Solitons in Bose-Einstein condensates

Auteur : Heuchamps, Alexandre
Promoteur(s) : Schlagheck, Peter; Vanderheyden, Benoit
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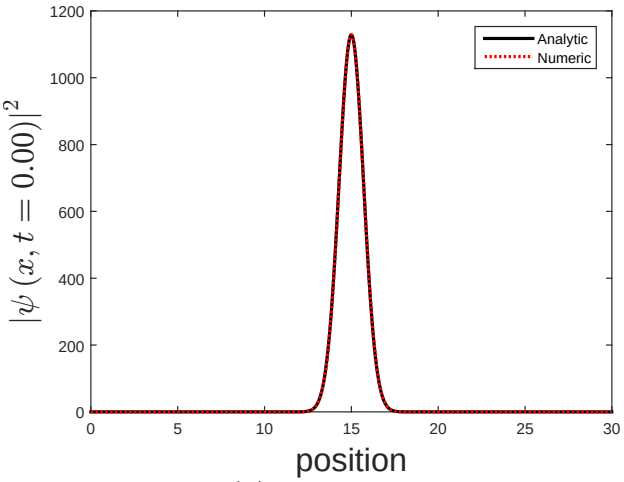
ABSTRACT

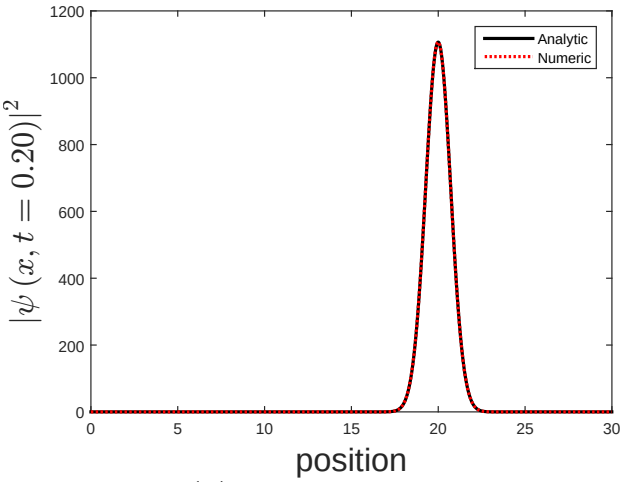
This work is aimed to study, both in an analytic and a numerical way, solitons in Bose-Einstein condensates. To reach that goal, a detour through the general theory of Bose-Einstein condensates in a mean-field picture is needed. Once the concepts established, the study of the solitons themselves can be undertaken.

In this work, these structures are studied in the framework of the Gross-Pitaevskii equation, which is a mean-field approximation and mainly consists in averaging the interactions between particles. Numerically, the discretization in time is performed through a spectral method, whereas the space discretization is done through finite difference. Even if the code that was developed is able to fit the analytic result for a free Schrödinger equation (for a sufficiently small timestep), the analytic solution of the full Gross-Pitaevskii equation cannot be fitted over long periods of time, even with a small timestep, which indicates that more elaborated numerical techniques have to be used.

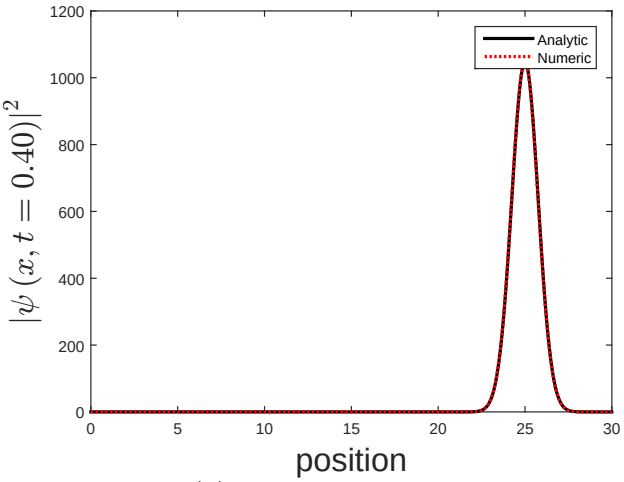
In order to check the validity of the code to fit to a free Schrödinger equation, fig. 1 can be consulted, which clearly shows that the results fit quite well. On the contrary, to see that the full Gross-Pitaevskii equation could not be simulated accurately, fig. 2 can be consulted. Even though the numerical results are not satisfying, the influence of the timestep on the numerical approximation can be seen in fig. 3, where it is clearly visible that a smaller timestep allows to come closer to the analytical solution.



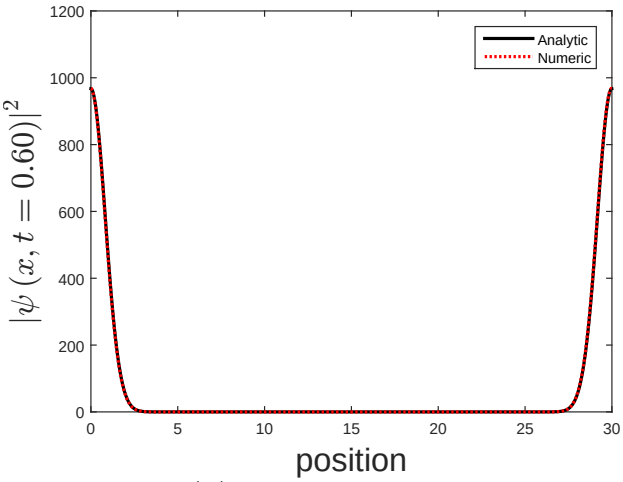
(a) $t = 0$.



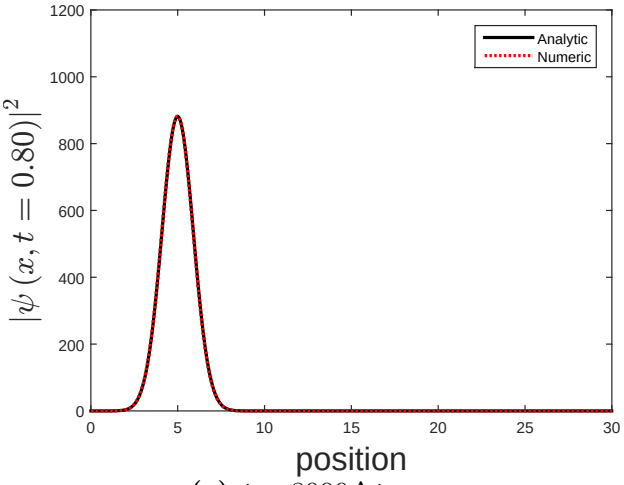
(b) $t = 2000\Delta t$.



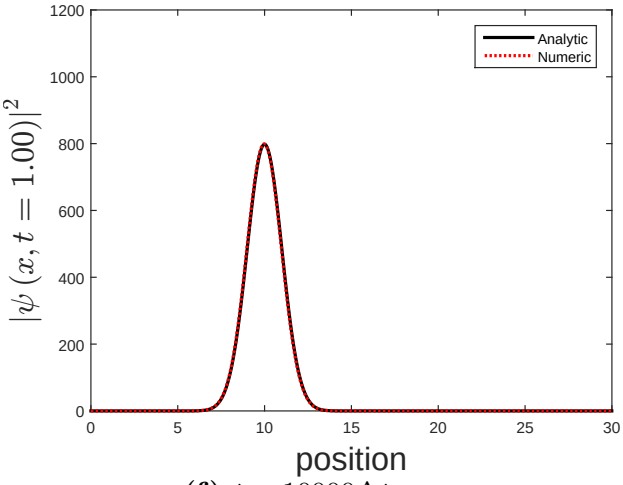
(c) $t = 4000\Delta t$.



(d) $t = 6000\Delta t$.

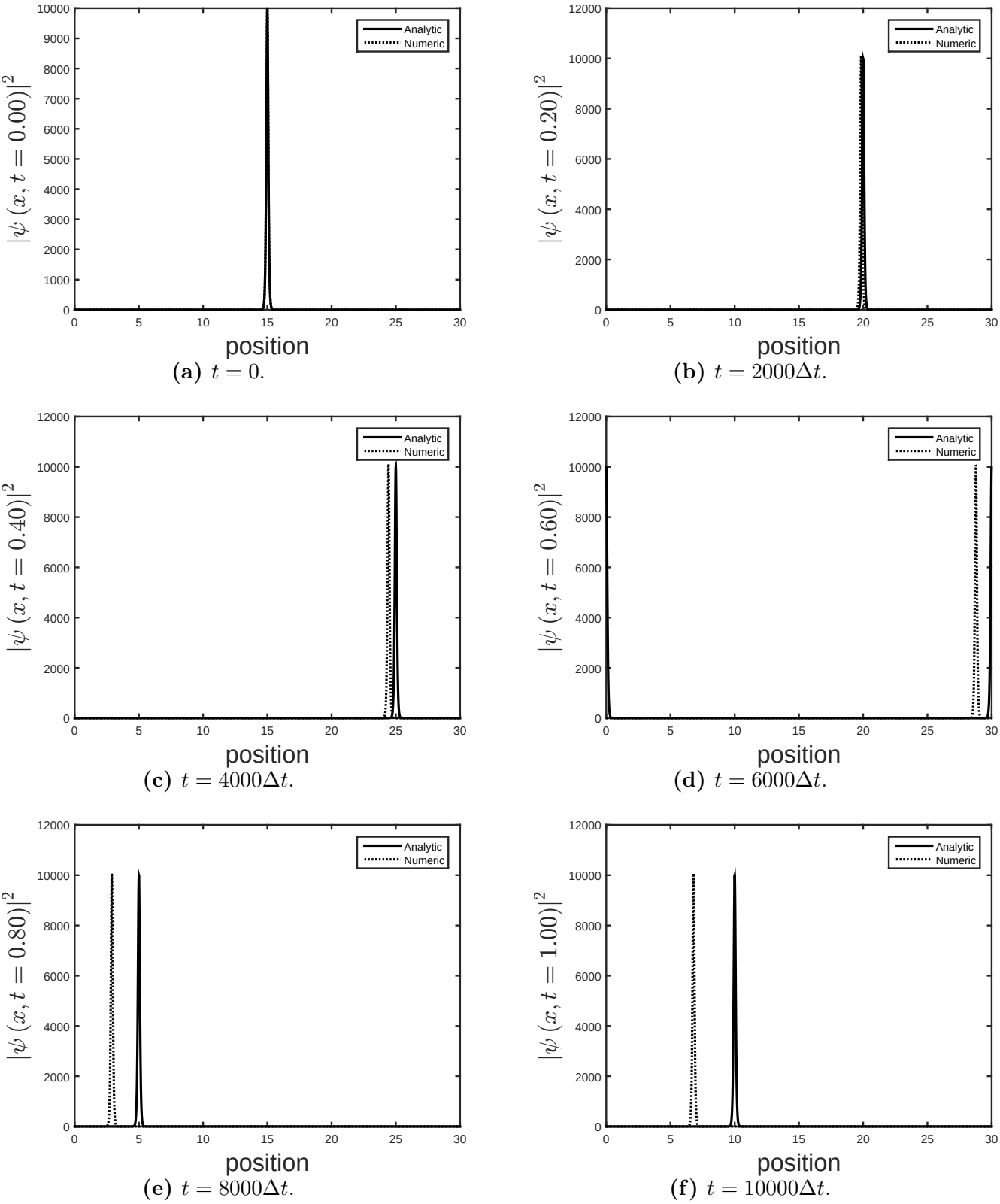


(e) $t = 8000\Delta t$.



(f) $t = 10000\Delta t$.

Figure 1 – Comparison between analytic and numeric solutions to the 1D free Schrödinger equation for a condensate of 2000 particles and discretization parameters $\Delta t = 10^{-4}$ and $\Delta x = 3 \cdot 10^{-3}$, showing the agreement between numerical and analytical results.



(a) $t = 0$. **(b)** $t = 2000\Delta t$. **(c)** $t = 4000\Delta t$. **(d)** $t = 6000\Delta t$. **(e)** $t = 8000\Delta t$. **(f)** $t = 10000\Delta t$.

Figure 2 – Comparison between analytic and numeric solutions to the 1D GPE for a condensate of 2000 particles and discretization parameters $\Delta t = 10^{-4}$ and $\Delta x = 6 \cdot 10^{-3}$, showing that the numerical solution propagates at a (wrong) decreasing speed.

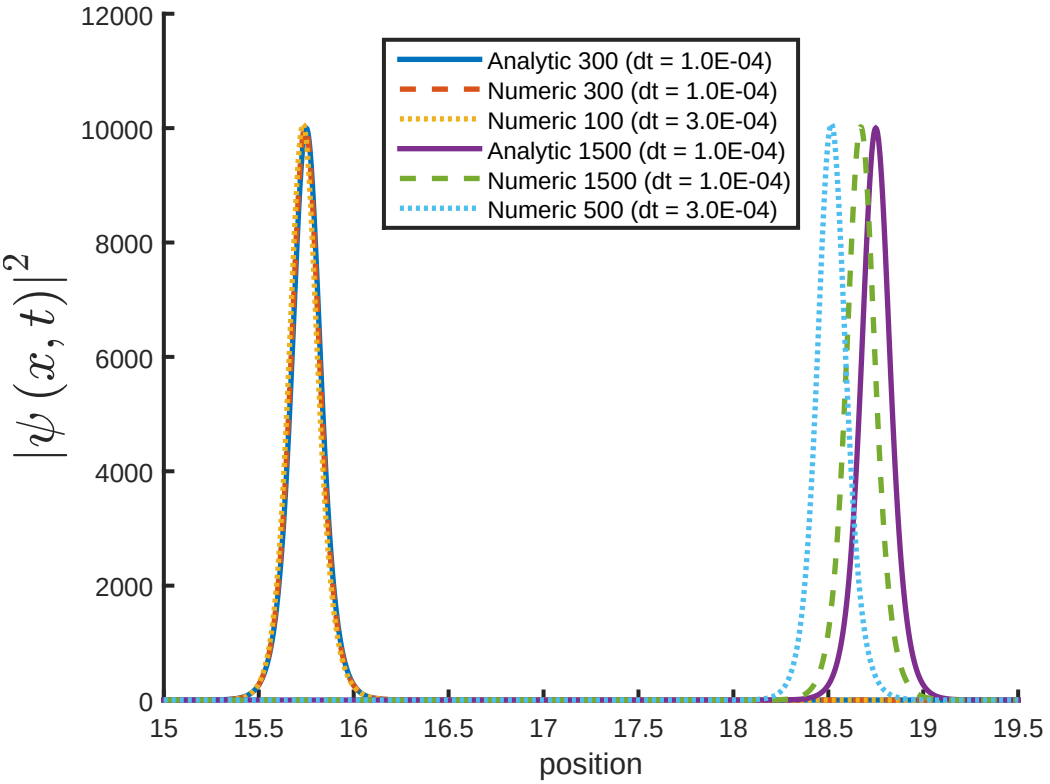


Figure 3 – Comparison between the analytic and numerical results for different timesteps and $\Delta x = 3 \cdot 10^{-3}$. The influence of the timestep is clearly visible: a smaller timestep allows to come closer to the analytic solution.