Understanding the volatility of European REITs : An approach based on GARCH models

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Understanding the volatility of European REITs : An approach based on GARCH models

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1 Introduction

The attraction of investors for real estate has been known for years. Whether in Europe or abroad, the real estate market is reaching record levels, especially in this unusual year. For example, the French residential market has increased by 6% compared to last year and by an average of 17% over the last five years (Savills report). According to Savills, the net rental yields are between 3 and 3.5% according to the International Real Estate Council. Moreover, the cost of buying a property in Europe seems to be generally increasing but at the same time the rental income seems to be stagnating. This is where real estate investment trusts could be of interest to the investors or portfolio managers. Indeed, due to their legal and fiscal structure, REITs are high-dividend yield assets. REITs are also available to many more "little" investors since one can buy a single REIT stock for 100 euros as well as 1,000 shares for 100,000.00 euros. However, these are companies listed on the stock exchange and therefore more prone to fluctuation than a single personal property. REITs must deal with risk and therefore it is critical for investors to understand the behavior of this risk, whether in times of economic stability or in times of crisis such as the GFC or COVID-19.

A lot of research on REITs has been conducted in the United States, Australia and Japan because these markets are mature. It seems that the European REIT market is much less developed; less than 20 years old. It therefore seemed crucial to me to study the European market through different indices specific to each country of my sample: FTSE Belgium REIT, S&P France REIT, S&P Netherlands REIT, FTSE Germany REIT, S&P Spain REIT and FTSE United Kingdom REIT. My analysis focused on a well-known measure of risk, which is volatility. But not just any volatility, I was interested in **dynamic and past-conditional volatility**. Through three different models ($GARCH$, $EGARCH$ and $GJR-GARCH$) that capture what is called heteroscedasticity, I have attempted to model volatility over time. The period studied is from Q3-2007 to endQ3-2021 and is divided into three subperiods: the Global Financial Crisis period going from 8^{th} November 2007 to 25^{th} March 2009, the normal economic cycle period going from 19^{th} September 2014 to 31^{st} January 2020 and the COVID period going from 1^{st} February 2020 to 18^{th} November 2020.

How do REITs behave, both in times of crisis and in normal times in terms of volatility? Which GARCH model seems to best fit my REITs data, knowing that Hansen and Lunde (2005) showed that GARCH (1,1) is often the best one? Would it be interesting for me to have European REITs in my portfolio? All these questions are compelling to portfolio managers and risk managers. Indeed, the former will need to have a clear idea of the volatility behavior in order to effectively pick stocks that will maintain the level of portfolio risk[1]. And, the latter will be able to compute relative measures of risk instead of absolute measures since the risk is becoming time-varying (i.e., Value at Risk is now time-varying).

The paper is structured as follows: the next section contains a literature review and some

[1]The portfolio manager's client may be risk averse or risk-seeking.

information on Real Estate Investment Trusts as it remains a little-known financial asset by investors. Afterwards, the following section includes a comprehensive methodology that will encompass different GARCH models; namely three univariate models (GARCH, EGARCH, GJR-GARCH) and one multivariate model (DCC-GARCH for correlation purpose). All the methods used in this master thesis and all the theoretical foundations are anchored. Then, the data are presented as well as descriptive statistics. The next section contains the analysis and the results. A conclusion and potential suggestions for further research finalize this master thesis.

2 Literature review and paving the way for European REITs

2.1 Definition of REITs

First of all, let's define what REITs are. The term "REIT" stands for "Real Estate Investment Trust". These are companies that own or finance income-producing real estate across a range of property sectors. These real estate companies have to meet a number of requirements to qualify as REITs. Most REITs trade on major stock exchanges, and they offer a number of benefits to investors. REITs allow anyone to invest in portfolios of real estate assets the same way they invest in other industries – through the purchase of individual company stock or through a mutual fund or exchange traded fund (ETF). The stockholders of a REIT earn a share of the income produced through real estate investment – without actually having to go out and buy, manage or finance property. However, during this master thesis I will focus on six European REITs: BE-REITs, SIIC, G-REITs, SOCIMI, FBI and UK-REITs. Respectively, these are real estate investment trusts from Belgium, France, the Netherlands, Germany, Spain and the United Kingdom.

As said in CFA level II, real estate investment trusts have specific advantages. One can mention exemption from taxation at the corporate level if some well-defined requirements are fulfilled or even the predictability of the earnings generated since REIT's rental income is fixed by contracts. Nevertheless, the major drawback is that the REIT's price is determined by the stock market. “While the appraisal-based value of a REIT may be relatively stable, the market-determined price of a REIT share is likely to be much more volatile”. Moreover, volatility seems to be poorly reflected in the appraisals and those appraisals tend to be scarce and backward-looking whereas the stock market is evidently continuous and forward-looking.

In a general way, REITs display a number of key features in comparison to direct real estate. The first one is the superior liquidity. REITs shares can be sold continuously on a stock exchange. Then, it requires a lower minimum investment whereas one may need several thousand euros to buy a real estate property. Afterwards, one can have access to premium properties, at least a tiny share of those properties. By acquiring a REIT share, one possesses a very small part of the whole properties' portfolio. That is a way to benefit from diversification. Another one comes from the fact that REITs employ active professional managers to control the whole process of acquiring, renting or selling properties. Finally, REITs stockholders boast of high dividend yield, which is truly a reality since Real Estate Investment Trusts must pay out most of their income as dividends to maintain their advantageous status. It should be noted that each country has sometimes specific rules but that the overall logic behind REIT is homogenized.

There are currently 17 BE-REITs, 27 SIICs, 6 G-REITs, 76 SOCIMIs, 5 FBIs and 57 UK-REITs[2] . They are active in various sectors such as residential real estate, retail real estate, industrial real estate, office real estate, storage real estate or hotel real estate. A REIT can be

[2]https://prodapp.epra.com/media/EPRA-Global-REIT-Survey-2020-1597930925323.pdf. Notice that UK left the EU on 31 January 2020.

active in a single sector, such as Befimmo[3] , which is only active in office real estate, or it can diversify and invest in warehouses as well as in local shops or residential real estate – one will call it diversified REIT.

2.2 An Overview of Related Literature

In November 2020, I attended a short webinar organized by Nareit[4] and I wrote some insights about the current situation. One of the speakers, Nicole Funari, NAREIT's vice president of research, discussed the current market conditions in the REIT industry. She said that REITs entered the COVID-19 crisis with strong balance sheets and have shown resiliency. However, the pandemic has impacted each REIT sector differently. "Almost two-thirds of REITs aren't affected as much by the health crisis, although they are affected by the overall economic downturn," said the speaker. Lodging/resorts and retail sectors have been the most negatively impacted by the pandemic, while some sectors, such as data centers and infrastructure, have seen a positive impact due to changing consumer habits. Those figures are quite nice but are they reliable? We have to be careful when we look at some figures on NAREIT website since they do promote REITs as a perfect investment and it is essentially dedicated to U.S real estate. The question is: what about the scope I chose, i.e., the six European countries? Indeed, we often hear during conferences, seminars or in the financial news that REITs are resilient. Is this really the case or is it just an argument to push investors to buy some REITs to diversify their portfolio?

There is no denying that the research and papers on the behavior of REITs were mainly concerned with the US and Australian markets as they are mature and have provided this property investment vehicle for more than 40 years. It is therefore important to try to expand the scope of research on European REITs. As a first step, thanks to Sotelo and McGreal (2013), we need to lay the foundations for the history and development of the European REIT market. First, the expansion of the European REIT market has taken place in the post-2000 period[5], prompted into action by the relatively strong performance of US and Australian REITs. Due to population growth, the need for real estate is increasingly becoming a crucial point of interest for every single country. Real estate is the baseline for economic growth and fulfilment. Moreover, it is a crucial source of employment. The European commercial real estate is estimated to represent EUR 7.27 trillion[6] of assets, 5.58% of which is devoted to listed real estate. This trend shows no sign of abating in light of its strong long-term performance compared to other European assets and its moderate long-term correlation with financial sector stocks. A recent study of Oxford Economics (2019) demonstrated the added value of LRE in a risk-adjusted returns multi-asset

[3]One of the 17 belgian REITs.

[4]Nareit serves as the worldwide representative voice for REITs and real estate companies with an interest in U.S. real estate.

[5]Except FIB (Dutch REIT) that has been introduced in 1969.

[6]« Features and trends in European listed real estate », EPRA (November 2020).

portfolio. Indeed, an optimal portfolio in a mean-variance framework could be obtained by allocating 10% to %25 to LRE – depending on the risk aversion. Nevertheless, one must be aware that the short-term correlation of LRE[7] with equity markets is relatively high. We will have the opportunity to investigate this latter assertion in the empirical part of this master thesis.

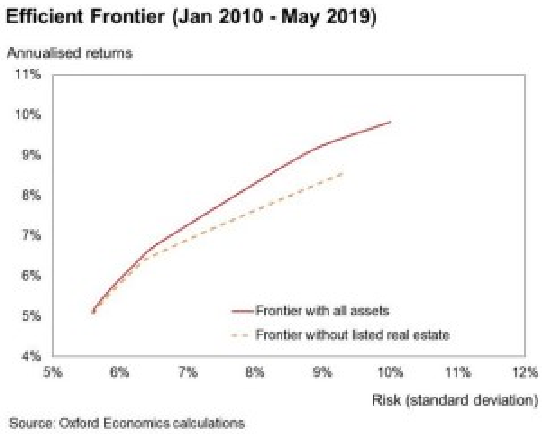

Figure 1: Graph showing the efficient frontier of a portfolio with and without LRE.

In addition, one of the distinctive elements of REITs compared to traditional stocks is that they are halfway between real estate and the stock exchange market. In the introduction of their book, Sotelo and McGreal (2013) inform us about the ongoing debate about whether REITs reflect the behavior of the underlying direct real estate market or not. They draw attention to the fact that the main difference between direct real estate and listed real estate is the basis of valuation. Indeed, the former uses different valuation techniques such as DCF, cost approach or comparable approach but the latter uses simply the quotation of the REIT on the stock exchange with a fundamental underlying notion that is expectation. If investors expect good news, they will be eager to buy the stock, which in turn will drive up the stock price. This is the whole dynamic of the financial markets: how the release of news differs from market expectations. Although, on the long run, private real estate and securitized real estate seem to display a common trend (Campeau, 1994; Glascock et al., 2000), one must be aware of the potential sudden turmoil[8] that can affect the broad financial equity markets and consequently the REITs stock prices. Moreover, a more severe and clear-cut view has been stated by Kizer and Grover (2017). After performing several statistical methods, they found tangible evidence against the legitimacy of real estate investment trusts as a different asset class. While knowing that the following criteria are not flawless, they listed four key criteria that prove and define that an asset is beyond the scope of a particular asset class: low correlation with established asset classes, statistically significant positive alpha, inability to be replicated on a comovement basis by a long-only portfolio of established assets and improvement of the mean-variance frontier when

[7] Listed Real Estate= LRE.

[8] I am referring to the global financial crisis (GFC) or the COVID-19 crisis for example.

added to a portfolio of traditional stocks and bonds. They point out that REITs deserve their place in a diversified portfolio of assets but that they behave more like the stock market than direct real estate market. All this leads us to study and understand the risks linked to REITs on the financial markets and therefore in an inseparable way to inquire about the **volatility** of my European REITs sample.

During the years surrounding the financial crisis, Real Estate Investment Trusts in the U.S. have shown a towering volatility, much higher than direct real estate for example. In the *Real Estate Economics*, Sun and al. (2015) highlighted that the riskiness of REITs largely increased during the global financial crisis. Between 2007 and 2009, the beta of NAREIT index fluctuated between 1 and 1.2, which indicates that REIT stock prices swing more widely in comparison to the overall market. The REITs experiencing a high debt-to-asset ratio, more variable interest rate debt and more debt coming due in 2008-2009 were the most affected by the GFC. This article bears out the time-varying risk faced by REITs in the sense that volatility will not be the same in periods of turmoil and in periods of relatively steady growth.

The subprime crisis has considerably weakened the REITs industry. Indeed, they are legally bound to pay out 80 to 90 percent of their net income (depending on the country-specific regulation), which limits their ability to use retained earnings to finance their acquisitions. During the 2007-2009 financial crisis, it was complicated to raise fund on the market (Case and al., 2012). In their paper, Huerta and al. (2016) argue that the financial crisis helped explain the significant increase in REITs volatility. They assert that the lack of funds available on the markets or from banks and the lack of opportunities have plunged the whole REITs industry in a period of disorder. By adding the behavioral finance viewpoint, one can say that investor perception and expectations can have huge influence on the behaviors of REITs volatility since a positive change in aggregate sentiment (herding behavior) will affect volatility negatively.

In the figure 2, one can observe the V2TX on a daily basis, which is the volatility benchmark for EU.

"The VSTOXX Indices are based on EURO STOXX 50 real-time options prices and are designed to reflect the market expectations of near-term up to long-term volatility by measuring the square root of the implied variance across all options of a given time to expiration. The VSTOXX Indices are part of a consistent family of volatility indices: VSTOXX based on the EURO STOXX 50 and VDAX-NEW based on the DAX."

Nonetheless, REITs are subject to risks. One of the major threats is the mismatch between supply and demand. During COVID-19, one can imagine that the occupancy rates of hotels drastically decrease and lead to a lack of rental incomes. For retail stores, it is slightly dissimilar because they are contractually obliged to pay rent – but obviously the slowdown of activities can lead to bankruptcies and consequently a higher uncertainty on the market. Since early March 2020, European financial markets are experiencing high volatility with the V2TX reached a peak of 84.79 on March 16th 2020 (data displayed on a daily basis).

In their paper entitled "Dynamic correlations between REIT sub-sectors and the implica-

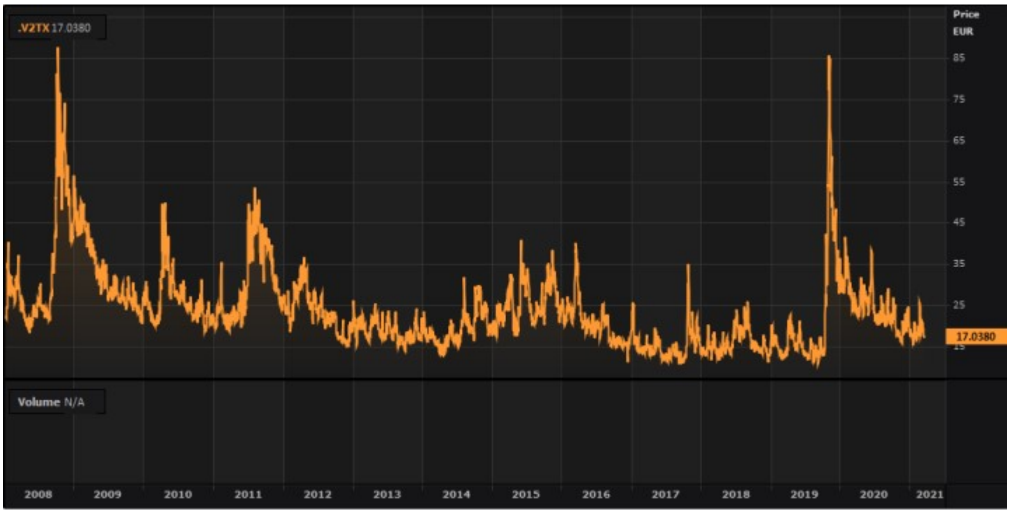

Figure 2: VSTOXX gaph and evolution through time

tions for diversification", Chong and al. (2012) focus on the existing relationship between REITs, depending on their membership to a particular property sector (i.e., residential, retail, health, office). Through the use of a Generalized Autoregressive Conditional Heteroscedasticity Dynamic Control Correlation (GARCH-DCC) framework, the daily conditional correlations disclose that since the 1990's there has clearly been a progressive upward trend in terms of correlation between US REITs sub-sectors. More generally, this means quite precisely that, regardless of their preferred sector type, REITs would behave in an increasingly homogeneous way. This could lead to significant consequences for asset managers, individual investors or risk managers. As far as REITs are concerned, one could imagine two axes of diversification: diversification by property type and geographical diversification. Let us remember that for this master thesis I have favored a scope of 6 European countries with multiple Real Estate Investments Trusts engaged in several sectors. After studying volatility and drawing conclusions, it would be interesting to study the correlation among REITs sub-sectors, both during the global financial crisis and the COVID-19. Notice that a correlation based on geographical position could also emerge.

As of December 2020, in the table 1 below one will find, for each country of my scope, the market capitalization and the number of REITs. Notice that my wish to examine those six countries in order to best represent the European market is motivated by the values of the different market capitalizations.

	Number of REITs	Market cap. ($ Bln.)
Belgium	17	23.16
France	27	51.90
The Netherlands	5	13.71
Germany	6	5.56
Spain	76	26.36
UK	57	83.19
Total	**188**	**203.88**

Table 1: Overview of my six European REITs

At the tip of 2020, all European markets unveiled recoveries with positive market cap growths. Germany was one of the best performers with a 24% annual market capitalization growth. On the other hand, Belgium proved to be rather resilient as it reached its pre-COVID19 market cap level. However, some countries are still lagging behind as of at the end of 2020: the UK, France and the Netherlands experienced an annual market decrease of respectively -13.5%, -19.6% and -46.4%. Health care sector has benefited from the population growth. Moreover, the ongoing frightening expansion of e-commerce has impacted both retail and industrial (warehouse and storage) assets – the former in a negative way and the latter in a positive way. Periods of economic distress can generate a broad variability in geographical and sector performance. During both GFC and COVID-19, real estate seemed exposed and displayed different level of volatility. Although these two slowdowns are not of the same nature, can we identify volatility patterns? Do those two crises look alike? Through my different readings I noticed a gap in the field of research on European REITs, in contrast to American, Australian or Japanese Real Estate Investment Trust which benefit from a quite broad and comprehensive analysis.

3 Methodology

3.1 Elementary notions

In this master thesis, several volatility models are explored. One cannot imagine starting this part without defining volatility in a fairly simple way. According to Corporate Finance Institute[9], volatility is "a measure of the rate of fluctuations in the price of a security over time. It indicates the level of risk associated with the price changes of a security. Investors and traders calculate the volatility of a security to assess past variations in the prices to predict their future movements." Yet volatility can be distinguished into two forms: implied volatility and realized volatility. The former is calculated from the price of an option, this is the volatility options' traders expect to be realized in the period from now until the expiration of the option. The latter is calculated from underlying price changes over a certain period of time.

The standard deviation is frequently used in order to determine the volatility of an asset or a dataset. It gives the magnitude of deviations between the observed returns around the mean:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - \overline{r})^2} \tag{1}$$

Notation:

r_t:	return at time t
$\overline{r}$:	mean return
σ:	standard deviation
n:	number of observations

The notion of return is also essential in this research. The arithmetic rate of return (simple net return) is defined as the capital gain divided by the initial price:

$$R_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} \tag{2}$$

Notation:

R_t :	simple net return
D_t :	potential dividends at time t
P_t :	stock price at time t

Notwithstanding this correct computation, there exists another manner that is more convenient to use: the geometric rate of return (also called continuously compounded return or log return). It is the natural logarithm of the simple gross return of an asset.

[9] Definition of volatility: https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/volatility-vol/

$$r_t = \ln(1+R_t) = \ln\frac{P_t}{P_{t-1}} \tag{3}$$

There are several advantages to manipulating geometric returns (Jorion, 2006). Firstly, if geometric returns are normally distributed, then the distribution can never lead to a price that is negative since the *ln* function is undefined when $\frac{P_t}{P_{t-1}}$ is negative. Meaningful here for asset prices that logically cannot be negative. The second major advantage is the additivity property of the compounded returns when it comes to compute multiperiod returns. Meaning that the 2-month return is simply the sum of the two monthly returns.

$$\begin{aligned} r_t[k] &= \ln(1+R_t[k]) = \ln[(1+R_t)(1+R_{t-1})....(1+R_{t-k+1})] \\ &= \ln(1+R_t) + \ln(1+R_t) + ... + \ln(1+R_{t-k+1}) \\ &= r_t + r_{t-1} + ... + r_{t-k+1} \end{aligned} \tag{4}$$

So far, the investor gets an overall indication of the volatility and the risk of a particular investment. Nevertheless, there is another paramount concept that derives from the value of the volatility: Value at Risk[10]. "The VaR is the maximum amount expected to be lost over a given time horizon, at a pre-defined confidence level"[11]. The three components – loss size, probability and time frame – makes VaR a practical and helpful metric in order to determine the risk of an asset. Indeed, risk managers may be obliged to maintain a certain limit of uncertainty in a portfolio and thus they will not engage in investments that risk exceeding the limit allowed.

$$q_t(\alpha) = \Phi^{-1}(\alpha)\sigma_t \tag{5}$$

where Φ^{-1} is the quantile function of the Gaussian distribution.

One cannot be satisfied by a fixed volatility. That is the reason why some volatility models emerged in order to model volatility more accurately. At the dawn of the eighties, volatility and consequently VaR became time-varying. Both metrics evolve through time and so this time-varying characteristic opened the road to more precise predictions. This is the beginning of the GARCH models and the volatility modelling of financial assets such as REITs; which are the purpose of this master thesis. In the following paragraphs, I am going to explain different GARCH models.

[10] VaR = Value at Risk

[11] https://www.risk.net/definition/value-at-risk-var

3.2 Foundations of the GARCH models

First, GARCH stands for Generalized AutoRegressive Conditional Heteroscedasticity. A far-reaching understanding of these terms is crucial.

3.2.1 The Autoregressive models

The sample Autocorrelation function (*SACF*) answers the question: do past returns tell us something about future returns? We can visualize it graphically as done in my MATLAB script but for ease I can perform Ljung-Box to test serial dependence between returns or, even more interesting, between the squared returns. We will then reject the null hypothesis of no serial correlation if p-value is less than or equal to the significance level. Moreover, in the case where the Ljung-Box test on the squared returns display even stronger rejection of $H(0)$, it means that the squared returns of the past (that represent the variance in volatility models I would say) tell us something about the future returns. It is important to note that this power of prediction decreases when the number of lags increases.

The starting point of the GARCH model is found in the autoregressive time series model[12]. The fact that a return has a statistically significant autocorrelation of $lag1$ stipulates that the lag return r_{t-1} is consistent in predicting r_t.Indeed, by taking monthly returns[13], it means that the last month return is useful in predicting the current monthly return. This statement could be formulated by an $AR(1)$ model such as:

$$r_t = \phi_0 + \phi_1 r_{t-1} + a_t \tag{6}$$

where a_t is assumed to be a white noise series with mean equal to zero and a variance of σ_a^2.

This model implies that the expected current return, conditional on the past return r_{t-1}, could be written as follows:

$$E(r_t|r_{t-1}) = \phi_0 + \phi_1 r_{t-1} \tag{7}$$

One could legitimately argue that there are situations in which r_{t-1} alone cannot determine the conditional expectation of r_t. Therefore, one can dig deeper and a meaningful generalization of the $AR(1)$ model is the $AR(p)$ model:

$$r_t = \phi_0 + \phi_1 r_{t-1} + ... + \phi_p r_{t-p} + a_t \tag{8}$$

This general model states that the lagged p returns jointly determine the conditional ex-

[12] Autoregressive time series model = AR.

[13] We can also take other frequencies of data such as weekly or daily data. Higher frequency data such as intraday returns are a possibility also.

pectation of r_t given the past data. The current return is the dependent variable and the lagged returns are the explanatory variables. As stated by researchers: "Of course, linear $AR(1)$ models provide a limited class with which to model real data. However, they can be used as building blocks for more complex models." (Grunwald and al, 1996). The difficulty is obviously to find the right order of the AR models (i.e., the right p order). The order determination has been broadly studied in time series literature (de Gooijer and al, 1985). Two main approaches are available: Partial Autocorrelation Function ($PACF$) or Information Criteria (IC). In the following pages I will explain into details the two information criteria; namely AIC (Aïkake, 1973) and BIC (Schwarz, 1978).

Some properties are inherent to autoregressive models but it is beyond the scope of this master thesis.

3.2.2 The Moving Average models

Moving-average models are another class of simple models that are also convenient in modelling return series in finance. This approach treats the model as an infinite-order AR model with some parameter constraints. In theory an AR model with infinite order is written as follows:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 r_{t-2} + ... + a_t \tag{9}$$

Nevertheless, it is clearly not realistic. One solution to make this model convenient is to suppose that the coefficients ϕ_i satisfy some constraints so that they are determined by a finite number of parameters. That is accomplished in formula (13).

$$r_t = \phi_0 - \theta_1 r_{t-1} - \theta_1^2 r_{t-2} - \theta_1^3 r_{t-3} - .... + a_t \tag{10}$$

In this formula, the many coefficients depend on a single parameter θ_1 via $\phi_i = -\theta_1^i$ for $i \geq 1$. For the purpose of stationarity, θ_1 must be lower than 1 in absolute value. The idea behind is that the dependence of the current return r_t on its lagged values r_{t-i} should decrease over time.

Let us rewrite the model for r_t in a compact form:

$$r_t + \theta_1 r_{t-1} + \theta_1^2 r_{t-2} + ... = \phi_0 + a_t \tag{11}$$

If we multiply this latter by θ_1 and then we subtract the result from the compact equation, we obtain $r_t = \phi_0(1-\theta_1) + a_t - \theta_1 a_{t-1}$ .The general form is stated as follows:

$$r_t = c_0 + a_t - \theta_1 a_{t-1} \tag{12}$$

Where c_0 is a constant and a_t is a white noise series. One can conclude that the time series r_t is a weighted average of shocks a_t and a_{t-1}. This is a $MA(1)$ model but we can easily extend to a $MA(q)$ model:

$$r_t = c_0 + a_t - \theta_1 a_{t-1} - .... - \theta_q a_{t-q} \tag{13}$$

In a nutshell, we want to keep the same general structure for moving average models as for *AR* models except that instead of looking at return previous values, we are interested in errors from the past (i.e., error lags). For example, a $MA(1)$ model shows that the actual target variable r_t depends on the error from the time period before r_{t-1} plus some current errors a_t. The unseen shifts we did not expect in the previous time period actually permeate into the next time period. That is the whole idea of Moving Average models. It would be fabulous to combine the short-term memory characteristic of *MA* models with the long-term memory characteristic of *AR* models. That is what we call AutoRegressive Moving Average models (*ARMA* models).

3.2.3 The AutoRegressive Moving Average models

Box, Jenkins, and Reisel (1994) introduced autoregressive moving average models. *ARMA* models answer the need for high-order model with many parameters in order to represent correctly the dynamic structure of the data. While *ARMA* models are not suitable for returns series in finance (or at least the likelihood to use this kind of models is kept low), the concept of *ARMA* models turns out to be relevant in volatility modelling. Actually, the generalized autoregressive conditional heteroscedastic (*GARCH*) model can be seen as an *ARMA* model but with nonstandard white noise series. A time series r_t following an $ARMA(1,1)$ can be stated as follows:

$$r_t - \phi_1 r_{t-1} = \phi_0 + a_t - \theta_1 a_{t-1} \tag{14}$$

where the left-hand side of the equation being the *AR* component and the right-hand side being the *MA* component.We need $\phi_1 \neq \theta 1$; otherwise the model is only represented by a white noise series.

An $ARMA(p,q)$ is the generalization model with p and q being non negative integers. Here the compact form:

$$r_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + a_t - \sum_{i=1}^{q} \theta_i a_{t-i} \tag{15}$$

3.3 The Autoregressive Conditional Heteroscedasticity models and Generalized Autoregressive Conditional Heteroscedasticity models

A bit of history first. Robert F. Engle is born in 1942 in Syracuse. He obtained a Ph.D. from Cornell University in 1969. On financial markets, fluctuations – volatility - appear through time and it became a major concern for financial professionals since the value of shares (REITs in our case) depends on their risk. Despite such time-varying volatility, early-stage researchers used to assume constant volatility in their different models. In 1982, Robert Engle came out with a masterpiece in terms of innovation. Indeed, he found that the concept of autoregressive conditional heteroscedasticity (*ARCH*) accurately captures the behavior of many time series and he implemented methods to model **time-varying volatility**. He was awarded the Nobel prize in Economics by his peers: « *ARCH* models have become indispensable tools not only for researchers but also for analysts on financial markets, who use them in asset pricing and in evaluating portfolio risk. » (The royal Swedish Academy of Sciences, 2003). Afterwards, the Danish economist T.P. Bollerslev (1986) proposed the Generalized ARCH in order to allow for past conditional variances (lagged variances) in the current conditional variance equation.

3.3.1 Autoregressive Conditional Heteroscedasticity

As said previously and in order to deal with the foolish assumption of constant variance, Engle (1984) proposed a new stochastic process (*ARCH*) with zero mean, serially uncorrelated with non-constant variances conditional on the past, but constant unconditional variance. Engle's purpose was to capture volatility clustering. He wanted to model the assertion that periods characterized by large changes are followed by further large changes and, conversely, periods characterized by small changes are followed by further small changes. Since this milestone, heteroscedasticity is a well-known concept in time series modelling. It refers to the fact that the variance of error terms is not constant over time.

$$r_t = \mu_t + \varepsilon_t \Leftrightarrow \qquad r_t = \sigma_t z_t \tag{16}$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 \tag{17}$$

$$\varepsilon_t \overset{iid}{\sim} N(0,1) \tag{18}$$

$$r_t|\Omega_{t-1} \sim N(0, \sigma_t^2) \tag{19}$$

Where:

r_t : return at time t

z_t : follows a N(0,1)

σ_t^2 : current conditional variance

ω : unconditional variance

α : the alpha captures the effect of yesterday's returns on today's volatility.

The first equation(19) refers to the return of a stock that is composed of its expected value plus an error term (i.e., the shock). Nevertheless, the assumption of stationarity is usually assumed, which means that the expected value of stock returns is equal to zero. In this latter case, the stock return equals the error term. The equation (20) displays the ARCH formula of order 1.

But ARCH models display a major drawback and it is what times series analysts called "bursty". By including the conditional past variance (volatility of yesterday) into the model, it will make it less "bursty".

3.3.2 The Generalized Autoregressive Conditional Heteroscedasticity models

In the paper entitled "Generalized Autoregressive Conditional Heteroscedasticity", it is argued that a simple GARCH model provides better fit than ARCH model (Bollerslev, 1986). He decided to add a term to the initial ARCH model which is the previous variances of the time series. GARCH models describe financial markets in which volatility can change, becoming more volatile during crisis or during the switch from a bull to a bear market (conversely) and less volatile during periods of relative serenity and steady economic growth. On a plot of time series, for example, stock time series may look relatively uniform (at least not abrupt) for the years leading up to a crisis such as the COVID-19. Nevertheless, in the period of turmoil, returns may swing wildly from negative to positive region. Moreover, the increased volatility may be predictive of volatility going forward since we condition the future volatility on all the information we have up to now. Volatility may then return to levels similar to pre-crisis levels or be more homogenous. A simple regression model does not account for this variation in volatility exhibited in financial markets. In a nutshell, *GARCH* was a milestone for risk managers. $GARCH(1,1)$ model can be stated as follows:

$$\sigma_t^2 = \omega + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{20}$$

where σ_{t-1}^2 is the conditional variance of the day, week or month before.

Therefore, the formula of the $GARCH(p,q)$, taking into account q lags of conditional variances and $ARCH(p)$ effect looks like:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 \tag{21}$$

Where:

r_t :	return at time t
σ_t^2 :	current conditional variance
ω :	unconditional variance
α :	the alpha captures the effect of the returns of yesterday on today's volatility.
$(\alpha+\beta)$:	this parameter measures the volatility persistence: how fast the current shock to the volatility wil
σ_{t-i}^2 :	the conditional variance of the i^{th} day, i^{th} week or i^{th} month before.

The conditional mean return at time t is equal to zero because:

$$\begin{aligned} E(r_t|\Omega_{t-1}) &= E(\sigma_t z_t|\Omega_{t-1}) \\ &= \sigma_t E(z_t|\Omega_{t-1}) \\ &= \sigma_t \times 0 = 0 \end{aligned} \tag{22}$$

The unconditional mean return equals zero also. The conditional variance at time t, knowing the information up to $(t-1)$ is σ_t^2:

$$\begin{aligned} V(r_t|\Omega_{t-1}) &= V(\sigma_t z_t|\Omega_{t-1}) \\ &= \sigma_t^2 V(z_t|\Omega_{t-1}) \\ &= \sigma_t^2 \times 1 = \sigma_t^2 \end{aligned} \tag{23}$$

It is essential to take a closer look at the parameters, which are the quantities of interest, omega alpha beta. Then, the unconditional variance can be described, by using *GARCH* parameters, as follows:

$$V(r_t) = E(r_t^2) = \frac{\omega}{1-\alpha-\beta} \tag{24}$$

As a remainder:

Parameters:

ω :	Constant term
α :	*ARCH* effect coefficient measuring the shock disturbance
β :	*GARCH* effect parameter measuring the volatility persistence

Some constraints are imposed to these parameters: ω must be greater than zero, α and β must be greater or equal than zero. Furthermore, the sum of parameters alpha and beta must be less than unity in order to guarantee stationarity. $(\alpha+\beta)$ is called persistence. The nonlinearity of *GARCH* models is a stumbling block to econometricians because the parameters must be estimated by maximization of the likelihood function and consequently by choosing the best potential distribution (Engle, 2001). Hopefully, there exist packaged programs in MATLAB in order to find out the value of these parameters.

$$maxF(\omega,\alpha,\beta|r) = \sum_{i=1}^{T} \ln f(r_t|\sigma^2) = \sum_{i=1}^{T} (\ln \frac{1}{\sqrt{2\pi\sigma^2}} - \frac{r_t^2}{2\sigma^2}) \tag{25}$$

Where f is the normal density function and T is the number of observations.

In the sight of heteroscedasticity (*ARCH* effects) in the residuals of the daily returns of REIT stocks, the ordinary least square estimation (OLS) is not appropriate. On this account, ARCH-type models cannot be estimated by simple techniques such as OLS. The method of maximum likelihood estimation will be brought into play when facing GARCH models, regarding this master thesis. In addition, one must be aware that the true distribution is not always normal and it is even more true with time series returns that mostly display leptokurtic distributions.

Since the breakthrough initiated by Engle (1982) and then by Bollerslev (1986), many researchers tried to improve the *GARCH* quality of volatility modelling by adding some terms in order to capture well-known financial effects of volatility behaviors. Indeed, an important weakness of the *GARCH* model is that it does not make the difference between positive and negative movements in the market; this is because the returns are squared in the formula. One of the first extensions is the *EGARCH* model (Nelson,1991). This model has the capacity to model an empirically observed fact in finance, which is that negative shocks of yesterday have a stronger impact in the variance today than positive shocks. We call it asymmetry and it is modelized by an additional leverage term that captures this asymmetry. Another major characteristic is that this model has the property to remove the non-negativity constraint of the parameters. Then, an additional extension is the $GJR-GARCH$ model (Glosten, Jagannathan & Runkle, 1993). It is also a non-linear model for the volatility but it denotes a different specific parametric for the conditional heteroscedasticity. These two models are explored and explained in depth in the following two paragraphs.

3.3.3 EGARCH - Exponential Generalized Autoregressive Conditional Heteroskedasticity model

In 1976, the researcher Black found evidence that returns are negatively correlated with changes in returns volatility. Indeed, volatility will have a tendency to increase when excess returns are lower than expected. Conversely, volatility will have a tendency to decrease when excess returns are higher than expected. Nevertheless, in the classical *GARCH* model, only the magnitude of the past returns is taken into account while the sign of the past excess returns (i.e., positivity or negativity) is ignored. In other words, only the size and not the sign[14] of the lagged residuals determines the conditional variance (Nelson, 1991). Thus, a model in which the conditional variance captures this asymmetry could be fruitful. This is accomplished through the leverage effect parameter "gamma". If $\gamma < 0$ negative shocks will increase the volatility more than positive shocks. If $\gamma > 0$ positive shocks increase the volatility more than negative shocks.

Another characteristic is that *EGARCH* formula models the natural logarithm of the conditional variance and it has a huge impact on the parameters' constraints. Indeed, since the value logarithm function can be either positive or negative, the parameters α, β, γ and ω can be potentially of any sign. If one turns this characteristic the other way round, it means that even if the parameters are negative, the variance will still be positive. $EGARCH(1,1)$ is defined as follows:

$$\ln \sigma_t^2 = \omega + \alpha(|r_{t-1}| - E(|r_{t-1}|)) + \gamma r_{t-1} + \beta \ln \sigma_{t-1}^2 \tag{26}$$

3.3.4 GJR - Generalized Autoregressive Conditional Heteroskedasticity model

Another regularly used model to capture asymmetric volatility was developed by Glosten, Jagannathan & Runkle (1993). This is called $GJR-GARCH$ and has the advantage to straightforward model the conditional variance. Indeed, it does not make use of the natural logarithm anymore, like it was the case for the $EGARCH$ formula.

$$\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1}) r_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{27}$$

In $GJR-GARCH$ models, the leverage effect is translated through the parameter γ. One denotes the emergence of I_{t-1} which is the identity matrix or indication function. In a $GJR-GARCH(1,1)$, it can be seen as a dummy variable. It will take the value 1 if the previous shock is negative (i.e., r_{t-1} is lower than zero) and it will take the value 0 if the last shock is positive.

$$I_{t-1} = \begin{cases} 1 & \text{if} \quad r_{t-1} < 0 \\ 0 & \quad \text{otherwise} \end{cases} \tag{28}$$

[14] This is due to the **squaring** imposed to past returns in the GARCH formula proposed by Bollerslev.

To be more pernickety, bad news will be represented by $(\alpha + \gamma)$ and good news will be represented by α itself. Notice that the equation becomes a simple $GARCH(1,1)$ model if γ is equal to zero (Asgharian, 2016).

3.4 Multivariate model

3.4.1 Dynamic Conditional Correlation – GARCH model

First of all, DCC-GARCH stands for Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroscedastic model. The concept of correlation is of paramount importance for portfolio risk managers who manage large multiple stocks portfolios or for hedgers who try to reduce the risks associated with uncertainty. GARCH methods can potentially be used in order to capture time variation in correlation. Moreover, correlations can be derived from multivariate GARCH models and it provides us with a better understanding of the exposure (Jorion, 2007). Furthermore, DCC-GARCH is modelling the conditional variances and the conditional correlations in order to find out the conditional covariance matrix ; if one attempts to directly model the conditional covariance matrix, it could lead promptly to the "curse of dimensionality" phenomenon.

The DCC GARCH method is designed in three steps. The first step consists in estimating the conditional variance H_t of each portfolio from a univariate GARCH process. Moreover, from the information criteria analysis, one must be able to infer the best univariate model for a particular REIT, resulting in a comprehensive insight of the parameters needed for the first step. Then, the second step is to construct the diagonal matrix containing the previously calculated conditional variances (from previously computed univariate GARCH model). Then, by taking the square root of this matrix, we obtain the matrix of standard deviations, noted D_t. Finally, in the third step, the residuals obtained by the regressions of the first step are used to construct the correlations in an autoregressive manner, which results in a conditional correlation matrix that evolves as a function of time.

The density of the returns is characterized by the absence of serial correlation in the mean returns, and by the presence of time-varying second-order moments.

$$r_t | I_{t-1} \sim B(\mu, H_t) \tag{29}$$

where B is a generic multivariate density function.

The model can thus be written as follows:

$$H_t = D_t R_t D_t$$

where H_t stands for the dynamic conditional covariance matrix. D_t stands for the dynamic conditional standard deviations matrix and R_t stands for the dynamic conditional correlations matrix.

D_t is decomposed as stated below and its elements can be seen as univariate GARCH models:

$$D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & .... & 0 \\ 0 & \sqrt{h_{2t}} & ... & 0 \\ .... & .... & .... & .... \\ 0 & .... & 0 & \sqrt{h_{nt}} \end{bmatrix} \tag{30}$$

with $\mathrm{h}_{it} = \omega_i + \sum_{p=1}^{P_i} \alpha_{ip} r_{i,t-p}^2 + \sum_{q=1}^{Q_i} \beta_{iq} h_{i,t-q}$

R_t is a symmetric correlation matrix and can be seen as follows:

$$R_t = \begin{bmatrix} 1 & \phi_{12,t} & .... & \phi_{1n,t} \\ \phi_{12,t} & 1 & ... & \phi_{2n,t} \\ .... & .... & .... & .... \\ \phi_{1n,t} & .... & \phi_{n-1,n,t} & 1 \end{bmatrix} \tag{31}$$

Consequently, the elements that form H_t are : $[H_t]_{ij} = \sqrt{h_{it} h_{jt} \rho_{ij}}$ with $\rho_{ij} = 1$ if $i = j$.

The dynamic correlation matrix, R_t, is not explicitly driven by a dynamic equation, but is derived from a standardization of a different matrix, Q_t, which has a dynamic structure. The form of Q_t determines the model complexity and feasibility in large cross-sectional dimensions. Moreover, some requirements exist in order to ensure R_t to be equal or less than one and not inferior to -1 (by definition of the correlation) and H_t to be positive definite.

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{32}$$

where $\mathrm{Q}_t = (1-\alpha-\beta)\overline{Q} + \alpha \varepsilon_{t-1} \varepsilon_{t-1}^T + \beta Q_{t-1}$(33)

$$\text{and } Q_t^{*-1} = \begin{bmatrix} \sqrt{q_{11t}} & 0 & .... & 0 \\ 0 & \sqrt{q_{22t}} & ... & 0 \\ .... & .... & .... & 0 \\ 0 & .... & 0 & \sqrt{q_{nnt}} \end{bmatrix} \tag{34}$$

And $\overline{Q}$ is the unconditional covariance matrix of the standardized residuals. Q_t^{*-1} is a diagonal matrix with the square root of the diagonal elements of Q_t.

The parameters α and β are quite similar to the ones presented in *GARCH* models but this time they have to deal with covariance matrix and correlation structure. Moreover, α and β must satisfy three conditions to certify the positivity of H_t:

$$
\begin{aligned}
a &\geq 0 \\
b &\geq 0 \\
(a+b) &< 1
\end{aligned}
$$

Dynamic conditional correlation (DCC) estimators entail the flexibility of univariate *GARCH* models but not the convolution of multivariate *GARCH* models.

3.5 Goodness of fit

3.5.1 Information Criteria

The three different *GARCH* models adopted in this master thesis are now fully explained. They will be used to analyze the volatility of the European REITs of my sample and we will find out the volatility patterns. One must be aware that the performance of each model varies and it would be useful to determine the best fitter among the three models. Moreover, Georges E. P. Box argued that “All models are wrong, but some are useful”. Knowing the goodness of fit of a particular model is convenient in order to assess how accurate a model is and, consequently, how useful a model can be in order to forecast the future behaviors of European REITs’ time series.

Firstly, if one faces nested models, the model selection is achieved through the likelihood-ratio test (i.e., LR). In this study, I encountered only non-nested models[15] and so, in this case, Information Criterion[16] formula is used. Furthermore, IC can be divided in two close concepts: AIC[17] and BIC[18]. The former, AIC, is a single number score that can be used to determine which of multiple models is most likely to be the best model for a given dataset. It estimates models relatively, meaning that AIC scores are only useful in comparison with other AIC scores for the same dataset. The latter, BIC, has the same background as AIC but it adds a larger penalty term for the complexity of the model than AIC. The smaller the IC, the better the fitting. One must not forget that it is all about trade-off between over-fitting and under-fitting. Intuitively, one can

[15]Basically, two models are said to be ’non-nested’ if one of the models can be obtained from the other one; otherwise, they are said to be ’nested’.

[16]IC = Information Criterion

[17]Akaike Information Criterion

[18]Bayesian Information Criterion

easily understand that if the number of parameters increases, one seeks a much better likelihood of his data.

- AIC mathematical expression

$$AIC = -2\log(L) + 2k$$

with k is the number of parameters and L is the maximum likelihood.

- BIC mathematical expression

$$BIC = -2\log(L) + klog(n)$$

with k is the number of parameters, L is the maximum likelihood and n is the number of observations in the dataset.

These information criteria will thus be used to determine the most apposite model - among the three *GARCH* models.

4 Presentation of the dataset and descriptive statistics

4.1 Data retrieval

I carried out some research on Eikon to retrieve the required data. Refinitiv Eikon is a software system used to monitor and analyze financial information. Thanks to HEC Liège, students can have access to a number of market data ranging from stocks time series to financial derivatives products. Firstly, I retrieved the close share prices of the 6 stock market indices[19] needed to perform a Markov-switching model (see next paragraph) - it will produce a big picture of each national economy. The whole study period goes from January 2^{nd} 2007 until June 10^{th} 2021. Then, I computed the log-returns of each REIT index as mentioned at the beginning of the methodology part of this master thesis. Afterwards, I stored them in a type "double" matrix with the intention of doing a number of computations.

In the next step, I used Thomson Reuters DataStream extension which is only available on the main computer in the trading room at HEC Liège. This is designed for economists and

[19] Indices retrieved from Eikon: BEL20 for Belgium, CAC40 for France, AEX for the Netherlands, DAX for Germany, IBEX35 for Spain and FTSE100 for the United-Kingdom.

research communities as it offers the world's most comprehensive financial historical database. Thanks to a dialogue I had with Georges Milunovich[20], I knew that DataStream provided REITs' country-specific indices (one single index for Belgian REITs, another one single index for G-REITs, etc.). It will be useful to investigate the volatility of certain countries as a whole, as studying all the REITs individually would be too laborious. Thus, I chose 6 country-specific indices[21] which are the following: FTSE Belgium REIT, S&P France REIT, S&P Netherlands REIT, FTSE Germany REIT, S&P Spain REIT and FTSE United Kingdom REIT. The Global Financial Crisis took root in the summer of 2007 in the US-based financial institutions and outspreaded throughout Europe by the first half of 2008. That is why I decided to select a reasonable timeframe going from November 8^{th} 2007 to June 10^{th} 2021 for the REIT indices; with the exception for Spain REIT index for which I did not find any consistent indices starting before September 19^{th} 2014. Although the SOCIMI regime was enacted in 2009, a substantial change of the SOCIMI regime was approved in December 2012, with effects as of the first quarter of 2013. As a result, the new SOCIMI regulation has been assimilated to other European REITs, in which the main feature is the elimination of direct taxation on the SOCIMI, transferring such taxation to the final investors.

4.2 REIT indices graphs

In figure 3, REIT indices time series by country are displayed.

Log-returns of each country-specific REIT time series are presented in figure 4.

4.3 Descriptive statistics

Then I decided to draw up descriptive statistics for two sub-periods for the 6 country-specific REIT indices: the first from Q32014 to 31-01-2020 and the second from 3-02-2020 to 10-06-2021. The latter being the period I will call the "COVID crisis period" and the former representing an economic cycle (i.e., expansion-recession-expansion). From the descriptive statistics, one can clearly notice leptokurtosis[22] and negative skewness of COVID crisis and-after log-returns; meaning that we expect more negative returns than normality and, in addition, it will occur in many more situations since the log-returns are leptokurtic. Periods of turmoil disrupt totally the normality of returns. And the standard deviation of each index has increased during crisis.

[20] Associate Professor of Business Analytics at Macquarie University (Sydney, Australia). He wrote the paper entitled "Speculative bubbles, financial crises and convergence in global real estate investment trust."

[21] These indices are constructed by Thomson Reuters DataStream.

[22] « A state in which the volatility of a security is itself not volatile. That is, leptokurtosis is a state in which the volatility of a security changes at a relatively low rate. This is shown on a chart by a distribution line with data points resembling fat tails and a higher mean, with an even distribution. » (Leptokurtosis. (n.d.) Financial Glossary. (2011). Retrieved August 4 2021 from https://financial-dictionary.thefreedictionary.com/Leptokurtosis)

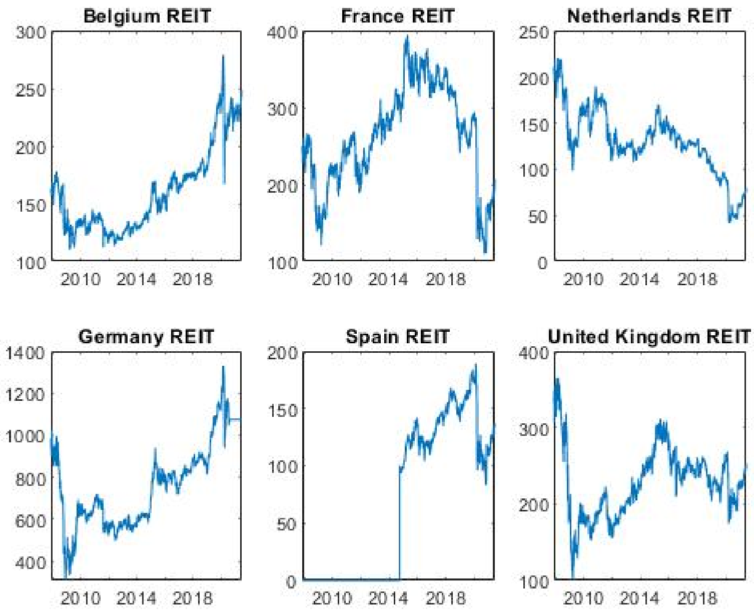

Figure 3: Graph showing the time series of the six EU indices time series

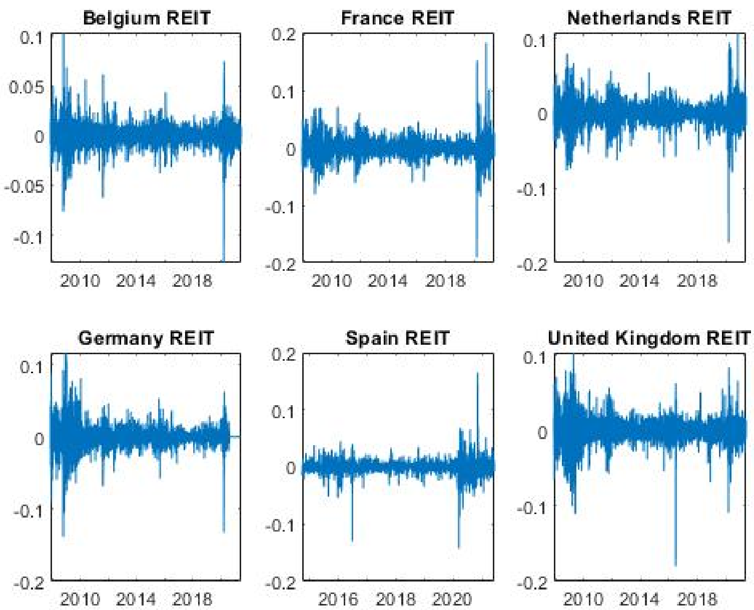

Figure 4: Graph showing the *log* returns of each index during the whole period.

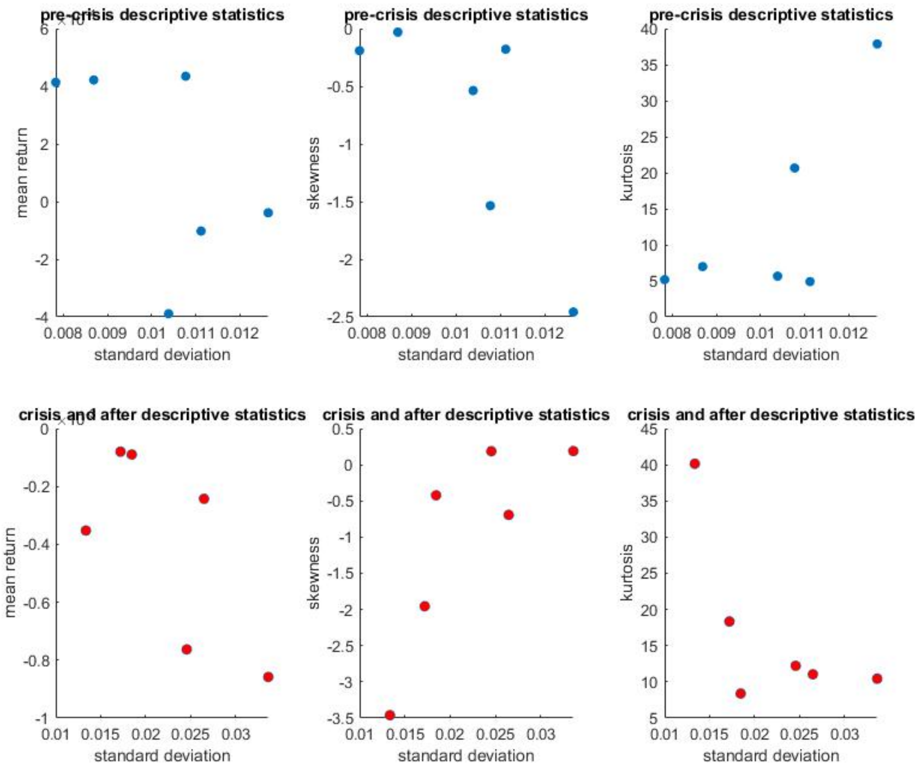

Figure 5: Standard deviation of each index compared to the mean, the skewness and the kurtosis.

In the table 2, one will find the values of the descriptive statistics in order to complement my graph.

To support my assertions concerning the non-normality of returns during crisis period, I drew a common graph on MATLAB with the normal distribution and the distributions of returns during COVID time for France REIT Index. I can definitely assert that the true distribution of returns is left-skewed and platykurtic. I decided to deliver the example for France REIT index returns but it remains true for the other indices of my European sample.

	Skewness		Kurtosis		Mean return		Std deviation	
	pre crisis	covid	pre crisis	covid	pre crisis	covid	pre crisis	covid
Belgium REIT	-0.19	-1.96	5.2	18.4	0.41	-0.1	0.0078	0.0172
France REIT	-0.18	0.19	4.9	10.5	-0.1	-0.86	0.01	0.03
The Netherlands REIT	-0.54	-0.69	5.6	11.1	-0.39	-0.25	0.01	0.013
Germany REIT	-0.03	-3.46	-3.46	40.14	0.004	-0.003	0.008	0.013
Spain REIT	-1.54	0.19	20.6	12.2	0.43	-0.76	0.01	0.02
UK REIT	-2.5	-0.42	37.8	8.38	-0.00	-0.00	0.013	0.0184

Table 2: Values of the descriptive statistics of the six REIT indices.

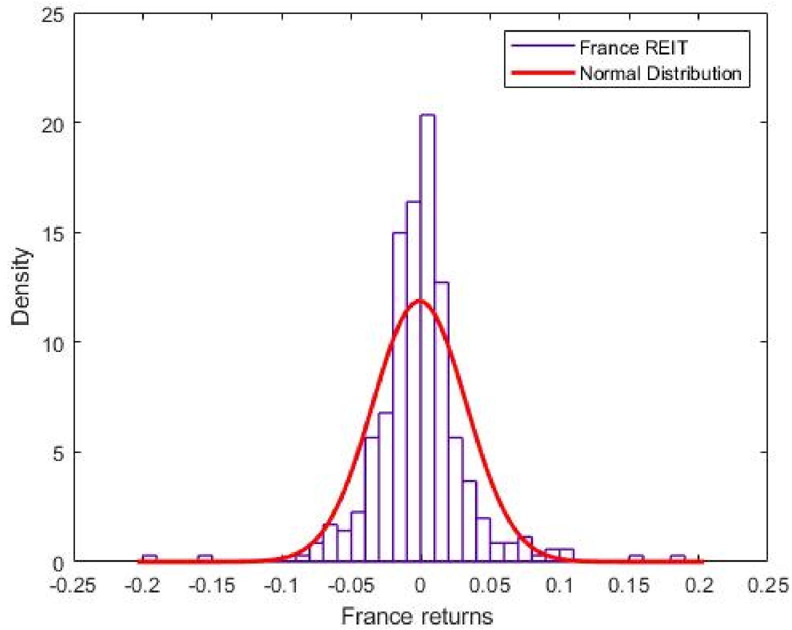

Figure 6: Graph showing the true distribution of France index returns vs. the normal distribution.

4.4 Testing ARCH effects for the six European REIT time series

I can perform Ljung-Box to test serial dependence between returns or, even more interesting, between the squared returns. In essence, the ARCH effect test is a white-noise test, but for the squared time series. In other words, we are investigating a higher order (non-linear) of auto-correlation[23]. We will then reject the null hypothesis of no serial correlation if p-value is less than or equal to the significance level. I chose a confidence level of 95% - a significance level of 5% - to run the Ljung-Box test of lag 1 on the squared returns. In the table 3, the results of the test on the squared REIT returns series are exhibited and they plainly display strong evidence against $H(0)$. It means that the squared returns of the past (which represent the variance in volatility models since $E(r_t^2) = V$) tell us something about the future returns.

Many sub-period time series unveiled a p-value lower than 0.05 and a rejection parameter $H = 1$. Nevertheless, three times series (UK Covid, UK GFC and Spain Covid) do not display enough statistical evidence to reject the null hypothesis. One must be aware that by taking a larger lag ($r = 5$ for example) when performing the Ljung-Box test, I can override this issue and end up with a relevant rejection of $H(0)$ for each single time series. So, I can prove that ARCH effects are a reality for my European REIT indices and, consequently, it seems suitable to plead in favor of making the analysis going further by implementing three well-defined GARCH models.

[23]Autocorrelation in the squared REIT return proves the phenomenon of conditional heteroscedasticity.

	H decision			
	for $r=1$	for $r=5$	$p-value$	$Q-statistics$
Belgium GFC	1	1	0.0064	7.44
Belgium NEC	1	1	0	38.88
Belgium Covid	1	1	0.0004	12.7
France GFC	0	1	0.065	3.39
France NEC	1	1	0	46.96
France Covid	1	1	0.0113	11.11
The Netherlands GFC	1	1	0	6.41
The Netherlands NEC	1	1	0	52.12
The Netherlands Covid	1	1	0.0003	12.81
Germany GFC	1	1	0	49.9
germany NEC	1	1	0	27.6
Germany Covid	1	1	0.011	6.47
Spain GFC	/	/	/	/
Spain NEC	1	1	0	27.26
Spain Covid	0	1	0.61	0.25
UK GFC	0	1	0.209	1.58
UK NEC	1	1	0	113.04
UK Covid	0	1	0.602	0.27

Table 3: Table with the decision value for Ljung-Box test (0 or 1), p-values and Q-statistics.

4.5 Markov switching model

Financial analysts are often concerned with recognizing when markets "change": a market's particular behavior over months or even years can suddenly transition to a completely new behavior (Hamilton, 1988). Investors would prefer to be able to notice these shifts as they occur so that they may adapt their strategy accordingly, but this is difficult to achieve. Markov switching models are a popular class of models that incorporate time-variation in parameters in the form of state- or regime-specific values. On one hand, a bull market is defined as a period of time during which the markets increase in value on average with little volatility. On the other hand, a bear market is defined as a market that is trending downhill and has a high level of volatility. By simply observing the daily changes in price, it can be hazardous to capture a change in state (i.e., from a bull market to a bear market; and conversely). It is understandable that this model will be used to divide my study period (from x to x: a determiner plus tard) into distinct sub-periods in order to finally compare my GARCH models; according to whether my REITs follow a steady economic cycle or a period of turmoil. Thus, I will perform a quick Markov-switching model on each stock market index (AEX for the Netherlands, IBEX35 for Spain or Bel20 for Belgium). Consequently, I will find out very accurately the timeframe when the market is in a crisis phase and when it is in a steady economic growth for each European country.

For the sake of this master thesis, I am going to write some simple mathematical expressions to briskly shed light on the major concepts.

We make the assumption that the stock return r_t follows a distribution that depends on a latent process S_{t+1}. The distribution in regime 1 and in regime 0 can obviously be different.

$$r_t \sim \begin{cases} N(\mu_0, \sigma_0^2) & \text{if} \quad S_t = 0 \\ N(\mu_1, \sigma_1^2) & \text{if} \quad S_t = 1 \end{cases} \tag{35}$$

The latent process S_t follows a first order Markov chain[24]. It means that the probabilities are defined in terms of transition from one state to the other state (remember that in our case we have to deal with only two different regimes). The probability for regime 0 to occur at time $(t+1)$ depends exclusively on the regime at time t.

Transition probabilities look like $\rho_{ij} = Pr[S_t = i | S_{t-1} = j]$

So, the transition matrix is the following:

$$P = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = \begin{pmatrix} \rho_{00} & 1-\rho_{11} \\ \rho_{00} & \rho_{11} \end{pmatrix} \tag{36}$$

Finally, by replicating the Markov-switching model on country-specific indices, I will point out accurately the time periods during which the different markets were mainly up and mostly down.

From a practical point of view, below you will find the results I found by applying a Markov-switching model on each stock market index for the whole period going from January 2^{nd} 2007 to June 10^{th} 2021. Thus, I have now the ability to discern the bull and bear periods. Moreover, I am delighted to conclude that the Markov analysis corroborates the primary conclusions I had by simply looking at the time series of each index (figure 7).

According to these charts, there were three major crises that strongly affected financial markets. First, the well-known global financial crisis[25] of 2007-2008 that has been considered by many economists as the most severe economic crisis since the Great depression in 1929. The GFC has been initiated by the bursting of the United States housing bubble and the subprime mortgage crisis which followed. Then came the shock of the European debt crisis that has been translated on the financial markets by the end of 2011 and 2012. Some countries such as Spain, Greece, Cyprus and Portugal were unable to repay or refinance their debt. Finally, the horrific health crisis we are facing today and its terrible virus COVID-19 which is a contagious disease. Since the moment the first case was revealed in December 2019 in Wuhan, the coronavirus disease has been spreading worldwide.

[24]"A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event." (Wikipedia)

[25]Global Financial Crisis = GFC.

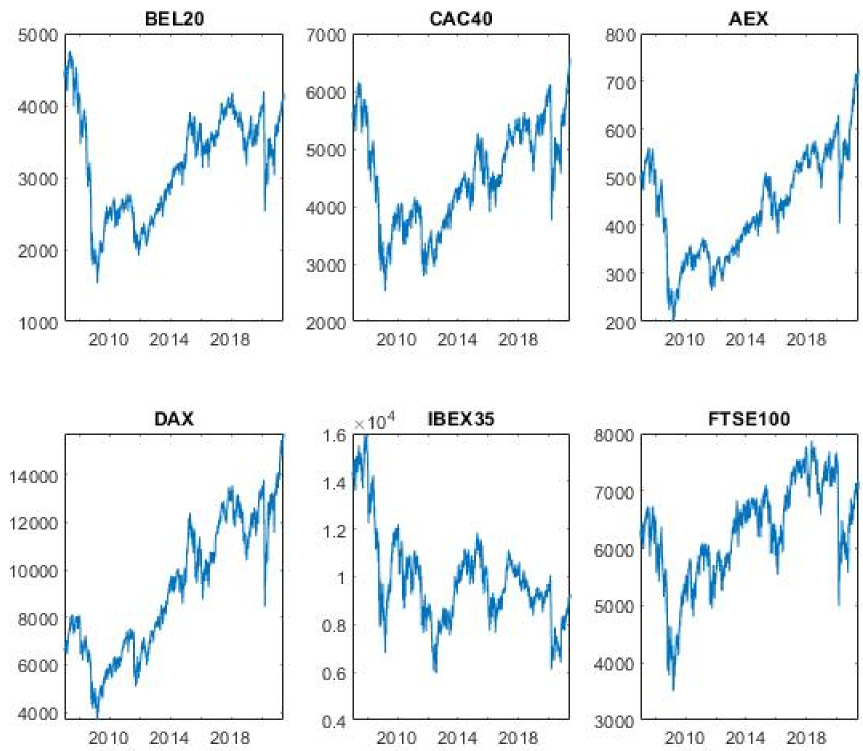

Figure 7: Graph showing each country market index.

4.5.1 Belgium

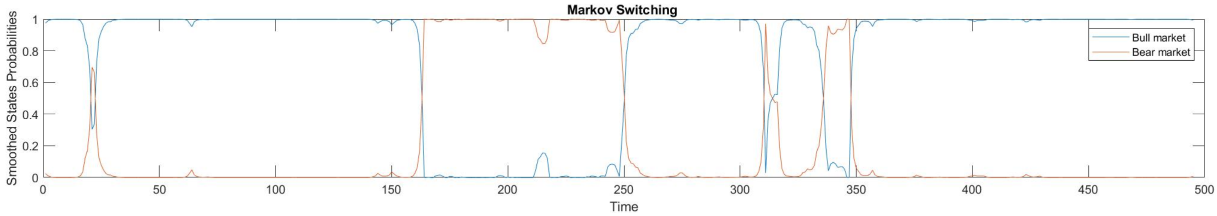

Figure 8: Graph exhibits changes from bull to bear market.

For the Belgian stock market, one can notice a change in regime - from bull to bear market - at the 162^{nd} observation. This observation corresponds to the date of February 20_{th} 2020. Then, at the 248^{th} observation, corresponding to June 24^{th} 2020 (the end of the first quarantine) and regaining a little freedom, a switch in regime takes place – from a period of turmoil to a period of economic growth even if a second COVID-19 wave is expected. Afterwards, some back and forth movements are following until the 348^{th} observation, corresponding to November 11^{th} 2020. From that time, Belgium would have presumably followed a steady economic growth (i.e., no collapse of index prices that could have led to a change in regime). To conclude, the crisis period would extend from February 20^{th} 2020 to November 11^{th} 2020.

In the next paragraphs, I am going to perform the same Markov analysis for the 5 stock market indices left.

4.5.2 France

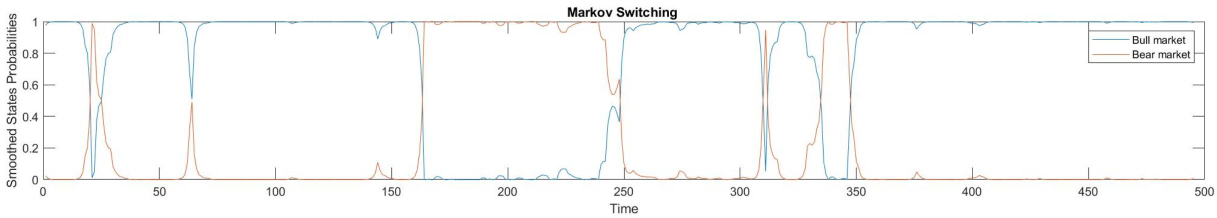

Figure 9: Graph exhibits changes from bull to bear market.

For the French stock market, one can notice a change in regime - from bull to bear market - at the 163^{rd} observation. This observation corresponds to the date of February 21st 2020. Then, at the 250^{th} observation, corresponding to June 26^{th} 2020 (the end of the first quarantine) and regaining a little freedom, a switch in regime takes place – from a period of turmoil to a period of growth even if a second COVID-19 wave is expected. Afterwards, some back and forth movements are following until the 346^{th} observation, corresponding to November 9^{th} 2020. From that time, France would presumably follow a steady economic growth. To conclude, the crisis period would extend from February 21st 2020 to November 9^{th} 2020.

4.5.3 The Netherlands

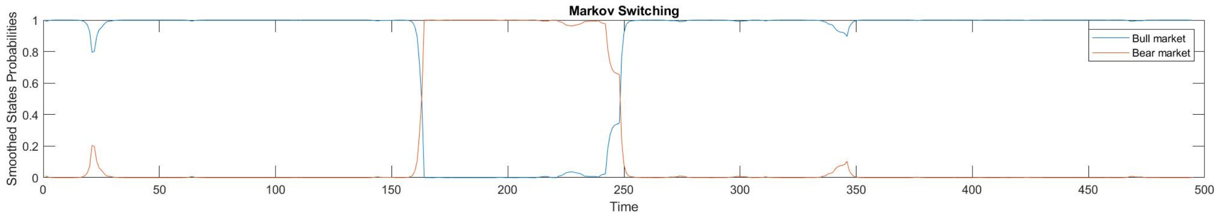

Figure 10: Graph exhibits changes from bull to bear market.

For the Dutch stock market, one can notice a change in regime - from bull to bear market – at the 163rd observation. This observation corresponds to the date of February 21^{st} 2020. Then, at the 248^{th} observation, corresponding to June 24^{th} 2020 (bars and restaurants are allowed to open) and regaining a little freedom, a switch in regime takes place – from a period of turmoil to a period of growth even if a second COVID-19 wave is expected. Nevertheless, from that time the Netherlands would apparently follow a steady economic growth. To conclude, the crisis period would extend from February 21st 2020 to June 24^{th} 2020.

4.5.4 Germany

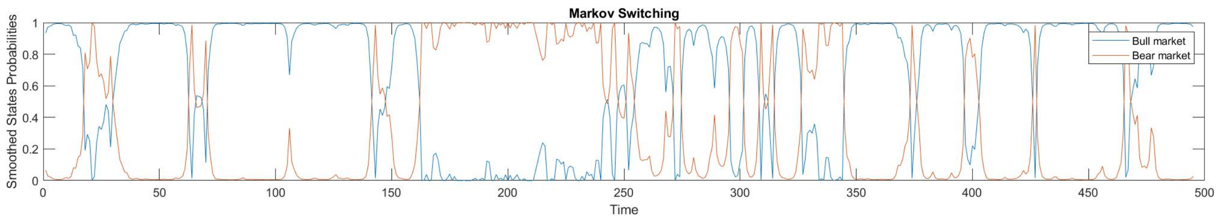

Figure 11: Graph exhibits changes from bull to bear market.

For the German stock market, one can notice a continuous change in regime – from bull to bear market – at the 162^{nd} observation. This observation corresponds to the date of February 20^{th} 2020. Then, at the 254^{th} observation, corresponding to July 2^{nd} 2020 (by this time most of the lockdowns of the country districts have ended up) and regaining a little freedom, a switch in regime takes place – from a period of turmoil to a period of growth even if a second COVID-19 wave is expected. Afterwards, many back and forth movements are following until the end of my sample period – that is to say June 6^{th} 2021. Albeit the German economy tipped drastically into a recession, it recovered well and DAX experienced an upward trend since the end of the first restrictions. To conclude, the crisis period would extend from February 21^{st} 2020 to July 2^{nd} 2020.

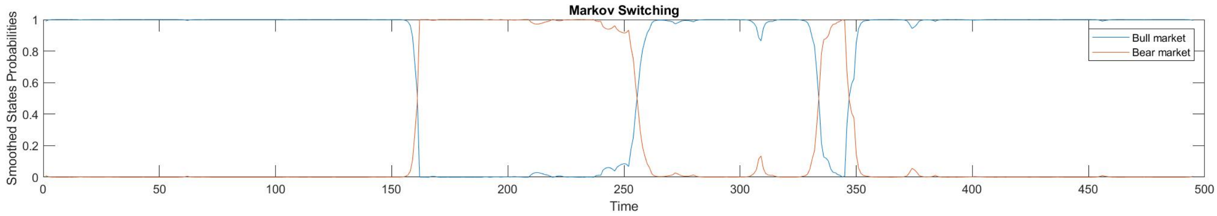

Figure 12: Graph exhibits changes from bull to bear market.

4.5.5 Spain

For the Spanish stock market, one can notice a change in regime – from bull to bear market – at the 161st observation. This observation corresponds to the date of February 19th 2020. Then, at the 256th observation, corresponding to July 6th 2020 (the end of the first quarantine) and regaining a little freedom, a switch in regime takes place – from a period of turmoil to a period of economic growth even if a second COVID-19 wave is expected. Afterwards, the state of emergency has been reimposed in October 2020 and the national market reacted negatively to this information (334th observation, corresponding to October 22nd 2020). Nonetheless, by mid-November, Spain has apparently followed a steady economic growth. To conclude, the crisis period would extend from February 19th 2020 to July 6th 2020.

4.5.6 The United Kingdom

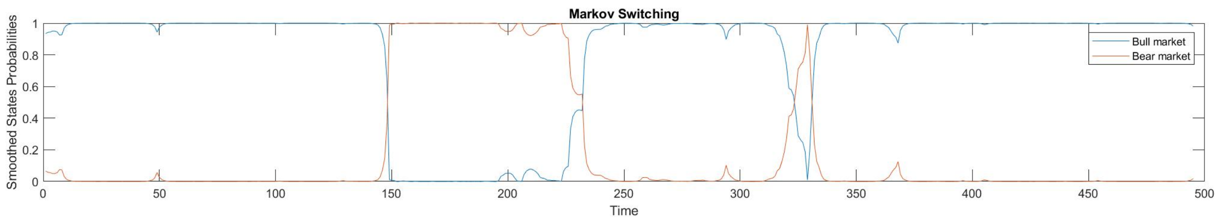

Figure 13: Graph exhibits changes from bull to bear market.

For the British stock market, one can notice a change in regime – from bull to bear market – at the 148th observation. This observation corresponds to the date of January 31st 2020. Then, at the 232nd observation, corresponding to June 2nd 2020 (announce of an easing of the lockdown with people able to meet friends and family outside in groups of not more than eight) and regaining a little freedom, a switch in regime takes place – from a period of turmoil to a period of economic growth even if a second COVID-19 wave is expected. Afterwards, a temporary bear market took shape at the 322nd observation, corresponding to October 6th 2020. From that time, the United Kingdom would have presumably followed a steady economic growth (i.e., no collapse of index prices that could have led to a change in regime). To conclude, the crisis period would extend from January 31st 2020 to June 2nd 2020.

5 Empirical results : volatility analyses

The first purpose in this master thesis is to be acquainted with the volatility structure of my 6 European REIT indices. As explained in the methodology chapter, three different GARCH models will be used to model the volatility of these time series: GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1). I divided the whole period into three distinct episodes: the Global Financial Crisis period going from 8^{th} November 2007 to 25^{th} March 2009, the normal economic cycle period going from 19^{th} September 2014 to 31^{st} January 2020 and the COVID period going from 1^{st} February 2020 to 18^{th} November 2020. As a first step, I am going to implement these models on each subperiod and analyze the parameters of each model. Then, I am going to deduce which one best captures the volatility patterns by means of the Akaike Information Criterion and the Bayesian Information Criterion.

5.1 Results in graphical form

To introduce this section, three graphs(figure14 , figure 15 and figure 16) are exhibited below. Each graph represents one of the three subperiods I am investigating. On each graph, the Belgium REIT index returns and the three respective conditional volatilities, computed with GARCH model, EGARCH model and GJR-GARCH model, are displayed. Note that the graphs for the other country indices can be found in the appendices.

5.2 Estimation of the parameters

The estimation of GARCH (1,1) models for the Global Financial Crisis period display highly significant ARCH parameters and GARCH parameters for the six European REIT indices. Alpha stands for short-run persistency of shocks, while beta stands for long-run persistency of shocks. Moreover, the alpha answers the following question: “Does the volatility from yesterday have an explanatory power for the current volatility?”. In addition, the beta parameter will answer the question: “Does the innovation from yesterday have an explanatory power for the current volatility?”. Regarding our results, long-term shocks seem to generate larger impacts on REITs volatility compared to short-term shocks ($\beta > \alpha$). This result holds for the overall scope of my study (i.e., each REIT index and each subperiod studied).

When it comes to asymmetric models, one must be aware that EGARCH does not lay down any restriction on the estimated parameters, whereas GJR-GARCH imposes $(\alpha+\beta)$ to be positive as well as ω, α and β. The stylized EGARCH fact is the following: if $\gamma < 0$, negative shocks will increase the volatility more than positive shocks. On the contrary, if $\gamma > 0$positive shocks increase the volatility more than negative shocks. Finally, the specificity of GJR-GARCH models is that bad news will be represented by $(\alpha+\beta)$ and good news will be represented by α. This suggests that negative returns from yesterday will have a bigger impact than positive returns on today’s volatility.

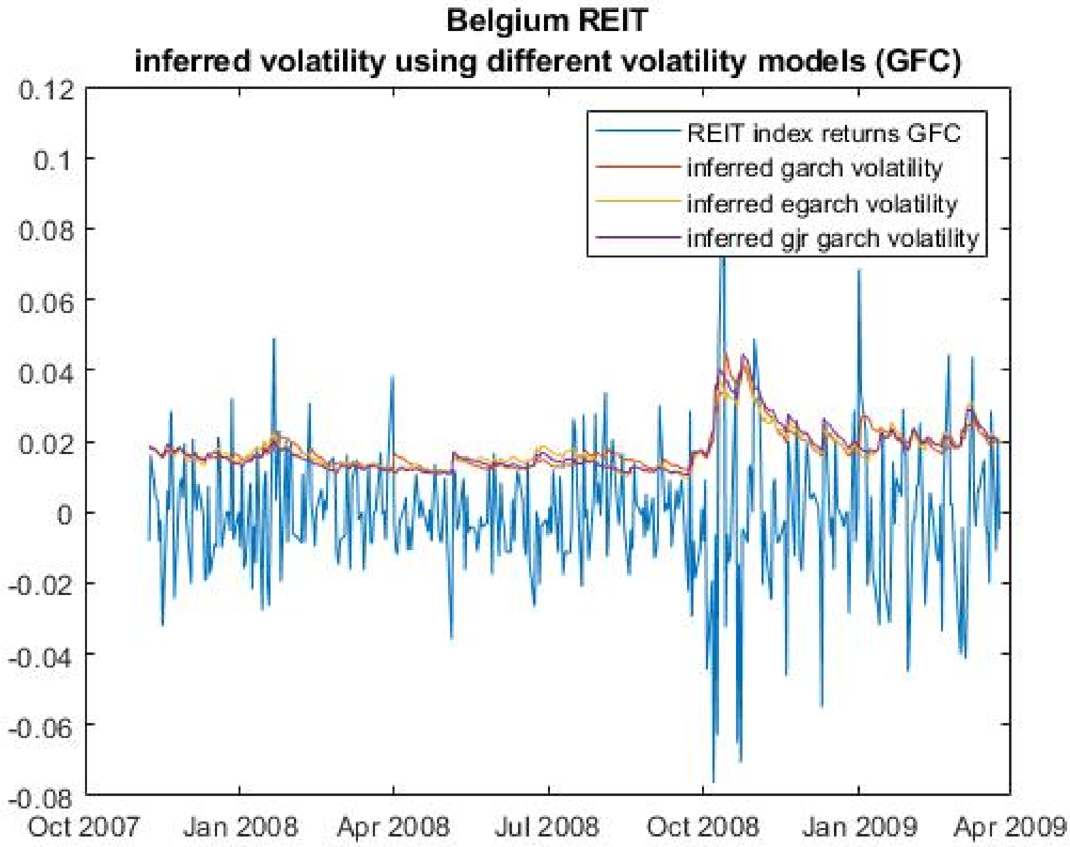

Figure 14: Volatility modelling during GFC.

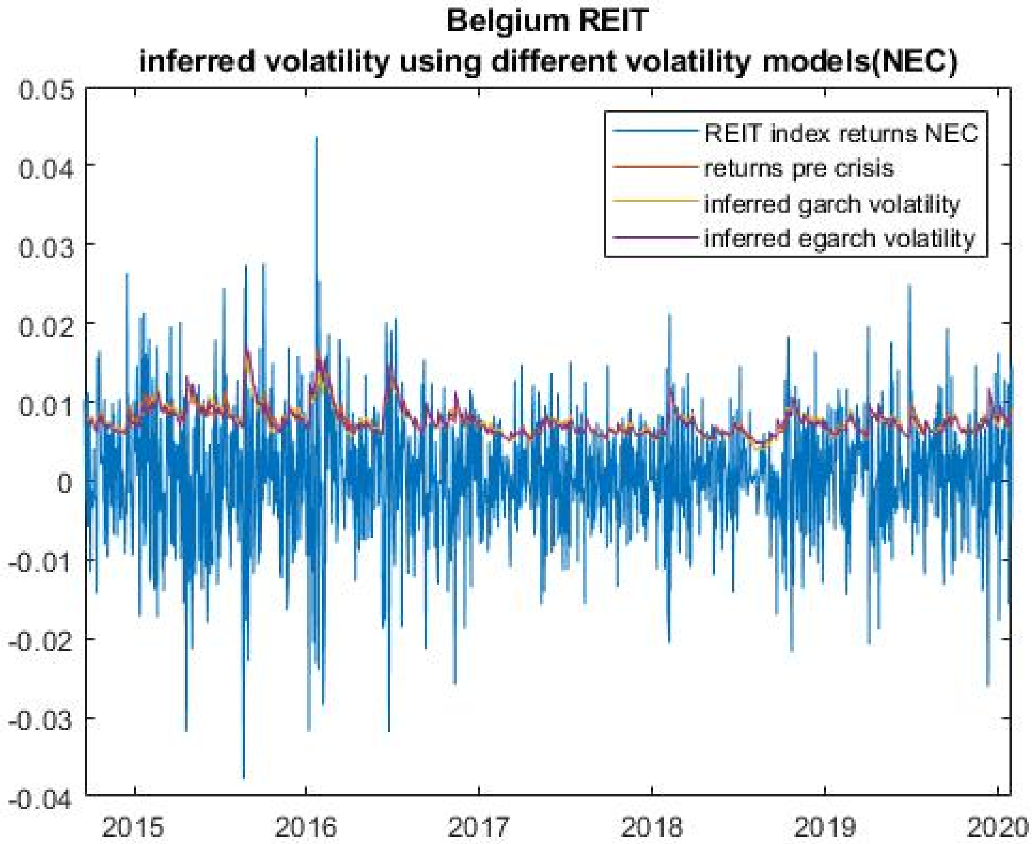

Figure 15: Volatility modelling during NEC.

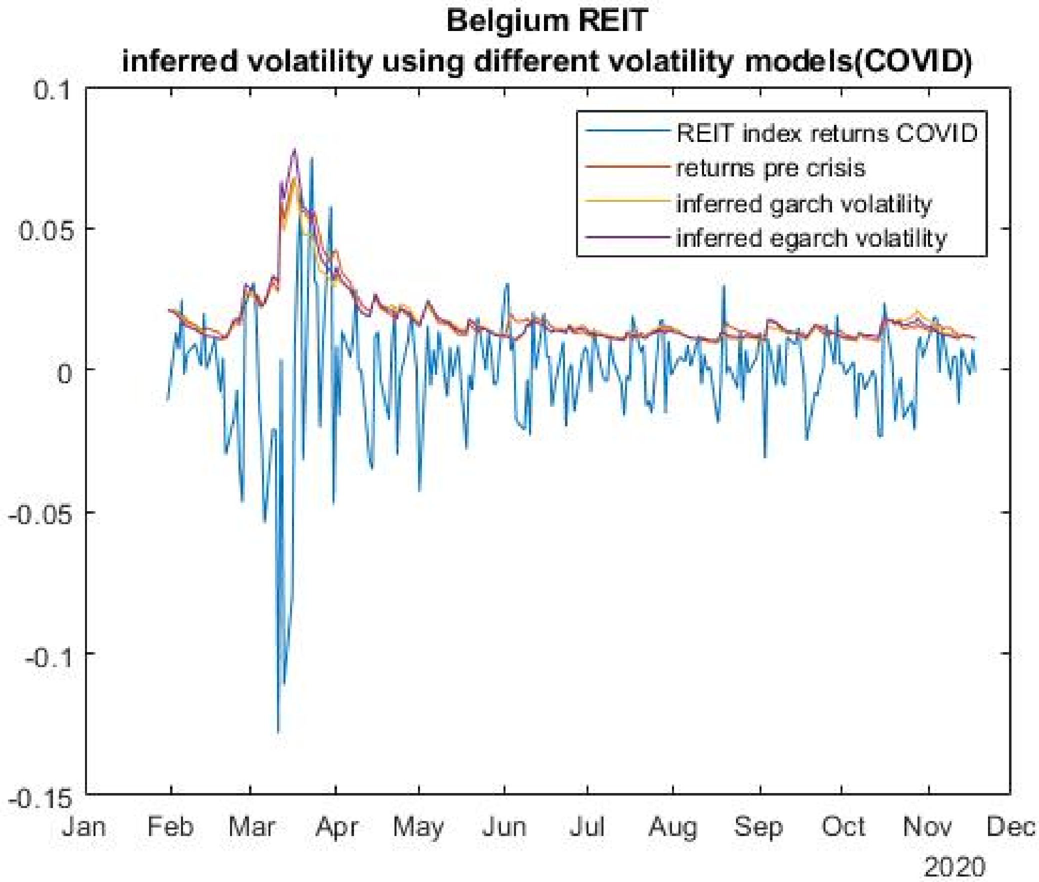

Figure 16: Volatility modelling during Covid.

5.3 Study of the volatility behaviour during Global Financial Crisis

The results are provided in the table 4. Note that Spain index has been excluded from the GFC period analysis because no information could have been retrieved before year 2014.

The GARCH model has very low ω but it does not seem significant because of the high p-values. Regarding the α and β paraemeters, they are all strongly significant. Belgium REIT index GARCH (1,1), for example, indicates a value of 0.0931 for α and a value of 0.8785 for β. It shows that the volatility of yesterday is an important piece of evidence in the explanation for the current volatility. In addition, the returns from yesterday also has a role to play; but to lesser extent. Furthermore, the persistence $(\alpha+\beta)$ is very high for each single country ; between 0.97 and 0.995. The "half-life"[26] of volatility is calculated through $(\log(0.5)/\log(\alpha+\beta)$. Note that if the sum of the *arch* and *garch* parameters is equal to 1, the half-life becomes infinite and so, the persistence. For example, the half-life of the Dutch REIT index is 172.94, meaning that it would take 173 days to move back to the mean volatility whereas the Belgian REIT index would recover faster (i.e.,24 days) due to the lower value of its α and β.

The asymmetric EGARCH model shows statistically significant leverage effects for every REIT index, except for UK and Germany REIT indices. The estimate of γ in almost all time series is negative. It indicates that negative shocks will increase the volatility more than positive shocks. The UK index is an outlier since γ is positive and consequently, past positive returns would increase the volatility more than negative-returns shocks (but this is tremendously insignificant with a 0.85 p-value).

The second asymmetric model, GJR-GARCH, shows statistically significance for parameter β but demonstrates insignificance for many of the other parameters. These are some clues to expect that GJR-GARCH will not be chosen as the best fitter, at least for the GFC period. Considering the GJR-GARCH(1,1) model for the Netherlands, the estimate of γ is positive. It stipulates the presence of the asymmetric effect of the past returns on the current conditional variance[27].

5.4 Study of the volatility behaviour during Normal Economic Cycle

The results are provided in the table 5. For the period going from September 2014 to January 2020 (NEC period), the GARCH model is highly significant at a 1% significance level for all parameters and all indices (except the intercept of the variance ω for Belgium). In comparison to GFC period, the past returns in normal economic cycle time seem to play a bigger role than during the financial crisis (except for france and belgium REITs that denotes a very similar value for parameter α). Furthermore, the volatility persistence $(\alpha+\beta)$ is lower than during GFC ; between 0.92 and 0.987. The half-life of the Dutch REIT is now 9 days versus 173

[26] It can be defined as the time needed for the volatility to move halfway back towards its unconditional mean following a deviation from this mean.

[27] Since the indication function takes the value 1.

			Coefficient parameters				IC	
		Countries	ω (p-value)	α(p-value)	β (p-value)	γ (p-value)	AIC	BIC
GFC	GARCH	Belgium	0,0001 (0,0019)	0,0931 (0)	0,8785 (0)	/	-1,909	-1,898
		France	0,00007 (0,35)	0,1035 (0,002)	0,8906 (0)	/	-1,705	-1,693
		The Netherlands	0,00007 (0,26)	0,1255 (0)	0,8705 (0)	/	-1,734	-1,722
		Germany	0,000025 (0,01)	0,1857 (0)	0,8014 (0)	/	-1,574	-1,562
		United Kingdom	0,00009 (0,37)	0,0721 (0,007)	0,9203 (0)	/	-1,545	-1,534
	EGARCH	Belgium	-0,2105 (0,002)	0,1112 (0,001)	0,97 (0)	-0,1327 (0)	-1,918	-1,903
		France	-0,19 (0,22)	0,1580 (0,006)	0,9758 (0)	-0,0994 (0)	-1,709	-1,693
		The Netherlands	-0,1174 (0,21)	0.1464 (0)	0,9865 (0)	-0,1529 (0)	-1,744	1,729
		Germany	-0,1945 (0,051)	0,2953 (0)	0,9718 (0)	-0,0165 (0,59)	-1,571	-1,556
		United Kingdom	-9,39 (0,0048)	0,1938 (0,12)	-0,3339 (0,47)	0,0113 (0,85)	-1,506	-1,49
	GJR GARCH	Belgium	0,00007 (0)	NaN	0,9074 (0)	0,1396 (0)	-1,92	-1,91
		France	0,00006 (0,36)	0,0461 (0,21)	0,8938 (0)	0,1043 (0,028)	-1,708	-1,893
		The Netherlands	0,00003 (0,57)	0,0047 (0,8)	0,8972 (0)	0,1851 (0)	-1,743	-1,728
		Germany	0,000095 (0,03)	0,1280 (0,004)	0,8287 (0)	0,057 (0,24)	-1,572	-1,599
		United Kingdom	0,000008 (0,38)	0,0477 (0,17)	0,9267 (0)	0,0326 (0,49)	-1,544	-1,528

Table 4: GARCH models results during Global Financial Crisis

during GFC. It indicates that the volatility behaviour would go back to normal faster in steady economic growth situation than in periods of turmoil.

Concerning EGARCH model, the estimate of γ in almost all time series is negative. It indicates that negative shocks will increase the volatility more than positive shocks. The Netherlands depict an outlier with a γ value of0.23. Meanwhile, the presence of asymmetric effect on the conditional variance makes it more volatile than symmetric GARCH model (bigger β parameter).

The second asymmetric model, GJR-GARCH, shows statistically significance for parameters α and β. Both, the past returns and the past conditional variance play a role in the determination of the current conditional variance. Moreover, in case of negative last-period return, this negative returns will have an impact of $(\alpha+\gamma)$ on the current conditional variance. As one can see in table 5, the effect of a negative return on Dutch index conditional variance will be 0.1743 (i.e., 0.12134+0.053).

5.5 Study of the volatility behaviour during COVID

The results are provided in the table 6.

The results are provided in the table 6. For the period going from January 2020 to November 2020 (COVID period), the GARCH model is significant at a 1% significance level for parameters α and β (except for α France at 5% level and α Spain that is not significant). In comparison to GFC period, the past returns (i.e, immediate disturbance) during the health crisis seem to play a bigger role than during the financial crisis. This goes hand in hand with a lower β. Furthermore, the volatility persistence $(\alpha+\beta)$ does not give clear-cut results. Actually, I would have expected a greater persistence for COVID than for NEC but it is not necessary the case and it is essentially due to the fast recovery that took place in summer 2020 after the end of the first lockdown.

Looking at EGARCH model, the estimate of γ is negative in all time series and it is significant for all indexes, except for UK. It confirms that negative shocks will increase the volatility more than positive shocks. In addition, the absolute value of nearly all γ are superior to the ones exhibited during NEC; meaning that negative past disturbance would have a more substantial impact on the current conditional variance. Meanwhile, the presence of asymmetric effect on the conditional variance makes it more volatile than symmetric GARCH model (bigger β parameter). I have to point out an issue regarding the β value of Germany. Indeed, $|\beta|$ must be strictly lower than 1. I challenged my MATLAB code but did not manage to find where the problem could come from.

The second asymmetric model, GJR-GARCH, shows statistically significance for parameters β and λ. So, the past conditional variance plays a role in the determination of the current conditional variance. Moreover, in case of negative last-period return, this negative disturbance will have an impact of $(\alpha+\gamma)$ on the current conditional variance. As one can see in table 6,

			Coefficient parameters				IC	
		Countries	ω	α	β	γ	AIC	BIC
NEC	GARCH	Belgium	0 (0,026712)	0,080919 (0)	0,88698 (0)	/	-9,752	-9,736
		France	0 (0,0079209)	0,095095 (0)	0,86895 (0)	/	-8,774	-8,758
		The Netherlands	0 (0)	0,15067 (0)	0,77453 (0)	/	-8,959	-8,944
		Germany	0,000025 (0,01)	0,1857 (0)	0,8014 (0)	/	-1,5741	-1,5625
		Spain	0 (0)	0,15606 (0)	0,7775 (0)	/	-9,008	-8,992
		United Kingdom	0 (0)	0,14273 (0)	0,79642 (0)	/	-8,579	-8,563
	EGARCH	Belgium	-0,384785 (0)	0,16453 (0)	0,95983 (0)	-0,052 (0)	-9,759	-9,738
		France	-0,31782 (0)	0,15855 (0)	0,96469 (0)	-0,049 (0)	-8,785	-8,764
		The Netherlands	-0,71546 (0)	0,23985 (0)	0,9214 (0)	0,23 (0)	-8,973	-8,952
		Germany	-0,23416 (0)	0,17342 (0)	0,97433 (0)	-0,03 (0)	-9,462	-9,441
		Spain	-0,39123 (0)	0,21432 (0)	0,95625 (0)	-0,08 (0)	-9,011	-8,990
		United Kingdom	-0,43457 (0)	0,25277 (0)	0,94932 (0)	-0,005 (0,7)	-8,567	-8,546
	GJR GARCH	Belgium	0 (0,010548)	0,046712 (0)	0,88548 (0)	0,064 (0)	-9,759	-9,738
		France	0 (0,0049069)	0,052931 (0)	0,87065 (0)	0,076 (0)	-8,782	-8,761
		The Netherlands	0 (0)	0,12134 (0)	0,76892 (0)	0,053 (0,04)	-8,960	-8,939
		Germany	0 (0,18871)	0,048969 (0)	0,93241 (0)	0,025 (0,02)	-9,458	-9,437
		Spain	0 (0)	0,092496 (0)	0,79617 (0)	0,11647 (0)	-9,017	-8,996
		United Kingdom	0 (0)	0,1423 (0)	0,79643 (0)	0,001 (0,95)	-8,577	-8,556

Table 5: GARCH models results during Normal Economic Cycle

the effect of a negative return on France index conditional variance, for example, will be 0.2470 (i.e., 0.077+0.17).

		Countries	Coefficient parameters ω	α	β	γ	IC AIC	BIC
COVID	GARCH	Belgium	0 (0,079)	0.181 (0)	0,779 (0)	/	-1,116	-1,106
		France	0,00018 (0,169)	0,17982 (0,025)	0,726 (0)	/	-7,79	-7,69
		The Netherlands	0 (0,034)	0,15 (0)	0,813 (0)	/	-8,78	-8,68
		Germany	0 (0,5439)	0,416 (0)	0,583 (0)	/	-1,606	-1,596
		Spain	0,00012 (0,298)	0,067 (0,079)	0,80 (0)	/	-8,81	-8,71
		United Kingdom	0 (0,148)	0,122 (0,0004)	0,85482 (0)	/	-1,030	-1,020
	EGARCH	Belgium	-0,34 (0,007)	0,15 (0,049)	0,959 (0)	-0,16 (0,0001)	-1,122	-1,108
		France	-0,55427 (0,163)	0,341(0)	0,911 (0)	-0,092 (0,018)	-0,781	-0,768
		The Netherlands	-0,368 (0,087)	0,29146 (0)	0,945 (0)	-0,089 (0,01)	-0,881	-0,868
		Germany	-1,2162 (0)	1 (0)	1 (0)	-0,017 (0)	-5,836	-5,823
		Spain	-0,23 (0)	-0,16 (0)	0,97109 (0)	-0,079 (0,006)	-0,917	-0,904
		United Kingdom	-10 (0,109)	-0,049 (0,67)	-0,31 (0,7)	-0,076 (0,34)	-0,995	-0,981
	GJR GARCH	Belgium	0 (0,0061)	/	0,815 (0)	0,22 (0.0003	-1,125	-1,115
		France	0,000116 (0,18)	0,077 (0,1304)	0,78275 (0)	0,17 (0,011)	-0,781	-0,768
		The Netherlands	0 (0,021)	0,0813 (0,037)	0,80094 (0)	0,14 (0,021)	-0,88	-0,867
		Germany	0 (0,552)	0,31983 (0)	0,59462 (0)	0,17111 (0)	-1,608	-1,594
		Spain	0 (0,115)	/	0,89 (0)	0,08 (0,004)	-0,887	-0,877
		United Kingdom	0 (0,054)	/	0,91247 (0)	0,13 (0)	-1,036	-1,026

Table 6: GARCH models results during Covid

6 Model evaluation

6.1 Testing the no autocorrelation for residuals and squared residuals

Ljung-Box test and Engle's test are applied on both residuals and squared residuals for each subperiod. There is clearly no strong evidence to reject the null hypothesis. This is a more than appreciable result because it indicates that the serial dependency disappeared. As argued by Burns in 2002, the principle is that the GARCH models should capture the dynamics of the REIT returns which means that the squared standardized residuals should be independently distributed. To visualize these results effortlessly, I plotted the autocorrelation diagram up to lag 20. I did this manipulation for Belgium REIT index but for information, the other graphs are available in the appendices.

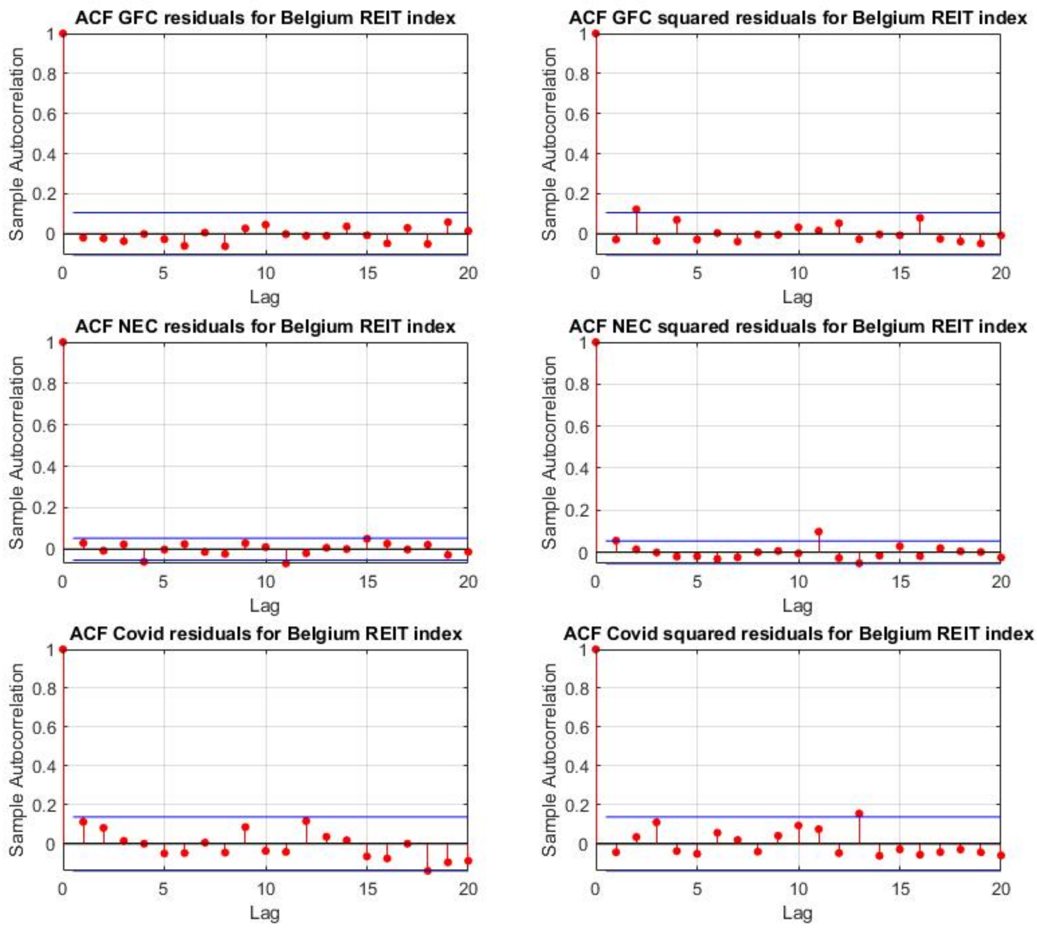

Figure 17: SACF of residuals and squared residuals for Belgium REIT index.

6.2 Model selection

Thanks to IC in tables 4, 5 and 6, we found out that EGARCH (1,1) model seems to be often the best one in order to capture the dynamic volatility. Nevertheless, GARCH (1,1) performed extremely well also. The main advantages of GARCH (1,1) sit in its simplicity and overall, its parsimony in terms of parameters needed. Moreover, the results of the goodness of fit are mixed and I was expecting more clear-cut results in favor of GARCH (1,1). Thus, I took the decision to opt for GARCH (1,1) but I want to point out that EGARCH (1,1) slightly outperformed GARCH (1,1). One can denote that the GJR-GARCH (1,1) is rarely considered as the best fitter.

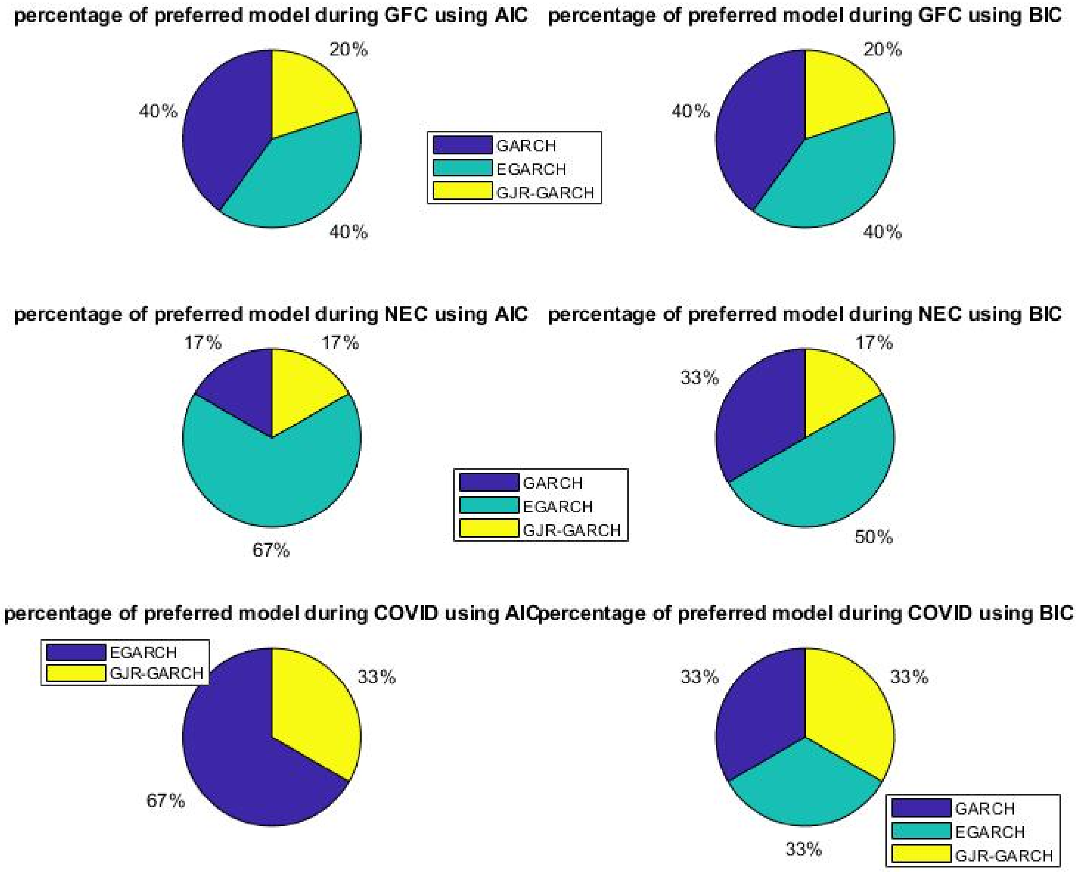

Figure 18: Easy way to get an overview of the best fitters according to BIC and AIC.

7 Usefulness of these models for portfolio or risk managers

In the past, portfolio managers used the volatility as a fixed value based on the past observations. Now, thanks to the GARCH models I built, portfolio managers can interpret volatility as a dynamic time-varying concept. Thus, financial analysts who have an interest in REITs, and in stocks broadly, have access to more accurate measures of REITs behaviors. It would be fruitful for a portfolio manager to detect precisely what is the volatility pattern of some stocks in comparison to other stocks, for example European REITs.

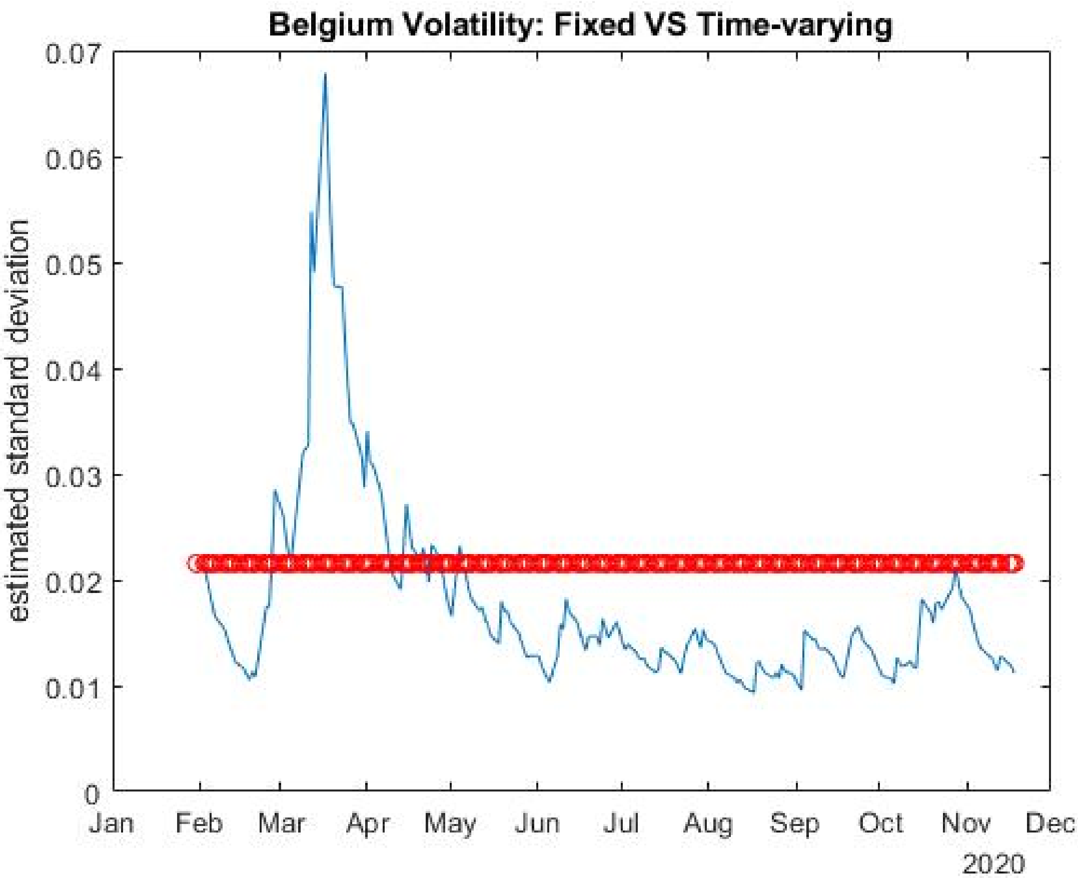

Figure 19: Fixed versus time-varying volatility for REIT index during Covid.

On the graph 19, on one hand, the fixed volatility underestimates the volatility explosion that occurred in March 2020. On the other hand, it overestimates the volatility for the months following the first wave of the health crisis. This awareness could have led to changes in portfolio strategies or hedging strategies.

But, a more relevant measure of risk used by risk managers is the Value at Risk[28]. VaR is essential in that it takes into account the direction of an investment's fluctuation. Indeed, a high volatility could be linked to a surge in REITs prices, which is positive for an investor. For

[28]Value at risk = VAR.

portfolio managers, the risk is about the probabilities of losing money due to a downside market movement and that is the essence of VaR. “What is the maximum loss?”. So, I computed the dynamic 1% VaR for each REIT index using my three conditional variance models and also the fixed VaR (i.e., homoscedasticity). I am pleased to prove that I always improve my VaR estimation (and so the forecasting) while using GARCH models estimates. To be consistent I plotted these measures for the Belgium REIT index on figure20 during the period of turmoil due to the health crisis. Furthermore, the unconditional VaR is only violated at the peak of the COVID period. The three dynamic models give similar VaR values that are time-varying. The hazardous belief that unconditional VaR is more restrictive than dynamic VaR is disproved. The well-informed investors could thenceforth review their investment strategies or their hedging strategies.

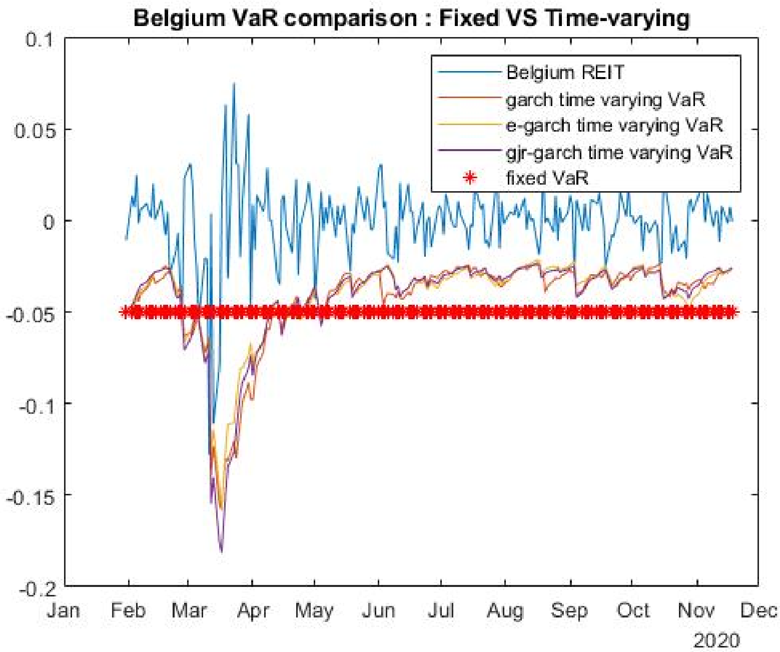

Figure 20: Value at risk is becoming time-varying.

8 DCC-GARCH model implementation

First, the correlations between REIT are exhibited in the tables 7, 8 and 9 for the REIT indices returns throughout all time periods analyzed. The correlation between REITs are mostly high (0.33-0.85). Correlations differ according to the time period studied. An apparent increase in correlation can be detected during the health crisis downturn- in comparison to the level of correlation observed during the "Normal Economic Cycle". During the global financial crisis, the correlation coefficients follow the same upward trend (GFC vs. NEC) even if the results are more contrasted than for the NEC-Covid period.

Correlation	Belgium REIT	France REIT	Netherlands REIT	Germany REIT	Spain REIT	UK REIT
BE-REIT	1					
FR REIT	0.69	1				
Neth REIT	0.77	0.83	1			
GE-REIT	0.38	0.41	0.42	1		
Spain REIT	/	/	/	/	1	
UK REIT	0.54	0.73	0.68	0.34	/	1

Table 7: Table – Global Financial Crisis period – REITs fixed correlation

Correlation	Belgium REIT	France REIT	Netherlands REIT	Germany REIT	Spain REIT	UK REIT
BE-REIT	1					
FR REIT	0.7	1				
Neth REIT	0.65	0.76	1			
GE-REIT	0.52	0.42	0.5	1		
Spain REIT	0.55	0.57	0.51	0.39	1	
UK REIT	0.59	0.69	0.63	0.33	0.54	1

Table 8: Table – Normal Economic Cycle period – REITs fixed correlation

Correlation	Belgium REIT	France REIT	Netherlands REIT	Germany REIT	Spain REIT	UK REIT
BE-REIT	1					
FR REIT	0.69	1				
Neth REIT	0.70	0.58	1			
GE-REIT	0.57	0.52	0.53	1		
Spain REIT	0.62	0.75	0.75	0.56	1	
UK REIT	0.70	0.70	0.72	0.61	0.72	1

Table 9: Table – Covid health crisis period – REITs fixed correlation

Moreover, one can be interested in comparing the correlation between REITs and EuroStoxx50[29] during the three subperiods. These findings highlight that the REIT market is correlated to the stock market in periods of turmoil. This is essentially true for the health crisis as well as for the GFC period but to a lesser extent. During the period 2014 - end 2019, it seems that the correlation is close to zero. The different correlations can be found in tables 10 ,11 and 12.

Correlation	Belgium REIT	France REIT	Netherlands REIT	Germany REIT	Spain REIT	UK REIT
EuroStoxx50	0.06	0.05	0.09	0.11	/	0.09

Table 10: Table – correlation EuroSTOXX50 vs. REITs - GFC

Correlation	Belgium REIT	France REIT	Netherlands REIT	Germany REIT	Spain REIT	UK REIT
EuroStoxx50	0.02	0.04	0.03	0.03	-0.03	0.03

Table 11: Table – correlation EuroSTOXX 50 vs. REITs – NEC

Correlation	Belgium REIT	France REIT	Netherlands REIT	Germany REIT	Spain REIT	UK REIT
EuroStoxx50	0.52	0.46	0.42	0.55	0.39	0.45

Table 12: Table – correlation EuroSTOXX 50 vs. REITs – Covid

But the preceding results do not take into account the time-varying characteristic of both the variance and the covariance. This is where DCC-GARCH comes into play. DCC-GARCH is implemented to detect the degree of volatility correlation changes between two or more variables, namely REIT returns. We want to investigate if there are links or comovements between REITs volatility pattern and also between REITs and EuroStoxx50 (representing the overall European stock market).

The figures 21 illustrates the dynamic correlation between Belgium REIT and the other five indices. Belgium REIT index seems to have the same correlation behaviour, except for France REIT which demonstrates a much higher correlation with Belgium REIT. Moreover, the increase of the correlation in periods of high correlation is even more marked with France, representing by the blue spikes. In figure 22, one can notice that the France REIT index appears more correlated to The Netherlands REIT and UK REIT than it was on graph 21.

For the sake of meticulousness, the figures corresponding to the correlation behaviour between ,respectively, The Netherlands, Germany, Spain and UK and the other REITs are displayed in the appendices.

Considering the correlation between EuroStoxx50 and REIT indices, each REIT index follows the same correlation behaviour. Indeed, the dynamic correlation is between -0.1 and

[29]"The EURO STOXX 50 is a stock index of Eurozone stocks designed by STOXX. It is made up of fifty of the largest and most liquid stocks." (Eikon software).

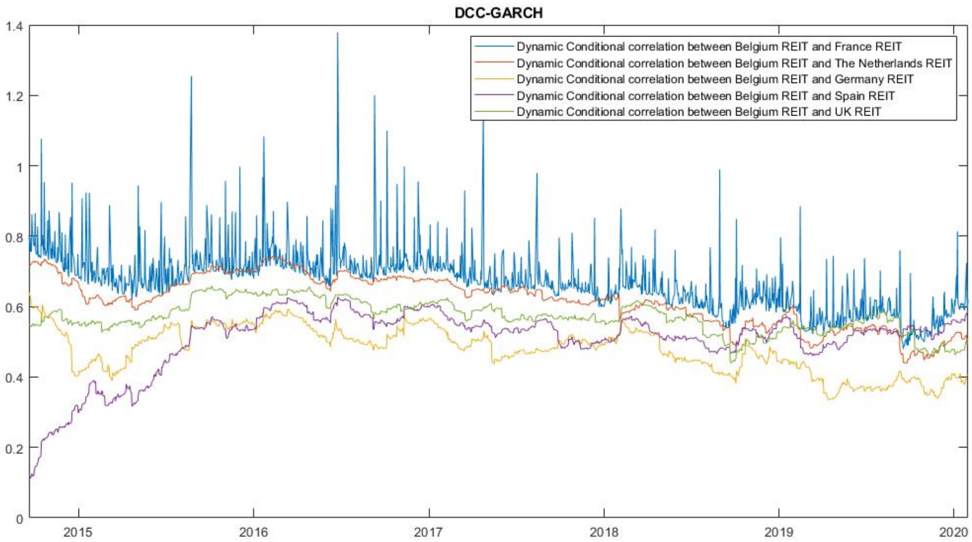

Figure 21: Dynamic Conditional Correlation for period Q42014 - Q42019 (Belgium point of view).

0.15/0.1 which is lower than the REIT vs. REIT correlation. Consequently, the REIT indices do not seem strongly correlated to the European stock market (figure 23).

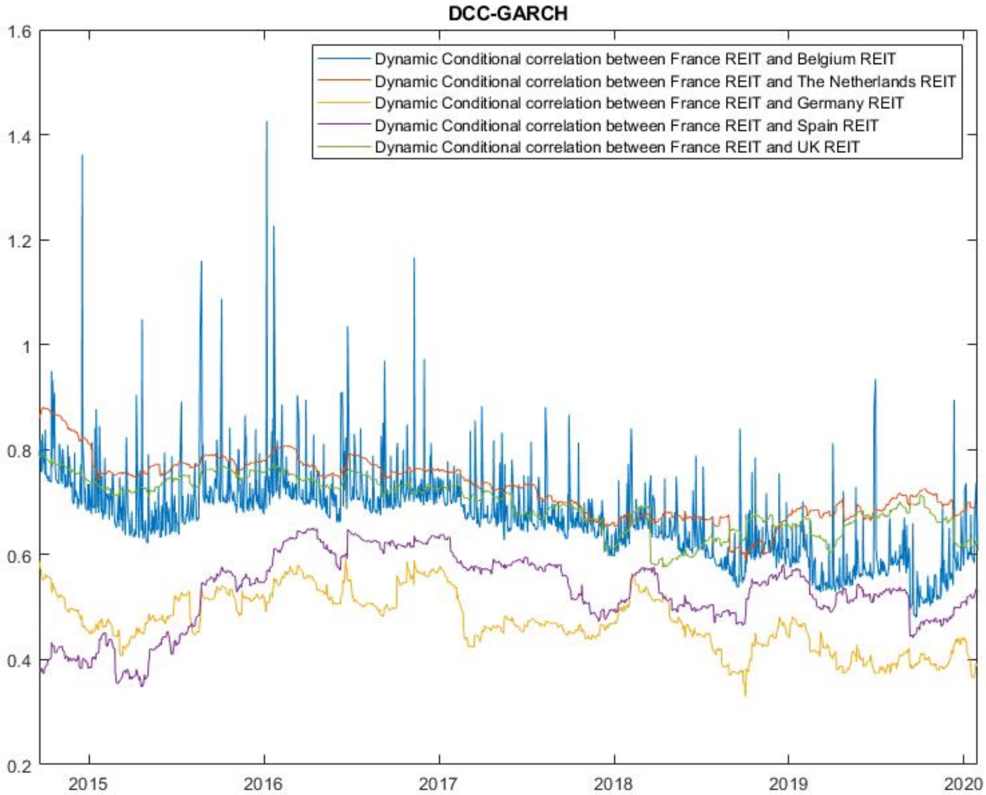

Figure 22: Dynamic Conditional Correlation for period Q42014 - Q42019 (France point of view).

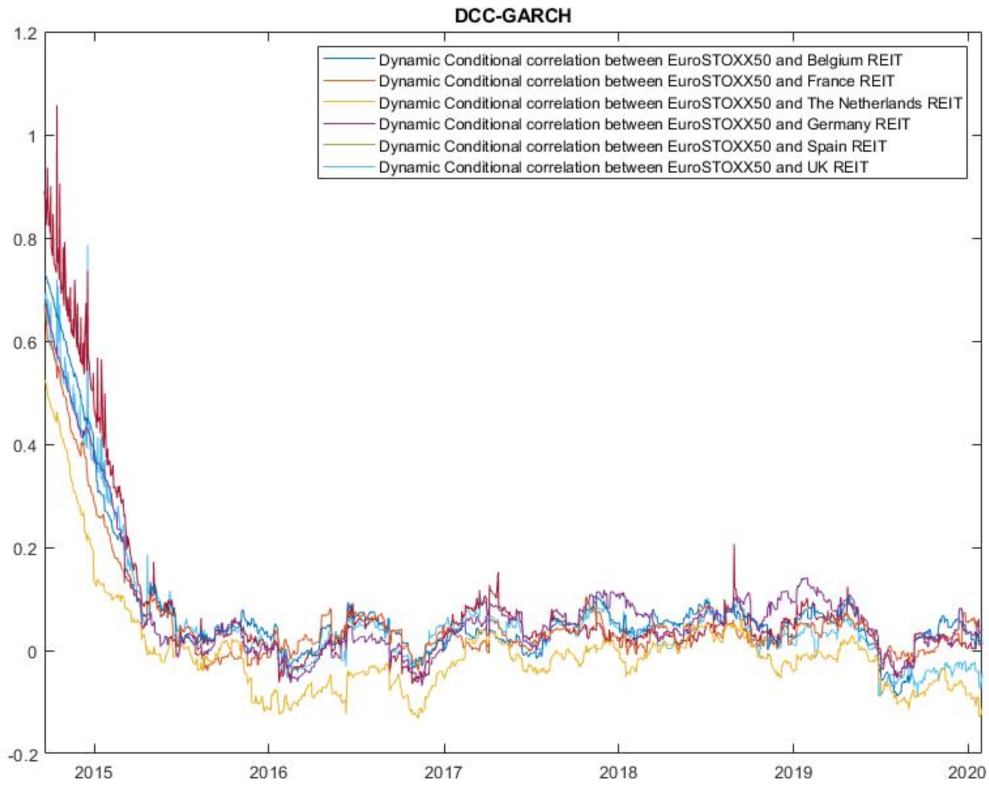

Figure 23: Dynamic Conditional Correlation between EuroStoww50 and REIT indices for period Q42014 - Q42019.

The spectrum is open for future research in the area of REIT volatility. One element that would be interesting to study is the behaviour of volatility no longer by distinguishing geographically but by distinguishing by property types (i.e., residential, warehouse, hotels). Now that the GARCH models are defined, it is possible to make forecasts in the future and to study the quality of these forecasts.

9 Conclusion

First of all, the theoretical models of GARCH, EGARCH and GJR-GARCH allowed me to have a precise idea of the structure of the volatility. More precisely, the first purpose in this master thesis is to be acquainted with GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1). Each conditional variance model gives me useful insights for the three subperiods studied : the Global Financial Crisis period going from 8^{th} November 2007 to 25^{th} March 2009, the normal economic cycle period going from 19^{th} September 2014 to 31^{st} January 2020 and the COVID period going from 1^{st} February 2020 to 18^{th} November 2020. Thanks to the estimation of the parameters of each model, I could see the specificity of the conditional volatility for each period. Both α and β parameters are most of the time significant. It means that the answer to the two following questions is YES : "Does the volatility from yesterday have an explanatory power for the current volatility?" and "Does the innovation from yesterday have an explanatory power for the current volatility?". In a nutshell, it shows that the yesterday's volatility is an important piece of evidence in the explanation for the current volatility. That was the idea behind the breakthrough initiated by Bollerslev in 1986.

Furthermore, the two asymmetric models, EGARCH(1,1) and GJR-GARCH(1,1), prove generally the usefulness of their respective leverage parameter γ. In this sense, considering EGARCH(1,1), γ is almost every time negative. It indicates that negative shocks will increase the volatility more than positive shocks.Meanwhile, It is also important to note that the presence of asymmetric effect on the conditional variance makes it more volatile than symmetric GARCH model (bigger β parameter). Considering GJR-GARCH, the study of the different coefficients and parameters shows that in case of negative last-period return, this negative returns will have an impact of $\alpha + \gamma$ on the current conditional variance. This is caused by the positive sign of the leverage effect that makes the indicator function being equal to 1 and, consequently, it would transfer to negative disturbance a power of $\alpha + \gamma$. This conclusion applies to all the subperiods studied and all the indices (i.e., Belgium REIT, France REIT, The Netherlands REIT, Germany REIT, Spain REIT and UK REIT; respectively for the three periods studied, except for Spain GFC period for which no information was available).

When it comes to volatility persistence, all REIT indices display larger and so longer persistence in crisis periods compared to economic cycle period. The uncertainty associated with periods of high stress or major changes affects the persistence of volatility, which may provide opportunities for some investors.

From a pragmatic point of view, all my models have efficiently captured the conditional variance. This is proved by the Sample Autocorrelation Function performed on the residuals of each model. There is clearly no more strong evidence of serial dependency.

Regarding the quality of the models and which one is the best fitter, the results show a tendency for the EGARCH(1,1) model which is opposite to Lunde's belief (" We find no evidence that a GARCH(1,1) is outperformed by more sophisticated models in our analysis of exchange rates, whereas the GARCH(1,1) is clearly inferior to models that can accommodate a leverage effect in our analysis of IBM returns."). Perhaps we can say that the EGARCH(1,1) is better suited to REITs? We have to be careful because we did this analysis with order p and order q equal to 1 and thus maybe a GARCH(3,1) or a GARCH(1,2) would have given a better AIC or BIC. Moreover, the differences of fitting between the three models are thin and so it will lead me to prefer GARCH model for its parsimony.

The study of the correlation between the REITs confirms the true belief that the correlation in times of crisis increases relative to its value in normal times. Furthermore, it is essentially for portfolio managers to say that the correlation between REITs and the European stock market, represented by the EuroStoxx50, is quite low or even zero during calm period. But, it will drastically increase during a crisis period such as COVID-19. The results are more tempered concerning the GFC. In a nutshell, the six REITs seem to have a similar dynamic correlation pattern and are correlated to the overall stock market in periods of turmoil.

To conclude, what is crucial to understand is that volatility is time-varying. Having the ability to model volatility over time gives huge advantages to portfolio managers and risk managers. It would lead them to take wiser decisions when it comes to hedging strategies or portfolio allocation. Indeed, the well-known fixed standard deviation tends to overestimate or underestimate the risk according to the periods that the markets go through because it is a fixed value that does not allow this essential nuance. Thanks to the models I described and applied on my data, the road is open to different forecasts, which is the ultimate goal for financial analysts.

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Appendices

I Graphs for each REIT index where the three volatility models are exhibited as well as the returns

I.1 France REIT index

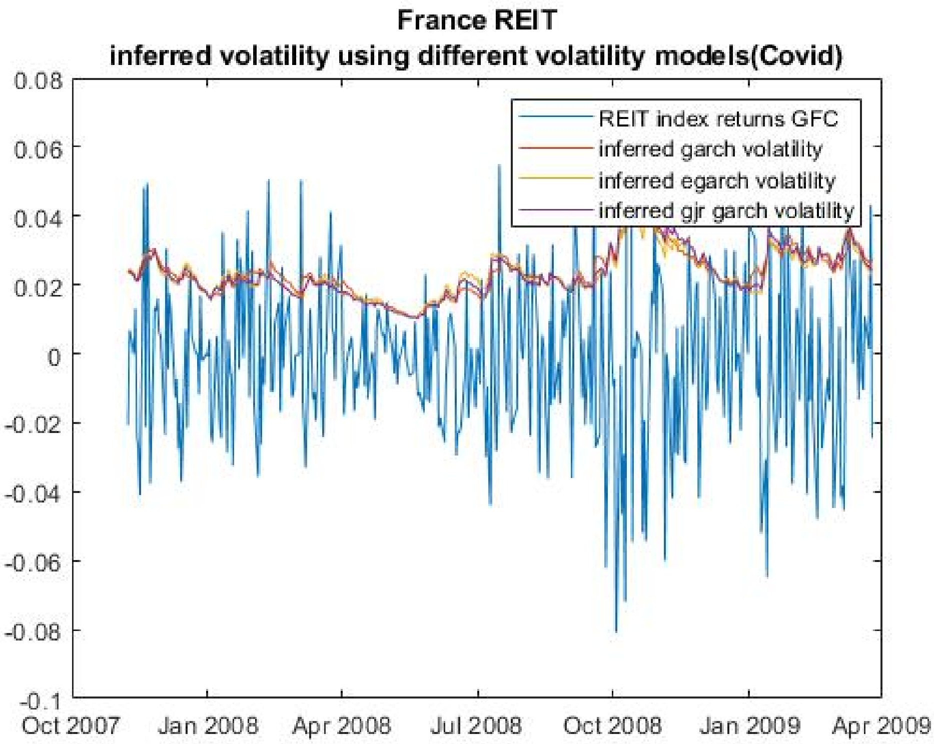

Figure 24: Volatility modelling during GFC.

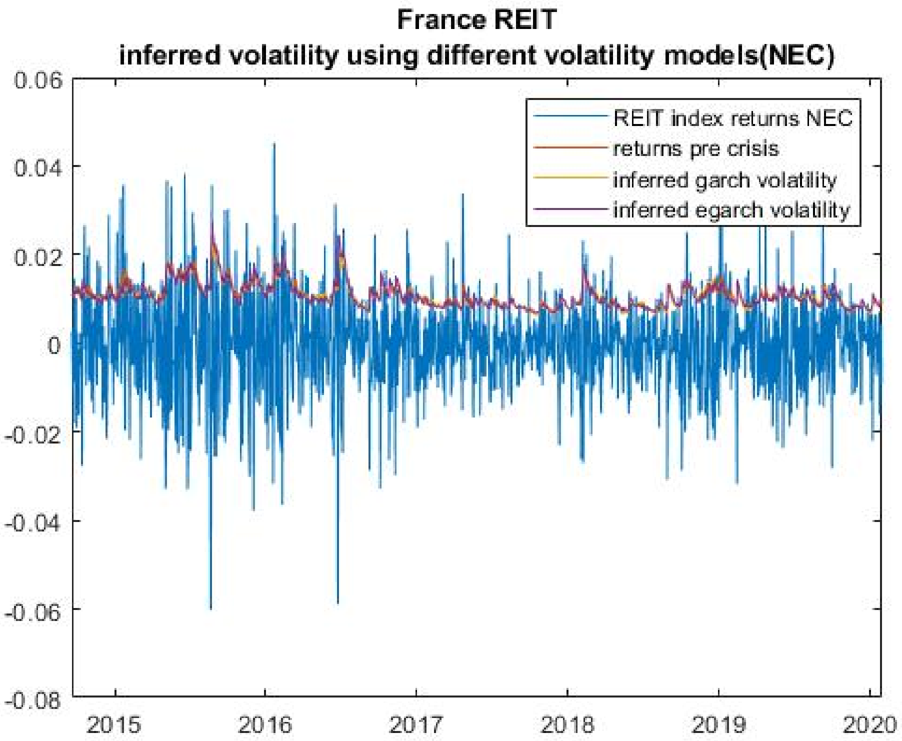

Figure 25: Volatility modelling during NEC.

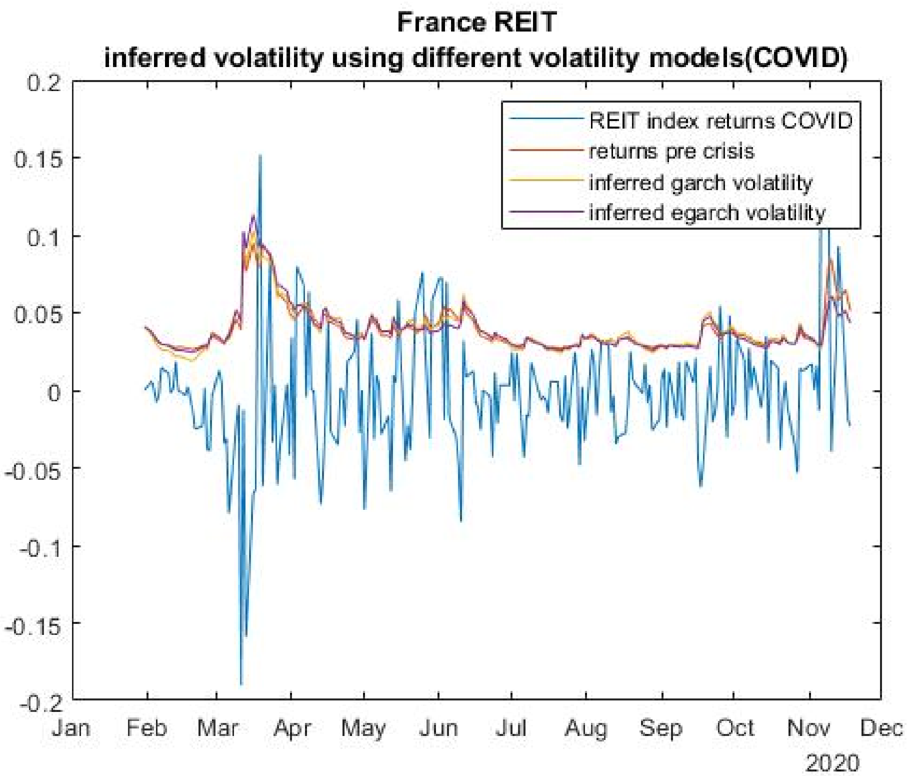

Figure 26: Volatility modelling during Covid.

I.2 The Netherlands REIT index

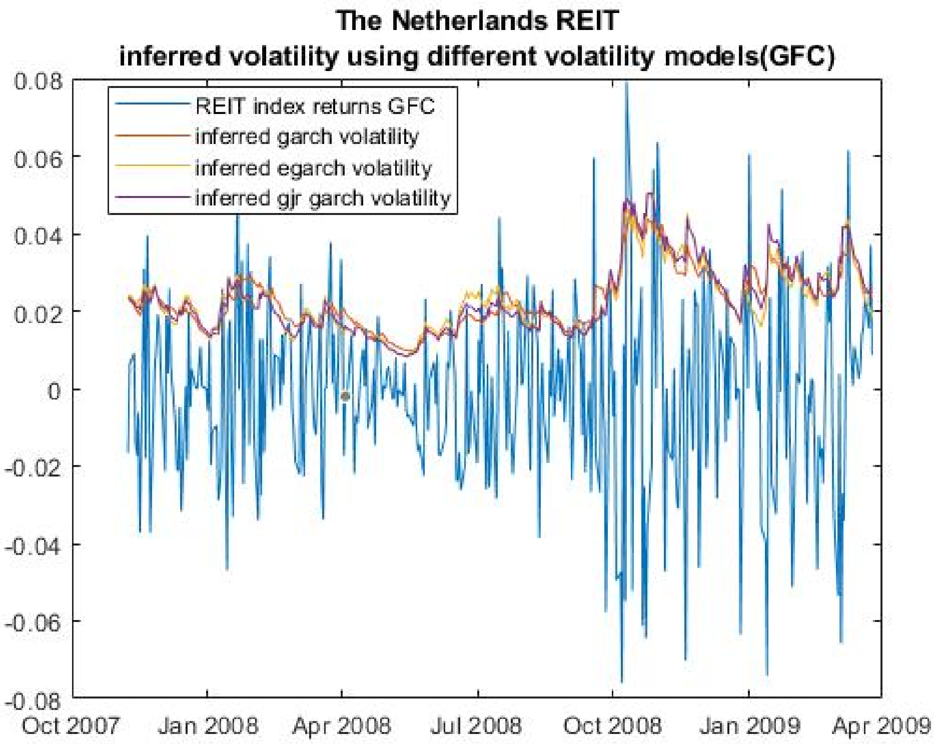

Figure 27: Volatility modelling during GFC.

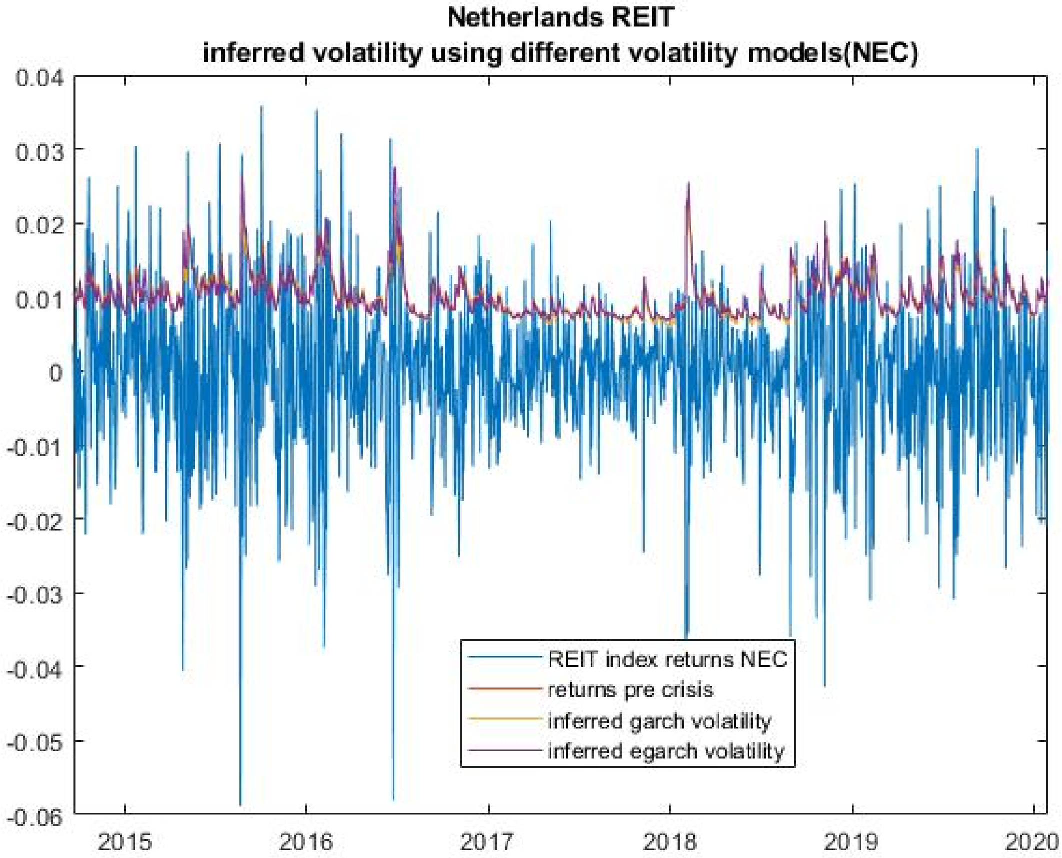

Figure 28: Volatility modelling during NEC.

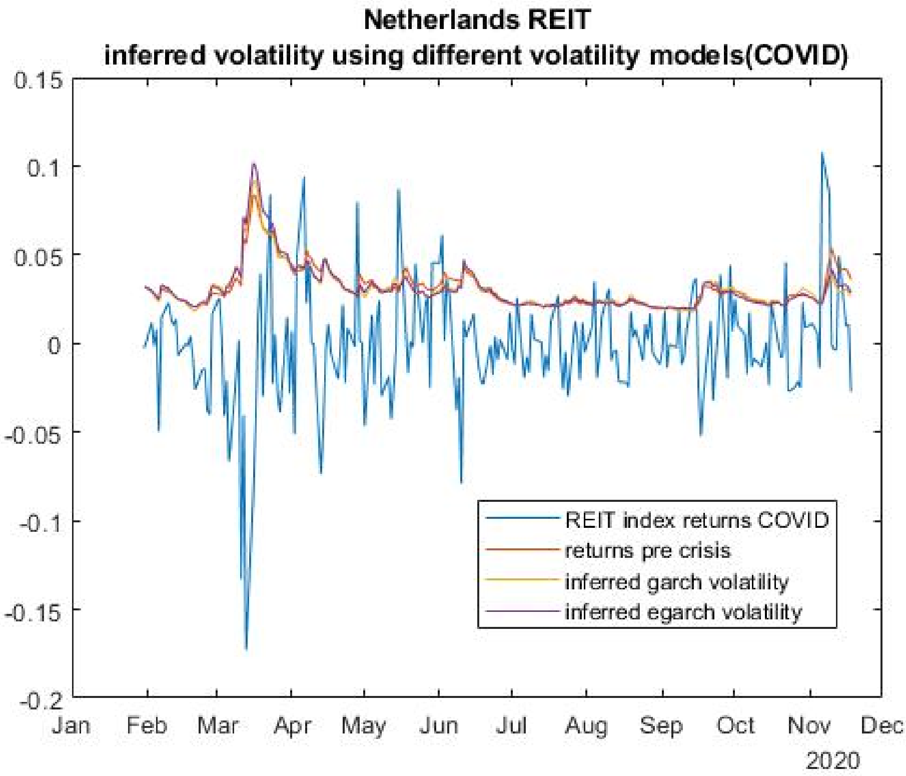

Figure 29: Volatility modelling during Covid.

I.3 Germany REIT index

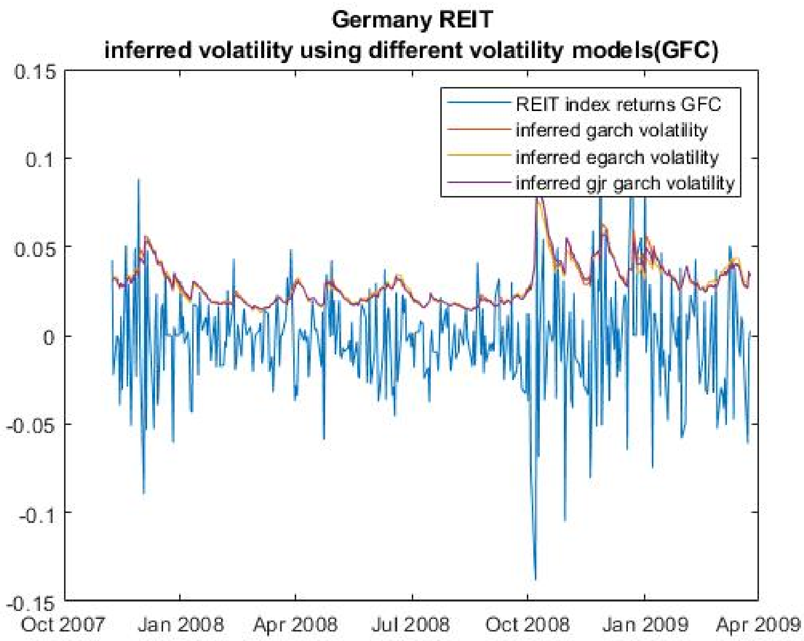

Figure 30: Volatility modelling during GFC.

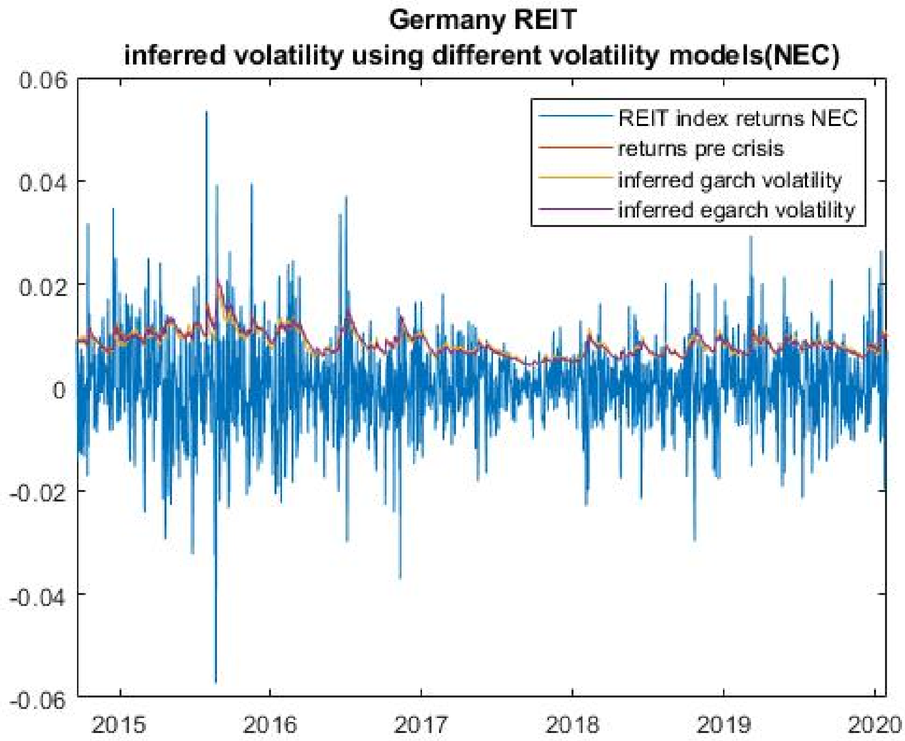

Figure 31: Volatility modelling during NEC.

I.4 Spain REIT index

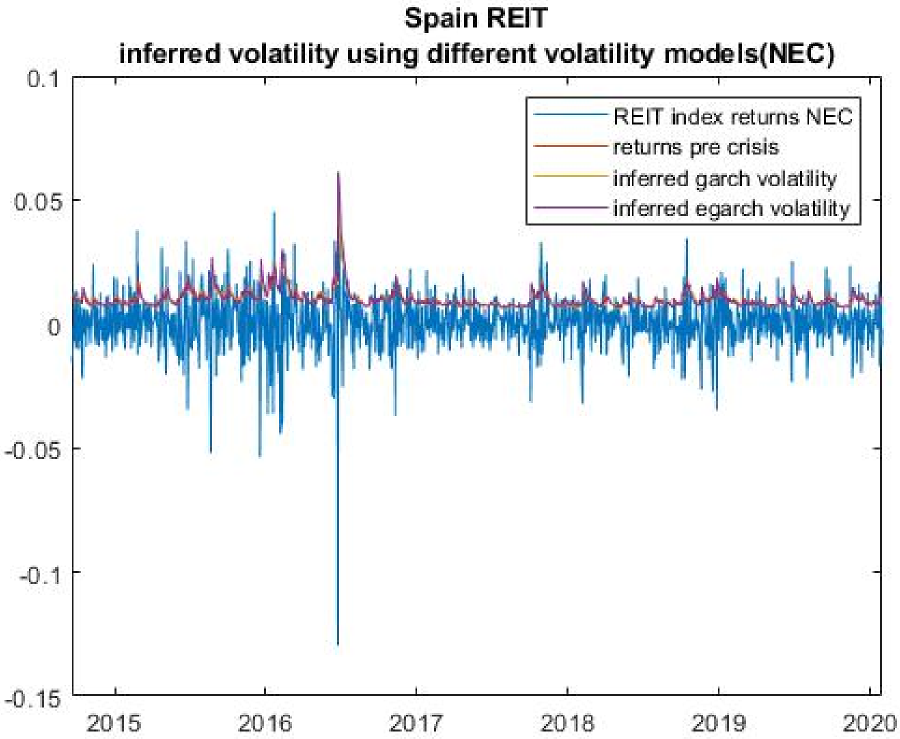

Figure 32: Volatility modelling during NEC.

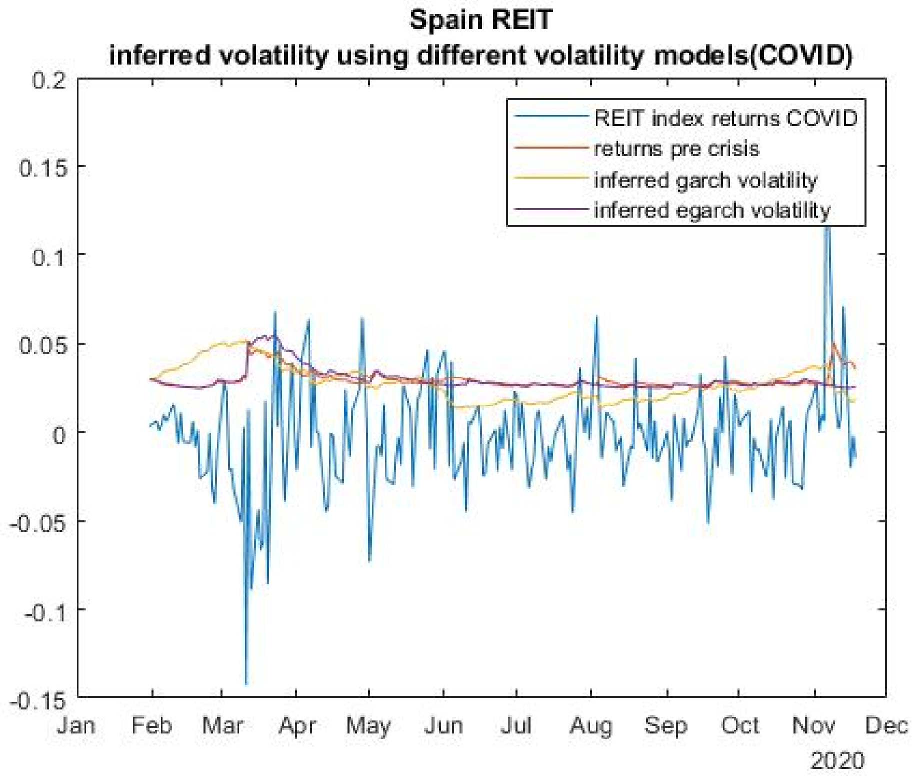

Figure 33: Volatility modelling during Covid.

I.5 UK REIT index

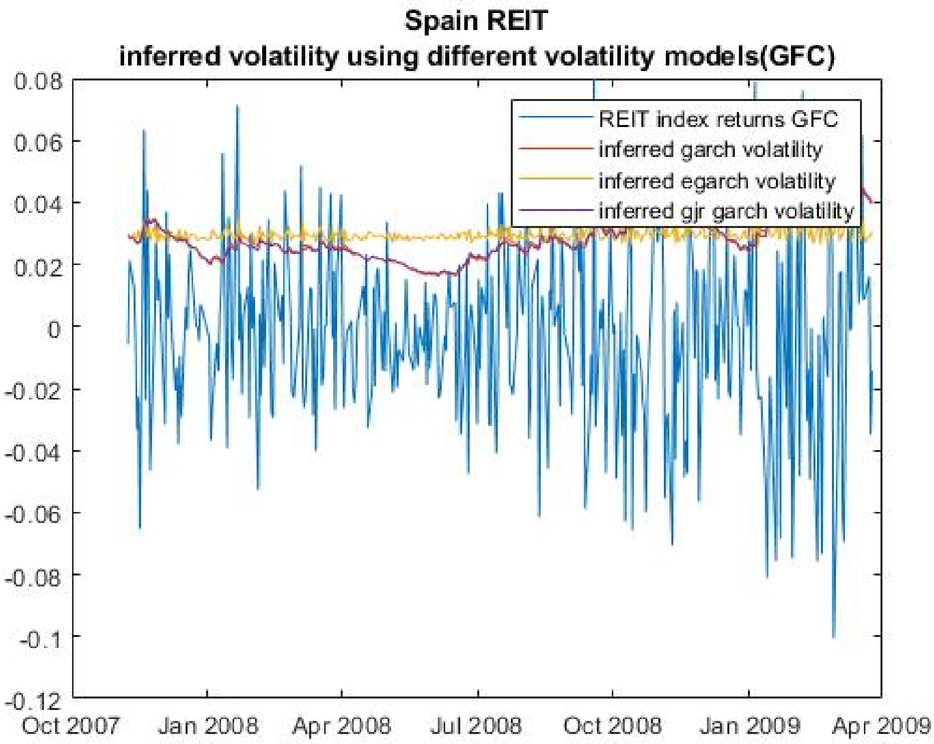

Figure 34: Volatility modelling during GFC.

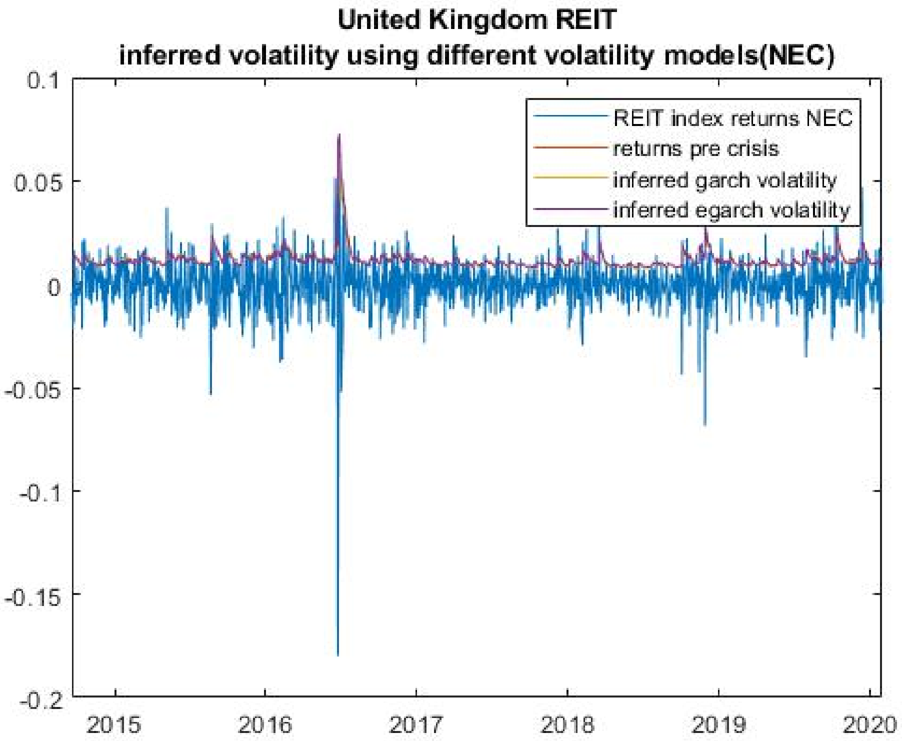

Figure 35: Volatility modelling during NEC.

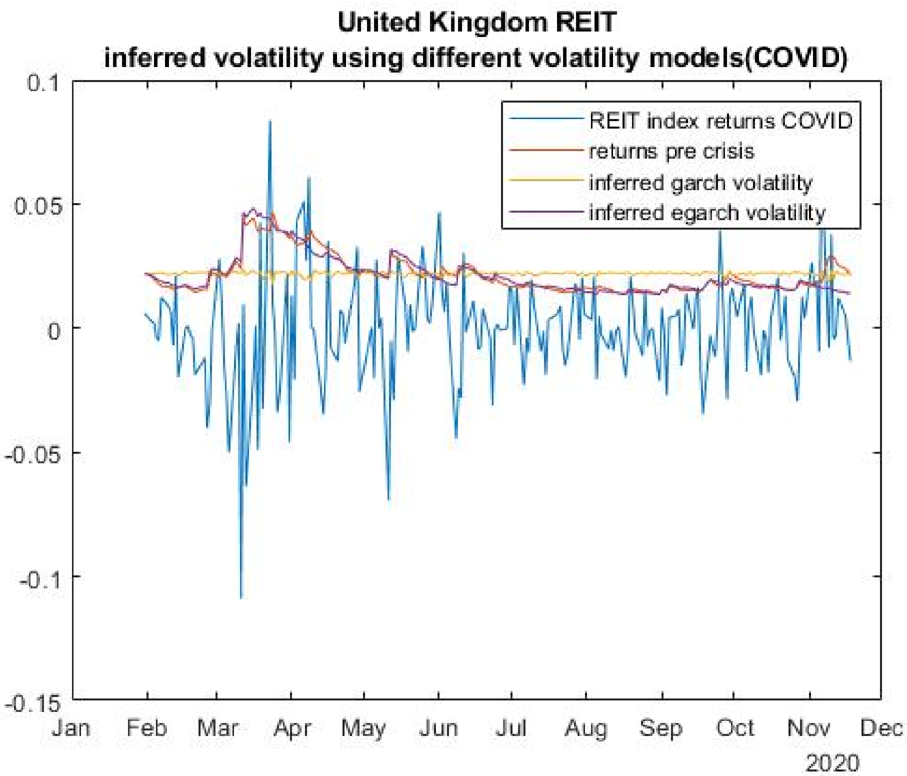

Figure 36: Volatility modelling during Covid.

II SACF for each REIT index

II.1 France

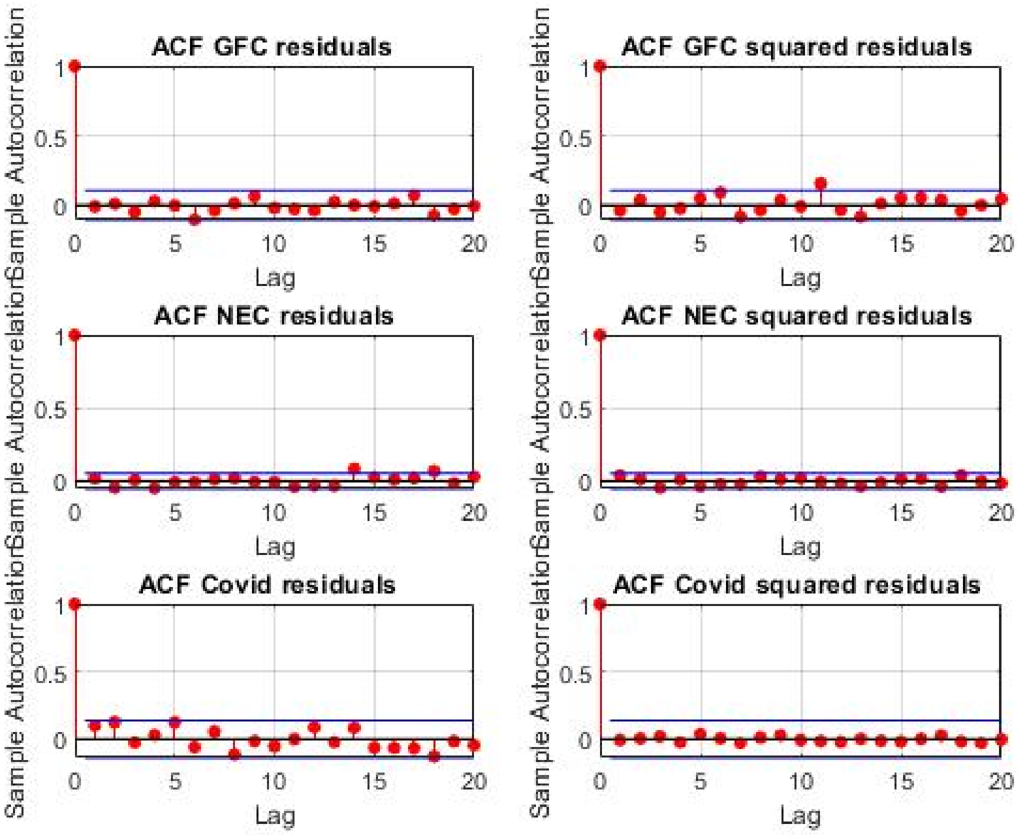

Figure 37: SACF of residuals and squared residuals for France REIT index.

II.2 The Netherlands

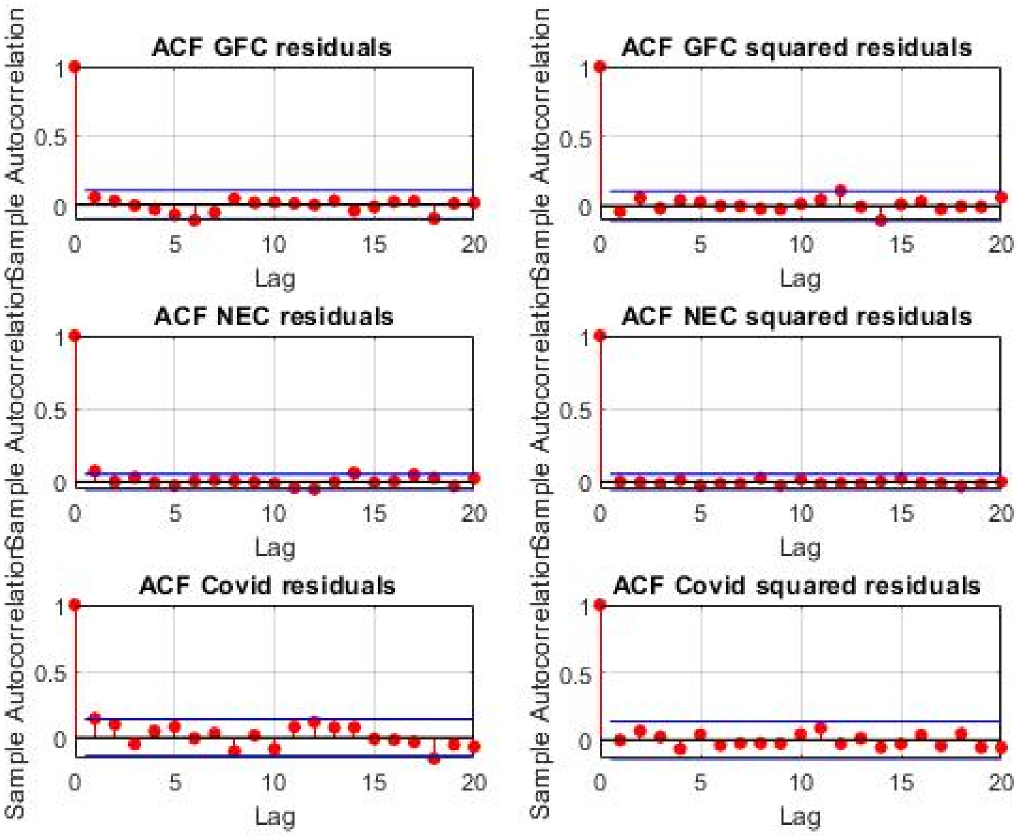

Figure 38: SACF of residuals and squared residuals for The Netherlands REIT index.

II.3 Germany

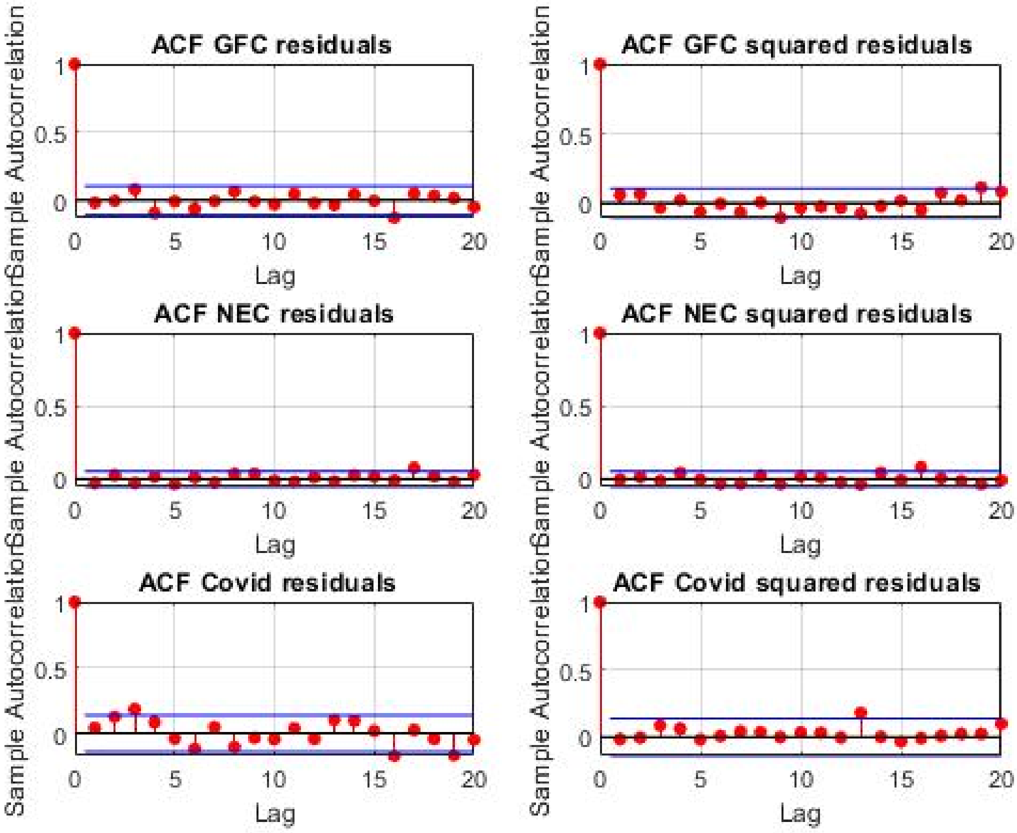

Figure 39: SACF of residuals and squared residuals for Germany REIT index.

II.4 Spain

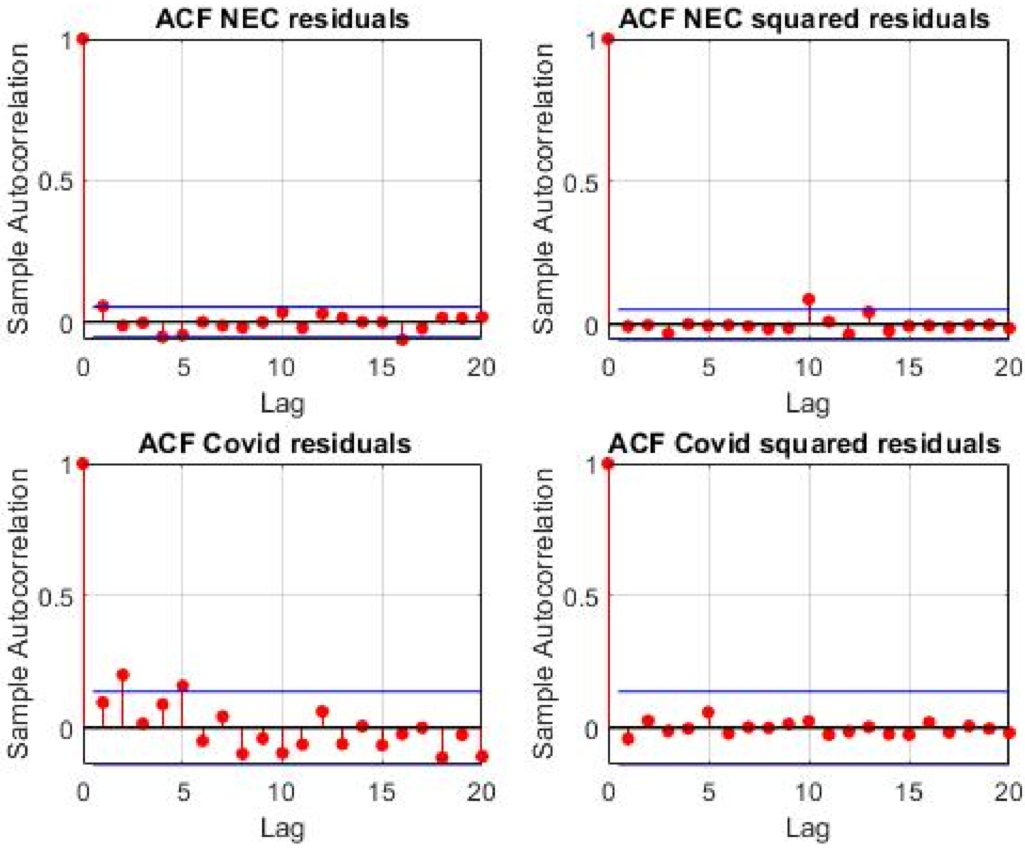

Figure 40: SACF of residuals and squared residuals for Spain REIT index.

II.5 UK

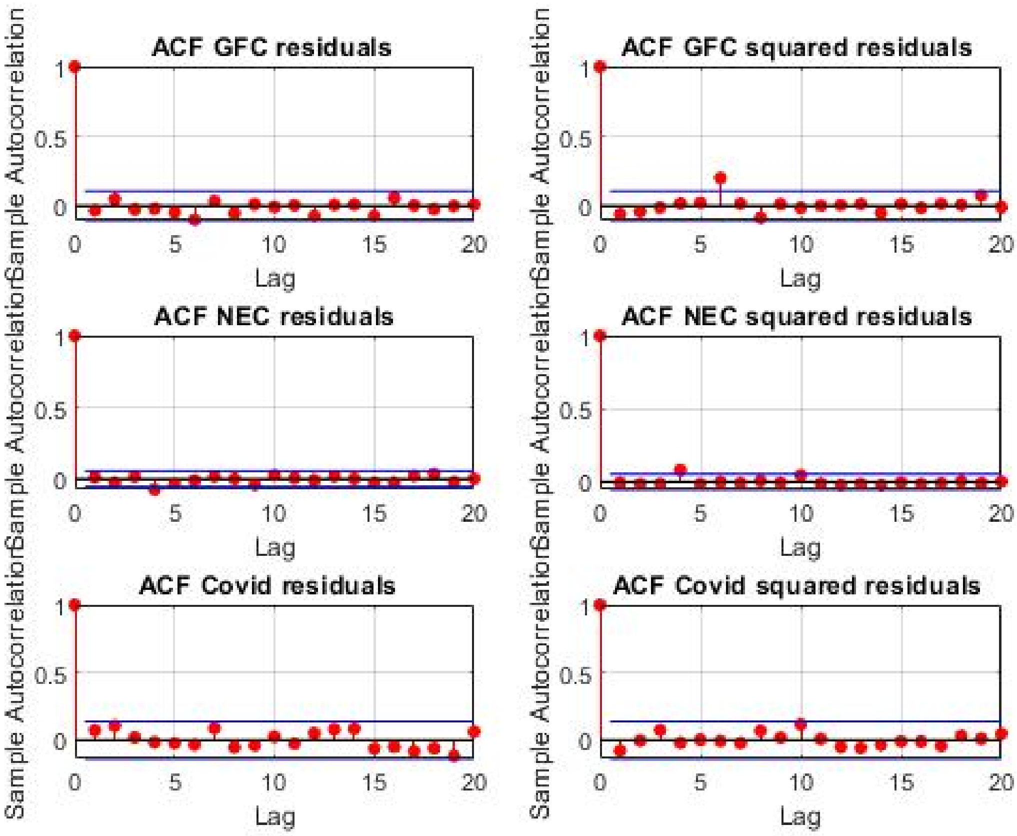

Figure 41: SACF of residuals and squared residuals for UK REIT index.

III Fixed vs. dynamic VaR for REIT indices during COVID

III.1 France

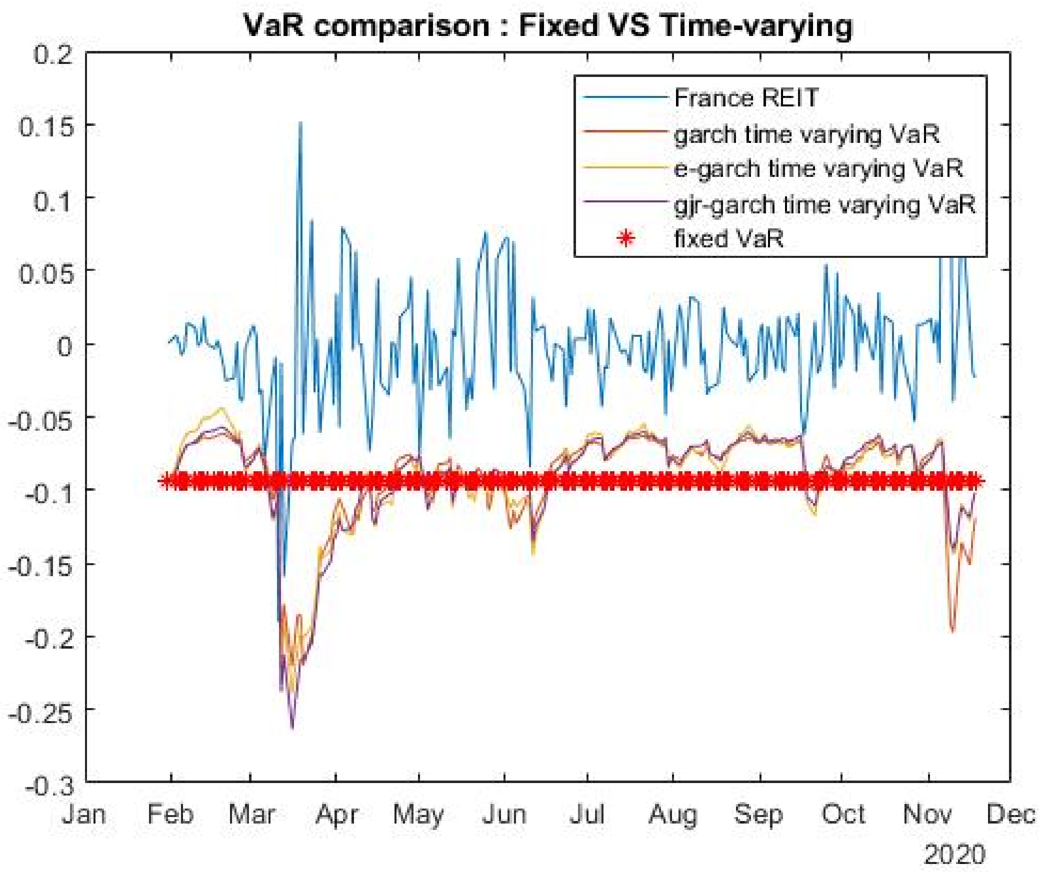

Figure 42: Value at risk for France REIT during health crisis.

III.2 The Netherlands

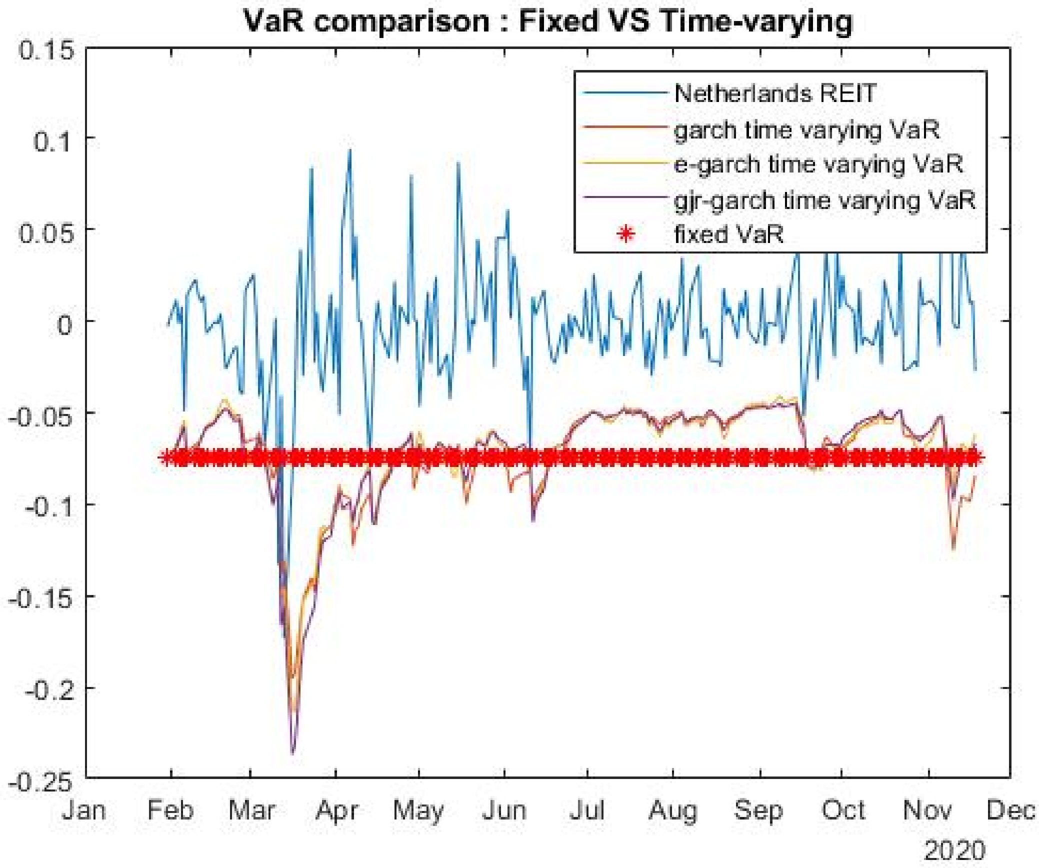

Figure 43: Value at risk for Dutch REIT during health crisis.

IV Fixed vs. dynamic VaR for REIT indices during Global Financial Crisis.

IV.1 Belgium

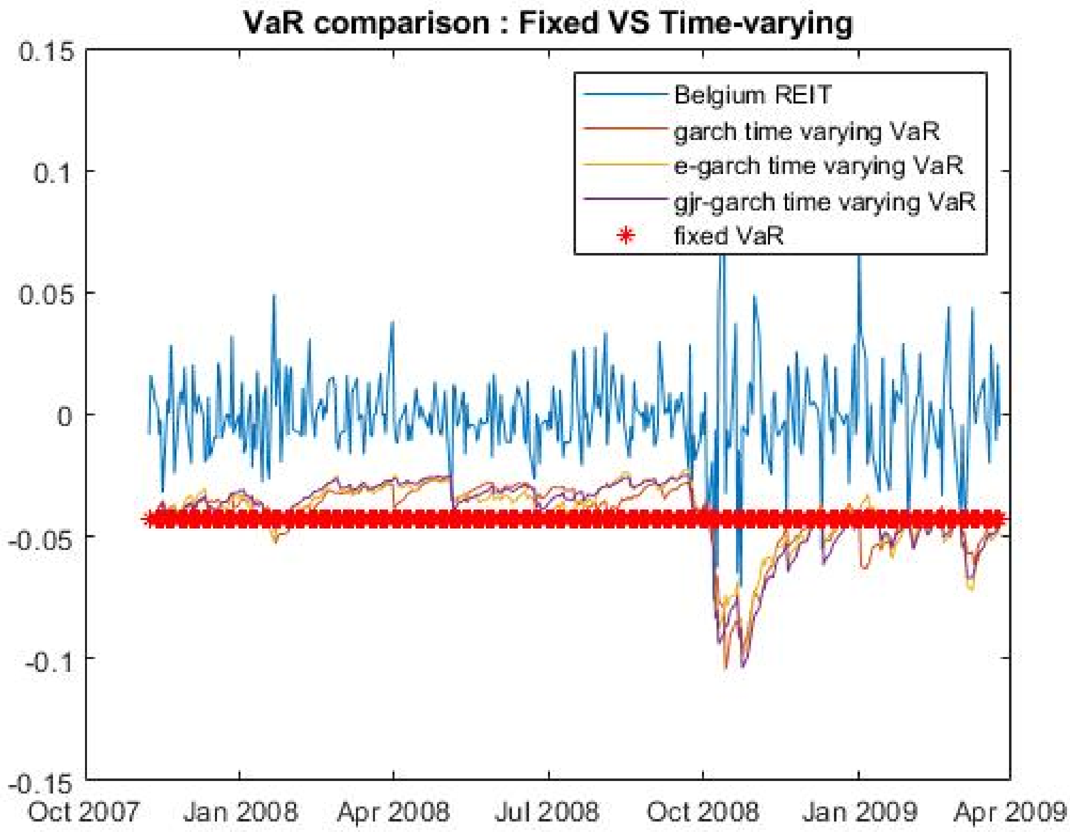

Figure 44: Value at risk forBelgium REIT during GFC.

V DCC-GARCH graphs that are not presented in the body of the text

V.1 The Netherlands versus the other REIT indices correlation

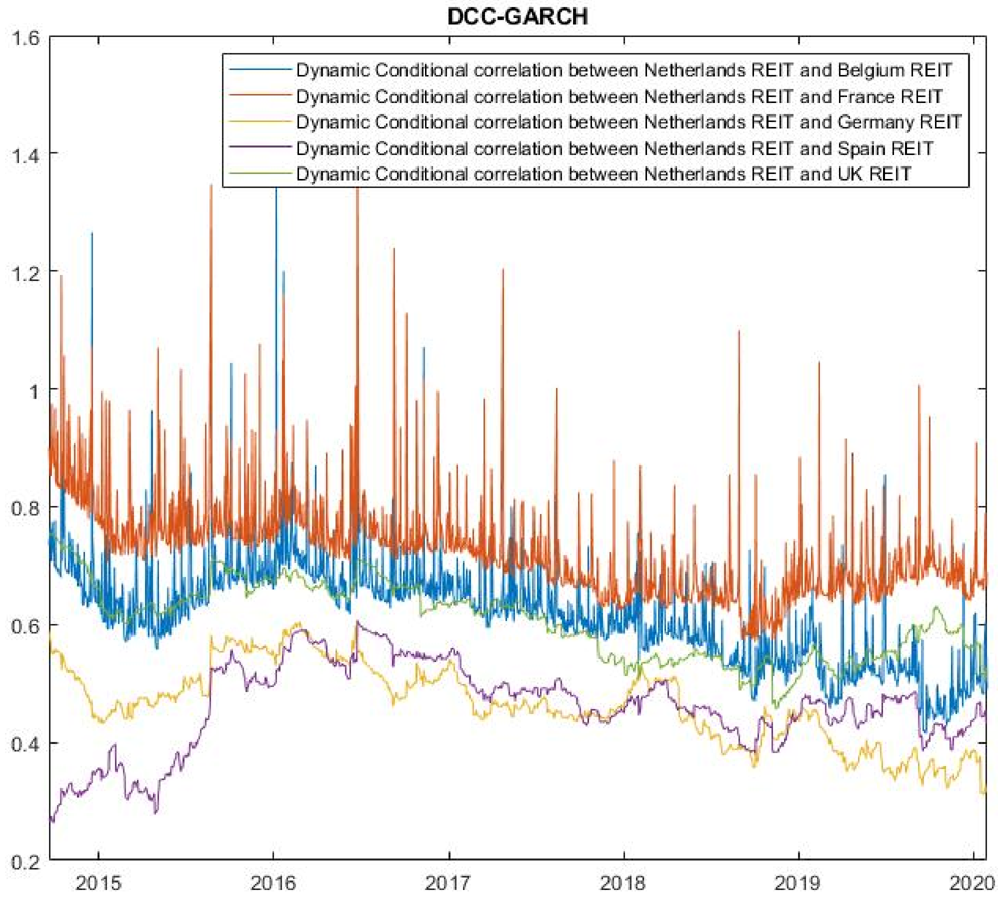

Figure 45: Correlation behaviour of The Netherlands REIT index versus the other indices.

V.2 Germany REIT versus the other REIT indices correlation

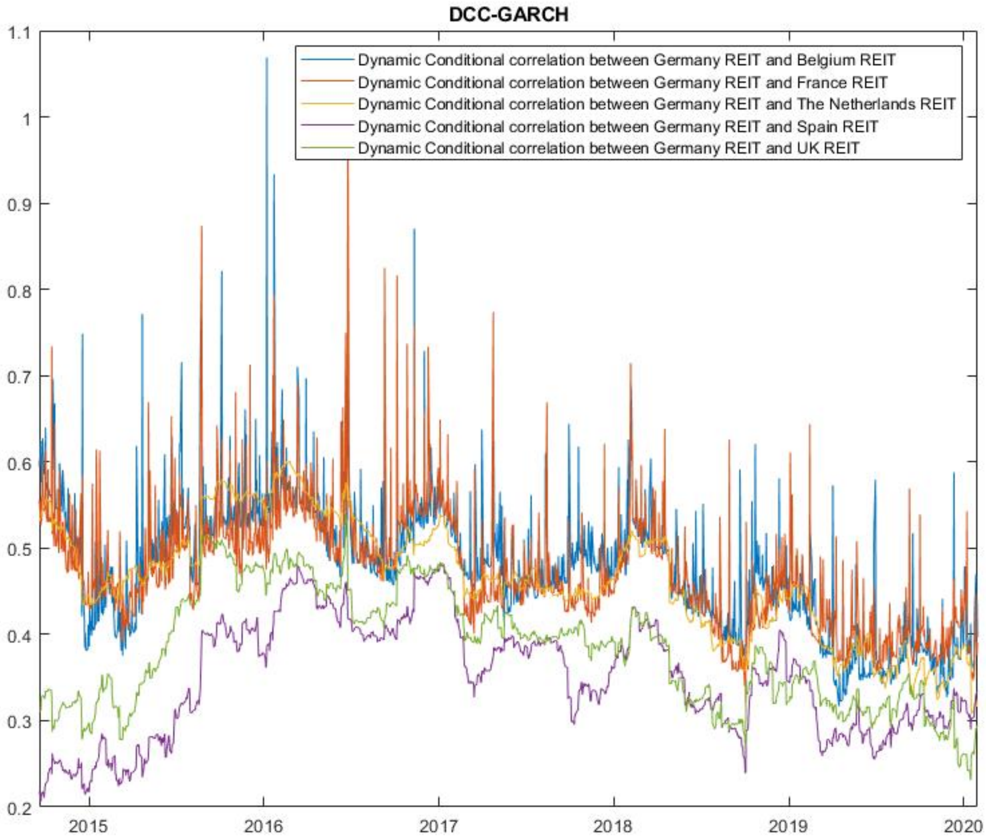

Figure 46: Correlation behaviour of Germany REIT index versus the other indices.

V.3 Spain REIT versus the other REIT indices correlation

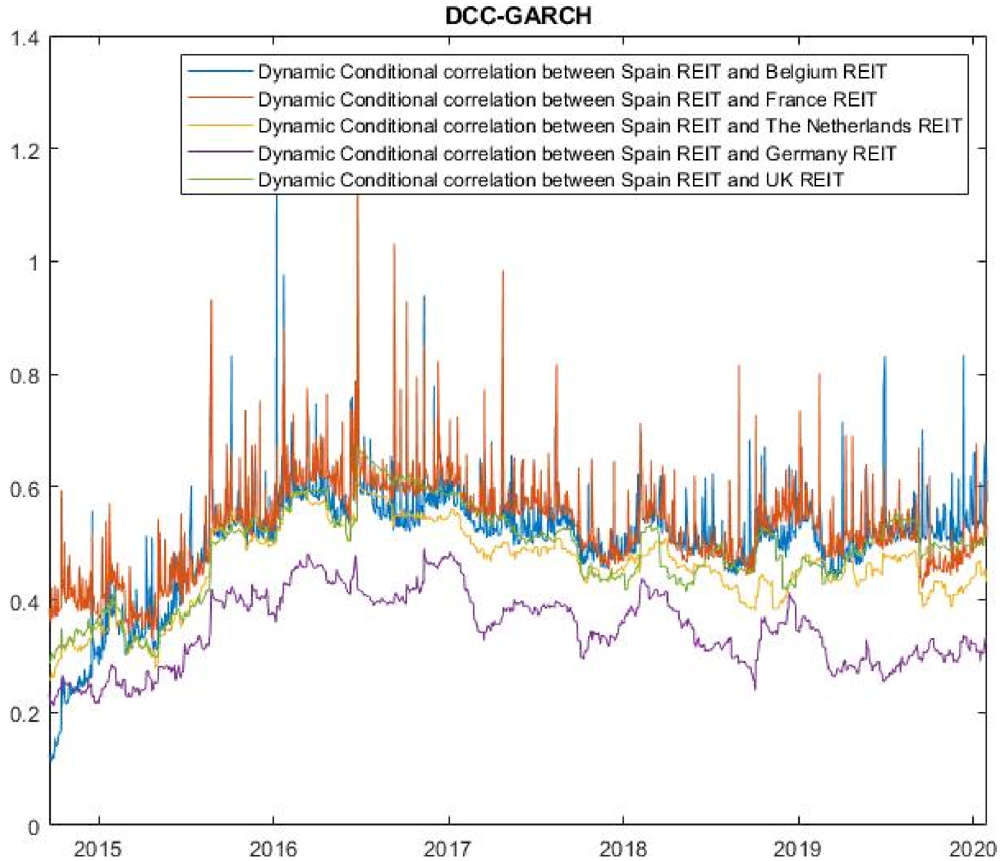

Figure 47: Correlation behaviour of Spain REIT index versus the other indices.

V.4 UK REIT versus the other REIT indices correlation

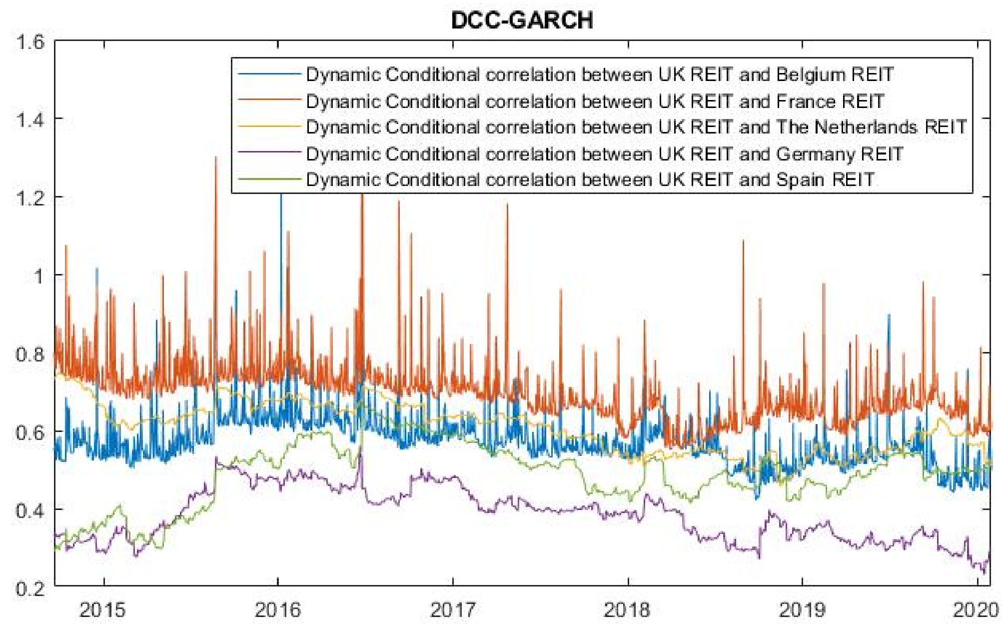

Figure 48: Correlation behaviour of UK REIT index versus the other indices.

Executive summary

The purpose of this master thesis is to provide an overview of the structure of conditional volatility for a sample of six European countries: FTSE Belgium REIT, S&P France REIT, S&P Netherlands REIT, FTSE Germany REIT, S&P Spain REIT and FTSE United Kingdom REIT. By introducing both univariate and multivariate GARCH models, the reader of this paper will be accustomed to understanding GARCH volatility structure and patterns regarding European REITs. Furthermore, he will discover the intrinsic parameters of theses models, namely α, β and γ. The way to interpret them will no longer be a secret. The inexperienced person will also understand the masterpiece that has been the discovery of conditional variance models and its usefulness for risk and portfolio managers. He will be warned that the best model for REITs seems to be the EGARCH but that the parsimonious nature of the GARCH also has its advantages. The time span being divided into three sub periods (Global Financial Crisis, Normal Economic Cycle and COVID-19), the reader of this master thesis will receive the keys to understand the differences between each subperiod in terms of conditional volatility structure and in terms of correlation.