Stress test analysis on ratings quality over the business cycle

Auteur : Mercenier, Pierre

Promoteur(s) : Platania, Federico

Faculté : HEC-Ecole de gestion de l'ULg

Diplôme : Master en sciences de gestion, à finalité spécialisée en Banking and Asset Management

Année académique : 2015-2016

URI/URL : http://hdl.handle.net/2268.2/1202

STRESS TEST ANALYSIS ON RATINGS QUALITY OVER THE BUSINESS CYCLE

Jury :
Promoter :
Federico PLATANIA
Readers :
Stéphanie HECK
Julien HAMBUCKERS

Dissertation by
Pierre MERCENIER
For the acquisition of a MA in
Banking and Asset Management
Academic year 2015/2016

Acknowledgments

I would like to express my gratitude to Federico Platania for his support and valuable guidance throughout my research. I would also like to thank my two readers (Stéphanie Heck and Julien Hambuckers) for the time devoted to my master's thesis. Moreover, I want to thank Elaine Brook for correcting and enhancing the quality of my written English.

Table of contents

1. Introduction

During the first semester of my second year of my master's degree, I had the opportunity of doing an internship at the Euroclear Bank in Bruxelles. Whilst I was there, I was responsible for calculating the ratings of commercial and investment banks based on the Standards and Poor's model. I became fascinated by the subject and it is for this reason that I decided to write a master's thesis with this subject in mind.

A credit rating agency (CRA) is a company that calculates credit ratings. In a nutshell this means that a CRA assesses the creditworthiness of a debtor and rates the ability of an issuer to pay back a debt and its likelihood of default, but never of individual consumers.

A CRA is remunerated by entities in two ways: by those who want to receive a rating and by those using it. The rating agencies have an enormous amount of power in the economy, as their assessments are widely used all over the world. In real terms a rating is nothing more than the opinion of a debtor made by a CRA. What is more it does not take mistakes or human error into account.

The first credit rating agency was founded in 1841 in New-York under the name "The Mercantile Agency". This grew out of a need to assess the solvency of companies after the Panic of 1837. Nowadays the three principal agencies operating in the field are Standards and Poor's, Moody's, and Fitch who control 95% of the ratings business worldwide.

Since their creation, CRA have been implicated in a number of shocks or financial scandals including a number of well-publicised ones. In 2001 Moody's and Standard & Poor's rated Enron as excellent, and the company went bankrupt a few days later. The same pattern was repeated some years later for Lehman Brothers. The CRA also overrated the CDO (collateralized debt obligation) for some years before coming to the conclusion that they should reduce the ratings. This error in judgement led to the subprime crisis.

Business cycles are periods of fluctuation in business activity that follow one another. There are four periods: boom, recession, slump and recovery. In the theoretical model analysed for the purposes of this thesis, we will only be dealing with two periods. They are the boom which is used to describe periods of relatively rapid economic growth and the recession which is used to describe periods of relative decline. The first business cycle theory appeared in 1819 when

Jean Charles Léonard de Sismondi published the book "Nouveaux Principes d'Economie Politique". Business cycles are classified according to the length of time they cover:

- Kitchin inventory (cycle of 3 to 5 years).
- Juglar fixed-investment (cycle of 7 to 11 years).
- Kuznets infrastructural investment (cycle of 15 to 25 years).
- Kondratiev wave or long technological (cycle of 45 to 60 years).

This master's thesis is based on the paper *Ratings quality over the business cycle* (Bar-Isaac, 2012). The theoretical model analyses how the CRAs' incentives to provide ratings of high quality vary depending on which business cycle (boom or recession) exists. A distinction is also made between the market in a monopoly and in a duopoly.

The aim of this paper is not to change or modify the model, but to analyse its sensitivity to a modification of what we consider to be the most interesting parameters, by carrying out a stress test analysis. These parameters are lambda, gamma and tau, they represent respectively the probability that a given investment is good, the probability that captures the labor-market condition and the probability of a transition from the current state to a different one.

In order to facilitate the reading and the understanding of this research thesis, we have decided to divide it into two parts which examine two different situations: first the model in a monopoly and then in a duopoly. In the first part we will start by explaining some of the most important principles of the model. After that we will explain why ratings quality are counter-cyclical and then proceed to a stress test analysis. Finally we will demonstrate how robust the model is, especially when there is a correlation between shocks and when naïve investors are added. In the second part we will analyse how the model will react when there are two credit rating agencies by focussing on two different punishment strategies and by proceeding to a stress test analysis.

The calculations that were used to make the graphs cannot be included in the appendices due to their size. It is possible, however, to access the Excel file on the on-line platform MateO.

The subject of this master's thesis being technical and complex and the fact that English is not my native language made the writing of this paper a real challenge for me.

2. Related theoretical literature

The aim of this literature review is to expose the papers that helped me to understand the model and to provide documentation for the reader who wishes to look into the subject in more detail. This master's thesis is based on the paper Ratings Quality over the Business Cycle (Bar-Isaac & Shapiro, 2013).

The closest paper to this theoretical model was written by Mathis, McAndrews, & Rochet, (2009). It analyses how ratings quality can be affected by a CRA's preoccupation for reputation. The paper published by Strausz (2005) is also very interesting. It is really similar in structure to the previous one but is more general in style.

There are several interesting papers analysing credit rating agencies and Faure-Grimaud, Peyrache, & Quesada, (2009) showed that the competition between CRAs could lead in less information disclosure. In this master's thesis, it is considered the issuer only has access to public information regarding the investment. Mariano, (2012) however, took another view and considered that private information is also available.

Competition among CRAs may reduce the welfare due to shopping around by issuers which means that the issuer does not pay a CRA for a bad rating as demonstrated in Bolton, Freixas, & Shapiro (2012). When there are more naïve investors and when exogenous reputation costs are lower, the possibility for conflicts of interest for the CRA increases.

Skreta & Veldkamp, (2009) have assumed that credit rating agencies are able to relay their information truthfully and showed that more information creates greater opportunities for issuers in order to take advantage of naïve investors through shopping. The theoretical model analyses interaction between the incentives and the business cycle. In Bond & Glode (2011), individuals have the right to become regulators or bankers and it appears that during booms banks attract the highest quality regulators leading to a more fragile regulation of the system. Similarly to the results of Bar-Isaac & Shapiro (2013) and Povel, Singh & Winton (2007) showed that fraud is far more likely to happen during booms than during recessions.

Two further papers written by Mattarocci & Gianluca (2013), Bar-Isaac & Shapiro (2011) and Rhee (2015) may also help the reader to understand the theoretical model as was the case for me.

3. The model in a monopoly

In order to facilitate a better understanding of this master's thesis, a list has been prepared of the Greek letters that have been used, together with their meanings in English:

α	Alpha
λ	Lambda
π	Pi
σ	Sigma
ω	Omega
γ	Gamma
δ	Delta
τ	Tau

3.1. First principles

In the first instance, the different states of the model and the notion of quality of an investment will be presented. For this part it will be assumed that the credit rating agency (CRA) holds the monopoly. There are two states that have been allowed for in the model:

$s \in \{R, B\}$

R = Recession

B = Boom

An issuer makes a new investment during each period, which can be either good (G) or bad (B):

	Probability of default	Pay-out
G	0	1
B	p_s	0 if default ; 1 if not default

The probability that the investment is good: λ_s

The probability that the investment is bad: 1-λ_s

The issuer only has access to public information regarding the investment (private information is never divulged). By assessing and identifying the quality of the investment, a CRA will take on the role of information producer for the issuer.

At the beginning of each period, the issuer approaches a CRA in order to evaluate its investment.

The amount paid by the issuer to the CRA for a good rating: π_S

The amount paid by the issuer to the CRA for a bad rating: 0

Where their staff are concerned, each period, the CRA pays a wage to an analyst:

$W_s \in [0, \overline{w}]$

The ability of this analyst is represented by: $z(w_s, \gamma_s) \in [0,1]$

The parameter that captures the labor-market condition is: γ_s

The argument no longer applies when there is no confusion or ability: z_s

Higher-performing analysts are much harder to attract and retain. This is even truer for the CRA at the top of the wage distribution, so: $\partial z / \partial w_s > 0$ and $\partial^2 z / \partial w_s^2 < 0$

We can therefore assume that: $\partial z / \partial w_s \rightarrow \infty$ as $w_s \rightarrow 0, z(0, \gamma_s) = 0$ and $\partial z / \partial w_s | \overline{w} = 0$

We can also consider that when γ_s is larger, the labor-market becomes tighter and as a result it is more problematic to find and hire high-quality analysts. As a consequence, the CRA has to offer higher salaries to maintain quality: $\partial z / \partial \gamma_s < 0$ and $\partial^2 z / \partial \gamma_s \partial w_s < 0$

The performance of an analyst is based on their ability to gather information efficiently and to evaluate whether an investment looks good or bad. A good quality analyst can correctly identify a good investment: $(p(G|G) = 1)$

The analyst may, however, make error or mistake regarding bad investments (with a probability of 1-z): $(p(B|B) = z)$

The ability of analysts is: z_s

It means that the CRA will decide on the percentage of errors or mistakes it will allow based on the incentives for accuracy and the cost of hiring analysts (labor-market).

Where the incentives for accuracy are concerned, these are awarded when investors suspect the CRA is not investing enough in ratings quality, so when: $z < \bar{z}$.

The reverse is true if investors think the CRA is investing sufficiently in ratings quality: $z > \bar{z}$

Where $\bar{z}$ represents the investors' decision to allocate money for an investment. The CRA maintains its reputation with respect to the constraint: $z \geq \bar{z}$. If this were not the case, the investors would fail to purchase the investment and issuers would not seek ratings.

It can therefore be said that there are three possible situations:

- A good report with an investment return of 1.
- A bad report.
- A good report where the investment defaults.

A boom period, by its very definition, means that everything increases and as a result there are higher fees, tighter labor-market competition, a higher number of interesting projects and lower probabilities of default:

$$\pi_B > \pi_R$$

$$\gamma_B > \gamma_R$$

$$\lambda_B > \lambda_R$$

$$p_B < p_R$$

It goes without saying, but is important to note that the state of the economy in a given period is affected by the state of the economy in the preceding period. The probability of a transition from the current state “s” to the other state is: τ_S

This means that:

- The probability of moving from a “boom” state to a “recession” state is: τ_B
- The probability of moving from a “recession” state to a “recession” state is: 1-τ_R
- The probability of moving from a “recession” state to a “boom” state is: τ_R
- The probability of moving from a “boom” state to a “boom” state is: 1-τ_B

If τ_B=1-τ_R or τ_R=1-τ_B, it means that the state of each period is an independent, identically distributed draw from the same distribution. It further means that the probability of transitioning to a recession or to a boom is the same (regardless of whether the current period is actually a boom or a recession).

When τ_B<1-τ_R, it means that there is a positive correlation or persistence among states: a state of boom is more likely to follow a state of boom than a state of recession.

The same applies to a state of recession. When τ_R<1-τ_B, it means that there is a positive correlation or persistence among states: normally speaking, a state of recession is more likely to follow a state of recession than a state of boom.

The situation is reversed when τ_B>1-τ_R and when τ_B>1-τ_R. In fact, the higher the value of τ_S, the shorter the duration for the state s and the faster the change from one to the other is likely to take place.

3.2. Counter-cyclical ratings quality

3.2.1. Theoretical part

In order to prove that ratings quality is counter-cyclical, it is necessary to consider a situation in which economic shocks are independent of each other and identically distributed.

In order to maximize its revenues, a CRA will look at current salary levels and calculate the continuation values based on the available information. This is crucial as potential investors will only purchase after checking that a particular analyst has an ability high enough: $z(w_s, \gamma_s) > \bar{z}$ for s=R or s=B.

This can be verified by the equilibrium wages that are: w_R^* and w_B^*

The probability that an investment gets a good rating, when state $s \in \{B, R\}$ is:

$$\alpha_s := \lambda_s + (1 - \lambda_s)(1 - z_s) \qquad (1)$$

A CRA will only attribute a favourable rating when the investment is good, or when it has been misreported as being bad (i.e. when the investment is actually good).

The probability that the investment is good: λ_s

The probability that the investment is misreported as being bad: (1-λ_s)(1-z_s)

The probability of an analyst making an error is (1-z_s).

The probability that the CRA survives into the future is the probability that the CRA doesn't give a good rating to an investment that subsequently defaults. In other words, the CRA gives a bad rating to an investment that is, indeed, bad because it defaults. This probability is:

$$\sigma_s := 1 - (1 - \lambda_s)(1 - \lambda_s) \qquad (2)$$

where s $\in \{B, R\}$

A rating is bad when an investment is rated as good and then, at a later date, defaults. This is in complete contrast to the investment that is bad and misreported.

The probability that the investment is bad: 1-λ_s

Probability that the investment was rated as good: (1-z_s)

Probability of default: p_s

σ_s and α_s are endogenous because they are dependent on z_s (i.e. ability of analysts), which depends on the CRA's strategy for deciding whether to invest or not based on ratings quality w_s.

The value functions from the beginning of a period can be written as V_s (where s is the state it can be either of a boom or a recession). The value from the end of a period in state s can be written EV_s

The probability of moving from the state s to another state is: τ_s

$$EV_B := (1 - \tau_B)V_B + \tau_B V_R \quad (3)$$

$$EV_R := (1 - \tau_R)V_R + \tau_R V_B \quad (4)$$

The value functions for each state are:

$$V_B = \max_{W_B} \pi_B \alpha_B - W_B + \delta \sigma_B EV_B$$

$$V_R = \max_{W_R} \pi_R \alpha_R - W_R + \delta \sigma_R EV_R \quad (5)$$

When a CRA gives a project a positive rating (with a probability of α_s), it earns a fee and subsequently pays a wage w_s. When the reverse happens, the probability that the project is bad, that the agency misreports, and that the project later defaults is: $1 - \sigma_s$. In this case scenario, no issuer returns to the CRA because it anticipates that the rating agency would set $w_s = 0$. In this situation, the CRA's continuation value is zero. What can also happen is that the CRA will earn the expected continuation value EV_s with a probability of σ_s.

We now make the lemma that there is a unique solution (V_B^*, V_R^*) to equation 5, which is associated with w_B^* and w_R^*. The asterisk "*" is used to denote the equilibrium values.

In order to demonstrate the difference between accuracy during a recession and a boom, we will begin by writing the first-order conditions for decision variables w_B and w_R:

$$\frac{\partial z}{\partial w}(w_B^*, \gamma_B) = \frac{1}{1-\lambda_B}\frac{1}{\delta p_B EV_B^* - \pi_B} \tag{6}$$

$$\frac{\partial z}{\partial w}(w_R^*, \gamma_R) = \frac{1}{1-\lambda_R}\frac{1}{\delta p_R EV_R^* - \pi_R} \tag{7}$$

Given (6) and (7), $w_B^* \leq w_R^*$ and the fact that there is a higher accuracy in recessions than in booms when:

$$(1-\lambda_B)(\delta p_B EV_B^* - \pi_B) \leq (1-\lambda_R)(\delta p_R EV_R^* - \pi_R) \tag{8}$$

As previously mentioned, when τ_B=1-τ_R or τ_R=1-τ_B, the state of each period is an independent identically distributed draw from the same distribution. Therefore the likelihood of transitioning, either to a recession or a boom, is the same (irrespective of whether there is a boom or a recession at the present time).

As a result, $EV_B^* = EV_R^*$, represents the fact that continuation values from both a recession and a boom are identical. Consequently, ratings quality are usually lower in a boom state than in a recessionary state.

We surmise that where states are independent over time (when $\tau_B = 1 - \tau_R$), there is a higher investment in ratings quality during a recession than during a boom.

Taking into account the assumption that economic shocks are independent of each other and evenly distributed, it leads a CRA to essentially treat the future as a cyclic pattern of recession and boom. The only element that changes the probability of it surviving in the future are current investments and payoffs.

The knock-on result of this is that ratings quality are counter-cyclical, with a lower accuracy in boom states. This is really intuitive and in a boom there are several results:

- The fees are higher and a CRA may be tempted to pay less attention to accuracy in order to collect more fees.
- A larger percentage of investments are good, which makes it less important to invest in ratings quality.
- There are lower probabilities of default, making it less likely to be caught-out for reduced accuracy.
- There is less unemployment, making good analysts more difficult to find and more expensive to employ.

3.2.2. Stress test analysis

The two first equations we will take a look at are alpha and sigma which are endogenous. Indeed, alpha is dependent, not only on the lambda parameter, but also on the performance and ability of analysts $z(w_s, \gamma_s)$. Sigma is only dependent on the gamma parameter.

In the first instance, we must consider the ability of analysts as being a function $z(w_s, \gamma_s) \in [0,1]$ where γ_s is the parameter that captures labor-market conditions and where $W_s \in [0, \bar{w}]$ is the wage that a CRA has to pay to an analyst for each period. We will assume that when gamma is greater, the labor-market becomes tighter, and it is therefore more difficult to take-on high-quality analysts. This means that a CRA will have to offer higher salaries to attract high-quality analysts in order to maintain quality. In the theoretical paper (Bar-Isaac, 2012), an numerical example is made and the authors used this equation for the ability of analysts: $z(w_s, \gamma_s) = \sqrt{w_s}/\gamma_s$. For the purposes of this analysis, we will use the same equation, making alpha $\alpha_s := \lambda_s + (1 - \lambda_s)(1 - \sqrt{w_s}/\gamma_s)$. In addition to the previous constraints, gamma cannot equal 0 because the result would be an infinite. The aim of the chart below is to determine which values gamma and wages can take in respect of the constraints:

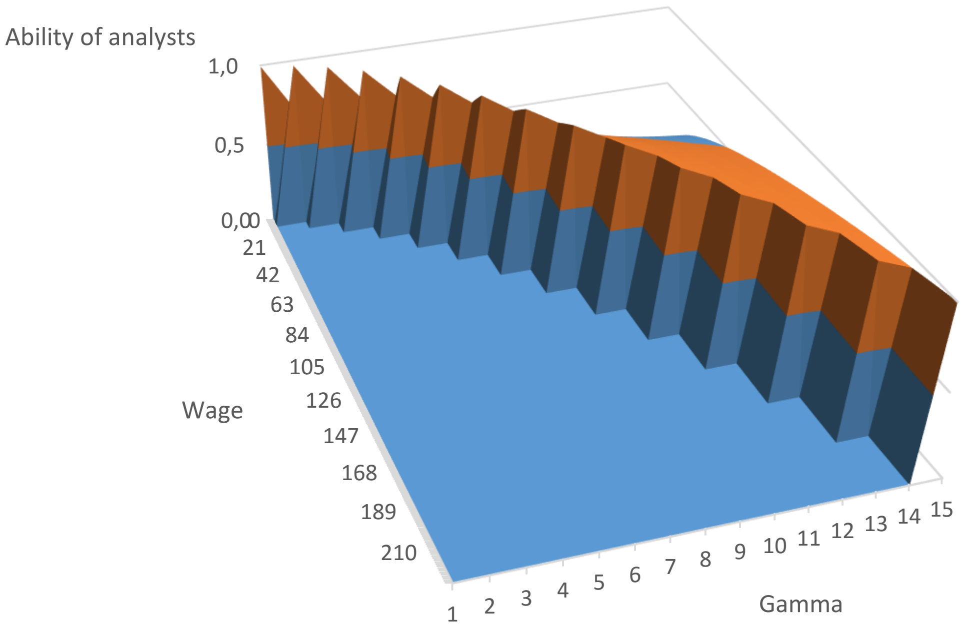

Figure 1. The correlation between the wage and gamma in the equation of the ability of analysts.

As it would be interesting to establish how the two parameters are correlated, we have given gamma different values in order to workout which values the wages could take. The above chart represents the sensitivity of the equation $z(w_s, \gamma_s)$ when gamma and the wages are modified. All the values superior to 1 have been suppressed because they must be between 0 and 1 as that corresponds to the flat left front corner of the graph. As it demonstrates, the values for gamma were examined between 1 and 15. The graph shows that the wages can vary from 0 to 225.

The graph clearly shows that the two parameters are positively correlated. It can be seen, that as gamma increases, the wages also increase in order to maintain the same level of performance. For example: with a gamma of 5, the best performing analyst is hired by offering wages of between 23 and 25. If, however, we look at a gamma of 10, the values for the wage that would be necessary to take on an excellent analyst range between 91 and 100. In conclusion, this confirms our supposition that when the labor-market becomes tighter (i.e. when gamma becomes larger), the wages also become higher, because a CRA will have to pay a higher wage to attract high quality analysts in order to maintain quality.

Lambda is an important parameter of the model and for that reason it was decided to analyse it. Lambda represents the probability that an investment is good. In the theoretical model we considered that the issuer only has public information about an investment and that private information is withheld. By identifying the quality of an investment, a credit rating agency will play the role of information provider for the issuer. As the parameter increases in value then the quality of the information given by a CRA should also increase. Due to the fact that lambda is a probability, it can only be given values of between 0 and 1. We also know that $\lambda_B > \lambda_R$ because the probability of finding good investment projects is higher during a boom than during a recession.

After having defined lambda and the ability of analyst, we can now analyse the sensitivity of alpha where those parameters are modified and its equation is $\alpha_s := \lambda_s + (1 - \lambda_s)(1 - z_s)$. This means that two points will be looked at: the impact on the probability that an investment gets a good rating if we vary the probability that an investment is good and then the ability of analyst. For that we varied lambda and the ability of analysts from 0 to 1. The charts below show the results under tow perspectives:

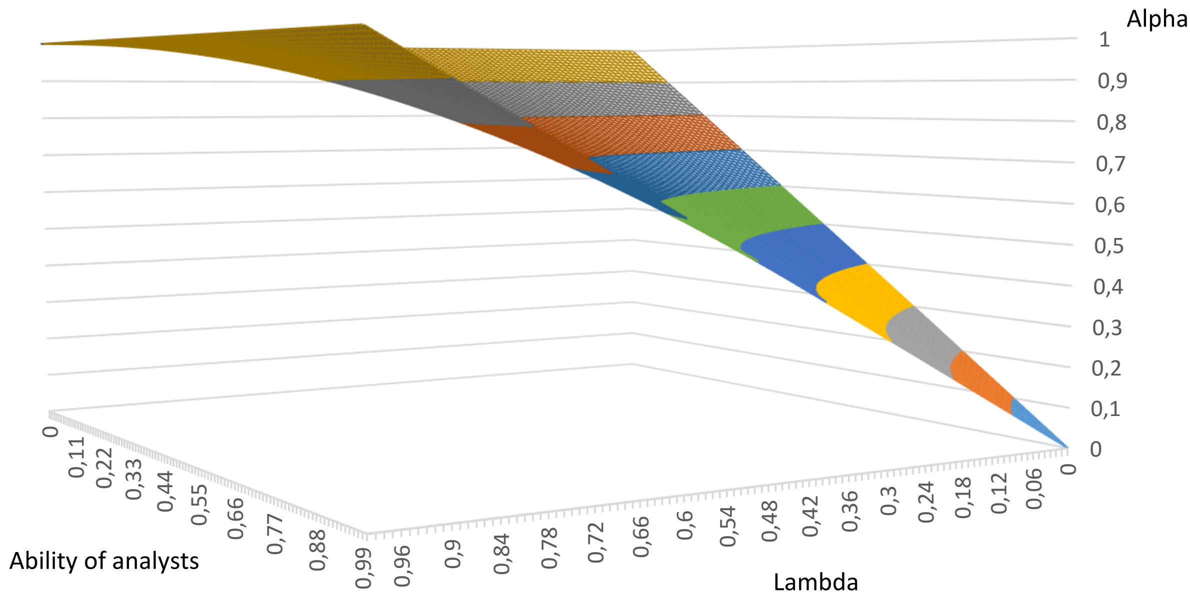

Figure 2. Perspective 1: Sensitivity of alpha to a modification lambda and the ability of analysts.

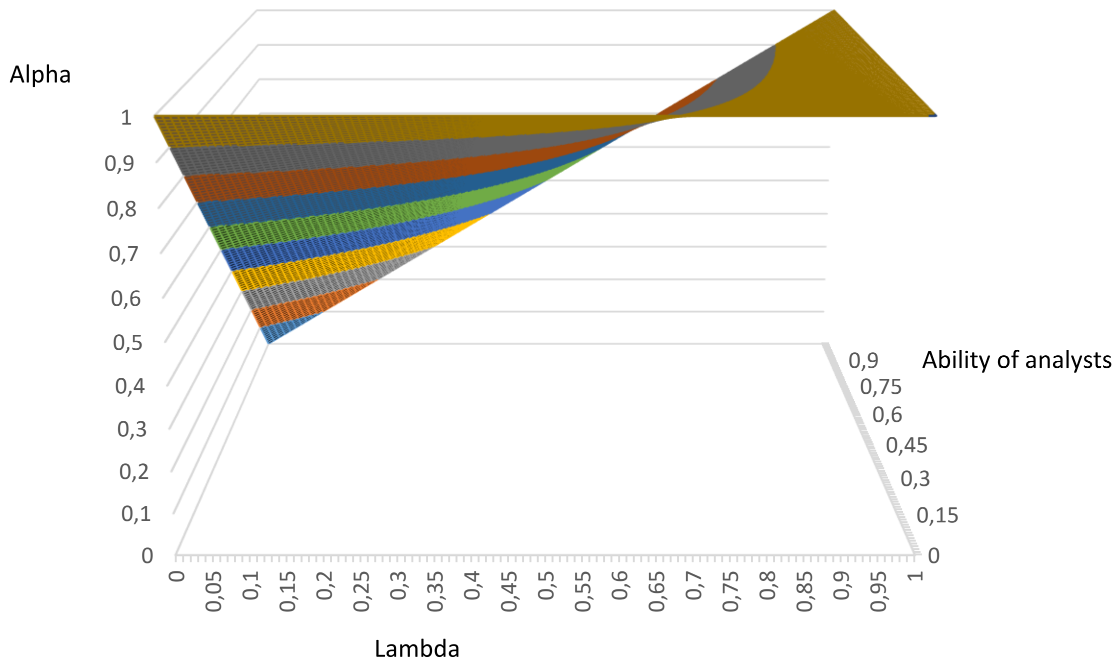

Figure 3. Perspective 2: Sensitivity of alpha to a modification lambda and the ability of analysts.

We already know that alpha can vary between 0 and 1 because it is the probability that an investment will get a good rating. We can see from the charts that alpha equals 0 when the ability of analysts is at its highest value and when lambda is at its minimum value. Alpha equals 1 for any values taken by lambda when the ability of analysts strictly equals 0 and for any values taken by the performance of analysts when lambda is strictly equal to 1. For a fixed value of z_s it was noted that as lambda increased then so did the alpha linear, which clearly demonstrated that they are positively and proportionally correlated. However, for a fixed value of lambda, it was noted that as $z_s$increased then the alpha linearly decreased which proves that they are negatively and proportionally correlated. As a conclusion, the probability that an investment gets a good rating is positively and proportionally correlated to the ability of analysts and is negatively and proportionally corralled to the probability that the investment is good.

As it is one of the parameters in the equation of sigma, gamma is an important element in the model. As we have previously stated, it captures the labor-market conditions. More precisely this means that it expresses whether the labor-market is either tight or slack. Moreover we know that $\gamma_B > \gamma_R$ because a boom produces tighter labor-market conditions.

We will now focus on the equation concerning the sigma $\sigma_s := 1 - (1 - \lambda_s)(1 - \lambda_s)$ which represents the probability that a CRA will survive into the future (i.e. the probability that a CRA does not give a good rating to an investment that subsequently defaults). In fact lambda is the only parameter that can be varied. The aim here is to analyse the impact of a modification on the probability that an investment is good against the probability that a CRA will survive into the future. Lambda is a probability so once again it can only vary between 0 and 1. The chart below shows the results:

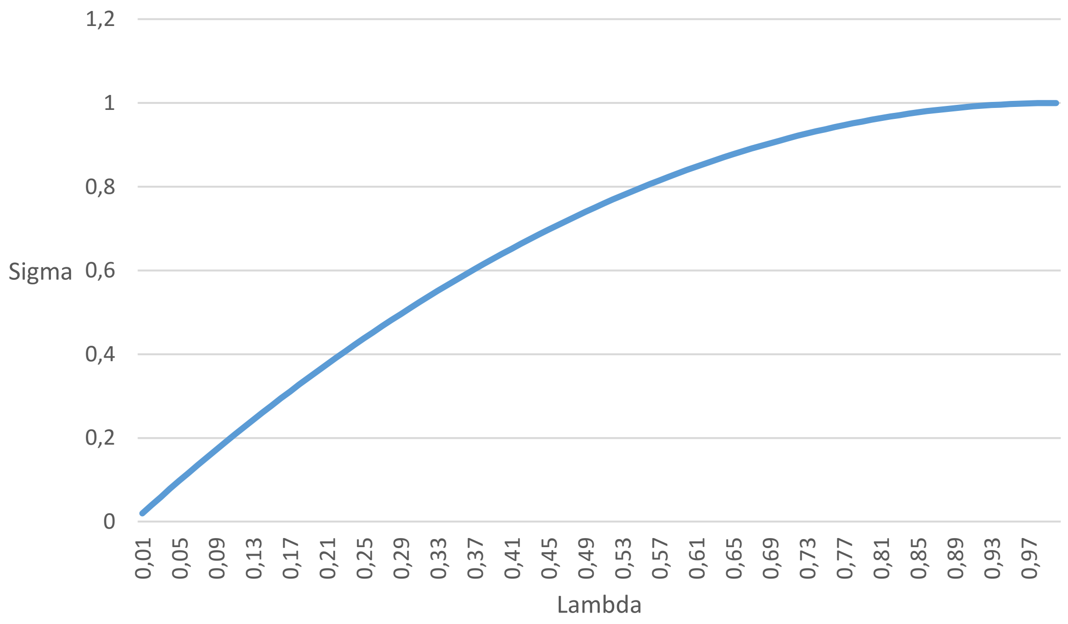

Figure 4. Sensitivity of sigma to a modification of lambda.

Here we can see that the curve is growing but is not linear. It is a concave curve. When lambda is modified, it has a greater impact on sigma if the parameter is low than if the parameter is already high. Sigma is positively but not proportionally correlated to lambda. In conclusion, the probability that a CRA survives into the future increases when the probability that an investment is good becomes higher.

We will next move onto the parameter tau (i.e. τ_S) which is used to define the probability of a transition from a current state to another state. Indeed, as we defined in the theoretical part, the state of the economy (i.e. boom or recession) in a given period is dependent from the state of the economy in the previous period. The state of each period can be an independent identically distributed draw, meaning that the probability of progressing to a recession or to a boom is the same, regardless of what the state of the economy is today. There can also be a positive correlation or persistence among states, so a boom state is more likely to follow a boom state than a recession state and inversely. In the same way as with the other graphs, tau is a probability and can therefore only have a value of between 0 and 1. The higher the τ_S the shorter the duration for the state s and the faster it moves towards the other state.

We would like to analyse the sensitivity of the value function of a CRA in a monopoly. More precisely we would like to see how the expected value functions to a modification of some parameters will vary. As we previously said, in state s the value functions from the beginning of a period can be written as V_s and the value from the end of a period can be written as EV_s. These are the value functions for each state (boom and recession):

$$EV_B := (1-\tau_B)V_B + \tau_B V_R$$

$$EV_R := (1-\tau_R)V_R + \tau_R V_B$$

$$V_B = \max_{W_B} \pi_B \alpha_B - W_B + \delta\sigma_B EV_B$$

$$V_R = \max_{W_R} \pi_R \alpha_R - W_R + \delta\sigma_R EV_R$$

We can therefore deduce that for a boom:

$$EV_B := (1-\tau_B)V_B + \tau_B V_R$$

$$EV_B := (1-\tau_B)\,[\max_{W_B} \pi_B \alpha_B - W_B + \delta\sigma_B EV_B] + \tau_B V_R$$

$$EV_B := (1-\tau_B)\,[\max_{W_B} \pi_B \alpha_B - W_B] + (1-\tau_B)\delta\sigma_B EV_B + \tau_B V_R$$

$$EV_B[1-(1-\tau_B)\delta\sigma_B] := (1-\tau_B)\,[\max_{W_B} \pi_B \alpha_B - W_B] + \tau_B V_R$$

$$EV_B := \frac{(1-\tau_B)\,[\max_{W_B} \pi_B \alpha_B - W_B] + \tau_B V_R}{1-(1-\tau_B)\delta\sigma_B}$$

And for a recession:

$$EV_R := (1-\tau_R)V_R + \tau_R V_B$$

$$EV_R := (1 - \tau_R)\,[\max_{W_R} \pi_R \alpha_R - W_R + \delta \sigma_R EV_R] + \tau_R V_B$$

$$EV_R := (1 - \tau_R)\,[\max_{W_R} \pi_R \alpha_R - W_R] + (1 - \tau_R)\delta \sigma_R EV_R + \tau_R V_B$$

$$EV_R[1 - (1 - \tau_R)\delta \sigma_R] := (1 - \tau_R)\,[\max_{W_R} \pi_R \alpha_R - W_R] + \tau_R V_B$$

$$EV_R := \frac{(1 - \tau_R)\,[\max_{W_R} \pi_R \alpha_R - W_R] + \tau_R V_B}{1 - (1 - \tau_R)\delta \sigma_R}$$

For the following calculations we will use these two equations and we will fix the values so that $\delta = 0.5$, $V_B = 20$, $V_R = 20$, $\pi_B = 30$, $\pi_R = 30$, $W_B = 10$ and $W_R = 10$. As a result EV_B and EV_R will show exactly the same results so we can use the notation EV.

First of all the sensitivity of the expected value function EV to a modification of tau and alpha was measured. As the chart clearly shows, values from 0 to 1 were used. In addition, the value for sigma was fixed at 0.75. The aim was to analyse how the expected value function EV would evolve after modifying the probability of a transition from the current state to the other state as well as the probability that an investment would receive a good rating. The graph below shows the results:

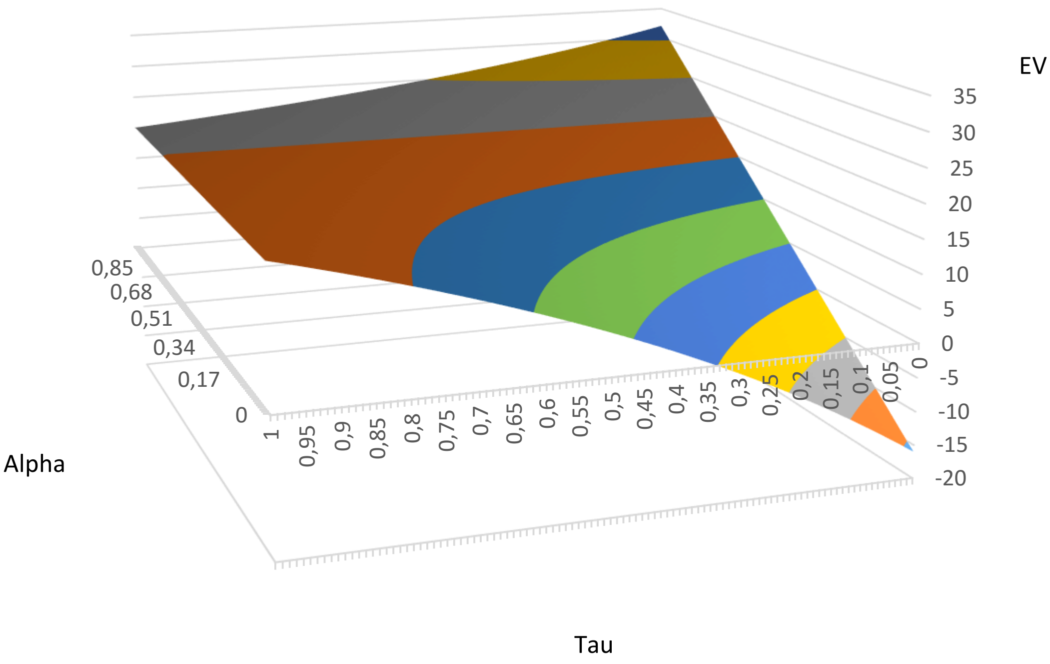

Figure 5. Sensitivity of the expected value function to a modification of alpha and tau.

We can see from the graph that the results for the expected value EV are all between -16 and 32. The minimum value is obtained when both alpha and tau equal 0, and the maximum value is reached when tau equals 0 and alpha equals 1. When alpha is less than 0.75 and tau increases, the expected value function EV also increases, therefore they are positively correlated. This tendency reverses when alpha rises above 0.75. That is to say when tau increases, the expected value function EV decreases and they are consequently negatively correlated. The pivot point corresponds to an alpha of 0.75, which is the exact value that was fixed for sigma. The expected value function is positively correlated to the probability of a transition from the current state to the other state until the point where alpha is equal to 0.75, at which point it becomes negatively correlated. As we previously said, the higher the τ_S the shorter the duration for the state s and the faster it moves toward the other state. As far as alpha is concerned, for a given value of tau, when it increases, the expected value function EV increases linearly. This demonstrates that they are both positively and proportionally correlated. In terms of a conclusion, it appears that the expected value function EV is positively and proportionally correlated to the probability that an investment gets a good rating.

In the next section, we will analyse the sensitivity of the expected value function EV to a modification of tau and sigma and once again they were given values ranging from 0 to 1. We also fixed the value for alpha at 0.75. The aim was to analyse how the expected value function EV evolved following a modification of the probability of a movement from a current state to the other state and the probability that a CRA survives into the future is the probability that a CRA does not give a good rating to an investment that defaults at a later date. The chart below shows the results:

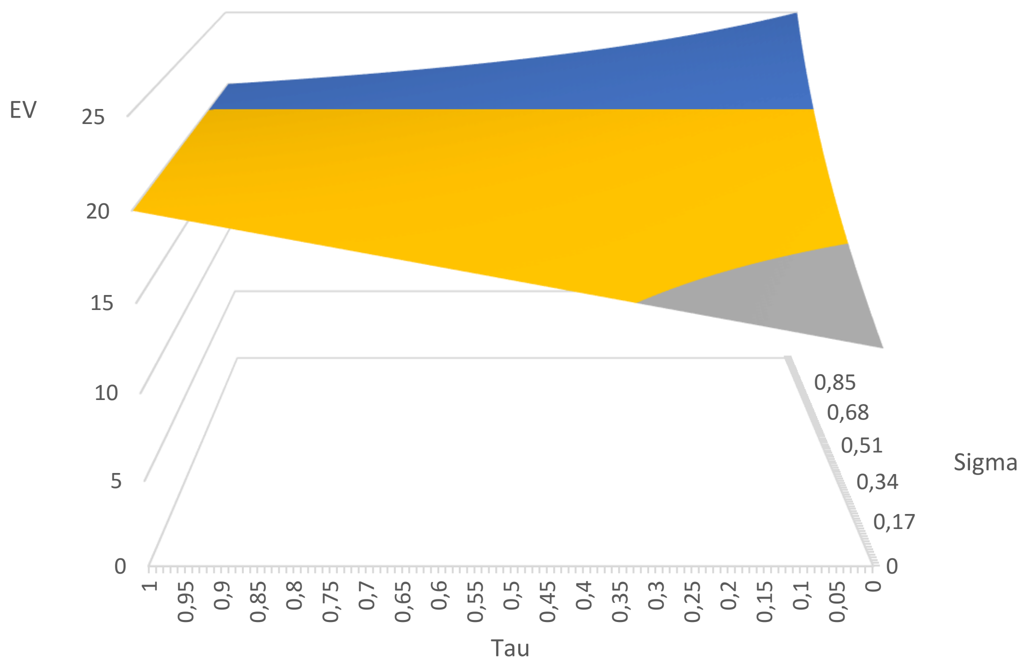

Figure 6. Sensitivity of the expected value function to a modification of sigma and tau.

We can see on the chart that the results for the expected value EV all stand between 12.5 and 25. The minimum value is reached when sigma and tau equal 0, and the maximum value is reached when tau equals 0 and sigma 1. In a similar way to the previous chart, for a sigma less than 0.75, when τ_S increases, the expected value function EV also increases and we can therefore conclude that they are positively but not proportionally correlated. This tendency reverses when sigma becomes higher than 0.75. Furthermore, when tau increases, the expected value function EV decreases, which means that they are negatively correlated. The pivot point corresponds to a sigma of 0.75, which is exactly the same as the value that was fixed for alpha. The expected value function is positively correlated to the probability of a transition from the current state to the other state until the point where sigma equals 0.75, then it becomes negatively correlated. If we take sigma for a given value of tau, we can see that when it increases, the expected value of function EV also increases but not proportionally. This demonstrates that they are positively correlated. As a conclusion, we can say that expected value function EV is positively correlated to the probability that a CRA will survive into the future.

3.3. Robustness of the model

In this third part, the counter-cyclical ratings quality results are studied in the case of a monopoly where a CRA is pushed in two directions: first the correlation between economic shocks and then the resulting influx of naïve investors among the sophisticated investors. The aim of this part is to see how robust the theoretical model is.

3.3.1. Correlation between shocks

When recessions and booms do not arise independently, equation 2 cannot be applied directly. Since then, it is not necessarily easy to verify equation 8 because the continuation values EV_R^* and EV_B^* are endogenously determined. We previously stated that being in a boom or in a recession has a different impact on a CRA's investment decisions. Indeed, in a boom, there are lower default probabilities, higher fees and a larger proportion of good projects. This leads us to consider that a boom period is much more profitable for a CRA than a recession. This is in spite of the tighter employment market and resulting higher salaries that could eliminate or reduce some of the abovementioned advantages.

We made the following suppositions that the difference between the value of being in a boom rather than in a recession$(\mathrm{V_B^*} - \mathrm{V_R^*})$:

- Decreases in the probability of default in a boom (p_B) and also decreases the competitiveness of labor-market conditions (γ_B). But increases in the proportion of good projects (λ_B) and as well as the fees (π_B).

- Increases in the probability of default in a boom (p_R) and the competitiveness of labor-market conditions (γ_R). But decreases in the proportion of good projects (λ_R) and the fees (π_R).
- If, and only if, it is more advantageous to be in a boom ($V_B^* > V_R^*$), the benefits of being in a boom rather than in a recession decrease with the probability of transitioning from a boom to a recession (τ_B) but then increase along with the probability of transitioning from a recession to a boom (τ_R).

This leads us to make a first assumption (A1): we will assume that $V_B^* > V_R^*$, meaning that the value of a CRA in a boom is larger than it is during a recession.

We make the proposition that if there is a mean reversion (i.e. when shocks are negatively correlated) between the two states, then there is a higher investment in ratings quality during a recession than during a boom. This is demonstrated by equation 8. Indeed, if we consider a mean reversion, we can assume that in a recession the future expected value is larger than in a boom because of the higher probability of transitioning from recession to boom. Interestingly, it is generally during a period of recession that a CRA earns a good reputation and then reaps the benefits during the boom that generally follows. During a boom, the incentive is to exploit this reputation because it is presumed that a recession will follow. The result is that ratings are more accurate in a recession than in a boom.

In the case of persistence or of an equivalently positive correlation, we can see that ratings may also be counter-cyclical. This means that when states are independent over time, condition 8 is slack suggesting that the condition would not be violated, at least for the "small" levels of positive correlation. This results in a situation where ratings are counter-cyclical.

We can now move on to study continuous changes in the correlation between states, which could be increases or decreases in the extent of correlation. As already stated, changing the magnitude of correlation between the two states has an empirical counterpart in the duration of a recession or of a boom. Indeed, decreasing the probability of transitioning from a boom to a recession (meaning reducing τ_B) is equivalent to conceiving that, on average, a boom lasts longer.

This leads us to presume two scenarios: firstly, longer booms (reduction in τ_B) increase the investment in ratings quality in both states. Secondly, longer recessions (reduction in τ_R) decrease the investment in ratings quality in both states. This result shows that changing expectations of the probable gravity of recessions, or of the extent of moderation, can impact the quality of ratings. As already stated, longer boom periods ultimately increase ratings quality in both a recession and a boom. This is for the simple reason that during a boom, there is a lower probability that the good times will come to an end, which means that there is also less need to exploit a reputation. During a recession, the pay-off of moving to a boom period increases, meaning that it is an excellent time to build up a good reputation.

3.3.2. Insertion of naïve investors

In this part, we will study the introduction of naïve investors into the theoretical model. The majority of investors are sophisticated, meaning that they can judge when the time is right to withdraw one of their rated investments if any evidence of poor rating quality persists. However, a smaller proportion of investors are naïve. Through lack of experience, they will also buy investments with good ratings, but without taking into account the quality of those ratings. They are more likely to invest in investments with good ratings regardless of what evidence about poor accuracy tells them. The existence of these naïve investors can be explained by a lack of incentives to exercise due diligence in order to check the quality of the ratings.

The proportion of fees that are generated by a CRA from the good ratings they give to issuers who sell to naïve investors is: ω

We will assume that this proportion remains constant, whether in boom or recession.

The continuation value for a CRA when naïve investors continue to purchase rated products even when sophisticated investors have stopped is: $\bar{V}_s$

As the CRA retains the trust of naïve investors regardless of performance, it pays a salary of $w_s^* = 0$ for s $\in \{B, R\}$.

So, $\bar{V}_s = (1-\tau_s)(\omega\pi_s + \delta\bar{V}_s) + \tau_s(\omega\pi_{-s} + \delta\bar{V}_{-s})$

This results in

$$\bar{V}_s = \frac{1-\tau_s-\delta(1-\tau_s-\tau_{-s})}{(1-\delta)(1-\delta(1-\tau_s-\tau_{-s}))}\omega\pi_s + \frac{\tau_s}{(1-\delta)(1-\delta(1-\tau_s\tau_{-s})}\omega\pi_{-s} \qquad (9)$$

The value function when sophisticated investors still want to purchase rated investment is:

$$V_s = \max_{w_s} \pi_s \alpha_s - w_s + \delta\sigma_s\big((1-\tau_s)V_s + \tau_s V_{-s}\big) + \delta(1-\sigma_s)\bar{V}_s \qquad (10)$$

With the knowledge that sophisticated investors are still purchasing rated investments, the more the percentage of naïve investors increases, the more the amount of effort invested by a CRA in achieving accuracy decreases. Indeed, such reductions in market discipline by investors result in a reduction in the accuracy of investments.

We can make the proposition that investment in ratings quality in both states (where $\mathrm{s} \in \{\mathrm{B}, \mathrm{R}\}$) falls as the proportion of fees that are generated from naïve investors rises.

We will now analyse the effect of naïve investors on the counter-cyclicality of the accuracy of ratings. In order to do this we will begin by writing the first-order conditions for the decision variables$w_s, s \in \{B, R\}$:

$$\frac{\partial z}{\partial w}(w_s^*, \gamma_s) = \frac{1}{1-\gamma_s}\frac{1}{\delta p_s\big((1-\tau_s)V_s^* + \tau_s V_{-s}^*\big) - \delta p_s \bar{V}_s - \pi_s} \qquad (11)$$

This demonstrates that $w_B^* \leq w_R^*$, which in turn means that there is a higher accuracy in recessions than in booms when:

$$(1-\lambda_B)(\delta p_B(EV_B^* - \bar{V}_B) - \pi_B) \leq (1-\lambda_R)(\delta p_R(EV_R^* - \bar{V}_R) - \pi_R) \qquad (12)$$

We are already aware that $EV_s^* > \bar{V}_s$. As we have already seen: $\pi_B > \pi_R$ implies that $\bar{V}_B > \bar{V}_R$.

Given the first assumption that $V_B^* > V_R^*$, this implies that $EV_B^* - \bar{V}_B < EV_R^* - \bar{V}_R$ when there is a mean reversion between states or when states are independent over time. What is more, it shows that counter-cyclical accuracy of ratings remain robust, even in the presence of naïve investors.

4. The model in a duopoly

During the previous section where the theoretical model was explained, we considered a CRA in a monopoly situation. In this last part, however, we will evaluate how the model performs in a duopoly. In actual fact, this situation is more realistic because Standard & Poors, Moody's and Fitch are only three main competitors, and the reality is, in fact, an oligopoly. Each CRA exercises some degree of market power and competes for its slice of the market.

To replicate this as accurately as possible, we will consider a market with two CRAs and will model competition between them. We will imagine that the fee charged by the CRAs will depend on two things. That is to say, not only on the state of the economy and whether it is a boom or a recession, but also on the level of competition between the CRAs.

The fee charged by a duopolist in state s: $\pi_{D,s}$

The fee charged by a monopolist in state s: $\pi_{M,s}$

Where $\pi_{M,s} > \pi_{D,s}$ and $s \in \{B, R\}$

We will continue to assume that a CRA rates an individual product during each period and we will allow for a correlation between the products. The probability that the CRAs j and i are simultaneously rating the same product is: ρ

In the case where there is an issue with only one good rating, and even if investors are aware of ρ, they are unable to determine if only one CRA rated that issue or whether both CRAs rated the issue but only one of them gave a rating. Indeed, we view the absence of a rating, or a bad rating, as more or less the same thing for the purposes of the model.

The aim of this fourth part is to analyse the CRA incentives and the reputational equilibrium between CRAs i and j. When they detect ratings inflation, investors have a choice of two different ways in which to react (investors react when they observe a good rating that subsequently defaults):

- Firstly, there is the independent punishment strategy. We can consider this to be a grim-trigger-strategy equilibrium. In this case, the investors who observe the default of an issue with a good rating from CRA j will stop buying investments rated by this organisation.
- Secondly, there is the linked punishment strategy. In this case both CRAs give a positive rating to an issue that subsequently defaults. If both CRAs make the same error, however, they go unpunished. This results in a situation where investors are unsure if the joint error reflects a problem with investment in accuracy by both CRAs or if it just reflects a one-time shock that was not easy to predict.

4.1. The independent punishment strategy

As pointed out earlier, we are analysing the model in a duopoly in this section. In a situation where the investors lose confidence in one of the two CRAs, however, the market effectively becomes a monopoly. When this is the case, we can represent optimal salaries in either a period of boom or recession by $w^*_{M,s}$, the continuation value associated with each state $V^*_{M,s}$, and finally the expected continuation value $EV^*_{M,s} = (1-\tau_s)V^*_{M,s} + \tau_s V^*_{M,-s}$ (where –s represents the other state).

The value for CRA i being in a duopoly in state s and paying wages is $w_{i,s}$, bearing in mind that its rival, CRA j is also believed to be paying wages $w_{j,s}$ is:

$$V_{i,s} = \pi_{D,s}\alpha_{i,s} - w_{i,s} + \delta\left[\rho\hat{\sigma}^{IP}_{ij,s} + (1-\rho)\sigma_{i,s}\sigma_{j,s}\right]EV^*_{D,s} + \delta(1-\sigma_{j,s})\left[\rho z_{i,s} + (1-\rho)\sigma_{i,s}\right]EV^*_{M,s}, \qquad (13)$$

where $EV^*_{D,s} = (1-\tau_s)V^*_{i,s} + \tau_s V^*_{i,-s}$, $s \in \{B, R\}$, $\alpha_{i,s}$ and $\sigma_{i,s}$ are the expressions for CRA i of α_s and σ_s which reflect the probability of issuing a good rating and of making it through to the next period, respectively. It is the analogue of equation 5 but in the case of duopoly the CRA has an obligation to pay costs of analysts $w_{i,s}$ and if it gives a good rating (with a probability of $\alpha_{i,s}$), it earns fees of $\pi_{D,s}$.

We can therefore introduce a new notation:

$$\hat{\sigma}_{ij,s}^{IP} := 1 - (1 - \lambda_s)(1 - z_{i,s} z_{j,s}) p_s \qquad (14)$$

This is used to denote the probability that both CRAs, i and j, will survive when they both rate the same issue. However, this is not usually the case, and the two CRAs will not both survive when they do this. For the most part, this will only happen when the investment is bad. This means that at least one CRA rates it as good and it subsequently defaults.

The probability that the investment is bad: $(1 - \lambda_s)$

The probability that at least one CRA gives a good rating: $(1 - z_{i,s} z_{j,s})$

The probability of default: p_s

If CRA i succeeds in maintaining its reputation, there are two possible outcomes. Either CRA j also do the same, in which case the market remains a duopoly. Alternatively, the rival firm fails by assigning a good rating to a bad investment that subsequently defaults, in which case the market becomes a monopoly for CRA i.

It goes without saying that the market only remains a duopoly when the two CRAs survive after rating either different or the same investment.

The probability that both CRAs survive when rating different investments: $(1 - \rho)\sigma_{i,s}\sigma_{j,s}$

The probability that both CRAs survive when rating the same investment: $\rho\hat{\sigma}_{ij,s}$

The market becomes a monopoly when only one of the two CRAs identifies a bad investment or when they rate different investments, and only one of them survives.

The probability that one CRA pinpoints a bad investment and the other fails to do so: $\rho(1 - \sigma_{j,s}) z_{i,s}$

The probability that the two CRAs rate different investments and only one of them survives: $(1-\rho)(1-\sigma_{j,s})\sigma_{i,s}$

We make the lemma that the CRAs' investments in ratings quality are strategic substitutes. This means that if one of the CRAs decides to raise its investment in rating quality, the other one will reduce its investment in response to the action. The reverse situation is also true. Indeed, when CRA i raises its investment in ratings quality, this increases the possibility that it will survive through the following periods. This further demonstrates that this will reduce the future pay-off for the CRA j, whilst at the same time creating an incentive for CRA j to reduce its quality in ratings. The fact that the CRAs' investments in ratings quality are strategic substitutes ensures that a unique symmetric equilibrium exists.

We put forward the proposition that when the states are independent over time (when $\tau_B = 1 - \tau_R$) and if labor-market conditions do not vary over time, there is less investment in ratings quality during a boom than during a recession. The effect of varying the labor-market conditions is, in actual fact, rather ambiguous.

We can consider that there are now two impacts on a CRA's incentives: a direct impact and a strategic impact. The direct impact has the same impact on incentives as has already been explained in the monopoly scenario to provide ratings quality. The strategic impact appears when a change in the parameters of CRA i affects the activity of the CRA j, which may in turn affect the probability of becoming a monopolist.

For three of the four parameters, the direct impact outweighs the strategic impact: default probability (p_s), fees (π_s) and the proportion of good investments (λ_s). This is not the case, however, for the fourth parameter: the labor-market (γ_s). Indeed tighter labor-market condition increase the cost of the rival CRA when all other conditions are constant. Therefore, when CRA i reduces its ratings quality, this gives CRA j an incentive to raise its own quality. This is in contrast to the direct effects of increased costs for accuracy. Once again, the result on labor-market tightness is ambiguous. We can assume that it is more beneficial for a CRA in the duopoly model with correlation, to be in a boom state than in a recessionary state. We therefore introduce a new notation for the first assumption (A1): $V^*_{M,B} > V^*_{M,R}$

We can also introduce a second assumption (A2): the value to a CRA in a duopoly of being in a boom is larger than the value of being in a recession ($V^*_{D,B} > V^*_{D,R}$). We also make the proposition that if there is a mean reversion between states ($\tau_B > 1 - \tau_R$), if A1 and A2 are remain steady, and if labor-market conditions do not vary over time, there is a lower level of investment in ratings quality during a boom than during a recession. The effect of varying labor-market conditions is, once again, rather ambiguous.

Counter-cyclical ratings quality can also be characteristic of a competitive ratings market. Interestingly, in that case, although competition changes the value of maintaining a CRA's reputation relative to a market which is dominated by a monopolist, the economic fundamentals will shift the incentives in a way very similar to that of the monopolist. There is an exception to this; tighter labor-markets during booms can bring about either counter-cyclical or pro-cyclical accuracy in ratings.

4.1.1. Stress test analysis

Next we will look at the equation $\hat{\sigma}_{ij,s}^{IP} := 1 - (1 - \lambda_s)(1 - z_{i,s}z_{j,s})p_s$ which is used to denote the probability that both CRAs i and j survive when they rate the same issue. We know that $z_{i,s}$ represents the ability of analysts of a CRA i in the state s, and that $z_{j,s}$ represents the ability of analysts of a CRA j in the sates s. As we defined earlier the function $z(w_s, \gamma_s) = \sqrt{w_s}/\gamma_s$, it becomes $\sqrt{w_{i,s}}/\gamma_{i,s}$ for the first CRA and $\sqrt{w_{j,s}}/\gamma_{j,s}$ for the second CRA in the duopoly. Knowing that $z_{i,s}$ and $z_{j,s} \in [0,1]$, the first thing that is required is to see what value $(1 - z_{i,s}z_{j,s})$ can take, as it represents the probability that at least one of the two CRAs will give a good rating. The graphs below show the results under two perspectives:

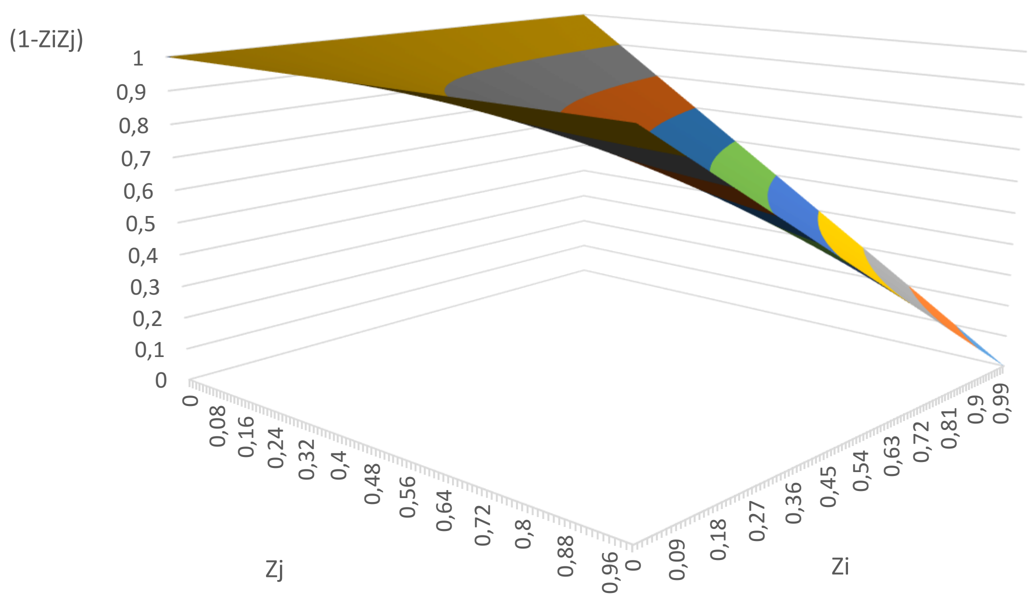

Figure 7. Perspective 1: The correlation between the ability of analysts of the CRA i and the CRA j in the independent punishment strategy.

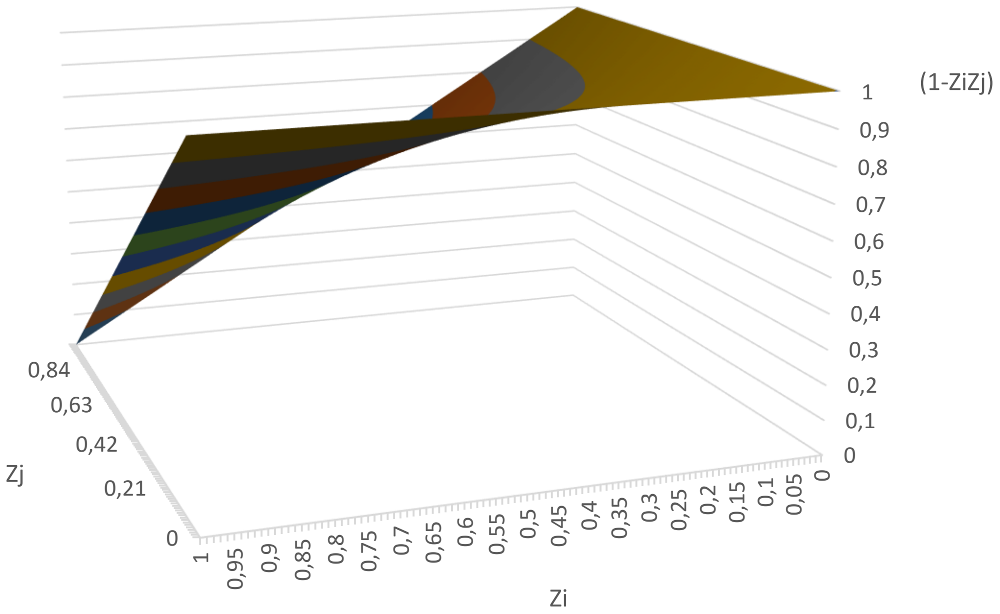

Figure 8. Perspective 2: The correlation between the ability of analysts of the CRA i and the CRA j in the independent punishment strategy.

The values for $z_{i,s}$ and $z_{j,s}$ were set between 0 and 1. As can be seen on the graph, the values of $(1 - z_{i,s}z_{j,s})$ are all between 0 and 1 and have a mean of 0.75. We can see on the graphs that as long as either $z_{i,s}$ or $z_{j,s}$ equals 0 then the result of $(1 - z_{i,s}z_{j,s})$ equals 1 and the maximum is reached. As far as the minimum is concerned, there is only one combination that lead to make the equation equal 0 and that is when $z_{i,s}$ and $z_{j,s}$ equal 1. If $(1 - z_{i,s}z_{j,s})$ equals 1, this means that there is 100% probability that at least one CRA will give a good rating. The probability becomes zero, however, when $(1 - z_{i,s}z_{j,s})$ equals 0. The shape of the surface is concave from the right angle of the maximum values to the point where $(1 - z_{i,s}z_{j,s})$ equals 0. The shape is convex between the combination $z_{i,s} = 0$, $z_{j,s} = 1$ and the combination $z_{i,s} = 1$, $z_{j,s} = 0$.

As a result of the above we can say that $(1 - z_{i,s}z_{j,s}) \in [0,1]$. We will therefore vary this equation and the lambda in order to analyse the sensitivity of $\hat{\sigma}_{ij,s}^{IP} := 1 - (1 - \lambda_s)(1 - z_{i,s}z_{j,s})p_s$. As defined earlier, λ_s is the probability that an investment is good. The equation $(1 - \lambda_s)$ is used to calculate the probability that the investment is bad. The aim of this analysis is to see how sensitive the probability that both CRAs i and j survive is when they rate the same issue. This will be a variation of the probability that at least one CRA gives a good rating and also a variation of the probability that the investment is good. The values for lambda and $(1 - z_{i,s}z_{j,s})$ ranged between 0 and 1 and a probability of default (i.e. p_s) of 10% was also allowed for in the equation. The graphs below show the results under two perspectives:

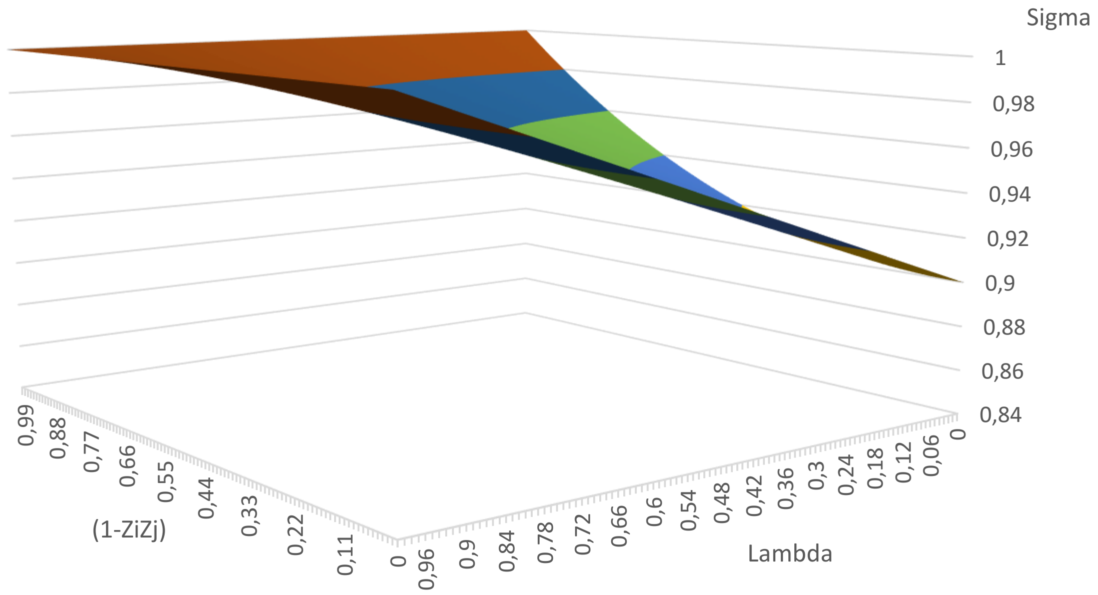

Figure 9. Perspective 1: Sensitivity of probability that both CRAs i and j survive when they rate same issue to a variation of the probability that at least one CRA gives a good rating and of the variation of lambda.

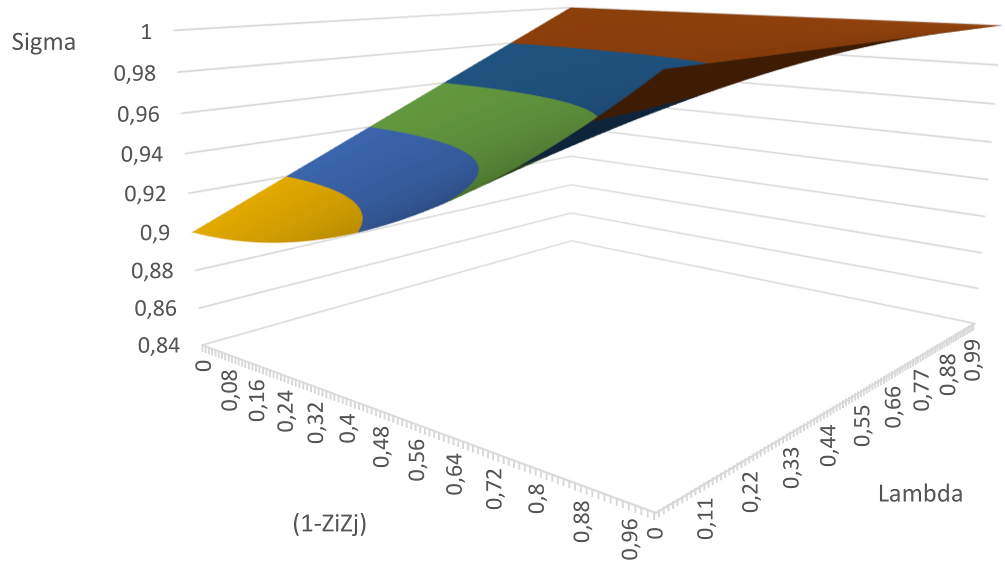

Figure 10. Perspective 2: Sensitivity of probability that both CRAs i and j survive when they rate same issue to a variation of the probability that at least one CRA gives a good rating and of the variation of the probability that the investment is good.

As we considered a probability of default of 10%, the minimum value that sigma can take is 0.9. This is only possible when lambda and $(1 - z_{i,s}z_{j,s})$ equal 0. The equation equals 1 when either lambda or $(1 - z_{i,s}z_{j,s})$ equal 1 regardless of the values of the other variable. If the value of $\left(1 - z_{i,s}z_{j,s}\right)$ is fixed, we can see that as the lambda increases then so does the sigma. The conclusion that can be drawn from this is that lambda and sigma are positively and also proportionally correlated. As far as $(1 - z_{i,s}z_{j,s})$ is concerned, we can see on the graph that the shape of the surface is convex. This means that for a given level of lambda, the more $(1 - z_{i,s}z_{j,s})$ is high, the more a modification of it has a huge impact on the equation of the sigma. These results showed that the probability that both CRAs i and j survive when they rate the same issue is proportionally and positively correlated to a modification of the probability that the investment is good and is positively correlated to a modification of the probability that at least one CRA gives a good rating.

4.2. The linked punishment strategy

In this second punishment strategy, we consider that if the two both CRAs give a positive rating to one particular issue and that issue subsequently defaults, then neither will be punished. However, if only one of the CRAs makes the mistake of positively rating an investment that subsequently defaults, the investors will no longer work with it.

In state s, where $s \in \{B, R\}$, the value function for a duopolist can be expressed in the following way:

$$V_{i,s} = \pi_{D,s}\alpha_{i,s} - w_{i,s} + \delta\left[\rho\hat{\sigma}_{ij,s}^{LP} + (1-\rho)\sigma_{i,s}\sigma_{j,s}\right]EV_{D,s}^{*} + \delta(1-\sigma_{j,s})\left[\rho z_{i,s} + (1-\rho)\sigma_{i,s}\right]EV_{M,s}^{*}, \quad (15)$$

where

$$\hat{\sigma}_{ij,s}^{LP} := 1 - (1-\lambda_s)(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})p_s \quad (16)$$

It is noteworthy that equations 13 and 15 are very similar. Indeed the only difference between them is that $\hat{\sigma}_{ij,s}^{LP}$ is replaced by $\hat{\sigma}_{ij,s}^{IP}$ in the second value function. The term $\hat{\sigma}_{ij,s}^{IP}$, is related to independent punishment, and expresses the probability of survival for the two CRAs when they both rate the same issue and live to tell the tale. The term $\hat{\sigma}_{ij,s}^{LP}$, is related to linked punishment, and expresses the probability of survival for the two CRAs when they both rate an issue that subsequently defaults. In this case, the CRAs would not survive in the case of independent punishments.

If $[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV_{D,s}^{*} - EV_{M,s}^{*}\right] + \rho EV_{D,s}^{*} < 0$, it means that the CRAs' investments in ratings quality are strategic substitutes. In that case, and if CRA i deceases the quality of its ratings, it will result in CRA j facing a higher probability of become a monopolist, which would then result in an increase in the investment of CRA j in terms of its ratings quality.

If $[\rho + (1-\rho)(1-\lambda_s)p_s]\left[EV_{D,s}^{*} - EV_{M,s}^{*}\right] + \rho EV_{D,s}^{*} > 0$, it means that the CRAs' investments in ratings quality strategically complement each other. In that case, and if CRA i decreases the quality of its ratings, it becomes more likely that CRA j will escape punishment for having made the mistake of referring to a bad investment as good. This effect would lead CRA j to reduce its investment in ratings quality.

To summarise, in the first (strategic substitutes) case, the decrease in CRA i's investment in ratings quality will lead CRA j to increase its own quality. In the second (strategic complement) case, reduction of CRA i's investment in ratings quality will also lead to a decrease in the quality of CRA j's ratings.

Strategic substitutes

There is a unique symmetric solution. When the punishments are linked, when investments in ratings quality are strategic substitutes and if states are independent over time (as we have already observed $\tau_B = 1 - \tau_R$), there will be a lower investment in ratings quality in a boom than in a recession in the case where labor-market conditions do not vary over time. The effect of varying the labor-market conditions is rather ambiguous.

When the punishments are linked, when investments in ratings quality are strategic substitutes, if the assumptions A1 and A2 are held, and if there is a mean reversion between states (when $\tau_B > 1 - \tau_R$), there will be a lower investment in ratings quality in a boom than in a recession in the case that labor-market conditions do not vary over time. The effect of varying the labor-market conditions is rather ambiguous.

Strategic complements

In the aforementioned case, it is possible to have a corner solution and/or multiple symmetric equilibria. This equilibrium will be symmetric, meaning that CRA i and j will invest the same amount of money in ratings quality in both a boom and a recession. Three conditions are sufficient to guarantee the existence and the uniqueness of an equilibrium:

(1) $EV^*_{M,s} > EV^*_{D,s}$ for $s \in \{B, R\}$

This first condition affirms that the expected value of a CRA is larger in a monopolist ituation than in that of a duopolist.

(2) $\pi_{D,s}$ is small for $s \in \{B, R\}$

The second condition is coherent with the first condition. Indeed both could represent a situation of Bertrand competition[1] for which $\pi_{D,s} = 0$ and $\pi_{M,s} > 0$ for $s \in \{B, R\}$.

(3) $\frac{\partial^3 z}{\partial w^3} \leq 0$

The last condition is the third derivative of $z(w)$.

A unique equilibrium exists when these three conditions are held, when the punishments are linked and when the investments in ratings quality are strategic complements.

When these three conditions are remain static, when the punishments are linked, when the investments in the quality of ratings are strategic complements and if states are independent over time (when $\tau_B = 1 - \tau_R$), the result is a reduced level of investment in ratings quality during a boom than during a recession. In the case of strategic substitutes, the results for probability of default, for fees, and for the fraction of good issues remain identical. Moreover, this results in an unambiguous result with respect to the lack of movement in the employment market. While the labor-market is tighter, the investment in ratings quality shows a sharp drop, as was shown to be the case in the monopoly market. Indeed, the strategic effect has switched, when tightness in the labor-market increases, it makes the competitor CRA lower its own investments. At the same time, on mean reversion, the results also hold.

When all three of these conditions remain constant, when the punishments are linked, when the investments in quality of ratings are strategic complements, when the assumptions A1 and A2 do not fluctuate, if there is a mean reversion between states (when $\tau_B > 1 - \tau_R$), the result is a lowering of investment in ratings quality during a boom than during a recession. There are also counter-cyclical ratings quality for all the variables. Moreover, the labor-market tightness is no longer ambiguous due to the strategic effect and the direct effect going in the same direction.

[1] It is a model of competition used in economics that describes interactions among sellers that set prices and the buyers that choose quantities at the prices set.

4.2.1. Stress test analysis

The next equation $\hat{\sigma}_{ij,s}^{LP} := 1 - (1 - \lambda_s)(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})p_s$ is used to denote the probability that both CRAs survive when they both rate the same issue that subsequently defaults and where $(1 - \lambda_s)$ is the probability that the investment is bad (because λ_s is the probability that it is good). In the second punishment strategy, we have considered that if both CRAs give a positive rating to the same issue that defaults then, no one will be punished. But if only one of both CRAs makes the mistake to positively rate an investment that later defaults, the investors will stop working with it. In a similar way to the independent punishment strategy, we will first analyse $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$. We already know that $z_{i,s}$ represents the ability of analysts of a CRA i in the state s, and that $z_{j,s}$ represents the ability of analysts of the CRA j in the state s. As described earlier, the equation $z(w_s, \gamma_s) = \sqrt{w_s}/\gamma_s$, becomes $\sqrt{w_{i,s}}/\gamma_{i,s}$ for the first CRA and $\sqrt{w_{j,s}}/\gamma_{j,s}$ for the second CRA in the duopoly. Knowing that $z_{i,s}$ and $z_{j,s} \in [0,1]$, the first thing that we have to do is to see what value $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ can take. The graph below shows the results:

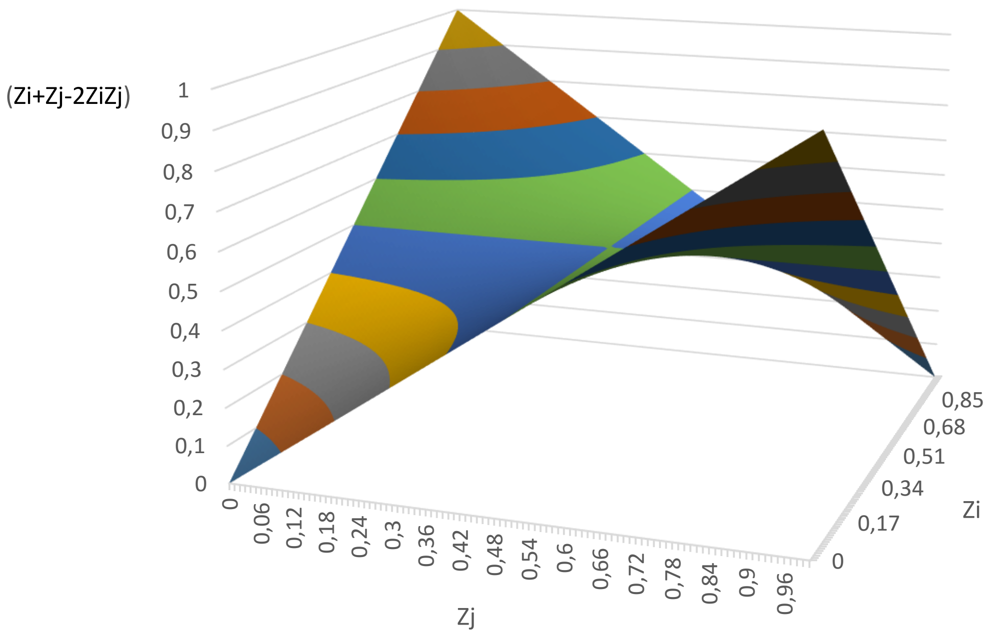

Figure 11. The correlation between the ability of analysts of the CRA i and the CRA j in the linked punishment strategy.

We varied the values of $z_{i,s}$ and $z_{j,s}$ between 0 and 1. On the graph, we can see that the values of $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ are all between 0 and 1, have a mean of 0.5, and the chart is consequently symmetrical. There are two minimum and two maximum values for $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$, and the minimum values are reached when $z_{i,s}$ and $z_{j,s}$ equal 0 and when they equal 1. The maximum values are reached when $z_{i,s} = 0$, $z_{j,s} = 1$ and the combination $z_{i,s} = 1$, $z_{j,s} = 0$. The shape of the surface is symmetrical, but it is concave between the two minimums and convex between the two maximums.

Now that we know that $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s}), \in [0,1]$, we will make this equation and the lambda vary in order to analyse the sensitivity of $\hat{\sigma}_{ij,s}^{LP} := 1 - (1 - \lambda_s)(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})p_s$. The aim of this analysis is to see what the sensitivity of the probability is that the two CRAs survive when they both rate the same issue that subsequently defaults to a modification of the probability that at least one CRA gives a good rating and of the probability that the investment is good. We gave values for lambda and $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ between 0 and 1. What is more we considered a probability of default (i.e. p_s) of 10% in the equation. The graphs below show the results under two perspectives:

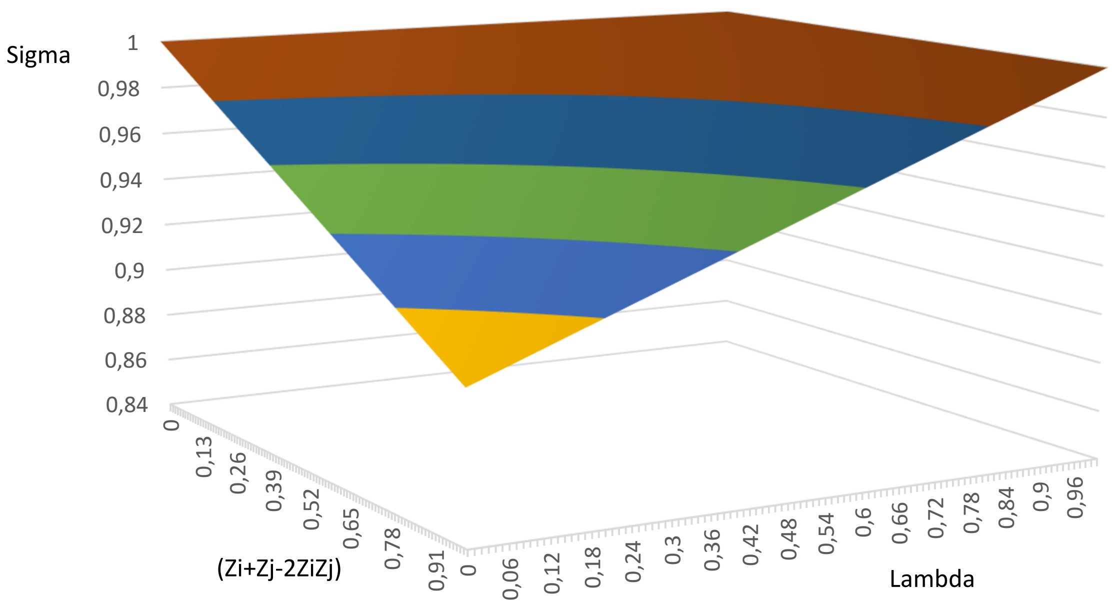

Figure 12. Perspective 1: Sensitivity of the probability that the two CRAs survive when they both rate the same issue that subsequently defaults to a modification of the probability that at least one CRA gives a good rating and of lambda.

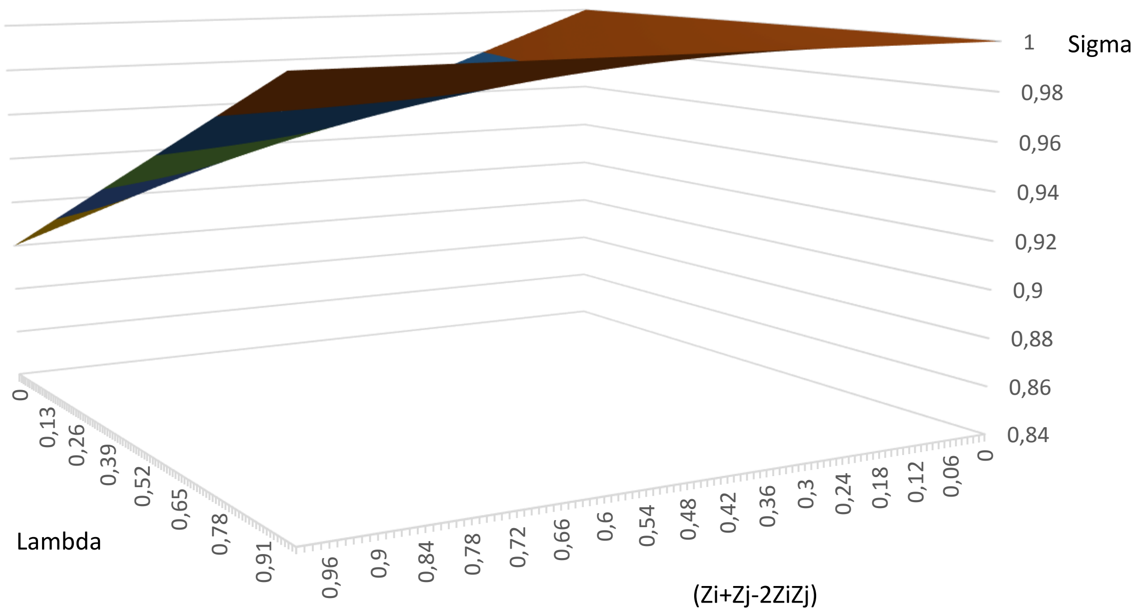

Figure 13. Perspective 2: Sensitivity of the probability that the two CRAs survive when they both rate the same issue that subsequently defaults to a modification of the probability that at least one CRA gives a good rating and of lambda.

As we considered a probability of default of 10%, the minimum value that sigma can take is 0.9. This is reached when lambda equals 0 and $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ equals 1. The result of the equation is 1 when lambda equals 1 regardless of the values of $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ and when $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})0$ regardless of the values of lambda equals. For a fixed value of $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ we can see as the lambda increases then so does the sigma and inversely, this is a linear rise/decrease. The conclusion is that lambda and sigma are positively and proportionally correlated. If we take $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ for a given value of lambda then we can see on the chart that as it increases, then so does the sigma and inversely. Once again this rise or decrease is linear. It also means that sigma and $(z_{i,s} + z_{j,s} - 2z_{i,s}z_{j,s})$ are positively and proportionally correlated. The results show that the probability of the two CRAs surviving when they both rate the same issue that defaults at a later date is positively and proportionally correlated to the probability that at least one CRA gives a good rating and to the probability that the investment is good.

5. Conclusion

The aim of this master's thesis was to analyse how the theoretical model *Ratings quality over the business cycle* (Bar-Isaac & Shapiro, Ratings quality over the business cycle, 2013) is sensitive to the modification of some of its most interesting parameters. These parameters were gamma which is the parameter that captures the labor-market conditions and that is used in equation of the ability of analysts; lambda that is the probability that the investment is good and tau that is probability of a transition from the current state to another one.

The theoretical model analysed how a CRAs' incentives to provide ratings of high quality vary depending on which business cycle (boom or recession) exists. Booms are characterised by larger revenues for the CRAs, lower levels of unemployment, a larger proportion of good projects and a lower average probability of default than during recessions. When the economic shocks are independent and identically distributed, the ratings quality is lower during booms than during recessions. This is called the counter-cyclicality of ratings and is due to incentives to milk a reputation. Those incentives increase when there is a mean reversion (i.e. when shocks are negatively correlated) and they decrease when shocks are positively correlated. In order to assess the robustness of the model naïve investors are added which does not appear to modify its qualitative results. A distinction was also made between a monopoly and a duopoly and it appeared that in both cases the counter-cyclical ratings quality holds.

First, we considered the market as being a monopoly, the stress analysis that we realised on the equation of alpha concluded that the probability that an investment gets a good rating is positively and proportionally correlated to the ability of analysts and is negatively and proportionally correlated to the probability that the investment is good. Then we analysed the equation of sigma and it resulted that the probability that a CRA survives into the future increases when the probability that the investment is good becomes higher.

After that, focussed on the value functions of the monopoly. We first wanted to analyse the sensibility of the expected value function to a modification of tau and alpha. The results showed that the expected value function is positively correlated to the probability of a transition from the current state to the other state until alpha reaches a certain level (which itself depends on the values that we fixed for the other parameters of the equation), then it becomes negatively correlated. They also demonstrated that the expected value function is positively and proportionally correlated to the probability that the investment gets a good rating.

Next we analysed the sensibility of the expected value function to a modification of tau and sigma. The results showed that the expected value function is positively correlated to the probability of a transition from the current state to the other state until sigma reaches a certain level (which itself depends of the values that we fixed for the other parameters of the equation), then it becomes negatively correlated. They also showed that the expected value function is positively correlated to the probability that the CRA survives into the future.

For the next stage, we considered the market in terms of being a duopoly and we identified two different punishment strategies. In the independent punishment strategy, the results of the stress test we carried out showed that the probability of both CRAs i and j surviving when they rate the same issue is proportionally and positively correlated to the probability that the investment is good. It is also positively correlated to the probability that at least one CRA will give a good rating. In the linked punishment strategy, our stress test showed that the probability of the two CRAs surviving when they both rate the same issue that is subsequently positively and proportionally correlated to the probability that at least one CRA gives a good rating and to the probability that the investment is good.

As a more general conclusion, I would like to say that working on this technical and complex subject was a real challenge for me. However it was an extremely interesting experience and taught me a great deal.

6. Literature references

Bar-Isaac, H., & Shapiro, J. (2011). Credit ratings accuracy and analyst incentives. *American Economic Review Papers and Proceedings 101*, 120-124.

Bar-Isaac, H., & Shapiro, J. (2013). Ratings quality over the business cycle. *Journal of Financial Economics 108*.

Bolton, P., Freixas, X., & Shapiro, J. (2012). The credit ratings game. *Journal of Finance 67*.

Bond, P., & Glode, V. (2011). Bankers and regulators. *Unpublished working paper. University of Minnesota and University of Pennsylvania*.

Faure-Grimaud, A., Peyrache, E., & Quesada, L. (2009). The ownership of ratings. *RAND Journal of Economics 40*, 234-257.

Mariano, B. (2012). Market power and reputational concerns in the ratings industry. *Journal of Banking & Finance 36*, 1616-1626.

Mathis, J., McAndrews, J., & Rochet, J. (2009). Rating the raters: Are reputational concerns powerful enough to discipline rating agencies? *Journal of Monetary Economics 56*, 657-674.

Mattarocci, & Gianluca. (2013). *The Independence of Credit Rating Agencies : How Business Models and Regulators Interact.* Burlington : Elsevier Science.

Povel, P., Singh, R., & Winton, A. (2007). Booms, busts, and fraud. *Review of Financial Studies 20*, 1219-1254.

Rhee, R. J. (2015). Why credit agencies exist. *Economic Notes*.

Sangiorgi, F., Sokobin, J., & Spatt, G. (2009). Credit-rating shopping, selection and equilibrium structure of ratings. *Unpublished working paper. Stockholm School of Economics, Securities and Exchange Commission, and Carnegie Mellon University*.

Skreta, V., & Veldkamp, L. (2009). Ratings shopping and asset complexity: a theory of ratings inflation. *Journal of Monetary Economics 56*, 678-695.

Strausz, R. (2005). Honest certification and the threat of capture. *International Journal of Industrial Organization 23*, 45-62.

7. Appendices

7.1. List of figures

Executive summary

This research thesis is based on a theoretical model that analysed how a CRAs' incentives to provide ratings of high quality vary depending on which economic cycle (boom or recession) exists. From the results it appears that ratings quality is lower during booms than during recessions. This is known as the counter-cyclicality of ratings and is due to incentives to milk a reputation. A distinction is also made between a monopoly and a duopoly and it is clear that in both cases the counter-cyclical ratings quality holds.

The aim of this master's thesis is to analyse by means of a stress test how this model is sensitive to a modification of three parameters. The three parameters are gamma which is the parameter that captures labor-market conditions, lambda that is the probability that an investment is good and tau that is the probability of a transition from the current state to another one. Again, a distinction is made between a monopoly and a duopoly.