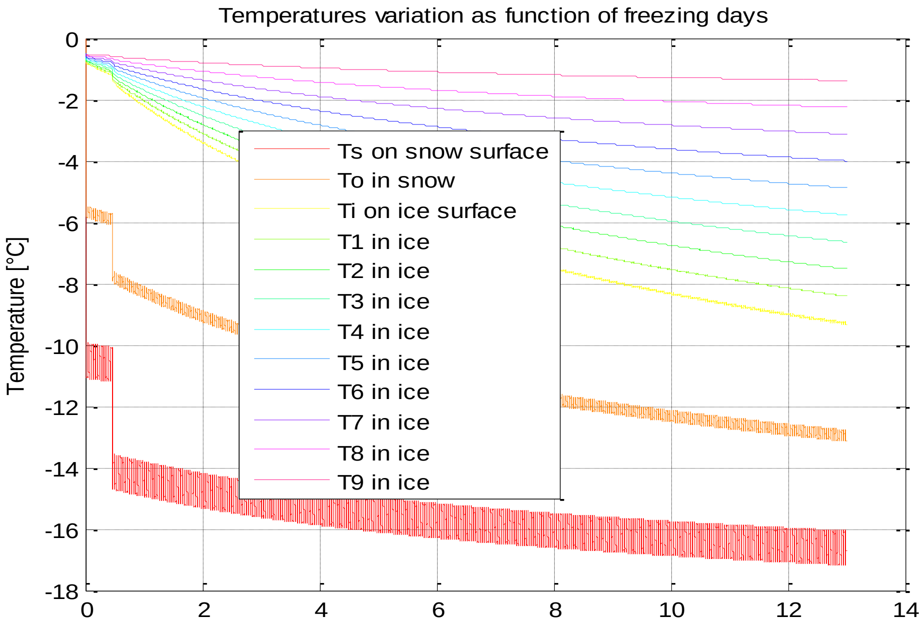


Figure 43 Temperature variation over Freezing Days _1.81 cm-snow-covered Ice

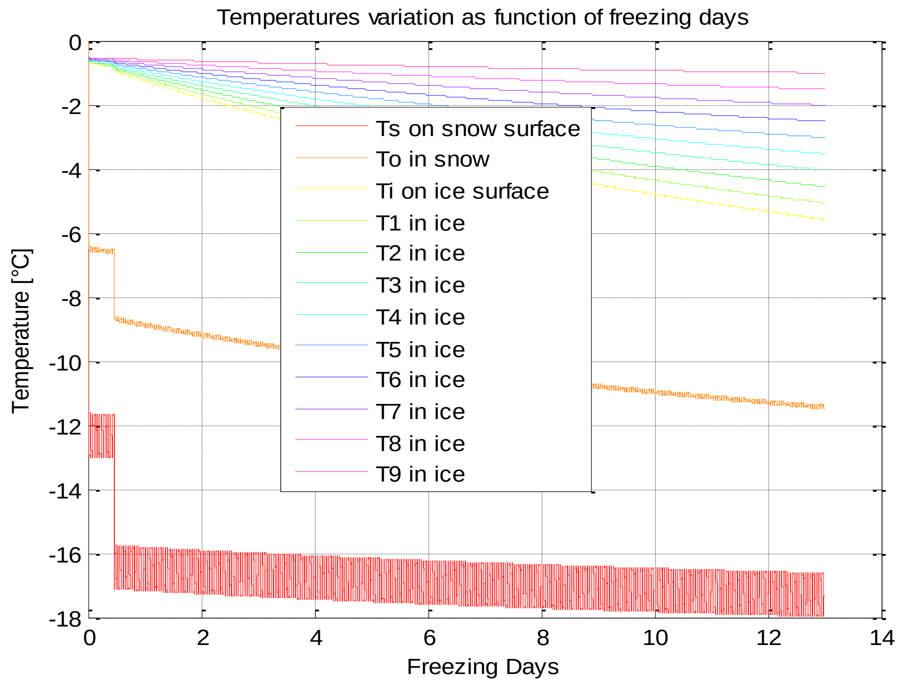


Figure 44 Temperature variation over Freezing Days _3.63 cm-snow-covered Ice

The remark that has been made when commenting on the temperature profiles in the previous paragraph is clearly and in a simpler way exposed on Figure 43 and Figure 44. Actually, in both latter figures, the red curve represents the temperature on the snow upper surface. The drop in the temperature when crossing the snow layer is very significant, that being much less than half the snow surface temperature. As previously, let us consider the outputs from the temperature sensors to see if the computations based on our assumptions are justified.

The Figure 45 contains the recorded temperatures for the 3.63cm-snow-covered ice. The curve we are mostly interested is the thick and green one, which is supposed to be on the surface of the extruded polystyrene. In fact, we were restricted by the high sensitivity of the sensors, i.e. any contact with a solid could damage the sensors. Then, as it can be seen from Figure 29, we made some holes to avoid contact between the sensors and the XPS layer. More precisely, the sensor is surrounded by the XPS layer, but beneath comes the ice layer. This could explain the differences in the results between the theoretical and the experimental temperatures.

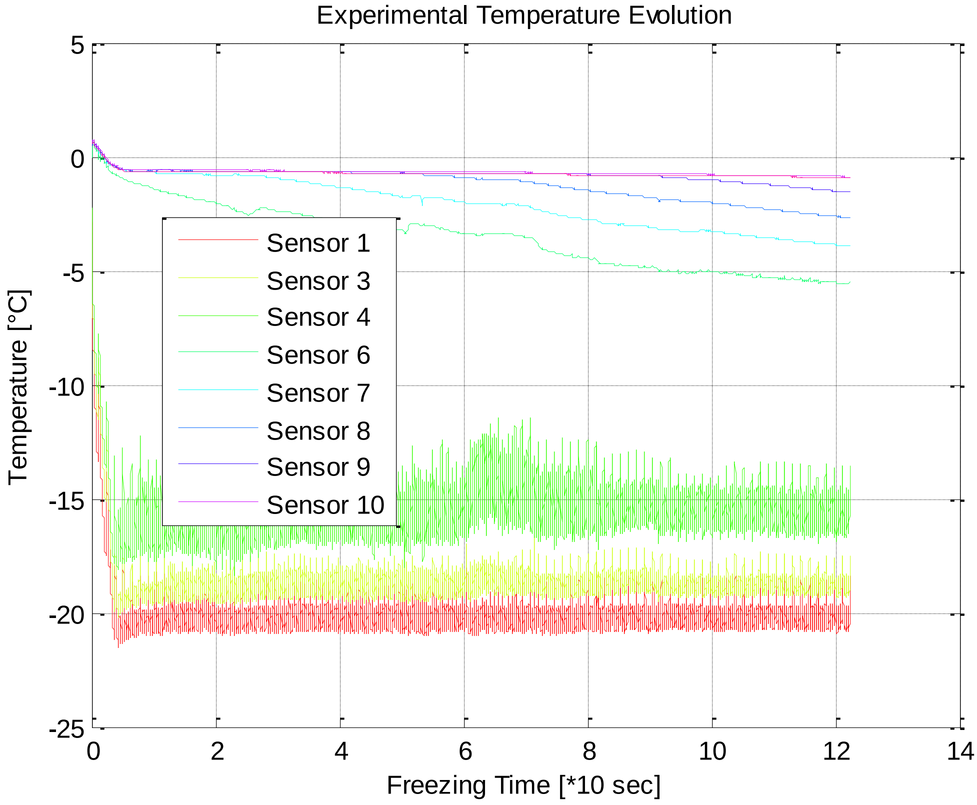


Figure 45_Experimental Temperature Variation_3.63 cm of snow

d. Growth Rate

Interpreting the results shown in Figure 46 and Figure 47, and comparing them to Figure 37 we can see that the order of magnitude of the growth rate per second is lower than in the case of snow-free ice, even if the cooling temperature is lower (-5ºC difference) in the case of snow-covered ice.

Additionally, we can see that we do not have surface growth as we had in the previous case. In nature, when the ice is covered by a snow layer, the surface growth does not occur, but on the contrary we might have a continuous growth of the snow layer, if of course the meteorological conditions are favourable. Finally, the ice with a thinner snow layer (1.81 cm) has higher growth rate than the one with the thicker snow layer (3.63cm). (See Figure 46 and Figure 47).

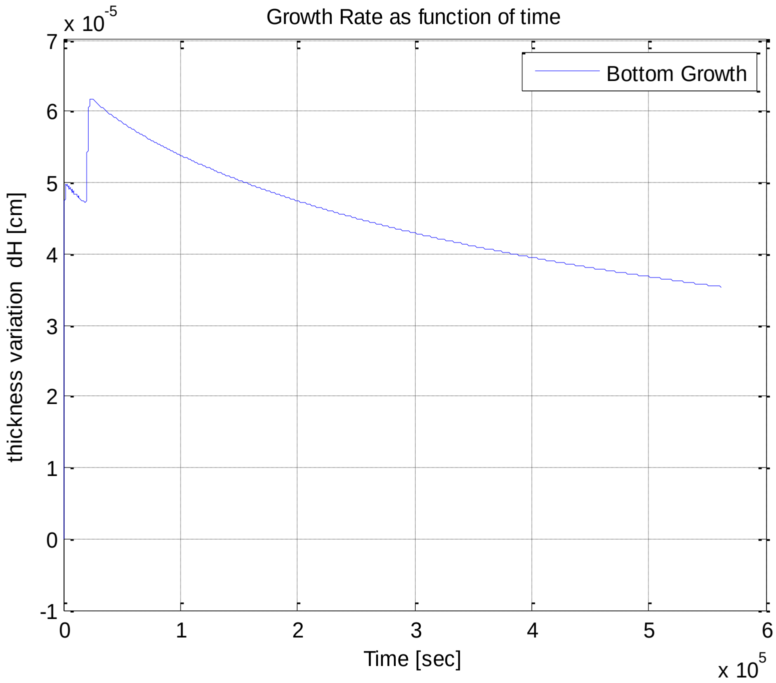


Figure 46 Growth Rate_1.81 cm -snow-covered ice

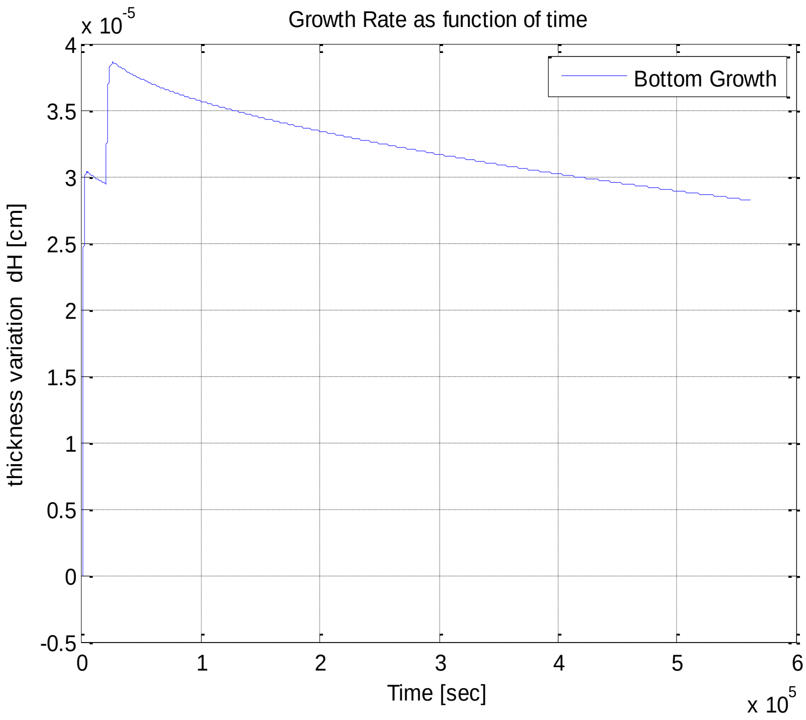


Figure 47 Growth Rate_3.63 cm-snow-covered ice

e. Stability

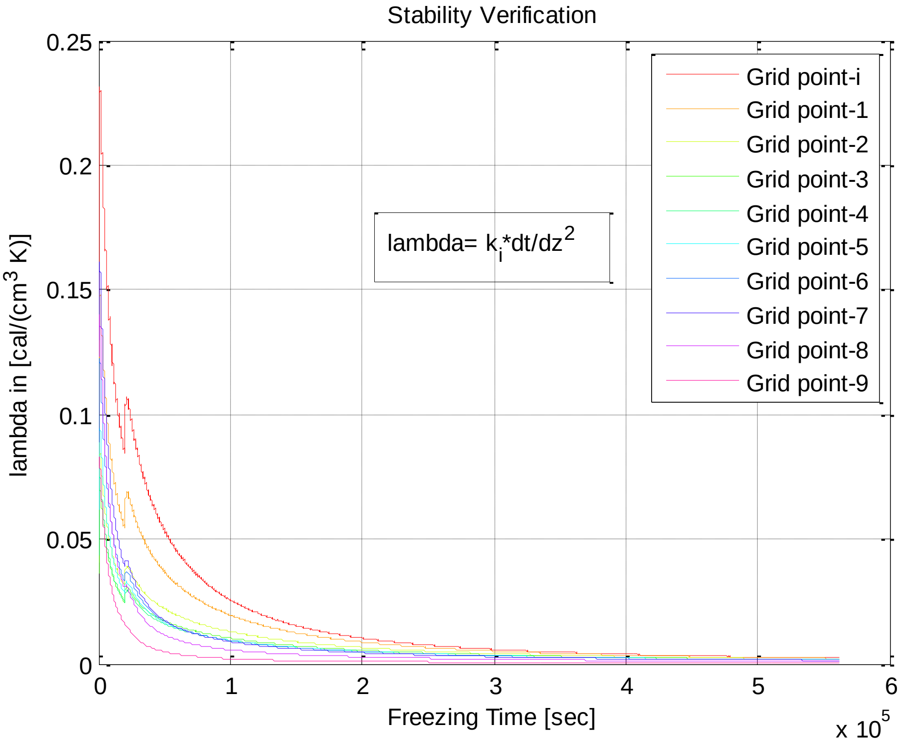


Figure 48 Stability Verification_1.81cm-snow-covered ice

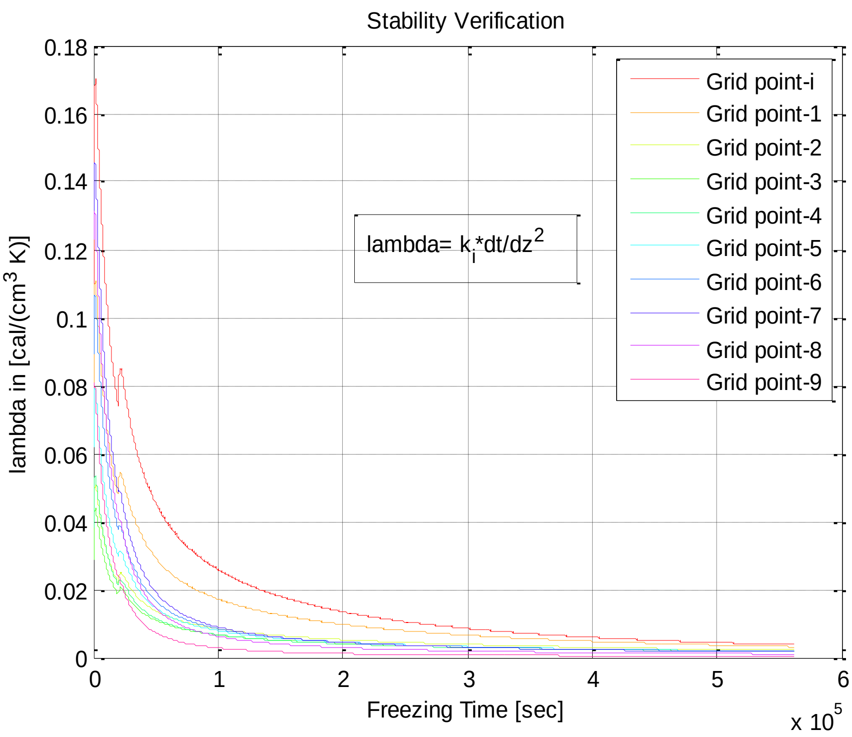


Figure 49 Stability Verification_3.63cm-snow-covered ice

The scheme is stable in both considered cases.

6.4.3 *Experiments in the Ice Tank*

After the above mentioned experiments in the ice laboratory, the method has been verified for smaller ice thicknesses. Most of the methods in the literature are built to deal with high values of ice thickness, generally, more than 30 cm; and they provide inaccurate results when predicting thinner ices. The challenge was to see if the method could be applicable in the large ice tank of the HSVA, where the targeted ice thickness for a couple of projects is about usually in the range of 15 to 60 mm.

Therefore, the ice tank of the *HSVA* was then cooled at around *-22ºC* for about 25 hours.

Table 9 Ice Tank Experiments Summary

Mean Wind Speed [m/s]	0.3
Cooling temperature [ºC]	-22
Mean Salinity of the Water [‰]	7.45
Mean Salinity of the Melted Ice [‰]	3.9

a. Ice Thickness

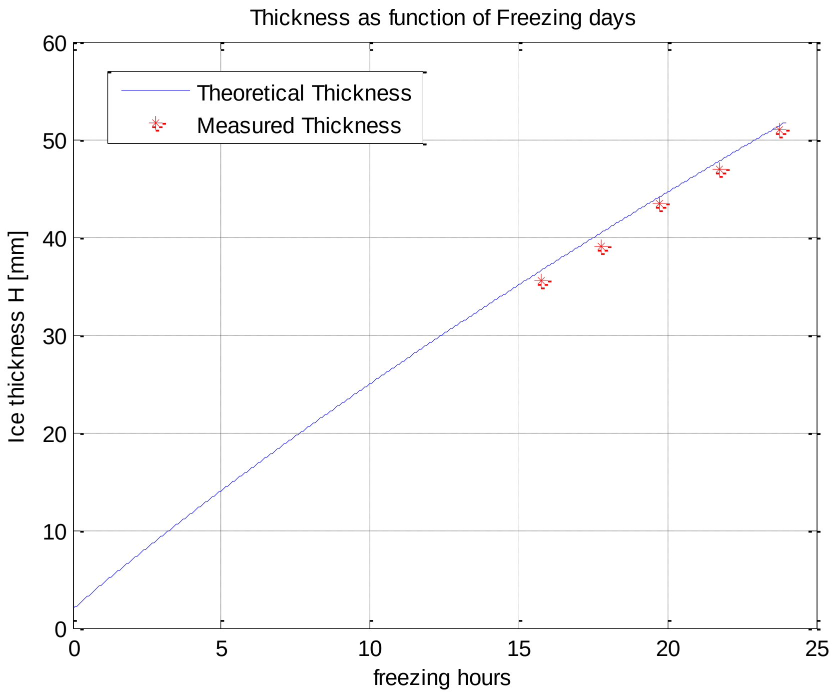


Figure 50 Tank Experiments - Thickness

The discrete values on the graph are measured in the middle of the ice tank. The differences between the theoretical and the experimental thicknesses are really insignificant and it is rather good that for this range, the theoretical curve appears as the upper bound with the maximum difference being less than 2 mm.

b. *Temperature Profile*

Figure 51 provides the theoretical temperature profile at each freezing hour. We can see that the changes are more important at the first hours, but they get less as the process progresses.

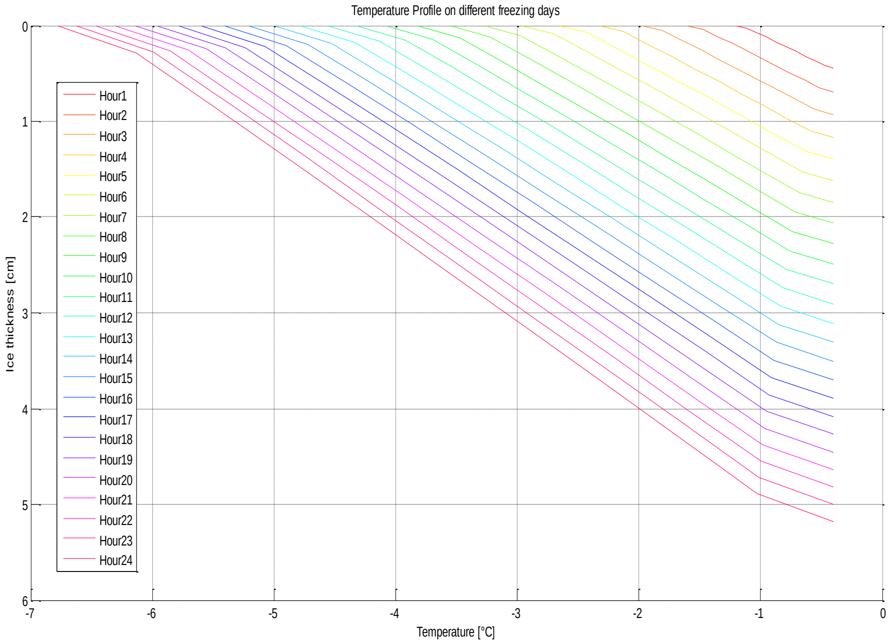


Figure 51 Temperature Profile on different freezing hours

c. *Growth Rate*

The change in the growth is not as significant as it was in the previous snow-free ice case. The reason is obvious: the ice is at the first stages of growth and it is not so thick to start impeding further heat exchange between the surrounding atmosphere and the water. This can be also clearly noticed through the Figure 50, where one could think that the curve is a straight line duo a low curvature.

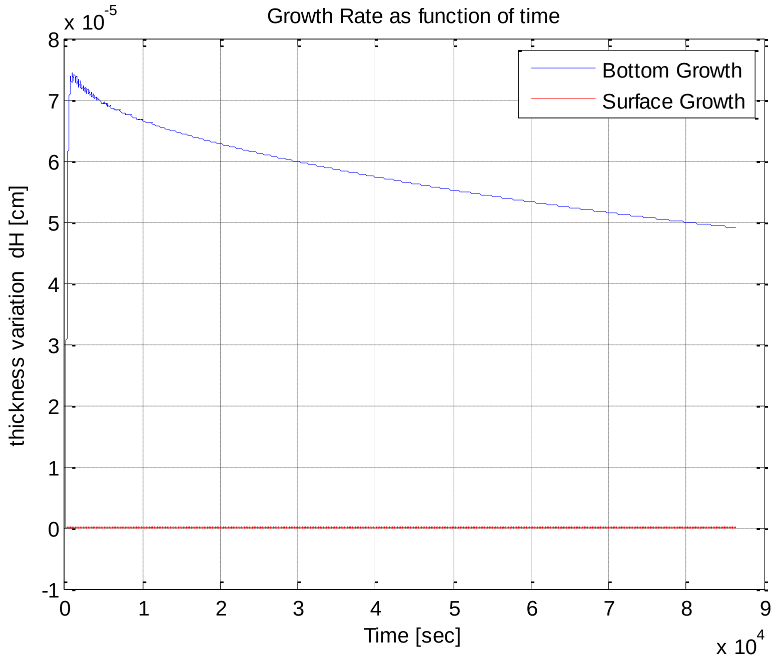


Figure 52 Tank Experiments - Growth Rate

6.5 Results Discussion

All the important results of the performed experiments have been presented in previous sections. In this section, we will discuss about their reliability, i.e. how reliable they are and to what extent.

First of all, the experimental results presented some deviations with the theoretical ones. On the table below, we present some statistical data to quantify these deviations.

Table 10 Results deviation from the theory

	Snow-free Ice		Snow-Covered Ice		
Deviation	Large Container	Small Container	Large Container h_{snow}=3.63 cm	Small Container h_{snow}=1.81 cm	ICE TANK experiments
Maximum [mm]	6.70	2.50	3.40	3.60	-0.30
Minimum [mm]	-7.70	-4.80	-4.70	-7.20	-1.80
Average [mm]	0.30	-2.80	-1.10	-2.50	-0.93

One could say that the numerical results are more than satisfactory, because they match very well with the experiments. On the one hand, to produce this theory, we made assumptions and neglect many features which in reality are not negligible; and on the other hand, the measurements from the experiments could also present some problems.

As far as the theory is concerned,

- The amount of heat coming from the bottom has not been considered. In the models we found in the literature, there is always an amount of heat coming from the ocean. Its contribution to the computed thickness is rather negative. In our case, if we knew the exact value of this quantity and used it, our curve would have been slightly lower than it is in the studied cases.

- The main assumption of this theory is that only the vertical heat transfer is considered. Nevertheless, as it has been demonstrated above, there is an amount of heat coming from the side walls of the containers. As a consequence, we could say that our theoretical curve should be higher than it is.
- We had to destroy the ice in order to measure its thickness. Our opinion is that this fact delayed the ice growth to a certain extent. Since we assumed horizontal uniformity while measuring the thickness, we took at least two samples from distanced locations and considered their average. The measurements made later might have been somehow influenced by this fact.
- Possible imperfections in the experimental set up maybe the reason for some inaccuracies in the results.

7. CONCLUSIONS AND SUGGESTIONS

The study performed for this Master thesis highlights the effects of the blowing air characteristics on the ice development process. Relevant experiments have been performed as to verify this theory and their results are exposed and analyzed as well.

In view of what has been said above, one could conclude that despite the problems mentioned in the previous sections, the experimental results are not so different from what we could expect based on the theory we developed. The issues we mentioned could be solved or at worst, their "damage" they cause could be reduced to improve the model and/or the experiments.

As far as the theoretical model is concerned, further studies should consider reducing the number of assumptions formulated as to make the model more realistic.

The exact determination of the different heat fluxes that contribute to the ice growth process is of major importance. The heat fluxes defined by Fletcher 1965 are widely used so far to model the heat exchange between the atmosphere and the ice, but in reality these should be function of seasons, geographic locations, etc. The same is also valid for the heat coming from the water, which constitutes one of the biggest uncertainties in this subject.

To treat the brine pockets, we used a defined salinity profile over the depth. As described in the theory, the brine pockets formation is part of the ice growth process, so the salinity has to be considered as a "real" parameter of the problem, instead of being used as an external parameter as it is done here. In fact, by "real" parameter, it is meant that the equation of salinity conservation law should be solved just as heat conduction equations.

As for the experiments, even if the model has been tested in two different environmental conditions, the outside-the-walls environment remains quite different from the HSVA ice laboratory and ice tank. It would be interesting to test the model and see its limits when there are some solar radiation and others atmospheric features. In addition, to measure the thickness of the ice, a non-destructive method is suggested to avoid any influence on the measurements.

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